

# Chaotic new inflation and formation of primordial black holes

Jun'ichi Yokoyama

*Department of Physics, Stanford University, Stanford, California 94305-4060  
and Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan*

(Received 17 February 1998; published 15 September 1998)

It is shown that in a number of scalar potentials with an unstable local maximum at the origin chaotic inflation is followed by new inflation if model parameters are appropriately chosen. In this model density fluctuation can have a large-amplitude peak on the comoving Hubble scale at the onset of the slow-roll new inflation and can result in the formation of an appreciable amount of primordial black holes on astrophysically interesting mass scales. [S0556-2821(98)09318-7]

PACS number(s): 98.80.Cq, 04.70.Bw

## I. INTRODUCTION

If overdensity of order of unity exists in the hot early universe, a black hole can be formed when the perturbed region enters the Hubble radius. While the properties of the primordial black holes (PBHs) thus produced were a subject of extensive study decades ago [1,2], there was no observational evidence of their existence and only observational constraints were obtained against their mass spectrum [3,4]. Recently, however, the possibility of their existence has been raised from a number of astrophysical and cosmological considerations and there is an increasing interest in it.

For example, they may be the origin of the massive compact halo objects (MACHOs) which are dark compact objects with a typical mass  $\sim 1M_{\odot}$  and make up about  $\mathcal{O}(10^{-2})$  of the critical density [5]. While the primary MACHO candidates are substellar baryonic objects such as brown dwarfs, it is difficult to reconcile such a large amount of these objects with the observed mass function of low mass stars [6] and with the infrared observation of dwarf components [7], unless the mass function is extrapolated to the lower masses in an extremely peculiar manner of Pop III stars are produced abundantly at the relevant mass scale [8]. Hence we should consider PBHs seriously as the second option. Furthermore this possibility can be experimentally tested by observing gravitational radiation from coalescing black hole MACHO binaries by laser interferometers [9].

Another interesting possibility is PBHs with mass  $M \sim 10^{15}$  g which are just evaporating now [10]. It has been argued that high energy phenomenon associated with evaporation can explain origin of a class of gamma-ray burst. For this purpose their abundance should be around  $\Omega = 10^{-8}$  in unit of the critical density [11].

One may also consider formation of much heavier black holes with mass  $M \sim 10^8 M_{\odot}$ . Such black holes are expected to exist in the center of AGNs and quasars [12] and act as an engine of their activity. Although it is generally believed that these black holes are formed after recombination with a specially arranged spectrum of density fluctuations [13], it might be interesting to consider the possibility that they were of primordial origin, which might lead to a new scenario of galaxy formation.

In any case, in order to produce PBHs on some specific scale, we must prepare density perturbation whose amplitude

has a high peak of  $\mathcal{O}(10^{-2})$  on the corresponding scale. It is difficult, however, to realize such a spectral shape in inflationary cosmology [14–17], which is not only indispensable to solve the horizon and the flatness problems but also the most sensible way to generate density perturbations [18], because usual models predict a scale-invariant spectrum. Given the smallness of the observed anisotropy of the cosmic microwave background radiation (CMB) [19], the amplitude of primordial density fluctuation turns out to be  $\mathcal{O}(10^{-5})$  on any observable scales in most common models.

It is possible, nevertheless, to realize a non-scale-invariant spectrum by choosing somewhat contrived forms of the inflaton's potential. While examples of the various perturbation spectrum realized in different potentials are found in [20], we must admit that few of the models giving non-scale-invariant spectrum with a single scalar field has a motivation in sensible particle physics. In particular, the model proposed by Ivanov *et al.* [21] in order to produce significant amount of PBHs employs a scalar potential with two breaks and a plateau in between. Several other toy potentials have also been studied in [22].

One can construct more natural inflation models for PBH formation if one allows multiple scalar fields. Some of them make use of primordially isocurvature fluctuations [23], others include multiple stages of inflation with each regime governed by different field [24–27]. We must assign appropriate form of coupling of the scalar fields in each model, which may not always be easy.

In the present paper, we propose a new scenario of multiple inflation which, unlike previous double inflation models [24–28], contains only one source of inflation. In this model we employ in the Einstein gravity an inflaton scalar field,  $\phi$ , with a simple potential,  $V[\phi]$ , which has an unstable local maximum at the origin, such as a quartic double-well potential. This is the same setup as the new inflation scenario [15], but chaotic inflation is also possible if the  $\phi$  has a sufficiently large amplitude initially. In fact Kung and Brandenberger [29] studied the initial distribution of a scalar field with such a potential and concluded that chaotic inflation is much more likely than new inflation. Thus we also start with chaotic inflation, but show that new inflation is also possible when  $\phi$  evolves towards the origin after chaotic inflation if the parameters of the potential is appropriately chosen so that the scalar field has the right amount of kinetic energy after cha-

otic inflation in order to climb up the potential hill near to the origin and start slow rollover there. Hence in this model the initial condition for new inflation is realized not due to the high-temperature symmetry restoration nor for a topological reason [30], but by dynamical evolution of the field which has already become sufficiently homogeneous because of the first stage of chaotic inflation. We shall refer this succession of inflation simply to chaotic new inflation.

With an appropriate shape of the potential, density fluctuations generated during new inflation can have larger amplitude than those during chaotic inflation. Furthermore, since the power spectrum of fluctuation generated during new inflation can be tilted, it can have a peak on the comoving Hubble scale when the inflaton enters the slow-rollover phase during new inflation. If the peak amplitude is sufficiently large, it results in formation of PBHs on the horizon mass scale when the corresponding comoving scale reenters the Hubble radius during radiation domination. We shall show below that such a scenario is indeed possible with a simple shape of the inflaton's potential.

The rest of the paper is organized as follows. In Sec. II we summarize basic features of chaotic inflation and new inflation scenarios and in Sec. III we calculate formation probability of PBHs in our model. Then we report the results of numerical analysis with various potentials in Sec. IV and Sec. V. Finally Sec. VI is devoted to conclusion.

## II. CHAOTIC INFLATION AND NEW INFLATION

First let us consider two potentials which have a global minimum at  $\phi = v < M_{Pl}$ :

$$V_{CW}[\phi] = \frac{\tilde{\lambda}}{4} \phi^4 \left( \ln \left| \frac{\phi}{v} \right| - \frac{1}{4} \right) + \frac{\tilde{\lambda}}{16} v^4, \quad (1)$$

$$V_{DW}[\phi] = \frac{\lambda}{4} (\phi^2 - v^2)^2. \quad (2)$$

The former is the Coleman-Weinberg potential [31], which was employed in the original version of the new inflation scenario [15], and the latter is a simple double-well potential. For  $\phi \gg M_{Pl} > v$ , the both potentials can be approximated at least locally by a simple quartic potential  $V[\phi] = (\lambda/4) \phi^4$  with  $\lambda \approx \tilde{\lambda} \ln|\phi/v|$  for the former case, and the evolution of the universe in this regime is practically the same as that in chaotic inflation with the quartic potential [16]. That is, from the slow-rollover equations of motion,

$$3H\dot{\phi} = -V'[\phi], \quad (3)$$

$$H^2 = \left( \frac{\dot{\phi}}{a} \right)^2 = \frac{8\pi}{3M_{Pl}^2} V[\phi], \quad (4)$$

we find

$$\phi(t) = \phi_i \exp \left( - \sqrt{\frac{\lambda}{6\pi}} M_{Pl} (t - t_i) \right), \quad (5)$$

$$a(t) = a_i \exp \left[ \frac{\pi}{M_{Pl}^2} (\phi_i^2 - \phi^2(t)) \right], \quad (6)$$

which are valid until the slow-rollover conditions  $|\dot{H}| \ll H^2$ ,  $\ddot{\phi} \ll 3H\dot{\phi}$  etc. break down at around  $\phi_e \approx \max(M_{Pl}/8\pi, 4v)$ . Here the suffix  $i$  implies the initial value at  $t = t_i$ . The  $e$ -folding number of (chaotic) inflationary expansion between  $\phi$  and  $\phi_e$ ,  $N(\phi \rightarrow \phi_e)$ , is therefore given by

$$N(\phi \rightarrow \phi_e) = \frac{\pi}{M_{Pl}^2} (\phi^2 - \phi_e^2) \approx \frac{\pi}{M_{Pl}^2} \phi^2, \quad (7)$$

the latter approximate equality being valid for  $\phi^2 \gg \phi_e^2$ .

The amplitude of linear curvature perturbation,  $\Phi$ , generated on the comoving scale,  $l = 2\pi/k$ , is given by

$$\Phi \left( l = \frac{2\pi}{k} \right) = f \frac{H}{|\dot{\phi}|} \delta\phi \Big|_{t_k} = f \frac{H^2}{2\pi|\dot{\phi}|} \Big|_{t_k} \equiv f \Delta(t_k). \quad (8)$$

Here  $t_k$  is the epoch when the wave number  $k$  satisfied  $k = a(t_k)H(t_k)$  during inflation [18], and  $\delta\phi = H(t_k)/(2\pi)$  is the root-mean-square amplitude of fluctuation accumulated during one expansion time around  $t = t_k$ , and  $f = 3/5(2/3)$  in the matter- (radiation-) dominated stage. The above expression is valid until the comoving scale  $l$  crosses the Hubble radius again. In the case with quartic inflaton potential, it reads

$$\Phi(l) = f \sqrt{\frac{2\pi\lambda}{3}} \left( \frac{\phi(t_k)}{M_{Pl}} \right)^3 \approx \frac{f}{\pi} \sqrt{\frac{2\lambda}{3}} N_{ke}^{3/2}, \quad (9)$$

with  $N_{ke} \equiv N(\phi(t_k) \rightarrow \phi_e)$ . The large-angular-scale anisotropy of CMB due to the Sachs-Wolfe effect [32] is then given by

$$\frac{\delta T}{T} = \frac{1}{3} \Phi = \frac{H^2}{10\pi|\dot{\phi}|} \Big|_{t_k}. \quad (10)$$

In the usual case with single stage of inflation one typically takes  $N_{ke} \approx 60$  and determine the value of  $\lambda$  using the Cosmic Background Explorer (COBE) data  $\delta T/T \approx 1 \times 10^{-5}$  to yield  $\lambda \approx 2 \times 10^{-13}$  [33]. In the case with second stage of inflation the value of  $N_{ke}$  corresponding to the scale observed by COBE can be significantly smaller and the normalized value of  $\lambda$  can be different. Taking  $N_{ke} = 20$  with another 40  $e$ -folds during new inflation, for example, we find  $\lambda \approx 6 \times 10^{-12}$ .

After chaotic inflation, the scalar field starts oscillation around  $\phi = v$  if  $v$  is too large, or it overshoots the symmetric state  $\phi = 0$  and approaches the other minimum  $\phi = -v$  if  $v$  is too small. If  $v$  is appropriately chosen at an intermediate value, on the other hand, it can spend a long enough time near the origin and then slow-rollover new inflation can set in, which ends up with either  $\phi = v$  or  $\phi = -v$  depending on the sign of  $\phi$  when classical slow-rollover regime begins.

We must resort to a precise numerical integration of equations of motion to find out what values of  $v$  to take.

Before doing this, let us review the basic properties of new inflation, assuming that the slow-rollover starts from  $\phi = \phi_s$  at the epoch  $t = t_s$ . Since the shape of the potential around the origin is different between the Coleman-Weinberg potential and the double-well potential, we must treat them separately.

First for the former, the potential near the origin can be approximated by

$$V_{\text{CW}}[\phi] \approx V_0 - \frac{\hat{\lambda}}{4} \phi^4, \quad (11)$$

with  $\hat{\lambda} = \tilde{\lambda} \ln|\phi_s/v|$  and  $V_0$  reads  $V_0 = \tilde{\lambda} v^4/16$ . The classical slow-rollover solution satisfies

$$H^2 = H_0^2 = \frac{8\pi V_0}{3M_{\text{Pl}}^2}, \quad (12)$$

$$\hat{\lambda} \phi^2 = \frac{3H_0^2}{2N(\phi \rightarrow \phi_f) + 1} \approx \frac{3H_0^2}{2N(\phi \rightarrow \phi_f)}, \quad (13)$$

the latter approximate equality being valid for  $N \gg 1$ . The curvature perturbation reads

$$\Phi(l) = \frac{f}{\pi} \sqrt{\frac{2\hat{\lambda}}{3}} N_{kf}^{3/2}, \quad (14)$$

where  $N_{kf}$  is defined by  $N_{kf} \equiv N(\phi(t_k) \rightarrow \phi_f)$ . Since  $\hat{\lambda}$  is no larger than  $\sim 20\lambda \sim 10^{-10}$  the resultant curvature perturbation is at most of order of  $10^{-4}$  for  $N_{kf} < 60$ . Hence chaotic new inflation with the Coleman-Weinberg potential is not suitable to produce large enough density perturbation on small scale to warrant significant formation of primordial black holes.

Next we consider new inflation with the double-well potential (2), which can be approximated as

$$V_{\text{DW}}[\phi] \approx V_0 - \frac{1}{2} m^2 \phi^2, \quad (15)$$

with  $V_0 = (\lambda/4) v^4$  and  $m^2 = \lambda v^2$ , but for later convenience we treat as if  $V_0$  and  $m^2$  were free parameters. The classical slow roll-over solution reads

$$\phi(t) = \phi_s \exp\left[\frac{m^2}{3H_0^2} H_0(t - t_s)\right],$$

$$H_0^2 = \frac{8\pi V_0}{3M_{\text{Pl}}^2}, \quad a(t) \sim e^{H_0 t}. \quad (16)$$

Inflation ends at  $\phi = \phi_f \equiv cv$  with  $c$  being a constant of order of 0.1, so we find

$$N(\phi \rightarrow \phi_f) = \frac{3H_0^2}{m^2} \ln \frac{\phi_f}{\phi_s} = \frac{3H_0^2}{m^2} \ln \frac{cv}{\phi_s}, \quad (17)$$

and curvature perturbation in the slow-rollover regime reads

$$\Phi(l) = f \Delta_{\text{SR}}(t_k) \equiv 3f \frac{H_0^2}{m^2} \frac{H_0}{2\pi\phi(t_k)}$$

$$= \frac{3f}{2\pi} \frac{H_0^3}{m^2 \phi_f} \exp\left(\frac{m^2}{3H_0^2} N_{kf}\right). \quad (18)$$

The above classical slow-roll solution (16) [and also Eq. (13) for the Coleman-Weinberg model] is valid only when classical potential force dominates diffusion due to quantum fluctuation. In other words, the classical motion during one expansion time,  $\Delta\phi = |\dot{\phi}|H^{-1}$ , should be larger than the amplitude of quantum fluctuation,  $\delta\phi = H/(2\pi)$ , accumulated in the same period. For the potential (15) this condition reads

$$|\phi| > \frac{3H_0^3}{2\pi m^2} \equiv \phi_q. \quad (19)$$

If  $\phi$  stays in the region  $|\phi| < \phi_q$  for more than one expansion time, quantum fluctuation will start to dominate the dynamics and the universe enters a self-reproduction regime which can last eternally [34]. Then the memory of chaotic inflation would be washed away and our double inflation scenario would not work.

Note that the amplitude of density fluctuation evaluated with the slow-roll approximation exceeds unity,  $\Delta_{\text{SR}}(t_k) > 1$ , for  $|\phi| < \phi_q$ . This means that if  $\phi$  entered the slow-roll regime in  $|\phi| < \phi_q$ , we would find the peak amplitude of density fluctuation larger than unity. Since our goal is to produce a spectrum of density fluctuation whose amplitude at the peak is at most  $\mathcal{O}(10^{-2})$ , we should arrange model parameters so that  $\phi$  crosses over the region  $|\phi| < \phi_q$  rapidly enough to ensure  $\Delta(t_k) \ll \Delta_{\text{SR}}(t_k)$  in this period and to avoid self-reproduction, or  $\phi$  should reverse its direction of motion before reaching  $\phi = \phi_q$ . In either case, as long as we choose the model parameters so that the peak amplitude of fluctuation is  $\mathcal{O}(10^{-2})$ , we can automatically avoid the dangerous self-reproduction in the new inflation regime.

Equation (18) implies the spectral index of power-law density fluctuation is given by

$$n = 1 - \frac{2m^2}{3H_0^2} < 1. \quad (20)$$

Thus the power spectrum can be significantly tilted. The linear perturbation has a peak amplitude

$$\max \Phi = 3f \frac{H_0^2}{m^2} \frac{H_0}{2\pi\phi_s}, \quad (21)$$

on the comoving scale  $l_s \equiv 2\pi/k_s$  where  $k_s \equiv a(t_s)H(t_s)$ . It can be large if  $\phi_s$  turns out to be small. Since the coupling constant is determined by COBE observation, we have essentially one parameter,  $v$ , which determines not only the ratio  $H_0^2/m^2$  but also  $\phi_s$ . Hence we must await the result of

numerical solution of the equations of motion in order to see if this model realizes large enough density perturbation on cosmologically interesting scales.

Before proceeding to the numerical analysis, we next calculate the abundance, if at all, of the PBHs produced due to the potentially large fluctuations generated in the new inflation regime with the potential (15).

### III. ABUNDANCE OF PRIMORDIAL BLACK HOLES

The initial fraction of the PBHs with mass  $M$ ,  $\beta(M)$ , produced during radiation dominated era has been estimated by Carr [3] as

$$\beta(M) \approx \delta(M) \exp\left(-\frac{1}{18\delta^2(M)}\right), \quad (22)$$

assuming the density fluctuation is Gaussian distributed with the root-mean-square at the horizon crossing given by  $\delta(M)$ . Since PBHs are formed at high density peaks with  $\delta \gtrsim 1/3$ , however, possible non-Gaussian effect on the tail may significantly affect the estimation of their abundance, as was first pointed out by Bullock and Primack [22]. Here we estimate the abundance of PBHs produced in our model by calculating the probability distribution function (PDF) of the coarse-grained field in terms of the stochastic inflation approach [35], following more recent analysis by Ivanov [36].

Since our model predicts a relatively sharp peak at the scale  $l_s$  corresponding to the onset of slow-rollover we estimate the abundance of PBHs on this particular scale with the potential

$$V[\phi] = V_0 - \frac{1}{2} m^2 \phi^2. \quad (23)$$

The scalar field coarse-grained over a super-Hubble scale,  $\hat{\phi}(t, \mathbf{x})$  satisfies the Langevin equation

$$3H_0 \dot{\hat{\phi}} + V'[\hat{\phi}] = q(t, \mathbf{x}), \quad (24)$$

with the temporal correlation function of the stochastic force,  $q(t, \mathbf{x})$ , given by

$$\langle q(t, \mathbf{x}) q(t', \mathbf{x}') \rangle = \frac{9H_0^5}{4\pi^2} \delta(t-t'). \quad (25)$$

Since the potential is a quadratic function, the PDF of  $\hat{\phi}$ ,  $\Pi[\hat{\phi} = \phi, t]$  remains Gaussian as long as the initial distribution is also Gaussian. For example, starting with the PDF  $\Pi[\phi, t_s] = \delta(\phi - \phi_s)$  and solving the Fokker-Planck equation

$$\frac{\partial \Pi}{\partial t} = \frac{1}{3H_0} \frac{\partial}{\partial \phi} V'[\phi] \Pi + \frac{H_0^3}{8\pi^2} \frac{\partial^2 \Pi}{\partial \phi^2} \equiv -\frac{\partial J}{\partial \phi}, \quad (26)$$

with the boundary condition  $\Pi[\phi \rightarrow \pm \infty, t] = 0$ , we find

$$\Pi[\phi, \tau] = \frac{1}{\sqrt{2\pi\sigma(\tau)}} \exp\left(-\frac{(\phi - \phi_{cl}(\tau))^2}{2\sigma^2(\tau)}\right), \quad (27)$$

$$\tau \equiv t - t_s,$$

with

$$\phi_{cl}(\tau) = \phi_s \exp\left(\frac{m^2}{3H_0^2} H_0 \tau\right), \quad (28)$$

$$\sigma^2(\tau) = \frac{3H_0^4}{8\pi^2 m^2} \left[ \exp\left(\frac{2m^2}{3H_0^2} H_0 \tau\right) - 1 \right]. \quad (29)$$

This does not imply, however, that the resultant abundance of PBHs are given in the form (22), because of the nonlinear dependence of the metric perturbations. Following Ivanov [36], we write the coarse-grained metric in the quasi-isotropic form

$$ds^2 = -dt^2 + \hat{a}^2(\hat{\phi}(t, \mathbf{x})) d\mathbf{x}^2, \quad (30)$$

where the scale factor  $\hat{a}$  now depends on the coarse-grained spatial coordinate as well, and quantify the metric perturbation in terms of

$$\hat{h} \equiv \frac{\hat{a}(t, \mathbf{x})}{a(t)} - 1, \quad (31)$$

which is frozen in the super-Hubble scales. In the limit  $\hat{h} \ll 1$ ,  $\hat{h}$  reduces to the gauge-invariant growing adiabatic perturbation [37].

We are interested in the statistical distribution of metric perturbation on the comoving scales that leave the Hubble radius in the period between  $\tau=0$  and  $\tau \approx H_0^{-1}$  when the classical solution has rolled down to  $\phi_{cl}(\tau = H_0^{-1}) = \phi_s \exp(m^2/3H_0^2) \equiv \phi_{s1}$ . While the amplitude of the metric perturbation is determined by the duration of inflation in each region, it is not straightforward to extract perturbation on these particular scales in the present approach in which nonlinear effects are at least partially taken into account. In this situation, in order to obtain the PDF for the duration of inflation in a specific regime, Ivanov [36] proposed to solve the Fokker-Planck equation (26) with the initial condition  $\Pi[\phi, 0] = \delta(\phi - \phi_s)$  and the absorptive boundary condition at  $\phi = \phi_{cl}(\tau = H_0^{-1})$ , and to identify the probability current  $J$  in Eq. (26) with the desired PDF, from which he finds the PDF of  $\hat{h}$ ,  $P[\hat{h}]$ , as

$$P[\hat{h}] = \frac{1}{\sqrt{2\pi\Delta^2 \hat{N}}} \frac{N_{cl}}{\hat{N}} \times \exp\left(-\frac{(\hat{N} - N_{cl})^2}{2\Delta^2 \hat{N}}\right) \frac{d\hat{N}}{d\hat{h}}, \quad (32)$$

in the case with a linear potential, where  $\hat{N}$  is the  $e$ -folding number of expansion while the classical solution allows the  $e$ -folding number of  $N_{cl}$ , to which we put unity here.  $\hat{h}$  and  $\hat{N}$  are related by

$$\hat{h} = \exp(\hat{N} - N_{cl}) - 1. \quad (33)$$

Since our potential is more complicated, it is cumbersome to adopt the same procedure. So we instead use the solution (27) and obtain the PDF of the duration of inflation while  $\hat{\phi}$  evolves from  $\phi_s$  to  $\phi_{s1}$ ,  $\hat{\tau}$ , simply assuming that  $\hat{\phi}$  satisfies the classical equation of motion at the moment  $\hat{\phi} = \phi_{s1}$ . Then we find the PDF of  $\hat{h}$  as

$$P[\hat{h} = h] = \frac{m^2}{3H_0^2} \frac{\phi_{s1}}{1+h} \times \Pi \left[ \phi_{s1}, \frac{1}{H_0} (1 + \ln(1+h)) \right]. \quad (34)$$

Before proceeding to the calculation of PBH abundance, we mention the relation between this approach and Ivanov's result (32). If we follow the same procedure as above for a linear potential we obtain explicitly

$$P[\hat{h}] = \frac{1}{\sqrt{2\pi\Delta^2\hat{N}}} \times \exp \left( -\frac{(\hat{N} - N_{cl})^2}{2\Delta^2\hat{N}} \right) \frac{d\hat{N}}{dh}. \quad (35)$$

Thus the only difference to Eq. (32) is the absence of a prefactor  $N_{cl}/\hat{N}$ . One may also be tempted to define the PDF for  $\hat{\tau}$  from the probability current  $J$  using a solution analogous to Eq. (27) but with a linear potential. In this case one would find

$$P[\hat{h}] = \frac{1}{\sqrt{2\pi\Delta^2\hat{N}}} \left( \frac{1}{2} + \frac{N_{cl}}{2\hat{N}} \right) \times \exp \left( -\frac{(\hat{N} - N_{cl})^2}{2\Delta^2\hat{N}} \right) \frac{d\hat{N}}{dh}. \quad (36)$$

Again we only have an extra prefactor of order of unity. Since the deviation between  $\hat{N}$  and  $N_{cl}$  is not expected to be too large in our case, unlike in the situation of eternally self-reproducing inflationary universe [34], all of the above three approaches will yield essentially the same result as far as the rare PBH formation is concerned. This also implies that our semi-nonlinear approach suffices to the present problem.

We now proceed to evaluation of the fraction of PBHs produced using Eq. (34), which is explicitly written as

$$P[h] = \frac{\gamma(1+h)^{-1}}{\sqrt{2\pi c^2} [(1+h)^{2\gamma} - e^{-2\gamma}]^{1/2}} \times \exp \left\{ -\frac{[(1+h)^\gamma - 1]^2}{2c^2 [(1+h)^{2\gamma} - e^{-2\gamma}]} \right\}, \quad (37)$$

with

$$c^2 \equiv \frac{3H_0^4}{8\pi^2 m^2 \phi_s^2}, \quad \text{and} \quad \gamma \equiv \frac{m^2}{3H_0^2}. \quad (38)$$

Since  $\phi$  is slowly rolling at  $\phi = \phi_s$  one can also write  $c^2$  in terms of  $\Delta$  as

$$c^2 = \frac{\gamma}{2} \Delta^2(t_s) \equiv \frac{\gamma}{2} \Delta_s^2. \quad (39)$$

In the limit of small  $h$  it properly reduces to Gaussian:

$$P[h] \rightarrow \frac{\gamma}{\sqrt{2\pi c^2(1 - e^{-2\gamma})}} \times \exp \left[ -\frac{\gamma^2 h^2}{2c^2(1 - e^{-2\gamma})} \right]. \quad (40)$$

If we further take the limit  $\gamma \rightarrow 0$ , it reduces to a more familiar form:

$$P[h] \rightarrow \frac{1}{\sqrt{2\pi\Delta_s^2}} e^{-h^2/2\Delta_s^2}. \quad (41)$$

From Eq. (37) the probability that  $h$  exceeds a threshold value  $h_{th}$  is estimated as

$$P[h > h_{th}] = \int_{h_{th}}^{\infty} P[h] dh \approx \frac{c}{\sqrt{2\pi}} \frac{[(1+h_{th})^{2\gamma} - e^{-2\gamma}]^{3/2}}{(1+h_{th})^\gamma [(1+h_{th})^\gamma - 1] [(1+h_{th})^\gamma - e^{-2\gamma}]} \times \exp \left\{ -\frac{[(1+h_{th})^\gamma - 1]^2}{2c^2 [(1+h_{th})^{2\gamma} - e^{-2\gamma}]} \right\}, \quad (42)$$

which is insensitive to the upper end of integral since the dominant contribution comes from  $h \approx h_{th}$ . The criterion for black hole formation has been numerically investigated by Nadegin, Novikov, and Polnarev [38] and by Biknell and Henriksen [39]. Although it depends on the shape of the perturbed region, the generic value of the threshold reads  $h_{th} > 0.75 - 0.9$ . We take the black hole threshold as  $h_{bh} = 0.75$ .

Putting  $h_{th} = h_{bh}$  we can identify Eq. (42) with the fraction of primordial black holes,

$$\beta(M_s) = P[h > h_{bh}], \quad (43)$$

where the typical black hole mass,  $M_s$ , is equal to the horizon mass when the comoving scale  $l_s$  reenters the Hubble radius. Note that since our model predicts density fluctuation which is highly peaked on the comoving scale  $l_s$  the resultant mass function of PBHs are sharply peaked at the mass around  $M_s$ .

#### IV. NUMERICAL ANALYSIS OF CHAOTIC NEW INFLATION WITH THE SIMPLEST POTENTIALS

We now proceed to the numerical analysis of the equations of motion for the homogeneous part:

$$\ddot{\phi}(t) + 3H\dot{\phi} + V'[\phi] = 0, \quad (44)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2(t) = \frac{8\pi}{3M_{Pl}^2} \left( \frac{\dot{\phi}^2}{2} + V[\phi] \right), \quad (45)$$

with either  $V[\phi] = V_{CW}[\phi]$  or  $V[\phi] = V_{DW}[\phi]$ . One of the nice features of chaotic inflation is that no matter what initial configuration is adopted, the system rapidly approaches the slow-rollover solution with exponential accuracy once inflation sets in with sufficiently large initial value of  $\phi$ . We stress in this sense that the dynamics of the second inflationary phase is independent of the choice of initial condition but is only sensitive to the model parameters of the potential, and unlike in the original version of new inflation fine-tuning of initial condition is not necessary. For our purpose, it is sufficient to take  $\phi \geq 3.5M_{Pl} - 4M_{Pl}$ .

First, for completeness we solve Eqs. (44) and (45) for new inflation with the Coleman-Weinberg potential (1). We find that if  $0.2201M_{Pl} < v < 0.2259M_{Pl}$  at least ten  $e$ -folds inflationary (accelerated) expansion with  $|\dot{H}| < H^2$  is realized independent of the value of  $\tilde{\lambda}$ . For  $0.2223M_{Pl} < v < 0.2239M_{Pl}$  we have more than 60  $e$ -folds new inflation and in this case the chaotic inflation regime would be inflated beyond our current Hubble radius. In any case, as we mentioned in Sec. II we cannot obtain large-amplitude fluctuations in the observable scale in this model.

Next we report the case of the double-well potential (2). In this case we find, independent of the value of  $\lambda$ , that the field settles down to the positive minimum if  $v \geq 0.16286751M_{Pl}$  and it overshoots the origin to a negative value for  $v \leq 0.16286750M_{Pl} \equiv v_{cr}$ . If  $v$  is much smaller than  $v_{cr}$ ,  $\phi$  will travel between positive and negative values several times before settling down to one of the minima, but we do not consider this possibility here.

For  $v \approx v_{cr}$ , we find the vacuum energy at the origin induces inflationary expansion which lasts only about five  $e$ -folds. This is because the curvature of the potential at the origin is so large that the universe cannot stay in the slow-roll phase as is seen from the fact that the ratio

$$\frac{m^2}{3H_0^2} = \frac{1}{2\pi} \left( \frac{M_{Pl}}{v_{cr}} \right)^2 \approx 6.00, \quad (46)$$

is substantially larger than unity. As a result the magnitude of density fluctuations produced in this regime remains small:  $\Delta = \mathcal{O}(10^{-5})$  with COBE-normalized self coupling  $\lambda = 3 \times 10^{-13}$ .

We thus find neither the Coleman-Weinberg potential (1) nor the double-well potential (2) lead to formation of large-amplitude density fluctuations on currently observable scales in the chaotic new inflation scenario. But the reasons of their failures are opposite; with the Coleman-Weinberg potential, the field moves too slowly and the scales with large-amplitude fluctuations are inflated away, while with the double-well potential,  $\phi$  moves so rapidly that  $\Delta$  remains too small.

#### V. PBH FORMATION IN CHAOTIC NEW INFLATION

The above observation naturally leads us to consider a different class of potential including another free parameter:

$$V_{ML}[\phi] = -\frac{1}{2}m^2\phi^2 + \frac{\tilde{\lambda}}{4}\phi^4 \left( \ln \left| \frac{\phi}{v} \right| - \frac{1}{4} \right) + V_0, \quad (47)$$

that is, typical one-loop effective potential with nonvanishing mass term at the origin. This type of the potential with a positive mass-squared at the origin was employed in the original inflation scenario [14], but for our purpose we adopt a negative mass term.

The potential (47) has four parameters, but one of them,  $V_0$  is fixed from the requirement that the vacuum energy density vanishes at the potential minimum  $\phi \equiv \pm \phi_m$ . While we numerically solve the equation  $V'[\phi_m] = 0$  and obtain the numerical value of  $V_0$  so that  $V[\phi_m] = 0$ , we can also calculate them perturbatively in the case  $m^2 \ll \tilde{\lambda}v^2$ , as

$$\phi_m = v \left[ 1 + \frac{m^2}{\tilde{\lambda}v^2} - \frac{3}{2} \left( \frac{m^2}{\tilde{\lambda}v^2} \right)^2 + \dots \right], \quad (48)$$

$$V_0 = \frac{\tilde{\lambda}}{16}v^4 + \frac{1}{2}m^2v^2 + \frac{m^4}{\tilde{\lambda}} + \dots. \quad (49)$$

Another parameter, say  $\tilde{\lambda}$ , can be fixed from the amplitude of large-scale CMB fluctuations using the COBE data as before. Hence we are essentially left with two free parameters,  $v$  and  $m$ . While  $v$  mainly controls the speed of  $\phi$  around the origin and its fate, i.e., to which minimum it falls, and  $m$  mainly governs the duration of new inflation, the entire dynamics is determined by a complicated interplay of the three parameters. For example, we cannot determine  $\tilde{\lambda}$  until we calculate the duration of new inflation which also depends on  $\tilde{\lambda}$  itself for fixed values of  $m$  and  $v$ . Hence we must numerically solve the equations of motion iteratively to find out appropriate values of parameters to produce PBHs at the right scale with the right amount.

Let us now consider a specific example of formation of MACHO-PBHs. For this purpose we must realize a peak with  $\beta \sim 10^{-10}$  on the comoving Hubble scale at  $N = 35$ . After some iterative trials we have chosen  $\tilde{\lambda} = 3 \times 10^{-12}$  and

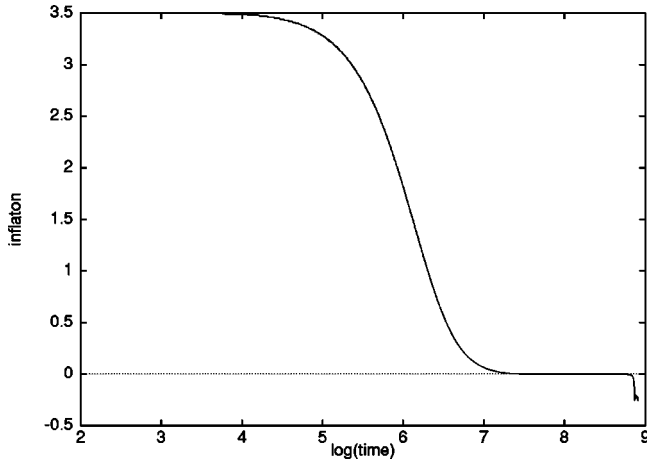


FIG. 1. Evolution of the inflaton in chaotic new inflation with  $\tilde{\lambda} = 3 \times 10^{-12}$ ,  $m = 6 \times 10^{-8} M_{Pl}$ , and  $v = 0.21384360 M_{Pl}$ . Time and  $\phi$  are displayed in units of the Planck time and  $M_{Pl}$ , respectively.

$m = 6 \times 10^{-8} M_{Pl}$ , and then solved the equation of motion for various values of  $v$ . In this choice of  $\tilde{\lambda}$  and  $m$  we find new inflation lasts for more than ten  $e$ -folds expansion if we take  $v$  in the range  $v = 0.2131 M_{Pl} - 0.2147 M_{Pl}$ . Hence we do not need much fine-tuning of the model parameters to realize a new inflationary stage itself. We also find that  $\phi$  settles down to  $\phi_m$  classically if  $v \geq 0.213843638 M_{Pl}$  and to  $-\phi_m$  if  $v \leq 0.213843631 M_{Pl}$ . If, on the other hand, we choose  $v$  in the range  $0.213843632 M_{Pl} < v < 0.213843637 M_{Pl}$ ,  $\phi$  spends more than one expansion time in the region  $|\phi| < \phi_q = 3.4 \times 10^{-8} M_{Pl}$  and the universe would enter the self-reproduction regime.

Figures 1 and 2 depict evolution of the scale factor and the inflaton  $\phi$ , respectively, for the case  $v = 0.21384360 M_{Pl} \equiv v_M$  with the initial condition  $a_i = 1$  at  $\phi_i = 3.5 M_{Pl}$ . The chaotic inflation ends at  $\phi = 0.89 M_{Pl}$  and new inflationary expansion sets in at  $\phi = 0.082 M_{Pl}$  but the slow roll-over phase starts only at  $\phi_s = -4.03 \times 10^{-7} M_{Pl} \equiv \phi_M$ . In this case  $\phi$  is found to stay in the region  $|\phi| < \phi_q$  for only about 0.1 expansion time. The linear perturba-

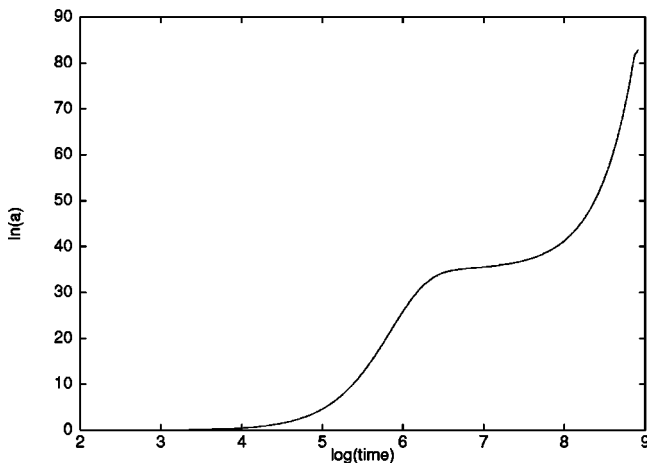


FIG. 2. Evolution of the scale factor with the same parameters.

tion  $\Delta$  has the right amplitude on large scales to meet the COBE observation, and it has a peak on the comoving Hubble scale at  $\phi = \phi_s$ . We find  $N(\phi_s \rightarrow \phi_f) = 35$ , which is the right scale for MACHO-PBHs.

In this case we find  $\gamma = 0.300590$  and the abundance of the PBHs at formation reads

$$\beta = 2.29c \exp(-0.0196994c^{-2}), \quad (50)$$

with  $h_{bh} = 0.75$ . For  $\phi_s = \phi_M$  we find the peak abundance of PBH,  $\beta = 6 \times 10^{-10}$  at the mass scale  $M \approx 1 M_{\odot}$ .<sup>1</sup> Using Eq. (39) we can also write it as

$$\beta = 0.888 \Delta_s \exp(-0.131072 \Delta_s^{-2}). \quad (51)$$

One can also obtain an approximate shape of the mass spectrum of the PBHs using Eq. (51) with  $\Delta_s$  replaced by  $\Delta$  at different epoch corresponding to different black hole mass. More specifically the mass of black holes,  $M$ , and their initial fraction,  $\beta(M)$ , can be written by an implicit function of  $\phi$  as

$$M = K \exp(2[N(\phi \rightarrow \phi_f) - 35]) M_{\odot}, \quad (52)$$

$$\beta(M) \approx 0.888 \Delta(\phi) \exp(-0.131072 \Delta^{-2}(\phi)), \quad (53)$$

where  $K$  is a factor of order of unity which depends on the expansion law of the post-inflationary universe but we put  $K = 1$  for simplicity below.

Figure 3 depicts the mass spectrum of black holes obtained from Eqs. (52) and (53). Thus the PBH abundance is sharply peaked. Note, however, that the shape of the large-mass tail is not exactly correct which corresponds to the regime where slow-roll solution is invalid. Nonetheless this figure correctly describes the location of the peak.

Apart from the effects of inflaton's detailed dynamics, however, we must say that the above spectrum is only qualitatively correct, because we have chosen a specific value of the threshold,  $h_{bh} = 0.75$ , and assumed the black hole mass is equal to the horizon mass when the perturbed region reentered the Hubble radius (see also [40,41]). In order to improve the calculation of the mass spectrum we must calculate the probability distribution functional of the configuration of the perturbed region and then calculate the final mass of the black hole, if formed, for each configuration, which is be-

<sup>1</sup>In obtaining Eq. (50) we have started with  $\Pi[\phi, t = t_s] = \delta(\phi - \phi_s)$  in order to extract information on a specific mass scale. This also corresponds to treating that  $\phi$  evolves along the classical trajectory until  $\phi = \phi_s$ , which is correct only as an average. In fact, due to quantum fluctuations generated during chaotic inflation and the early stage of new inflation,  $\phi_s$  itself takes different values in different domains. We have confirmed, however, that  $\phi_s$  shifts only about  $\pm 4H_0/(2\pi)$  even if we consider their effects. Since we find  $|\phi_M| \gg 4H_0/(2\pi)$ , it does not induce significant fluctuations in  $\beta$ . For the same reason we are free from the domain wall problem which could be present if  $\phi$  had a large fluctuation and different regions fell different minima.

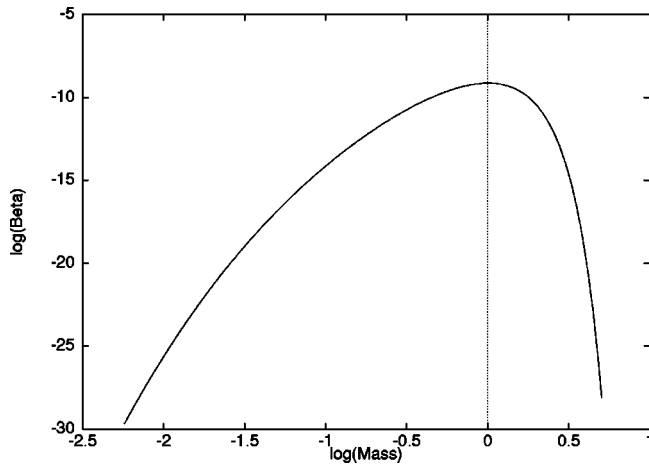


FIG. 3. Expected mass spectrum of PBHs in chaotic new inflation with the same model parameters. Mass is displayed in units of the solar mass.

yond the scope of the present analysis. Nevertheless from the above analysis we can convince ourselves that we can produce large enough amplitude of curvature fluctuation on a desired scale in the chaotic new inflation scenario.

Within the limit of our predictability of the mass spectrum, we can also apply our model for the formation of PBHs with different masses and abundance. For example, we may produce PBHs with  $M \approx 10^8 M_\odot$  which may act as a central engine of AGNs with the current density, say,  $n \sim 10^{-5} \text{ Mpc}^{-3}$  corresponding to  $\beta \sim 10^{-11}$  at formation. From Eq. (52) we find  $M = 10^8 M_\odot$  corresponds to  $N(\phi_s \rightarrow \phi_f) = 44$  and the desired spectrum is realized for  $\lambda = 3.7 \times 10^{-12}$ ,  $m = 6.1 \times 10^{-8} M_{pl}$ , and  $v = 0.21532324 M_{pl}$  under the COBE normalization.

Another interesting possibility is to produce a tiny amount of PBHs which are evaporating right now, with the initial mass  $M \approx 10^{15} \text{ g}$ . With the current abundance  $\Omega \approx 10^{-8}$  or  $\beta \approx 10^{-25}$  at formation, they may explain a class of gamma-ray bursts. In this case we should have only a short period of slow-roll new inflation,  $N(\phi_s \rightarrow \phi_f) = 14$ . We find  $\beta = 2 \times 10^{-25}$  at the right mass scale if we choose  $\lambda = 3 \times 10^{-13}$ ,  $m = 5 \times 10^{-8} M_{pl}$ , and  $v = 0.16557604828 M_{pl}$ . In this case a relatively large value of  $m$  is required in order to keep new inflation short, and we cannot necessarily rely on the stochastic inflation method which is valid only if  $|m^2| \lesssim H_0^2$ . This does not mean we cannot generate large enough density fluctuation on the relevant scale. The only problem is we do not have a reliable method to calculate magnitude of fluctuation or black hole abundance accurately in such situations.

## VI. CONCLUSION

In the present paper we proposed a new scenario of double inflation which contains only one inflation-driving scalar field, that is, we pointed out that a scalar potential which has an unstable local maximum at the origin can not only realize new and chaotic inflation separately but also accommodate both sequentially if its model parameters are appropriately chosen with natural initial conditions as employed in the chaotic inflation scenario. We have further shown that the spectrum of density fluctuation in this model can have a large-amplitude peak on the comoving Hubble scale at the onset of the slow-roll regime of new inflation and that this can be applied to formation of PBHs on a specific mass scale. This feature of the spectrum is realized naturally compared with other models with a single scalar field [21,22] because the scalar field is not slowly rolling at the onset of new inflation. On the other hand, we must specify the values of model parameters with many digits in order to produce an appropriate amount of PBHs on a desired scale. This feature, however, is more or less common to all the other models attempting to account for formation of PBHs in inflationary cosmology (see e.g. [22]), because both the peak amplitude of the fluctuations and its location must be specified with high accuracy due to the exponential dependence of the black hole abundance and its mass on the model parameters.

On the other hand, one could in principle claim that observation of PBHs can serve as a strong tool to determine the parameters in the inflaton's dynamics. Unfortunately, however, our ignorance of the detailed condition for PBH formation, such as the precise threshold amplitude of fluctuation as a functional of the shape of perturbed region, makes it impossible to link the mass spectrum of PBHs with the shape of the inflaton's potential precisely. In the present paper we have calculated the values of the model parameters rather precisely under the universal assumption of  $h_{bh} = 0.75$ . In fact, however, the values of the parameters would totally change had we chosen a different threshold. Hence the precise numbers we have quoted do not have much significance, but the number of digits simply indicates the sensitivity of the mass spectrum to the model parameters.

## ACKNOWLEDGMENTS

The author is grateful to Professor Andrei Linde for useful comments and his hospitality at Stanford University, where this work was done. This work was partially supported by the Monbusho.

- [1] Ya. B. Zel'dovich and I. D. Novikov, *Sov. Astron.* **10**, 602 (1967).  
 [2] S. W. Hawking, *Mon. Not. R. Astron. Soc.* **152**, 75 (1971).  
 [3] B. J. Carr, *Astrophys. J.* **201**, 1 (1975).

- [4] B. J. Carr, *Astrophys. J.* **206**, 8 (1976); S. Miyama and K. Sato, *Prog. Theor. Phys.* **59**, 1012 (1978); I. D. Novikov, A. G. Polnarev, A. A. Starobinsky, and Ya. B. Zel'dovich, *Astron. Astrophys.* **80**, 104 (1979).



- [5] C. Alcock *et al.*, *Nature* (London) **365**, 623 (1990); *Phys. Rev. Lett.* **74**, 2867 (1995); *Astrophys. J.* **486**, 697 (1997); E. Aubourg *et al.*, *Nature* (London) **365**, 623 (1993); *Astron. Astrophys.* **301**, 1 (1995).
- [6] H. B. Richer and G. G. Fahlman, *Nature* (London) **358**, 383 (1992).
- [7] S. P. Boughn and J. M. Uson, *Phys. Rev. Lett.* **74**, 216 (1995).
- [8] B. J. Carr, J. R. Bond, and W. D. Arnett, *Astrophys. J.* **277**, 445 (1984).
- [9] T. Nakamura, M. Sasaki, T. Tanaka, and K. S. Thorne, *Astrophys. J. Lett.* **487**, L139 (1997).
- [10] S. W. Hawking, *Nature* (London) **248**, 30 (1974); *Commun. Math. Phys.* **43**, 19 (1975).
- [11] D. Cline, D. A. Sanders, and W. Hong, *Astrophys. J.* **486**, 169 (1997).
- [12] E. L. Turner, *Astron. J.* **101**, 5 (1991).
- [13] A. Loeb, *Astrophys. J.* **403**, 542 (1993); M. Umemura, A. Loeb, and E. L. Turner, *ibid.* **419**, 459 (1993).
- [14] A. H. Guth, *Phys. Rev. D* **23**, 347 (1981); K. Sato, *Mon. Not. R. Astron. Soc.* **195**, 467 (1981).
- [15] A. D. Linde, *Phys. Lett.* **108B**, 389 (1982); A. Albrecht and P. J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [16] A. D. Linde, *Phys. Lett.* **129B**, 177 (1983).
- [17] For a review of inflation see, e.g. A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Academic, Chur, Switzerland, 1990).
- [18] S. W. Hawking, *Phys. Lett.* **115B**, 295 (1982); A. A. Starobinsky, *ibid.* **117B**, 175 (1982); A. H. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982).
- [19] C. L. Bennet *et al.*, *Astrophys. J. Lett.* **464**, L1 (1996).
- [20] H. M. Hodges and G. R. Blumenthal, *Phys. Rev. D* **42**, 3329 (1990).
- [21] P. Ivanov, P. Naselsky, and I. Novikov, *Phys. Rev. D* **50**, 7173 (1994).
- [22] J. S. Bullock and J. R. Primack, *Phys. Rev. D* **55**, 7423 (1997).
- [23] J. Yokoyama, *Astron. Astrophys.* **318**, 673 (1997).
- [24] J. Silk and M. S. Turner, *Phys. Rev. D* **35**, 419 (1987).
- [25] L. Randall, M. Soljacić, and A. H. Guth, *Nucl. Phys.* **B472**, 377 (1996).
- [26] J. García-Bellido, A. D. Linde, and D. Wands, *Phys. Rev. D* **54**, 6040 (1996).
- [27] M. Kawasaki, N. Sugiyama, and T. Yanagida, *Phys. Rev. D* **57**, 6050 (1998).
- [28] L. F. Kofman, A. D. Linde, and A. A. Starobinsky, *Phys. Lett.* **157B**, 361 (1985); R. Holman, E. W. Kolb, S. L. Vadas, and Y. Wang, *Phys. Lett. B* **269**, 252 (1991); D. Polarski and A. A. Starobinsky, *Nucl. Phys.* **B385**, 623 (1992); *Phys. Rev. D* **50**, 6123 (1994); S. Gottlöber, J. P. Muecket, and A. A. Starobinsky, *Astrophys. J.* **434**, 417 (1994).
- [29] J. H. Kung and R. H. Brandenberger, *Phys. Rev. D* **42**, 1008 (1990).
- [30] A. D. Linde, *Phys. Lett. B* **327**, 208 (1994); A. Vilenkin, *Phys. Rev. Lett.* **72**, 3137 (1994).
- [31] S. Coleman and E. Weinberg, *Phys. Rev. D* **7**, 788 (1973).
- [32] R. K. Sachs and A. M. Wolfe, *Astrophys. J.* **147**, 73 (1967).
- [33] D. S. Salopek, *Phys. Rev. Lett.* **69**, 3602 (1992).
- [34] A. Vilenkin, *Phys. Rev. D* **27**, 2848 (1983); A. D. Linde, *Phys. Lett. B* **175**, 395 (1986); *Mod. Phys. Lett. A* **1**, 81 (1986).
- [35] A. A. Starobinsky, in *Field Theory, Quantum Gravity, and Strings*, edited by H. J. de Vega and N. Sanchez, *Lecture Notes in Physics* Vol. 246 (Springer, Berlin, 1986), p. 107.
- [36] P. Ivanov, *Phys. Rev. D* **57**, 7145 (1998).
- [37] H. Kodama and M. Sasaki, *Prog. Theor. Phys. Suppl.* **78**, 1 (1984).
- [38] D. K. Nadezhin, I. D. Novikov, and A. G. Polnarev, *Sov. Astron.* **22**, 129 (1978).
- [39] G. V. Bicknell and R. N. Henriksen, *Astrophys. J.* **232**, 670 (1978).
- [40] J. C. Niemeyer and K. Jedamzik, *Phys. Rev. Lett.* **80**, 5481 (1998).
- [41] J. Yokoyama, *Phys. Rev. D* (to be published), gr-qc/9804041gr-qc.