Quantum creation of a universe with $\Omega \neq 1$: Singular and nonsingular instantons

Raphael Bousso and Andrei Linde
Department of Physics, Stanford University, Stanford, California 94305
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We propose two new classes of instantons which describe the tunneling and/or quantum creation of closed and open universes. The instantons leading to an open universe can be considered as generalizations of the Coleman–De Luccia solution. They are nonsingular, unlike the instantons recently studied by Hawking and Turok, whose prescription has the problem that the singularity is located on the hypersurface connecting to the Lorentzian region, which makes it difficult to remove. We argue that such singularities are harmless if they are located purely in the Euclidean region. We thus obtain new singular instantons leading to a closed universe; unlike the usual regular instantons used for this purpose, they do not require complex initial conditions. The singularity gives a boundary contribution to the action which is small for the instantons leading to sufficient inflation, but changes the sign of the action for small ϕ corresponding to short periods of inflation. [S0556-2821(98)08016-3]

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I. INTRODUCTION

It is well known that most of the models of inflationary cosmology predict $\Omega = 1 \pm 10^{-4}$. It is possible to have inflation with $\Omega \neq 1$, but it is rather difficult. The basic idea is to solve the homogeneity and isotropy problem not by the long stage of inflation, but by quantum tunneling to a state describing an open or closed universe. Then the universe will be homogeneous if the probability of tunneling is sufficiently and strongly suppressed. In this scenario an infinite number of open universes can be created in one of two ways. One may consider a purely classical evolution of an inflationary universe in the false vacuum and a subsequent creation of inflating open universes by tunneling to the true vacuum [1–3]. Alternatively, one may consider the quantum creation from nothing of a closed inflationary universe, which later decays into an infinite number of open universes by the process described above [4,5]. In all such models, it is necessary to assume a potential with a false vacuum.

Recently, Hawking and Turok claimed that open universes can be obtained without an intermediate stage involving false vacua. They described a process in which an open universe is created from nothing in the chaotic inflation scenario with a generic effective potential [6]. They used the standard deformed-four-sphere Euclidean solution, in which the inflaton field is constant on the lines of constant latitude. This solution generically has a singularity on one of the poles. Usually it is cut along the equator; the singular hemisphere is discarded, and the regular one is analytically continued to yield a closed Lorentzian universe.

Instead, Hawking and Turok cut the Euclidean solution through the poles, thus including the singularity on the hypersurface through which the Euclidean and Lorentzian sectors are joined. The hypersurfaces of constant inflaton field will then form infinite open spacelike sections in a part of the resulting Lorentzian universe.

This approach suffers from two problems. The first problem is that they obtain $\Omega = 10^{-2}$ for the ratio of the present density to the critical density. This contradicts observational data by almost two orders of magnitude, and even this result was obtained only after invoking the anthropic principle, without which one would get $\Omega = 0$. The prediction comes from the probability measure associated with the Hartle-Hawking wave function. In [5] it was argued that, according to [7,8], this wave function should not be used for the description of the creation of the universe and it was shown that the use of the tunneling proposal would typically lead to $\Omega \approx 1$. It thus appears that with either choice of the wave function of the universe we are not currently in a position to obtain a realistic value of $\Omega = 0.3$ unless customized potentials are employed.

The second problem is associated with the presence of a singularity on the nucleation surface. Vilenkin [9] argued that instantons of the Hawking-Turok type lead to vacuum instability and should therefore be excluded from the path integral. In [5] it was shown that not every instanton is permitted even if it is nonsingular. On the other hand, it was suggested that singular instantons are not necessarily forbidden, but one should be extremely careful about the analytical continuation involving singularities which was used in [6]. For other problems associated with this issue see also [10].

In this paper, we will perform a more detailed investigation of these issues. We will suggest two ways of avoiding the problems associated with the Hawking-Turok singularity. First, we will consider potentials with a local maximum, for which there are non-singular solutions. They include the Coleman–De Luccia instantons as well as some new, related solutions that we found. We will discuss the structure and application of these solutions in Sec. II. They describe the nucleation of open universes, and allow the correct prediction of Ω for suitable potentials. Of course, this means that the generality claimed by Hawking and Turok is lost, but as we pointed out above, generic inflaton potentials do not seem very promising in any case when one tries to predict universes which are both non-flat and non-empty.

We will allow generic potentials in Sec. III, where we will use variants of the deformed-four-sphere instanton to nucleate closed universes. We cut them along the equator and discard the *regular* hemisphere. The Hawking-Turok singularity will be present in this case. It will not, however, lie on

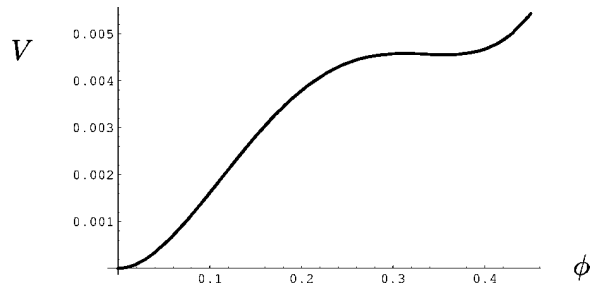

FIG. 1. Effective potential $V(\phi)=m^2/2[\phi^2(\phi-v)^2+B\phi^4]$ for $m^2=2$, $B=0.12$, and $v=0.5$. It has a shallow minimum at $\phi_0 \sim 0.357$ and a local maximum at $\phi_1=0.312$. All quantities in the figures in this section are in units of $M_p/\sqrt{8\pi}$.

the nucleation hypersurface. Therefore it can be ''surgically removed,'' or viewed as a small region of Planckian density. We calculate the boundary contribution to the action and show that it is small in all cases where sufficient inflation ensues. We discuss a possible interpretation of these solutions as the birth of a closed inflationary universe by tunneling from space-time foam. Finally, we construct instantons that are symmetric about the equator and contain two singularities. They allow the construction of nucleation paths on which all variables are everywhere real.

II. NONSINGULAR INSTANTONS

Suppose we have an effective potential $V(\phi)$ with a local minimum at ϕ_1, and a global minimum at $\phi=0$, where $V=0$ (see Fig. 1). In an $O(4)$-invariant Euclidean spacetime with the metric

$$ds^2=d\tau^2+a^2(\tau)(d\psi^2+\sin^2\psi d\Omega_2^2), \tag{1}$$

the scalar field ϕ and the three-sphere radius a obey the equations of motion

$$\phi''+3\frac{a'}{a}\phi'=V_{,\phi}, \quad a''=-\frac{8\pi G}{3}a(\phi'^2+V), \tag{2}$$

where primes denote derivatives with respect to τ.

These equations have several nonsingular solutions, the simplest of which are the $O(5)$ invariant four-spheres one obtains when field ϕ sits at one of the extrema of its potential. In this case the first of the two equations above is trivially satisfied, and $a(\tau)=H^{-1}\sin H\tau$. Here $H^2=8\pi V/3M_p^2$. Using the solution for which $\phi=\phi_1$, Hawkins and Moss [11] found the rate at which field ϕ in a single Hubble volume tunnels to the top of the potential, from which it can roll down towards the true vacuum. For a recent discussion of this instanton and its interpretation see [5]. The main other use of these trivial instantons is to find the action of the false vacuum background solution, which must be subtracted from the bounce action to obtain a tunneling rate.

We shall consider potentials for which $V_{,\phi\phi}\gg H^2$ in the region where the tunneling occurs. In this case, tunneling out of the false vacuum does not occur primarily on the scale of an entire Hubble volume via the Hawking-Moss instanton. Instead the transition will proceed via more complicated Euclidean solutions with varying field ϕ. These include the Coleman–De Lucia instanton, and related instantons that we found.

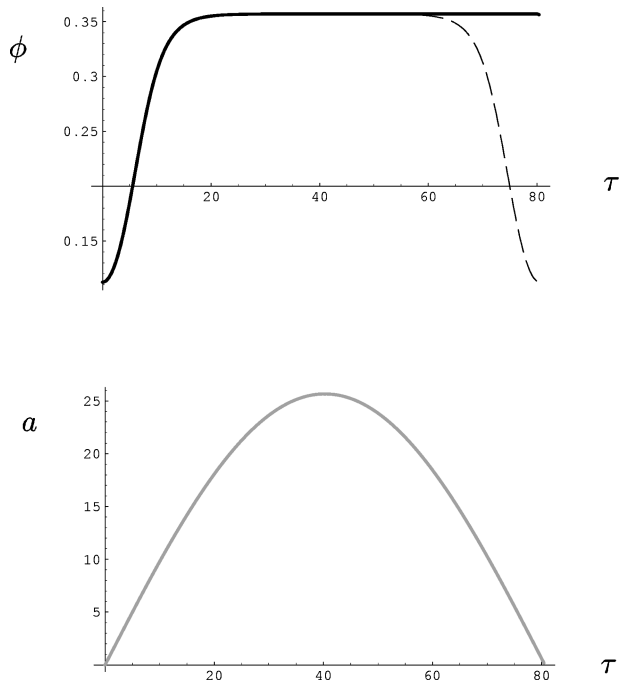

FIG. 2. The upper panel shows the behavior of scalar field ϕ for examples of the Coleman–De Luccia ''bubble'' instanton (solid line) and the new ''double-bubble'' instanton that we found (dashed line). For both instantons, the field is in the domain of the true vacuum at small τ, forming a bubble. For the bubble instanton, the field is closest to the false vacuum at the pole opposite the bubble. For the double-bubble instanton, this happens on the equator at the moment of the maximal expansion. The behavior of the three-sphere radius $a(\tau)$ shown in the lower panel is very similar for both instantons, though it is not identical.

A. Bubble instantons

A Euclidean solution that describes the creation of an open universe was first found by Coleman and De Luccia in 1980 [12]. It is given by a slightly distorted de Sitter four-sphere of radius $H^{-1}(\phi_0)$. Typically, field ϕ is very close to false vacuum ϕ_0 throughout the four-sphere except in a small region (whose center we may choose to lie at $\tau=0$), in which it lies on the ''true vacuum'' side of the maximum of V. The behavior of the field and scale factor for the potential in Fig. 1 is shown in Fig. 2. The scale factor vanishes at the points $\tau=0$ and $\tau=\tau_f\approx\pi/H$, which we will call the north and south pole of the four-sphere. In order to get a singularity-free solution, one must have $\phi'=0$ and $a'=\pm 1$ on the poles.

This solution can be cut in half along the line $\psi=\pi/2$, which removes half of each three-sphere. Then one can continue analytically to a Lorentzian spacetime [13,6] with the time variable σ, given by $\psi=\pi/2+i\sigma$. This gives region II of the Lorentzian universe (see Fig. 3)

$$ds^2=-a^2(\tau)d\sigma^2+d\tau^2+a^2(\tau)\cosh^2\sigma d\Omega_2^2. \tag{3}$$

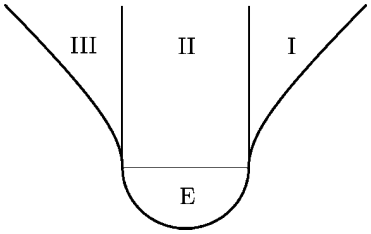

FIG. 3. The Lorentzian de Sitter-like spacetime obtained from the analytic continuation of Coleman–De Luccia instantons contains three regions. In regions I and III the hypersurfaces of constant field ϕ form open spacelike sections. Region II is a shell separating the two bubbles.

Field ϕ will still depend on τ in the same way as before, and will be independent of σ. This describes a shell of width H^{-1}, which is mostly near the false vacuum and expands exponentially. The shell separates two bubbles, regions I and III, in which the universe looks open.

Region I is obtained by taking $\sigma = i\pi/2 + \chi$ and $\tau = it$, giving the metric

$$ds^2 = -dt^2 + \alpha^2(t)(d\chi^2 + \sinh^2 \chi d\Omega_2^2), \tag{4}$$

where $\alpha(t) = -ia[\tau(t)]$. Its spacelike sections (defined by the hypersurfaces of constant inflaton field) are open. Thus, region I looks from the inside like an infinite open universe, which inflates while field ϕ slowly rolls down to the true vacuum. The evolution will then undergo a transition to a radiation or matter-dominated open Friedmann-Robertson-Walker universe.

In region III, which is obtained by choosing $\sigma = i\pi/2 + \chi$ and $\tau = \tau_f + it$, field ϕ rolls to the local minimum at ϕ_0, and one gets indefinite open inflation in the false vacuum.

The analytic continuations we have given support the interpretation of such solutions as the spontaneous nucleation of a bubble of true vacuum on the background of de Sitter space expanding in the false vacuum. For this reason we will call them ''bubble instantons.'' The nucleation rate is given by

$$\Gamma = e^{-\Delta S}, \tag{5}$$

where ΔS is the difference between the action of the full Euclidean bubble solution, and the action of a Euclidean solution describing the background spacetime. Except for near-Planckian potentials, both actions will be large and negative (about -2.6×10^4 in our example). The background solution is given by an exact Euclidean four-sphere on which field ϕ is constant and equal to ϕ_0, the false vacuum. Its action will be $-(3M_p^4)/8V(\phi_0)$. Subtracting this from the action of the bubble solution, one obtains a positive ΔS (≈ 4.9 in our example). This means that bubble formation by tunneling is suppressed, as it should be.

One usually requires instanton solutions to interpolate between the initial and final spacelike sections (in this case, a section of pure de Sitter space in the false vacuum and a similar section containing a bubble of true vacuum). The above description, which seems to use two disjoint instantons, is actually consistent with this formal requirement, since the instantons may be connected by virtual domain walls after small (Planck size) four-balls are removed. This will cause the background instanton to contribute to the total action with a negative sign. If one connects the background instanton to the region of the bubble instanton where ϕ is closest to its false vacuum, the discontinuity in ϕ will be small, so the volume contributions of the removed regions cancel almost exactly. Requiring continuous instantons, therefore, does not change the pair creation rate significantly [14].

Cosmological instantons have frequently been interpreted to describe the creation of a universe from nothing, i.e. without a pre-existing background. This case is considerably less well-defined than the quantum nucleation of structures on a given background solution. In particular, the sign with which the large, negative action enters the exponent in the path integral is subject to debate [6,5,15]. Leaving such questions aside for now, we will take the position that isolated cosmological instantons are indeed related to universe creation, independently of the formalism used to assign probabilities to such processes.

B. Double-bubble instantons

We have found a new instanton in which there are two bubbles, one on each pole. In this solution, ϕ is in the domain of the true vacuum in small regions near the poles, and near the false vacuum elsewhere; this can be seen from the dashed line in Fig. 2. The geometry is still approximately a four-sphere. As before, ϕ' vanishes on the poles; but now it also vanishes on the equator, at $\tau = \tau_{max}$. The Northern and Southern hemispheres are exactly symmetric.

Not surprisingly, the action of the double-bubble solution, after the background subtraction described above, is approximately twice that of the bubble (Coleman–De Luccia) instanton. For the instanton shown in Fig. 2 one has $\Delta S_2 \approx 9.8$.

The analytic continuations will be the same as before, with a different result. Region II will be mostly in the domain of the false vacuum. Regions I and III will be identical, each containing an open inflating universe in which the field rolls down to the true vacuum. Globally, therefore, we obtain two bubbles of true vacuum separated by a shell which inflates in the false vacuum.

This solution can be interpreted as the spontaneous pair-creation of bubbles of open inflation on the background of false vacuum inflation. Alternatively, one may view it as the creation from nothing of two open inflating universes separated by a metastable shell.

C. Anti-double-bubble instantons

In addition we have found another family of instantons, two examples of which are shown in Fig. 4. In these instantons, the field is in the domain of the false vacuum in two regions surrounding the poles. They are separated by a thin shell at the equator, where the field is in the true vacuum domain. These instantons have a much greater action difference to the background instanton, since the true-vacuum re-

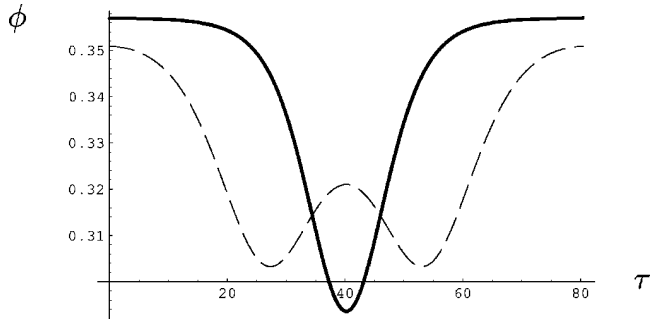

FIG. 4. Two examples of ''anti-double-bubble'' instantons in which the field is in the false vacuum domain near the poles and reaches into the domain of the true vacuum on a shell near the equator. It can be cut through the poles to describe shell nucleation, or across the equator, describing the tunneling to true-vacuum inflation in a closed universe.

gion is significantly larger than in the previous two cases. In particular, $\Delta S=93.6$ for the instanton shown by the solid line in Fig. 4, and $\Delta S=124.7$ for the instanton shown by the dashed line.

1. Open cut

With the analytic continuation used for the previous two instantons, regions I and III will become open inflationary universes in which the field rolls down to the false vacuum. They are separated by the region II, which contains a shell on which ϕ is in the domain of the true vacuum.

Therefore we may interpret this solution as the nucleation of a shell of true vacuum on a false vacuum inflationary background, or alternatively, as the creation of such a universe from nothing. Because of the larger action difference, spontaneous shell creation will be quite suppressed compared to bubble formation.

2. Closed cut

A more intriguing application of this instanton can be found by choosing a different analytic continuation. Instead of cutting at $\psi=\pi/2$, we may choose to leave the three-spheres intact, and cut across the equator. Lorentzian time will be defined by $\tau=\tau_{\max}+iT$, and we obtain a metric with closed spacelike sections:

$$ds^2=-dT^2+a^2(T)d\Omega_3^2. \tag{6}$$

The inflaton field is in the domain of the true vacuum on the nucleation surface (the equator), so it will start rolling down towards the absolute minimum. During this time, the spacelike three-spheres grow exponentially:

$$a(T)\approx H^{-1}(T)\cosh\int H(T)dT. \tag{7}$$

Thus we obtain a closed inflationary universe in which the scalar field rolls towards the true vacuum.

One could interpret this instanton as describing the creation of such a universe from nothing. But this would just add an alternative to the usual instantons on which the field

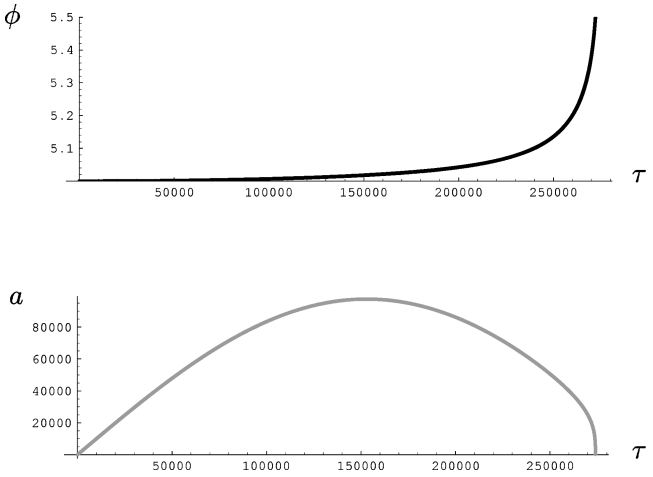

FIG. 5. The standard solution. The scalar field is in units of $M_{\rm p}$. Field ϕ as well as the curvature are singular at the south pole. Note that ϕ' does not vanish at the equator, so one should complexify the scalar field in order to make the analytical continuation to a closed universe. The analytic continuation to an open universe suggested by Hawking and Turok [6] involves the singularity on the nucleation surface, which makes it rather problematic.

is entirely in the domain of the true vacuum. A much more interesting interpretation is the one associated with a pre-existing background of false vacuum inflation. In this case, the anti-double-bubble instanton is seen to describe the spontaneous tunneling to the true vacuum in an entire Hubble-volume of de Sitter space. Unlike the Coleman–De Luccia bubbles, these regions will not contain an open universe. In the example we consider, for which Hawking-Moss tunneling is not possible, this shows that one can nevertheless nucleate true vacuum bubbles containing a closed universe.

III. SINGULAR INSTANTONS

A. Standard instantons

We now assume a generic effective potential of chaotic inflation, with a minimum, $V=0$, at $\phi=0$, and no other stationary points. The standard Euclidean solution used in quantum cosmology is obtained by requiring regularity at the north pole, at $\tau=0$. This means one must take $a'=1$ and $\phi'=0$ there. A Euclidean solution with these initial conditions is shown in Fig. 5 for a massive scalar field.

On most of the manifold, the solution will be almost a four-sphere, with ϕ increasing very slowly:

$$a(\tau)=H^{-1}\sin H\tau, \quad \phi(\tau)=\phi_{\rm N}, \tag{8}$$

where $\phi_{\rm N}$ denotes the value of the inflaton field on the regular pole. A first approximation to the Euclidean action of the standard instanton will therefore be given by the volume term for a four-sphere of radius $H^{-1}(\phi_{\rm N})$, $S\sim-(3M_{\rm p}^4)/8V(\phi_{\rm N})$. This will be a good approximation for large values of $\phi_{\rm N}$ leading to long periods of inflation.

Near the south pole, at $\tau=\tau_{\rm f}$, the anti-damping term $\sim\phi' b'/b$ starts to dominate the equation of motion for ϕ.

The field diverges logarithmically, and the potential terms can be neglected. Approximate solutions are given by [6,9]

$$a(\tau) = A(\tau_f - \tau)^{1/3}, \tag{9}$$

$$\phi(\tau) = -\frac{1}{\sqrt{12\pi}} \ln(\tau_f - \tau) + \phi_m, \tag{10}$$

where A and ϕ_m are constants.

There are many questions associated with this singularity. First of all, even though the divergence of the scalar field is only logarithmic, the energy density and curvature diverge according to a power-law. If the singularity is part of the nucleation geometry, i.e. if it is included in the Euclidean instanton and its Lorentzian analytic continuation, the corresponding method can no longer be called ''the no-boundary proposal.''

The boundary at τ_f contributes a Gibbons-Hawking [16] term to the action

$$S_{\tau_f} = -\frac{M_p^2}{8\pi} \int d^3x h^{1/2} K \Big|_{\tau=\tau_f}, \tag{11}$$

where h is the determinant of the three-metric h_{ij}, and K is the trace of K_{ij}, the second fundamental form. For an $O(4)$-invariant metric, we find $K = 3a'/a$. This yields

$$S_{\tau_f} = -\frac{\pi}{4} M_p^2 \frac{d(a^3)}{d\tau} \Big|_{\tau=\tau_f}. \tag{12}$$

By Eq. (9), $a(\tau)$ goes like $(\tau_f - \tau)^{1/3}$ near the singularity. Therefore, the boundary term will be positive and finite.[1] Note that for any power other than 1/3, the boundary term would either vanish, or diverge.

The contribution of this boundary term to the total action is relatively small if one considers the creation of an inflationary universe. For example, numerical investigation shows that in the theory $m^2\phi^2/2$ this correction, as compared to the action $-(3M_p^4)/8V(\phi_N)$, is suppressed by a factor $O(M_p/\phi_N)$ for $\phi_N \gg M_p$:

$$S_{std} \approx -\frac{3M_p^4}{8V(\phi_N)} \left(1 - \frac{M_p}{2\phi_N}\right). \tag{13}$$

There is also a correction to the volume term because ϕ is not exactly constant anywhere; we will not discuss this correction here.

As is obvious from this result, at small ϕ_N the total action including the boundary term may become positive. One can confirm numerically that this is indeed the case. This is a rather unexpected conclusion as it indicates that the absolute value of the action reaches its maximum not at $\phi_N = 0$ but at $\phi_N \sim M_p$.

[1]This result was also independently found by Vilenkin [9].

1. Closed cut

This Euclidean solution can be cut in different ways to obtain an instanton that allows an analytic continuation to a Lorentzian spacetime. The standard method is to cut along the equator, at $\tau = \tau_{max}$ and to discard the Southern hemisphere, which contains the singularity. With $\tau = \tau_{max} + iT$, one can join the regular hemisphere across the equator to Lorentzian de Sitter space with the metric given by Eqs. (6) and (7). The surfaces of constant inflaton field will be closed spacelike slices in the Lorentzian sector. Therefore this cut corresponds to closed inflation.

In general, ϕ' will be small but non-zero on the equator. This means that it will acquire an imaginary part in the Lorentzian sector. But at late Lorentzian times, when measurements are made, one must demand that all variables be exactly real. It is therefore necessary to compensate by adding a small imaginary part to the initial value of ϕ [17,18]:

$$\phi_N = \phi_N^{Re} - i \frac{M_p^2}{8\phi_N^{Re}} \tag{14}$$

where the superscript Re denotes the real part of a quantity.

For nucleation geometries which are everywhere real, the real part of the Euclidean action comes entirely from the Euclidean sector, since the Lorentzian sector contributes only a purely imaginary part. But in the current case, the Lorentzian sector will not be purely real for a time of order H^{-1} after the nucleation hypersurface. It will therefore give a further correction to the real part of the Euclidean action. It can be easily checked both analytically and numerically that the ratio of this term to the the total action is of order ϕ_N^{-2}. This corrects claims in [17] that the correction is of the same order as the total action.

2. Open cut

Hawking and Turok [6] have suggested using the same analytic continuation for the standard solution that had been traditionally used for the Coleman–De Luccia solution, which we gave explicitly in Sec. II. This involves cutting the Euclidean space through the poles, thus including half of the singularity on the nucleation surface. The resulting Lorentzian solution contains regions I and II of Fig. 3, while region III is cut off by the singularity. In region I the hypersurfaces of constant ϕ trace out infinite open spacelike sections.

The interpretation of this instanton and the analytical continuation proposed in [6] may be rather problematic. It was argued in [5] that even if instantons are nonsingular, they do not always describe tunneling. But in this case one has additional problems associated with cutting the singularity in half and performing the analytical continuation there [5].

Vilenkin [9] has argued that the admission of such nucleation geometries, in which there is a singularity on the hypersurface of the vanishing second fundamental form, leads to problems of vacuum instability. Such instantons exist for a Minkowski space background, where they may cause an almost unsuppressed nucleation of singular bubbles spreading out nearly at the speed of light. Clearly, this is physically unacceptable. It poses a problem for the prescription sug-

gested by Hawking and Turok unless one finds sound arguments why such instantons should be admitted for inflationary universes, but not for flat space.

According to [9], singular instantons may not be allowed at all because they are not true (nonsingular) solutions of equation of motion that would correspond to an extremum of action. However, the singularity is not really a part of the manifold. Moreover, one can sometimes cut it out, and consider a configuration that is nonsingular but coincides with a singular instanton everywhere except for a small vicinity of the singularity. If the action of the instanton converges, then for a sufficiently small size of the patch replacing the singular region, the action will differ from the instanton action by less than $\Delta S=1$. Such ''almost solutions'' are perfectly admissible and play the same role in the functional integral as the true solutions, see e.g. [19,14]. However, if one makes an analytical continuation through the singularity, one cannot easily remove it by the method described above, and then it may pose a real problem.

A possible way out of this problem would be to use nonsingular instantons of the type we described in Sec. II. They require special potentials with a false vacuum. This has the advantage that one can, in principle, obtain any given value $\Omega<1$. The price we pay is some loss of generality.[2]

An interesting result which appears after the boundary term is taken into account is that the action at very small ϕ changes its sign and becomes positive. This means that the maximal absolute value of the action is reached not at the point when $V(\phi)=0$, but at some other point, where inflation is still possible. It would be very interesting if this point were at a sufficiently large value of ϕ, which would provide a realistic value of Ω within the Hartle-Hawking approach without any use of the anthropic principle. Unfortunately, however, our numerical investigation of this question shows that in all realistic models with potentials $\sim\phi^n$, the absolute value of the action is maximal at $\phi<M_{\rm p}$, which does not lead to long inflation, and which, consequently, yields an exponentially small value of Ω.

3. Closed cut revisited

We will now turn to a different possibility. We wish to study the consequences of allowing the singularity to be part of the nucleation geometry but not of the hypersurface joining the Euclidean and Lorentzian section. The simplest such instanton is obtained by cutting the standard solution, once again, across the equator, but discarding the *regular* hemisphere, and keeping the *singular* hemisphere.

The instanton thus corresponds to the interval $\tau_{\rm max}\leqslant\tau\leqslant\tau_{\rm f}$. The Lorentzian section is obtained by taking $\tau=\tau_{\rm max}+iT$, so it will be the same as that in Sec. III A 1: a closed inflationary universe.

The Euclidean region looks mostly like a four-sphere of radius $\sim H^{-1}(\phi_{\rm N})$. But there will be a region near the singularity, where the curvature and energy density diverge. We may impose a Planck-scale cut-off here, and consider the singularity and its vicinity as a small Planckian region immersed in the large four-sphere. The presence of this region on the south pole is the crucial difference between this nucleation geometry and the one studied in Sec. III A 1.

How should we interpret this difference? The regular instanton, viewed in isolation, has often been interpreted as representing the creation of the universe from nothing. This was motivated by its self-contained nature; one might think of ''nothing'' as the vanishing of the scale factor a, which occurs on the regular north pole. In contrast, the interpretation of the singular hemisphere actually seems less vague. We can think of this instanton as an interpolation between a Planckian regime, and a large closed inflating universe. Therefore we propose that it describes the spontaneous ignition of inflation from a bubble of spacetime foam. In fact, this agrees with the interpretation of creation from ''nothing'' proposed in [7]. We are speaking about a state where the classical part of metric strongly fluctuates, so that one cannot measure distance using any measuring devices. This is what may happen at the Planckian epoch. But at the Planck time one would expect all physical fields to take large and strongly fluctuating values, rather than a definite value corresponding to the north pole of the usual nonsingular de Sitter instanton at $\tau=0$. In this sense the use of singular instantons seems quite appropriate for the description of quantum creation of a closed universe from space-time foam.

The main difference of this use of the singular instanton to that proposed by Hawking and Turok is that the singularity in our case does not reach into the Lorentzian sector. It is limited to a tiny region in the Euclidean regime, where it can easily be smoothed out, or removed. Since the neither the boundary term nor the volume term have divergences near the singularity, the action will be finite. For large $\phi_{\rm N}$ it will not differ noticeably from the action of the other hemisphere, or the action of the Hawking-Turok instanton.

B. Gondola instantons

In the previous subsection we argued that weak localized singularities inside the Euclidean sector of a tunneling geometry can be interpreted as interpolations to space-time foam and can thus be quite useful. Once this point of view is adopted, however, it is easy to see that the standard Euclidean solution is only a special case in a one-parameter family of solutions. Generically, these solutions will have singularities on both poles.

We will now focus on a particular member of this family that is exactly symmetric about the equator, shown in Fig. 6. It can be constructed by specifying very simple boundary conditions on the equator: one is free to choose the initial value of the field, $\phi=\phi_E$; the derivatives of all fields and metric components are set to zero. There will thus be identical singularities on the north and south pole. We will call this the ''gondola'' solution.

If we cut this solution along the equator, we obtain two identical hemispheres, each containing a small Planckian re-

[2]There is a second, more brute-force way of eliminating the singularity. One may take a spherical region around the regular pole and join it to its mirror image across a domain wall of positive energy density. This method, which will be described in a separate publication [20], does not require false vacua, but assumes the presence of fields supporting the topological defect.

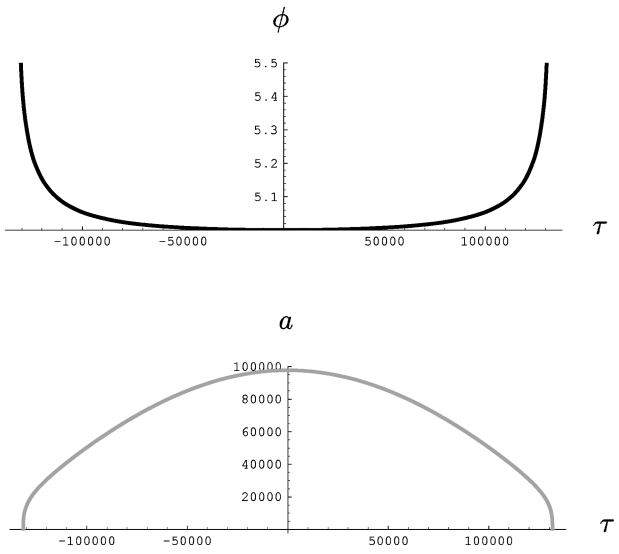

FIG. 6. Gondola solution. Field ϕ as well as the curvature are singular at the south and north poles. Unlike the standard solution, this one is symmetric about the equator, where all derivatives vanish.

gion at its pole. As we discussed above, we may consider this region to interpolate to spacetime foam. Performing the usual analytic continuation, $\tau = \tau_{\max} + iT$, we obtain, once more, a closed Lorentzian universe. But the gondola instantons have the great advantage that the second fundamental form, and all field derivatives, vanishes on the nucleation hypersurface by construction. This means that all variables will be perfectly real in the entire Euclidean and Lorentzian sectors. There is no need for introducing complex initial conditions in this case.

The gondola solution has two boundaries that contribute terms to the action. For comparable values of ϕ_E on the equator, we found numerically that these terms add up almost exactly to the contribution of the single boundary term in the standard solution. The instanton given by half of the gondola solution will contain only one Planckian boundary. Therefore, compared to the singular instanton studied in Sec. III A 3, the boundary contribution will be only half as large here.

For small values of ϕ_E, which give barely enough inflation, this means that the absolute value of the action is largest for the regular closed instanton of Sec. III A 1, followed by the gondola instanton, and the singular instanton of Sec. III A 3. For large values of ϕ_E, which lead to a long period of inflation and a very flat universe, the difference is completely negligible. Then the gondola instanton will be the most practical to use, since it requires no analysis of complex variables.

IV. SUMMARY

We have described a number of nonsingular instantons leading to open inflating universes. They include the Coleman–De Luccia solution, in which a bubble of true vacuum expands inside a universe inflating in the false vacuum. We found new solutions that contain two bubbles, or a shell of true vacuum.

We also constructed instantons with a singularity. If the singularity does not lie on the hypersurface of nucleation, it causes no problems in the Lorentzian region, and can be interpreted as a small region of Planckian density. Such instantons can be used to describe the quantum creation of a closed inflationary universe from space-time foam without the need to use complex solutions.

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