Nearest-neighbor interaction quark-lepton mass matrices in supersymmetric SU(5) grand unified theories

Toshiaki Ito*
Department of Childhood Education, Aichi Prefectural University, Nagakute 480-1198, Japan

Naotoshi Okamura†
Theory Group, KEK, Tsukuba, Ibaraki 305-0801, Japan

Morimitsu Tanimoto‡
Science Education Laboratory, Ehime University, Matsuyama 790-0826, Japan

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We propose the Fritzsch–Branco–Silva-Marcos–type fermion mass matrix, which is a typical texture in the nearest-neighbor interaction form, in SU(5) GUT. By evolution of the mass matrices with SU(5) GUT relations in the minimal SUSY standard model, we obtain predictions for the unitarity triangle of CP violation as well as the quark flavor mixing angles, which are consistent with experimental data, in the case of tan $\beta \simeq 3$. [S0556-2821(98)06717-4]

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One of the most important unsolved problems of flavor physics is the understanding of flavor mixing and fermion masses, which are free parameters in the standard model. The observed values of those mixing and masses may provide clues to solve this problem. Many works have been made to find *Ansätze* for quark-lepton mass matrices. The typical one is the Fritzsch *Ansatz* [1], which is called a texture zero analysis where some elements of mass matrices are required to be zero to reduce the degrees of freedom in mass matrices. As presented by Branco, Lavoura, and Mota, both up- and down-quark mass matrices could always be transformed to non-Hermitian matrices in the nearest-neighbor interaction (NNI) basis by a weak-basis transformation for the three and four generation cases [2]. Based on the NNI form, several authors have studied the quark masses and Cabibbo-Kobayashi-Maskawa (CKM) matrix [3] phenomenologically [4–7]. One of the authors (T.I.) proposed a texture in which the up-quark mass matrix is in the Fritzsch form and the down-quark mass matrix is in Branco–Silva-Marcos (BS) form [7]. In this Brief Report, we call this texture the Fritzsch–Branco–Silva-Marcos (F-BS) one. Recently, Takasugi has shown that quark mass matrices can be transformed in general to either one of the following two forms: the Fritzsch-type parametrization or the BS-type parametrization with retaining the NNI form for the other matrix [8]. Moreover, Takasugi and Yoshimura pointed out that it is reasonable to take the BS *Ansätze* for the down-quark mass matrix if the up-quark one is assumed to be the Fritzsch texture [9]. The F-BS texture reproduces the well-known empirical relations:

$$|V_{us}| \sim \sqrt{\frac{m_d}{m_s}}, \tag{1}$$

$$|V_{cb}| \sim \frac{m_s}{m_b}, \tag{2}$$

$$\frac{|V_{ub}|}{|V_{cb}|} \sim \sqrt{\frac{m_u}{m_c}}. \tag{3}$$

Thus, this texture is one of the simplest *Ansätze* of the quark mass matrices which is consistent with all experimental data [7]. In this Brief Report, we examine whether the Yukawa matrices of the F-BS texture set on the grand unified theory (GUT) energy scale reproduce the fermion masses and the CKM matrix in the low energy region.

In the case of SU(5), one may worry that the NNI forms are not taken in general for the Yukawa matrices of the up- and down-quarks because the left and right components of the up quarks are in the same representation and cannot be rotated independently. We can find that the NNI form is not a general one in SU(5) by studying the proof in Ref. [2]. However, as far as we restrict the up-quark Yukawa matrix to be symmetric ones, which are preferred in SU(5) and SO(10), the symmetric Yukawa matrix of the up-quark sector can be transformed to the Fritzsch-type one with retaining the NNI form for the down-quark Yukawa matrix. This proof is guaranteed by Takasugi's proof [8], in which the degrees of freedom of the rephasing [10] of the quark fields are used.

Following these investigations, we employ the F-BS-type Yukawa matrices for quarks and leptons at the SU(5) GUT scale. The evolution based on the supersymmetric (SUSY) renormalization group equations from the GUT scale to the M_Z scale gives predictions for the CKM matrix. Yukawa matrices are written as follows:

$$Y^U = \begin{pmatrix} 0 & a_u & 0 \\ a_u & 0 & b_u \\ 0 & b_u & c_u \end{pmatrix}, \tag{4}$$

*E mail address: toshiaki@eken.phys.nagoya-u.ac.jp
†E mail address: okamura@theory.kek.jp
‡E mail address: tanimoto@edserv.ed.ehime-u.ac.jp

$$Y^D=\begin{pmatrix} 0 & a_d e^{i\theta_1} & 0 \\ a_d e^{-i\theta_1} & 0 & b_d e^{i\theta_2} \\ 0 & c_d & c_d \end{pmatrix},$$

where Y^U and Y^D are matrices for the up quark and down quark, respectively. The up-quark Yukawa matrix is the Fritzsch texture, while the down-quark one is the BS texture. Since these Yukawa matrices should give the six quark masses in the low energy region, the generation hierarchy $a_{u(d)} \ll b_{u(d)} \ll c_{u(d)}$ is guaranteed from the phenomenological point of view. The charged lepton mass matrix is related to the quark one. The charged lepton Yukawa matrix Y^E is given by using some Higgs fields of SU(5) such as **5*** and **45***. We have found the desired charged lepton Yukawa matrix as follows:

$$Y^E=\begin{pmatrix} 0 & a_d e^{i\theta_1} & 0 \\ a_d e^{-i\theta_1} & 0 & -3b_d e^{i\theta_2} \\ 0 & c_d & c_d \end{pmatrix}. \tag{5}$$

Each entry of quark-lepton matrices is assumed to arise from the vacuum expectation value (VEV) of **5***'s of the Higgs field except for the (2,3) entries in Y^D and Y^E, which are assumed to arise from **45*** of the Higgs field. Therefore, the matrix Y^U should be symmetric while Y^D and Y^E are allowed to be nonsymmetric. So parameters $a_{u(d)}, b_{u(d)}, c_{u(d)}$ are taken to be real and the phase parameters appear in Y^D and Y^E with θ_1, θ_2. Including tan β, we have 9 parameters in the fermion mass matrix. On the other hand, there are 14 low energy observables, 9 charged fermion masses, 4 CKM mixing angles, and tan β. Thus, there are 5 predictions.

The mass eigenvalues of Y^D and Y^E are given in terms of those parameters as follows:

$$m_d=\frac{a_d^2}{b_d}, \quad m_s=\frac{b_d}{\sqrt{2}}, \quad m_b=\sqrt{2}c_d, \tag{6}$$

$$m_e=\frac{1}{3}\frac{a_d^2}{b_d}, \quad m_\mu=3\frac{b_d}{\sqrt{2}}, \quad m_\tau=\sqrt{2}c_d. \tag{7}$$

Eliminating parameters, we obtain the SU(5) GUT mass relations

$$m_\tau=m_b, \tag{8}$$

$$m_\mu=3m_s, \tag{9}$$

$$m_e=\frac{1}{3}m_d, \tag{10}$$

which is the same one in Georgi-Jarlskog texture [11]. The numerical result of the F-BS texture is compared with the Georgi-Jarlskog one in the latter.

In the minimal SUSY standard model, the renormalization group equations of one loop are [12]

$$\frac{d}{dt}Y^U=\frac{1}{16\pi^2}\Big\{\mathrm{tr}(3Y^UY^{U\dagger})Y^U+3Y^UY^{U\dagger}Y^U + Y^DY^{D\dagger}Y^U-\Big(\frac{13}{9}g'^2+3g_2^2+\frac{16}{3}g_3^2\Big)Y^U\Big\}, \tag{11}$$

$$\frac{d}{dt}Y^D=\frac{1}{16\pi^2}\Big\{\mathrm{tr}(Y^EY^{E\dagger}+3Y^DY^{D\dagger})Y^D+3Y^DY^{D\dagger}Y^D + Y^UY^{U\dagger}Y^D-\Big(\frac{7}{9}g'^2+3g_2^2+\frac{16}{3}g_3^2\Big)Y^D\Big\}, \tag{12}$$

$$\frac{d}{dt}Y^E=\frac{1}{16\pi^2}\{\mathrm{tr}(Y^EY^{E\dagger}+3Y^DY^{D\dagger})Y^E + 3Y^EY^{E\dagger}Y^E-3(g'^2+g_2^2)Y^E\}, \tag{13}$$

for Yukawa matrices with $t=\ln(M_G^2/\mu^2)$, and

$$\frac{d}{dt}\alpha_i=\frac{b_i}{2\pi}\alpha_i^2 \quad \Big(\alpha_i=\frac{g_i^2}{4\pi}, \; g_1^2=\frac{5}{3}g'^2, \; i=1,2,3\Big), \tag{14}$$

for gauge couplings, where

$$b_1=\frac{3}{10}n_H+2n_G, \tag{15}$$

$$b_2=\frac{1}{2}n_H+2n_G-6, \tag{16}$$

$$b_3=2n_G-9. \tag{17}$$

In our analysis, the GUT scale is fixed as $M_G=1.7\times10^{16}$ GeV by use of the experimental data of α_1 and α_2. Then we obtain $\alpha_s(M_Z)=0.114$, which is almost consistent with the experimental data, $\alpha_s(M_Z)=0.118\pm0.003$ [13]. The factors n_H and n_G are the number of Higgs doublets and fermion generations, respectively. We set $n_H=2$ and $n_G=3$. By numerical analysis of the renormalization group equations, the fermion mass matrices are obtained at the M_Z energy scale.

It is useful to comment on tan β. If tan β is less than 2, the Yukawa coupling of the t quark blows up under the GUT scale. A recent study of the proton decay suggests that tan β is less than 4 [14]. Thus, tan $\beta\simeq3$ is a reasonable region. Actually, our numerical results favor tan $\beta=3$. The fits with the experimental values become worse as tan β increases.

Since the two phases θ_1 and θ_2 in Y^D and Y^E hardly affect the running of Yukawa matrices, the matrix elements $a_{u(d)}, b_{u(d)}, c_{u(d)}$ can be adjusted by the following central values of six fermion masses at the M_Z energy scale [13,15]:

$$m_u(M_Z)=0.0022\pm0.0007 \text{ GeV},$$

$$m_c(M_Z)=0.59\pm0.07 \text{ GeV},$$

$$m_t(M_Z)=175\pm14 \text{ GeV},$$

$$m_e(M_Z)=0.486660328\pm0.000000143 \text{ MeV},$$

$$m_\mu(M_Z)=102.7288759\pm0.0000332 \text{ MeV},$$

$$m_\tau(M_Z)=1746.5^{+0.296}_{-0.266} \text{ MeV}.$$

Then the down-quark masses are obtained as output:

$$m_d(M_Z)=0.0032 \text{ GeV}, \quad m_s(M_Z)=0.081 \text{ GeV},$$

$$m_b(M_Z)=3.31 \text{ GeV},$$

which are compared with the experimental values [15]

$$m_d(M_Z)=0.0038\pm0.0007 \text{ GeV},$$

$$m_s(M_Z)=0.077\pm0.011 \text{ GeV},$$

$$m_b(M_Z)=3.02\pm0.19 \text{ GeV}.$$

Thus obtained values of down-quark masses are almost consistent with the experimental data. It is remarked that due to the running of Yukawa matrices, the (2,2) entries of the quark mass matrices develop in non-negligible finite ones, which are comparable with magnitudes of (1,2) and (2,1) entries. On the other hand, the (1,1), (1,3), and (3,1) entries are almost zero at the M_Z scale. So even if the Yukawa matrices of quarks are F-BS type at the GUT scale, they become of non-NNI form at the low energy scale due to renormalization effects. On the other hand, the lepton mass matrix keeps the NNI form in the renormalization running.

If θ_1 and θ_2 are fixed, we can predict the CKM matrix. We obtain the CKM matrix in the case of $\theta_1=253°$ and $\theta_2=60°$:

$$|V_{CKM}|=\begin{pmatrix} 0.9757 & 0.2190 & 0.0045 \\ 0.2189 & 0.9747 & 0.0458 \\ 0.0082 & 0.0453 & 0.9989 \end{pmatrix}. \tag{18}$$

This result is consistent with the present experimental one within error bars. In particular, we get $V_{cb}\simeq0.045$, which should be compared with the prediction $V_{cb}\geqslant0.05$ in Georgi-Jarlskog texture [16]. The CP violating phase of the CKM matrix is also fixed. The unitarity triangle of the predicted CKM matrix is shown in Fig. 1. The vertex of this unitarity triangle is on the point (0.263, 0.352) in the ρ-η plane [18]. Thus the vertex is in the first quadrant in the ρ-η plane as well as the case of Ref. [7]. Here in order to describe the experimentally allowed region, we have used the following parameters [17] and experimental data [13]:

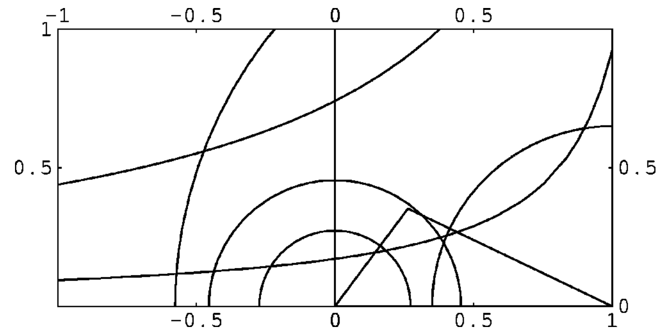

FIG. 1. The unitarity triangle of the CKM matrix in the case of $\tan\beta=3$. The allowed region is given by the experimental constraints of ϵ_K, B_d-$\overline{B}_d$ mixing, and $|V_{ub}|/|V_{cb}|$.

$$B_K=0.75\pm0.15, \quad f_{B_d}\sqrt{B_{B_d}}=0.20\pm0.04,$$

$$\frac{|V_{ub}|}{|V_{cb}|}=0.08\pm0.02.$$

It is noted that the predicted CKM matrix cannot reproduce the experimental data of $|V_{ub}|/|V_{cb}|$ in the case of $\tan\beta \geqslant 4$.

In this paper, we have predicted the CKM matrix at the M_Z energy scale by assuming the F-BS texture for Yukawa matrices at the GUT scale. In the case of $\tan\beta\simeq3$, the obtained CKM matrix is consistent with experiments. One may worry about our prediction $V_{cb}\simeq0.045$ because recent experiments favor $V_{cb}=0.040\pm0.003$ [17]. There is a plausible possibility to push down this predicted value, that is, to modify the SU(5) GUT relations by introducing other Higgs fields. The modification may be guaranteed by recent lattice calculations of light quark masses [19], in which the light quark masses are considerably reduced compared with the conventional ones.

We emphasize that if the B-factory experiments at KEK and SLAC restrict the experimentally allowed region of the unitarity triangle in the first quadrant of the ρ-η plane, our proposed simple model can be a candidate for Yukawa matrices at the GUT scale.

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