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Implications of $b \rightarrow s\gamma$ decay measurements on the Higgs sector of the S_3-NFC model

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Using the data from CLEO Collaboration on the inclusive $b \rightarrow s\gamma$ decay, we obtain the allowed regions of $\tan\beta$ of the S_3-NFC model. In contrast with the conventional NFC models, $\tan\beta$ can exist in two separate regions where one region is determined by the contribution from the top quark ($0.42 \leqslant \tan\beta \leqslant 5.75$) and the other one by the charm quark ($2.5\times 10^{-4} \leqslant \tan\beta \leqslant 2.06\times 10^{-2}$). Our results indicate that the S_3-NFC model is consistent with the current experimental data. [S0556-2821(98)06019-6]

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I. INTRODUCTION

In the standard model of electroweak interactions, spontaneous symmetry breaking of a gauge group is achieved through the Higgs mechanism with an SU(2) Higgs doublet to give masses to the gauge bosons and the fermions. Because of the lack of experimental inputs from the Higgs sector, the symmetry-breaking mechanism may also be provided by more than one Higgs doublet. However, it is well known that the extra Higgs doublet always leads to flavor-changing neutral-current (FCNC) processes at the tree level if both Higgs doublets couple to the same type of quarks. To resolve this difficulty, Glashow and Weinberg [1] suggested two approaches to ensure the vanishing of FCNC at the tree level. The simplest approach demands that only one Higgs doublet Φ_1 interact with fermions and that the extra Higgs doublet Φ_2 decouple from the fermionic sector. We will follow the convention to name this approach model I. In the second approach, named model II, both Higgs doublets interact with fermions. However, Φ_1 is restricted to interact solely to up-type quarks whereas Φ_2 only interacts with down-type quarks. For years these have been the only two known natural-flavor-conserving (NFC) models which are called the "conventional" NFC models hereafter. In order to address the problem of the NFC model, Branco *et al.* [2] have provided a necessary and sufficient condition of NFC models in terms of a commutation relation of the quark mass matrices. For example, for the up-quark sector, the condition is stated as

$$M_1 M_2^T - M_2 M_1^T = 0, \tag{1.1}$$

where M_1 and M_2 are the up-quark mass matrices obtained from the interactions of Φ_1 and Φ_2, respectively, and M_i^T is the transpose of M_i. A similar relation also holds true for the down quark with M_i denoting the down-quark mass matrices defined in a similar fashion.

In model I, since M_2 is always vanishing, the above condition is satisfied automatically. For model II, the NFC condition is satisfied due to the fact that either $M_1 = 0$ or $M_2 = 0$. However, for both models, Eq. (1.1) is realized by restricting it to having a zero mass matrix. Thus it is of great interest to explore the possibility of realizing the NFC condition in models with both M_1 and M_2 being nonvanishing. Recently, Cheng and Sher [3] have suggested a third type of two-Higgs-doublet model that is designated as "model III." Because of the fact that both Higgs doublets couple to all fermions, it is not natural flavor conserving at the tree level. However, one can obtain a NFC model by imposing an S_3 symmetry among the fermion generations and it is named the "S_3-NFC model" [4–6]. One interesting feature of the S_3 model is that both M_1 and M_2 are nonzero matrices and Eq. (1.1) is guaranteed by the S_3 symmetry. Moreover, this new type of NFC model also induces a more complicate Yukawa interaction among the Higgs doublets and fermions. As a result, one expects that the phenomenological consequences might be different from conventional NFC models. This work is aimed at exploring the theoretical consequences of the S_3-NFC models.

In the standard model, FCNC transitions occur in loop corrections through exchange of the W boson [7]. However, with an extra Higgs doublet, there exists a charged Higgs boson which can also contribute to the FCNC transitions at the loop level [8–14]. The recently observed exclusive $B \rightarrow K^*\gamma$ decay [15] and inclusive $b \rightarrow s\gamma$ decay [16] provide important tests of the standard model and its extensions. In this work, we discuss the implications of $b \rightarrow s\gamma$ decay on the constraint of the S_3-NFC model. By using the CLEO data [16] we have obtained the allowed region of $\tan\beta$ for different charged-Higgs-boson masses, where $\tan\beta$ is the vacuum expectation value (VEV) ratio of the neutral Higgs fields. Furthermore, our results indicate that, in contrast to models I and II, there exists a narrow region of parameter space where the charm quark contributions can become dominant in the S_3-NFC model.

This paper is organized as follows. The S_3-NFC model is briefly reviewed in Sec. II. A general discussion of the branching ratio of the $b \rightarrow s\gamma$ decay is presented in Sec. III. Predictions of the parameter space of three NFC models are given in Sec. IV, and the phenomenological consequences are discussed. Comparisons among the three models are also discussed.

II. S_3-NFC MODEL

It is known that the fermion generations are similar except for their masses. Owing to this similarity, an S_3 symmetry

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among generations has been imposed on the standard model by many authors [17–23]. In their works, the fermion generations are assumed to be indistinguishable before spontaneous symmetry breaking (SSB) since the fermions are massless and thus an S_3 symmetry among the generations exists naturally. When SSB happens, the fermions obtain their masses through the nonvanishing vacuum expectation value of the Higgs doublet and hence the S_3 symmetry is also broken spontaneously. Unfortunately, this idea does not work well in the S_3 model with one Higgs doublet since it implies that two of the fermion masses are degenerate. However, in Refs. [3–6], it is shown that with two Higgs doublets and the S_3 symmetry among fermion generations, the above fermion mass degeneracy can be removed.

The Yukawa interactions of the quark fields in the weak eigenstate can be written as

$$\mathbf{L}_y=\overline{Q'_L}(\mathbf{g}_1\Phi_1+\mathbf{g}_2\Phi_2)D'_R+\overline{Q'_L}(\mathbf{h}_1\tilde{\Phi}_1+\mathbf{h}_2\tilde{\Phi}_2)U'_R+\text{H.c.} \tag{2.1}$$

where $\tilde{\Phi}_i\equiv i\tau_2\Phi_i$ and τ_2 is the Pauli matrix. The factors $\mathbf{g}_1$, $\mathbf{g}_2$, $\mathbf{h}_1$, and $\mathbf{h}_2$ are 3×3 Yukawa coupling matrices and the primed fields $\overline{Q'_L}$, D'_R, and U'_R denote quarks in weak eigenstates. If one assumes that the theory is S_3 invariant, then the Yukawa coupling matrices are restricted to the following form:

$$M_{u1}=\langle\tilde{\Phi}_1\rangle\mathbf{h}_1=\begin{pmatrix} a & b & b\\ b & a & b\\ b & b & a\end{pmatrix},$$

$$M_{u2}=\langle\tilde{\Phi}_2\rangle\mathbf{h}_2=\begin{pmatrix} 0 & -c & c\\ c & 0 & -c\\ -c & c & 0\end{pmatrix}. \tag{2.2}$$

It is interesting to note that these matrices automatically satisfy the NFC condition (1.1) for arbitrary complex numbers a, b, and c. Therefore, at the tree level, there are no FCNC transitions. The matrices given in Eq. (2.2) also apply for d-type quarks. For more detailed discussions on this point, the readers are referred to Ref. [3]. Furthermore, it can be shown that both mass matrices can be diagonalized simultaneously:

$$\begin{aligned} M_u^d&=M_{u1}^d+M_{u2}^d\\ &=\text{diag}(a-b-\sqrt{3}c,\quad a-b+\sqrt{3}c,\quad a+2b). \end{aligned} \tag{2.3}$$

Hence all mass eigenvalues are nondegenerate if $c\neq0$. The diagonalized mass matrices of the down-type quarks can also be obtained by the same procedure.

III. MODEL-INDEPENDENT CALCULATION

The charged-Higgs-boson interaction is given in a general form as

$$\begin{aligned} L=\frac{g}{\sqrt{2}M_W}H^+\overline{U}V_{KM}[&(\tan\beta M_{u1}^d-\cot\beta M_{u2}^d)\gamma_-\\ &+(-\tan\beta M_{d1}^d+\cot\beta M_{d2}^d)\gamma_+]D+\text{H.c.}, \end{aligned} \tag{3.1}$$

where $\gamma_\pm\equiv(1\pm\gamma_5)/2$, and M_{q1}^d and M_{q2}^d are the *diagonalized* mass matrices of q-type ($q=u,d$) quarks corresponding to Φ_1 and Φ_2, respectively. The conventional NFC models are included as special cases with $M_{u2}^d=M_{d2}^d=0$ for model I and $M_{u2}^d=M_{d1}^d=0$ for model II. However, in general, all mass matrices of the S_3-NFC model are nonvanishing as discussed in the last section. The effective Lagrangian for $b\to s\gamma$ decay contains contributions from one-loop diagrams with exchanging the W boson and the charged Higgs boson [8–13,24–35]. The W-boson contribution was obtained by Inami and Lim [7]. The charged-Higgs-boson contribution depends on the particular coupling of the extra Higgs doublet with fermions, and we will parametrize the Higgs-boson contributions as follows:

$$\begin{aligned} L_H=-\frac{eg^2}{32\pi^2M_W^2}\Sigma V_{ib}V_{is}^*\{\overline{s}[&iq_\nu\sigma^{\mu\nu}m_bH(y_i)\gamma_+\\ &+(q^\mu\not{q}-q^2\gamma^\mu)(K_+\gamma_++K_-\gamma_-)G]b\}A_\mu, \end{aligned} \tag{3.2}$$

where A_μ is the photon field and the factors G, K_+, and K_- are given in Appendix A. In Eq. (3.2) we have neglected the term proportional to m_s. Here $y_i\equiv(m_i/m_H)^2$, and $i=u,c,t$ denote the up-type quarks. The remaining form factor $H(y_i)$ can be written as

$$H(y_i)=\left(\frac{y_s}{y_i}\right)A(y_i)H_1(y_i)+B(y_i)H_1(y_i)+C(y_i)H_2(y_i). \tag{3.3}$$

The factors $H_{1,2}$, $A(y_i)$, $B(y_i)$, and $C(y_i)$ are given in Appendix B. Even though we have written $A(y_i)$, $B(y_i)$, and $C(y_i)$ as generation-dependent quantities, however, it can be seen from Appendix B that $A(y_i)$ are generation independent in all models. Furthermore, in models I and II, both $B(y_i)$ and $C(y_i)$ are generation independent and $B(y_i)$ have the same form $\tan^2\beta$. It is noted that $H_{1,2}$ are model independent and always positive for arbitrary Higgs-boson mass. This fact has important consequences for the determination of $\tan\beta$. However, the factors $A(y_i)$, $B(y_i)$, and $C(y_i)$ are model dependent.

In the literature [8–13], only the top quark contribution is considered and $A(y_t)$ have been neglected since it is multiplied by a small factor $(m_s/m_t)^2$. Because of the fact that $A(y_i)=\cot^2\beta$ in model II, it can be dominant when β is small.

By adding the W-boson contribution and using the small-external-fermion-mass approximation, the decay width of $b\to s\gamma$ decay without QCD corrections can be obtained:

$$\Gamma(b\rightarrow s\gamma)=\frac{\alpha G_F^2 m_b^5}{128\pi^4}\left(1-\frac{m_s^2}{m_b^2}\right)^3 |\Sigma V_{ib}V_{is}^*[F_2(x_i)+H(y_i)]|^2$$
$$\cong \frac{\alpha G_F^2 m_b^5}{128\pi^4}|V_{cb}V_{cs}^*|^2$$
$$\times\left|[-F_2(x_t)-H(y_t)+H(y_c)+F_2(x_c)]\right.$$
$$\left.+\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*}[-F_2(x_t)-H(y_t)+H(y_u)+F_2(x_u)]\right|^2, \tag{3.4}$$

where $F_2(x_i)$ is the W form factor given in Ref. [7]. The second line in Eq. (3.4) is obtained by making use of the unitarity condition of the Cabibbo-Kobayashi-Maskawa (CKM) matrix and the small ratio m_s^2/m_b^2 in the brackets has been neglected. Numerically, for $M_W=80$ GeV, $m_t=174$ GeV, $m_c=1.5$ GeV, and $m_u=5$ MeV [36], the W form factors are determined: $F_2(x_t)=0.391$, $F_2(x_c)=3.51\times10^{-4}$, and $F_2(x_u)=2.28\times10^{-9}$. Hence $F_2(x_u)$ and $F_2(x_c)$ can be neglected. Because of the smallness of the ratio of the CKM matrix elements $|V_{ub}V_{us}^*/V_{cb}V_{cs}^*|\sim 0.022$, we can simply neglect the last term of Eq. (3.4) and obtain

$$\Gamma(b\rightarrow s\gamma)=\frac{\alpha G_F^2 m_b^5}{128\pi^4}|V_{cb}V_{cs}^*|^2|F_2(x_t)+H(y_t)-H(y_c)|^2. \tag{3.5}$$

In conventional NFC models, the top quark contribution to this process always dominates over the first two generations. However, this is not always the case in the S_3 model, and so we shall keep the charm contribution $H(y_c)$.

Following the approach of Refs. [11,27,31,39], the branching ratio $B(b\rightarrow s\gamma)$ is computed by making use of the branching ratio of the semileptonic decay $b\rightarrow ce\bar{\nu}$. In this way one may reduce the uncertainties inherited from the experimental values of the CKM matrix elements and the factor m_b^5 [36]. Thus,

$$B(b\rightarrow s\gamma)=\frac{\Gamma(b\rightarrow s\gamma)_{QCD}}{\Gamma(b\rightarrow ce\bar{\nu})_{QCD}}B(b\rightarrow ce\bar{\nu}), \tag{3.6}$$

where for semileptonic decay $B(b\rightarrow ce\bar{\nu})$ we use the averaged experimental value 0.105 [36].

The one-loop QCD corrections to $b\rightarrow ce\bar{\nu}$ have been evaluated in the literature [37],

$$\Gamma(b\rightarrow ce\bar{\nu})_{QCD}=\frac{G_F^2 m_b^5}{192\pi^3}\rho|V_{bc}|^2\left[1-\frac{2\alpha_s(m_b)}{3\pi}f\right], \tag{3.7}$$

where the phase-space factor ρ is 0.447 and the QCD correction factor f is 2.41. We have calculated α_s to two loops to obtain $\alpha_s(m_b)=0.22$ [36], such that the QCD scale Λ_{QCD} can be determined to fit the measurement of $\alpha_s(M_Z)$ at LEP, which corresponds to $\Lambda_{QCD}=150$ MeV and $m_b=4.5$ GeV. The leading-order QCD correction for $b\rightarrow s\gamma$ is given in Refs. [37–40] for $\mu\ll M_W$ and the decay width $\Gamma(b\rightarrow s\gamma)_{QCD}$ is then obtained:

$$\Gamma(b\rightarrow s\gamma)_{QCD}=\frac{\alpha G_F^2 m_b^5}{32\pi^4}|V_{cb}V_{cs}^*|^2|C_7(\mu)|^2, \tag{3.8}$$

with

$$C_7(\mu)=\left[\frac{\alpha_s(M_W)}{\alpha_s(\mu)}\right]^{16/23}\left\{C_7(M_W)-\frac{8}{3}C_8(M_W)\left[1-\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{2/23}\right]+\frac{232}{513}\left[1-\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{19/23}\right]\right\}, \tag{3.9}$$

where

$$C_7(M_W)=-\frac{1}{2}\left\{F_2(x_t)+\frac{y_s}{y_t}A(y_t)H_1(y_t)+B(y_t)H_1(y_t)+C(y_t)H_2(y_t)-\left[\frac{y_s}{y_c}A(y_c)H_1(y_c)+B(y_c)H_1(y_c)+C(y_c)H_2(y_c)\right]\right\},$$

$$C_8(M_W)=-\frac{1}{2}\left\{3M'(x_t)+\frac{y_s}{y_t}A(y_t)M'(y_t)+B(y_t)M'(y_t)+C(y_t)N'(y_t)\right.$$
$$\left.-\left[\frac{y_s}{y_c}A(y_c)M'(y_c)+B(y_c)M'(y_c)+C(y_c)N'(y_c)\right]\right\}. \tag{3.10}$$

The coefficients $A(y_i)$, $B(y_i)$, $C(y_i)$, M', and N' are given in Appendix B.

Substituting Eqs. (3.7) and (3.8) into Eq. (3.6), we have the QCD-corrected branching ratio $B(b\rightarrow s\gamma)$ as

$$B(b\rightarrow s\gamma)=0.648\times\frac{\alpha}{\pi\rho}\frac{|V_{cb}V_{cs}^*|^2}{|V_{cb}|^2}|C_7(m_b)|^2\left[1-\frac{2\alpha_s(m_b)}{3\pi}f\right]^{-1}=0.00362|C_7(m_b)|^2. \tag{3.11}$$

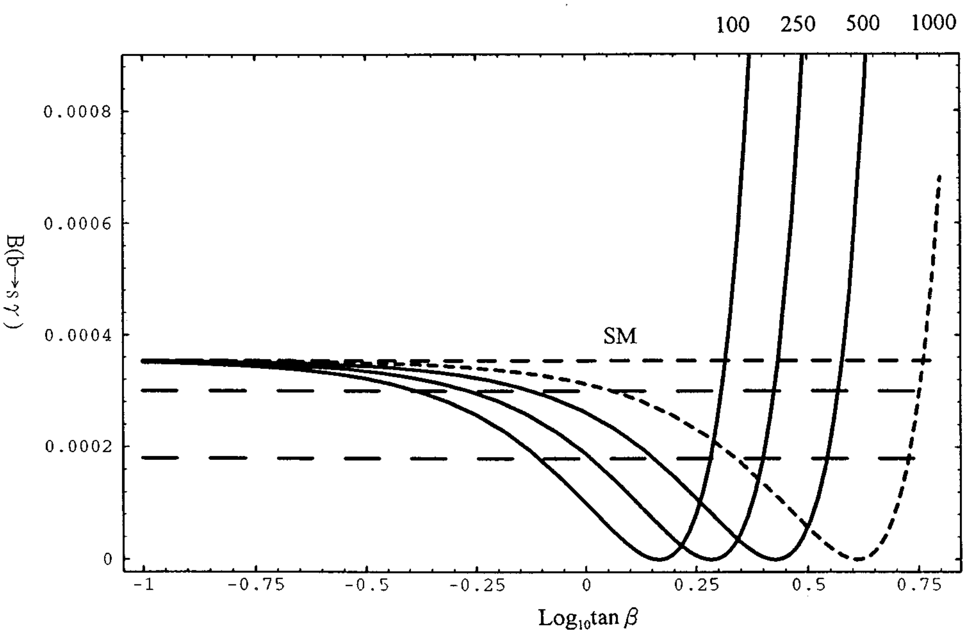

FIG. 1. The branching ratios $B(b \rightarrow s\gamma)$ predicted in model I with $m_H = 100$, 250, 500, and 1000 GeV are plotted. The horizontal long-dashed lines correspond to the experimental upper and lower bounds of $B(b \rightarrow \gamma)_{\rm expt} = 2.32 \pm 0.67 \times 10^{-4}$. The horizontal short-dashed line shows the prediction of the standard model. The prediction of $B(b \rightarrow s\gamma)$ with $m_H = 1$ TeV is indicated by the dotted curve.

IV. DISCUSSIONS AND CONCLUSIONS

In this article the prediction of the branching ratio $B(b \rightarrow s\gamma)$ is calculated in an S_3-NFC model. The predictions of the conventional NFC models are also calculated for comparison. The parameter space of $\tan\beta$ and m_H can be constrained by using the experimental branching ratio $B(b \rightarrow s\gamma)$. In 1993, the hadronic branching ratio $B(B \rightarrow K^* \gamma)$ was first reported in Ref. [15] and the inclusive quark level branching ratio $B(b \rightarrow s\gamma)$ was deduced from that by using the ratio $R \equiv B(B \rightarrow K^* \gamma)/B(b \rightarrow s\gamma)$ which is model dependent. However, a direct measurement of the exclusive branching ratio $B(b \rightarrow s\gamma) = 2.32 \pm 0.57 \pm 0.35 \times 10^{-4}$ has been made recently [16,36], in which the model-dependent error is contained in the second error which is at most 15%. However, this model-dependent error does not change our main conclusion on the range of $\tan\beta$; this is due to the same effects as higher order QCD corrections which will be discussed later. The branching ratios $B(b \rightarrow s\gamma)$ for various $\tan\beta$ and m_H in these NFC models are plotted in Figs. 1–3, respectively. In each figure, two horizontal long-dashed lines which indicate the upper and lower bounds of the experimental $B(b \rightarrow s\gamma)$ are plotted. The allowed parameter domain of $\tan\beta$ and m_H is constrained by these lines. For comparison the standard model prediction is also presented in our graphs.

The prediction of model I is plotted in Fig. 1 as a function of $\tan\beta$ for different m_H. It is noted that each curve crosses the experimentally allowed region twice. In model I, it can be shown that the contributions from the first two generations are negligible. Even though $A(y_i)$, $B(y_i)$, and $C(y_i)$ have the same absolute value, namely, $\tan^2\beta$ (see Appendix

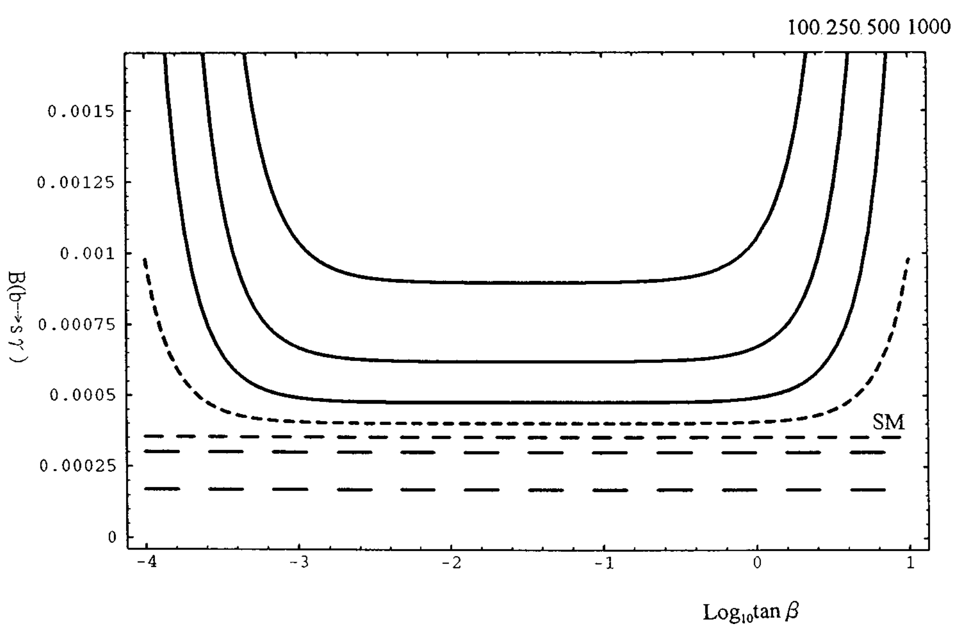

FIG. 2. The branching ratios $B(b \rightarrow s\gamma)$ predicted in model II with $m_H = 100$, 250, 500, and 1000 GeV are plotted. The horizontal long- and short-dashed lines are the same as in Fig. 1. The prediction of $B(b \rightarrow s\gamma)$ with $m_H = 1$ TeV is also indicated by the dotted curve.

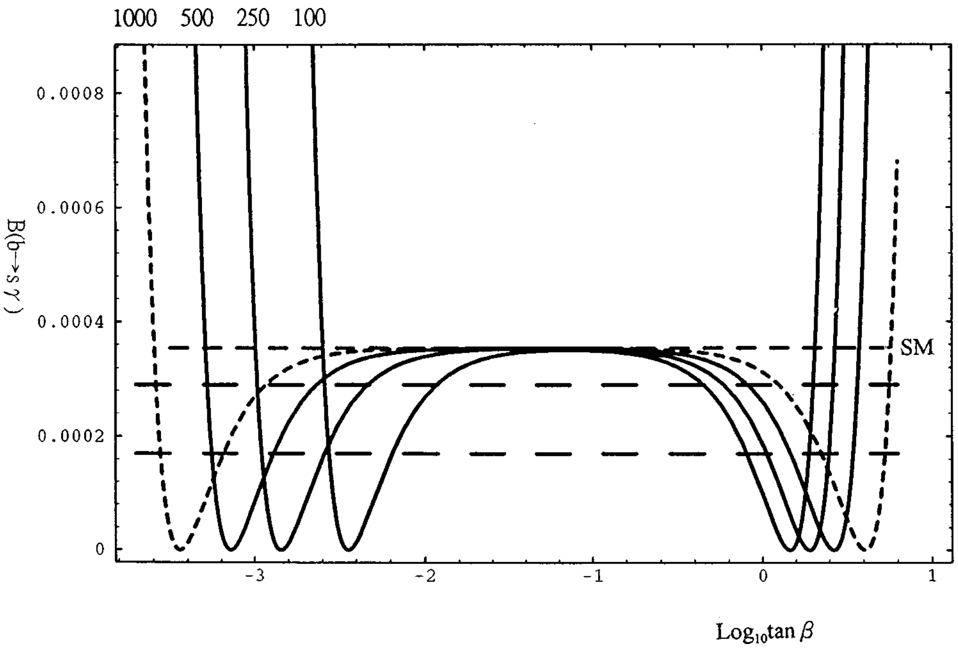

FIG. 3. The branching ratios $B(b\to s\gamma)$ predicted in the S_3-NFC model with $m_H=100$, 250, 500, and 1000 GeV are plotted. The horizontal long- and short-dashed lines are the same as in Fig. 1. The prediction of $B(b\to s\gamma)$ with $m_H=1$ TeV is also indicated by the dotted curve.

B), the contributions to $C_7(\mu)$ from terms containing $A(y_t)$ and $A(y_c)$ are suppressed by the small mass ratios m_s^2/m_t^2 and m_s^2/m_c^2, respectively. Moreover, even the contribution from the term of $A(y_u)$ is enhanced by a factor of m_s^2/m_u^2; the up-quark contribution is still negligible due to the smallness of $H_{1,2}(y_u)$. Thus, the form of C_7 and C_8 can be simplified. For example, one has

$$C_7(M_W)=-\frac{1}{2}\{F_2(x_t)+\tan^2\beta[H_1(y_t)-H_2(y_t)]\}. \tag{4.1}$$

Since $H_1(y_t)$, $H_2(y_t)$, and $F_2(x_t)$ are always positive for any m_H, a cancellation mechanism can occur. For $C_8(M_W)$, the same cancellation also occurs. This cancellation forces the curves to bend down through the allowed region and hence the constraint on $\tan\beta$ is obtained. If one takes the experimental lower bound of the charged Higgs boson, $m_H \geqslant 60$ GeV [36], into account, then a lower bound $\tan\beta \geqslant 0.42$ is obtained. There are general reasons to believe that m_H is less than or on the order of 1 TeV. In this work, we simply take $m_H\leqslant 1$ TeV as an upper bound for m_H which implies $\tan\beta\leqslant 5.75$.

In Fig. 2, we show the results of model II with the $A(y_t)$-dependent term included. In contrast to model I, $A(y_i)$ equals $\cot^2\beta$ instead of $\tan^2\beta$. Furthermore, $B(y_i)=\tan^2\beta$ and $C(y_i)=1$. Hence the dominant contribution is from $A(y_i)$ in contrast to the result of model I. As discussed earlier, $H_i(y_t)$ and $F_2(x_t)$ are positive for any m_H, and $A(y_t)$, $B(y_t)$, and $C(y_t)$ are also positive; thus the top quark contribution is positive definite. Furthermore, the charm quark contributes negligibly in model II. As a result, the charged-Higgs-boson contribution to this process always enhances the branching ratio. In addition, the $A(y_t)$-dependent term and its corresponding expression in $C_8(M_w)$ push the branching ratio upward at small $\tan\beta$. In the literature, such terms are always ignored and the predicted curve is flat at small $\tan\beta$ (see Fig. 1 in Ref. [27] and Fig. 1 in Ref. [28] for examples).

The prediction of the S_3-NFC model is shown in Fig. 3. We find that, for large $\tan\beta$, the results are very similar to that of model I. In fact, if the charm contribution and $A(y_t)$-dependent terms are neglected, then the prediction of the S_3 model is identical to that of model I and so is the constraint on $\tan\beta$. When the charm quark contributions are also included, it is interesting to note that the curves cross the experimentally allowed region of $B(b\to s\gamma)$ again at small $\tan\beta$, which is absent in conventional NFC models. In this case, the charm quark contribution dominates over that of the top quark. This phenomenon is due to the $\cot^2\beta$ dependence of $B(y_c)$ which is enhanced at small β, while the other factors are small compared to $\cot^2\beta$. If one applies the same bounds on m_H as done for model I, then the allowed region of $\tan\beta$ contains two separated intervals where $0.42 \leqslant \tan\beta \leqslant 5.75$ for the top-quark-dominant contribution and $2.5\times 10^{-4}\leqslant \tan\beta \leqslant 2.06\times 10^{-2}$ for the charm-quark-dominant contribution.

In the above discussion, we keep only the one-loop QCD effects. There remains a question on how higher order QCD effects contribute to the determination of $\tan\beta$. Recently results on two-loop QCD corrections has been obtained [41]. It is shown that the two-loop correction may give a 10–20 % reduction to the one-loop results. Based on their results we have estimated the corrections on $\tan\beta$. It turns out that the range of $\tan\beta$ is quite stable against the higher order QCD corrections. This is due to the fact that the boundaries of $\tan\beta$ are determined at the intersection points between the horizontal lines [the experimental bounds of $B(b\to s\gamma)$] and the theoretical curves. It is clear from the figures that the theoretical curves rise sharply at the intersection points and as a result the boundaries of $\tan\beta$ do not depend sensitively on the higher order QCD corrections.

There remains the uncertainty of quark masses related to our results. There are mainly two mass-dependent parts contained in Eq. (3.5), which are the phase-space factor ρ and $F_2(x_t)+H(y_t)-H(y_c)$. The phase-space factor ρ is a function of m_c/m_b and has usually taken the value 0.447 for $m_c/m_b=1/3$ with $m_c=1.5$ GeV and $m_b=4.5$ GeV. The errors from quark masses can make ρ as high as 0.694 with a maximum $m_b=4.5$ GeV and minimum $m_c=1.1$ GeV, which will reduce the predicted branching ratio. However, in this limit only the prediction of model II will be affected since the predicted region of model II can barely overlap with the experimentally allowed region for heavy Higgs-boson mass ($\geqslant 1$ TeV). On the other hand, a minimum $\rho=0.333$ ($m_b=4.1$ GeV and $m_c=1.6$ GeV) will give a larger branching ratio in all three models. As a result, it only narrows the allowed region of $\tan\beta$ in models I and III and does not change the conclusion of this work. The other quark mass dependence is contained in $F_2(x_t)$, $H(y_t)$, and $H(y_c)$ of Eq. (3.5). The first factor $F_2(x_t)=0.391$ is insensitive to the masses and it is dominant in the range $10^{-3}\leqslant\tan\beta\leqslant 10$. The mass-dependent $H(y_c)$ and $H(y_t)$ dominate only the rising parts of the curves. Hence, the errors of quark masses do not change the flat part of the curves so that our conclusion remains intact and is insensitive to the errors of quark masses.

In conclusion, we studied the branching ratio of $b\to s\gamma$ decay in all three NFC models. In model I, a constraint on the VEV ratio is obtained as $0.42\leqslant\tan\beta\leqslant 5.75$. In model II, one obtains an enhanced branching ratio, which is above the experimental bound. In the S_3 model, we find that the charm quark contribution is not negligible and the allowed region of $\tan\beta$ contains two intervals where $0.42\leqslant\tan\beta\leqslant 5.75$ for the top-quark-dominant contribution and $2.5\times 10^{-4}\leqslant\tan\beta\leqslant 2.06\times 10^{-2}$ for the charm-quark-dominant contribution. It is interesting to note that one of the intervals is the same as obtained in model I whereas the charm-quark-dominant region does not occur in the other models. Hence we expect that in the S_3-NFC model, due to this second possibility of $\tan\beta$, charm-quark-dominant contributions to other rare processes can also arise and further investigations along this direction are in progress.

APPENDIX A

The form factors in Eq. (3.2) are given in a general form:

$$K_+=(\tan\beta m_{b1}-\cot\beta m_{b2})(\tan\beta m_{s1}-\cot\beta m_{s2}), \tag{A1}$$

$$K_-=(\tan\beta m_{i1}-\cot\beta m_{i2})^2, \tag{A2}$$

$$G\equiv J+Q_iJ', \tag{A3}$$

where $Q=\frac{2}{3}e$ is the charge of the quarks in the loop with

$$J=\int_0^1 du\int_0^{1-u} dv(u-v)v/D, \tag{A4}$$

$$J'=\int_0^1 du\int_0^{1-u} dv(1-u-v)v/D', \tag{A5}$$

$$D=(1-u-v)(y_i-1-vy_b-uy_s)+1-uvs, \tag{A6}$$

$$D'=(1-u-v)(1-y_i-vy_b-uy_s)+y_i-uvs, \tag{A7}$$

with $s\equiv q^2/m_H^2$ and $y_i\equiv m_i^2/m_H^2$.

Since $(m_b/m_H)^2$ and $(m_s/m_H)^2$ are very small, the factors D and D' can be simplified as

$$D=1-(1-u-v)(1-y_i),\quad D'=y_i+(1-u-v)(1-y_i). \tag{A8}$$

APPENDIX B

The factors in Eq. (3.3) are defined as $H_1(y_i)\equiv M(y_i)+Q_iM'(y_i)$ and $H_2(y_i)\equiv N(y_i)+Q_iN'(y_i)$ with

$$M(y)=\frac{y}{12(1-y)^3}\left[1-5y-2y^2-\frac{6y^2\ln y}{(1-y)}\right], \tag{B1}$$

$$N(y)=\frac{y}{2(1-y)^3}[1-y^2+2y\ln y], \tag{B2}$$

$$M'(y)=\frac{y}{12(1-y)^3}\left[2+5y-y^2+\frac{6y\ln y}{(1-y)}\right], \tag{B3}$$

$$N'(y)=\frac{-y}{2(1-y)^3}[3-4y+y^2+2\ln y]. \tag{B4}$$

The model-dependent factors $A(y_i)$, $B(y_i)$, and $C(y_i)$ are given in a general form as follows:

$$A(y_i)=\frac{1}{m_bm_s}(\tan\beta m_{s1}-\cot\beta m_{s2})(\tan\beta m_{b1}-\cot\beta m_{b2}), \tag{B5}$$

$$B(y_i)=\frac{1}{m_i^2}(\tan\beta m_{i1}-\cot\beta m_{i2})^2, \tag{B6}$$

$$C(y_i)=\frac{-1}{m_bm_i}(\tan\beta m_{i1}-\cot\beta m_{i2})(\tan\beta m_{b1}-\cot\beta m_{b2}). \tag{B7}$$

The resulting expression for models I and II and the S_3 model are the following.

(1) Model I:

$$A(y_i)=\tan^2\beta,\quad B(y_i)=\tan^2\beta,\quad C(y_i)=-\tan^2\beta.$$

(2) Model II:

$$A(y_i)=\cot^2\beta,\quad B(y_i)=\tan^2\beta,\quad C(y_i)=1.$$

(3) S_3 model:

$$A(y_t)=\frac{1}{2}(\tan^2\beta-1),\quad B(y_t)=\tan^2\beta,\quad C(y_t)=-\tan^2\beta,$$

$$A(y_c)=\frac{1}{2}(\tan^2\beta-1),\quad B(y_c)=\frac{1}{4}(\tan\beta-\cot\beta)^2,$$

$$C(y_c)=-\frac{1}{2}(\tan^2\beta-1).$$

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