

## Large- $N$ and vacuum alignment in top-color models

R. Sekhar Chivukula\*

*Boston University, Boston, Massachusetts 02215*

Howard Georgi†

*Harvard University, Cambridge, Massachusetts 02138*

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Top-color and top-color-assisted technicolor provide examples of dynamical electroweak symmetry breaking which include top-quark condensation, thereby naturally incorporating a heavy top quark. In this paper we discuss the roles of the Nambu–Jona-Lasinio (NJL) and large- $N$  approximations often used in phenomenological analyses of these models. We show that, in order to provide for top-quark condensation but not bottom-quark condensation, the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to  $\mathcal{O}(1/N)$  in any theory in which the isospin-violating “tilting” interaction is a  $U(1)$  gauge interaction. A consequence of these considerations is that the potentially dangerous “bottom pions” are naturally light. We also show that the contributions to  $\rho - 1$  previously estimated are of leading order in  $N$ , are not included in the usual NJL analysis, and are the result of “vacuum alignment.” [S0556-2821(98)03319-0]

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### I. INTRODUCTION

Top-color and top-color-assisted technicolor [1,2] provide examples of dynamical electroweak symmetry breaking which include top-quark condensation [3–8], thereby naturally incorporating a heavy top quark. In this paper we discuss the roles of the Nambu–Jona-Lasinio (NJL) and large- $N$  approximations often used in phenomenological analyses of these models.

We begin by reviewing the usual analysis of top-color in the large- $N$  NJL approximation. From this analysis one finds immediately [9] that, in order to provide for top-quark condensation but not bottom-quark condensation, the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to  $\mathcal{O}(1/N)$ . We show that this is a general result in any theory in which the isospin-violating “tilting” interaction is a  $U(1)$  gauge interaction. A consequence of these considerations is that the potentially dangerous “bottom pions” [9,10] are naturally light. In the NJL approximation, the effects of the portion of the strong top-color interactions coupling left-handed currents to right-handed currents are presumed to dominate, and to give rise to chiral symmetry breaking. We show that other portions of the top-color interactions, in particular products of pairs of left-handed currents or right-handed currents, give rise to contributions to  $\rho - 1$ . These contributions, which have been estimated previously [11] by direct computation, are of leading order in  $N$  and are the result of “vacuum alignment” [12–15].

### II. TOP COLOR

In top-color models [1,2] all or part of electroweak symmetry breaking is due to the presence of a top-quark conden-

sate. In many models this condensate is driven by the combination of a strong isospin-symmetric top-color interaction and an additional isospin-breaking  $U(1)$  gauge boson which couples only to the third generation of quarks. These additional interactions are strong, but are spontaneously broken at a scale  $M \geq 1$  TeV. In the simplest model [2], the couplings of the third generation of quarks to the new  $U(1)$  interaction were taken to be proportional to weak hypercharge, and the masses of the heavy top color and  $U(1)$  gauge bosons were taken to be comparable. At low energies, the top-color and hypercharge interactions of the third generation of quarks could then be approximated by the four-fermion operators

$$\mathcal{L}_{4f} = -\frac{4\pi\kappa_{tc}}{M^2} \left[ \bar{\psi} \gamma_\mu \frac{\lambda^a}{2} \psi \right]^2 - \frac{4\pi\kappa_1}{M^2} \left[ \frac{1}{3} \bar{\psi}_L \gamma_\mu \psi_L + \frac{4}{3} \bar{t}_R \gamma_\mu t_R - \frac{2}{3} \bar{b}_R \gamma_\mu b_R \right]^2, \quad (2.1)$$

where  $\psi$  represents the top-bottom doublet,  $\kappa_{tc}$  and  $\kappa_1$  are related respectively to the top color and  $U(1)$  gauge couplings squared.

The analysis of the dynamics of this model usually proceeds in two steps. First, using a Fierz transformation, those left-handed–right-handed (LR) terms in Eq. (2.1) above which couple left-handed and right-handed currents and can be converted into products of color-singlet scalar/pseudoscalar bilinears are rewritten as

$$\mathcal{L}_{NJL} = +\frac{8\pi}{M^2} [\kappa_t (\bar{\psi}_L t_R) (\bar{t}_R \psi_L) + \kappa_b (\bar{\psi}_L b_R) (\bar{b}_R \psi_L)], \quad (2.2)$$

where

$$\kappa_t = \kappa_{tc} + \frac{8\kappa_1}{9N} \quad \kappa_b = \kappa_{tc} - \frac{4\kappa_1}{9N}, \quad (2.3)$$

and  $N(=3)$  is the number of top colors.

\*Email address: sekhar@bu.edu

†Email address: georgi@physics.harvard.edu

Next, this effective ‘‘NJL’’ model [Eq. (2.2)] [16,3–7] is analyzed to leading order in  $N$ . This can be conveniently done by introducing a complex  $2 \times 2$  matrix field  $\Phi$  and writing the NJL interactions in the form

$$\mathcal{L}_{NJL} \rightarrow \left( \bar{\psi}_L \Phi \begin{pmatrix} t_R \\ b_R \end{pmatrix} + \text{H.c.} \right) - \frac{M^2}{8\pi} \text{Tr} \left[ \Phi^\dagger \Phi \begin{pmatrix} \frac{1}{\kappa_t} & 0 \\ 0 & \frac{1}{\kappa_b} \end{pmatrix} \right]. \quad (2.4)$$

Note that, written this way, the interaction of  $\Phi$  with the fermions is  $SU(2)_L \times SU(2)_R$  symmetric. To leading order in  $N$ , the theory is now solved by computing the trace of the fermion propagator in the presence of a background  $\Phi$  field [7]. Computing the effective potential for the field  $\Phi$  using a momentum-space cutoff of order  $M$ , we find

$$V^{eff}(\Phi) \approx \frac{M^2}{8\pi\kappa_c} \text{Tr} \left[ \Phi^\dagger \Phi \begin{pmatrix} \frac{\kappa_c}{\kappa_t} - 1 & 0 \\ 0 & \frac{\kappa_c}{\kappa_b} - 1 \end{pmatrix} \right] + \frac{N}{16\pi^2} \text{Tr} \left[ (\Phi^\dagger \Phi)^2 \log \left( \frac{M^2}{\Phi^\dagger \Phi} \right) \right], \quad (2.5)$$

where  $\kappa_c = \pi/N$ .

The essential features of this model in the large- $N$  NJL approximation can now be determined from Eq. (2.5). For  $\kappa_{t,b}$  close to  $\kappa_c$ , the field  $\Phi$  yields four light complex scalar fields which have the quantum numbers of two independent 2-component ‘‘Higgs’’ fields ( $\phi_t$  &  $\phi_b$ ) (with hypercharges  $\mp 1$ , respectively). Choosing the values of  $\kappa_{tc}$  and  $\kappa_1$  such that

$$\kappa_t > \kappa_c > \kappa_b, \quad (2.6)$$

we obtain the phenomenologically desirable result that the doublet  $\phi_t$  develops a vacuum expectation value,

$$\langle \phi_t \rangle = \begin{pmatrix} \frac{f_t}{\sqrt{2}} \\ 0 \end{pmatrix}, \quad (2.7)$$

giving rise to a (potentially large) top-quark mass and leaving  $\langle \phi_b \rangle \equiv 0$ . Taking into account the necessary wave-function renormalization for the scalar field [7] we find [17]

$$f_t^2 \approx \frac{N}{8\pi^2} m_t^2 \log \left( \frac{M^2}{m_t^2} \right). \quad (2.8)$$

For  $m_t \approx 175$  GeV and  $M \approx 1$  TeV, this yields  $f_t \approx 64$  GeV. From the equations of motion for  $\phi_t$  derived from Eq. (2.4), we see that this expectation value can be interpreted as a top-quark condensate. To the extent that  $\kappa_t$  and  $\kappa_b$  are close to the ‘‘critical value’’  $\kappa_c$  for chiral symmetry breaking, the fields  $\phi_t$  and  $\phi_b$  have masses (and expectation values) small compared to  $M$  and are composite Higgs fields [7,18].

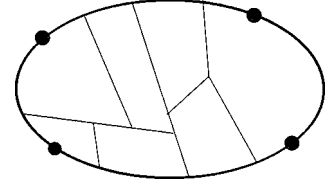


FIG. 1. A ‘‘typical’’ planar contribution, of leading order in  $1/N$ , to the effective potential for  $\Phi$ . Fermions are represented by the thick solid lines, gluons by the thin solid lines, and insertions of  $\Phi$  by the dots.

At this level of approximation, for fixed  $M$ , the effective mass squareds of the Higgs fields change smoothly from positive to negative as the respective  $\kappa$ 's vary from below to above the ‘‘critical value’’  $\kappa_c$ . That is, in this approximation the chiral phase transition (for fixed  $M$  and viewed as a function of  $\kappa_{tc}$ ) is second order. For  $\kappa$ 's close to  $\kappa_c$  the effective scalar lagrangian is a Landau-Ginzburg theory of the chiral phase transition with order parameter  $\Phi$ .

### III. LARGE- $N$ AND THE CHIRAL PHASE TRANSITION

From Eq. (2.3), we see that the difference  $\kappa_t - \kappa_b$  is of order  $\kappa_1/N$ . We will argue shortly that the ratio  $\kappa_1/\kappa_{tc}$  is independent of  $N$ . Therefore, in order to provide for top-quark condensation but not bottom-quark condensation, the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to  $\mathcal{O}(1/N)$ . In this section we show that this is a general property of the large- $N$  limit, independent of the NJL approximation. We will also see that it is independent of the assumption that the top color and strong  $U(1)$  gauge boson masses are equal—it persists even if these masses are very different.

Consider the top-color theory in large- $N$  [19]. As in QCD, in order to have a well-defined high-energy theory, we must choose a top-color coupling

$$g_{tc} = \frac{\tilde{g}_{tc}}{\sqrt{N}}, \quad (3.1)$$

and hold  $\tilde{g}_{tc}$  fixed as  $N \rightarrow \infty$ . The chiral symmetries of the top-color theory are  $SU(2)_L \times SU(2)_R$ , under which the left-handed top-bottom doublet transforms as a (2,1) and the right-handed top and bottom transform together as a (1,2). The behavior of the chiral symmetries is governed by the effective potential for an order parameter  $\Phi$ , which transforms as a  $(2, \tilde{2})$  under  $SU(2)_L \times SU(2)_R$ . To leading order in  $1/N$ , this potential comes from the sum of all *planar* diagrams involving one fermion loop as shown in Fig. 1, and is  $\mathcal{O}(N)$ . The flavor structure of this class of diagrams insures that the vectorial subgroup,  $SU(2)_V$ , remains unbroken<sup>1</sup> [21].

<sup>1</sup>In fact, for any QCD-like vector gauge theory, this remains true exactly [20].

Because  $g_{tc}$  varies with  $N$ , we must be careful about what we mean by the scale of top-color breaking. It will be most useful to define this scale as the Lagrangian mass of the top-color gauge boson. This is appropriate because it is this mass which acts as the cut-off for the effective theory below the symmetry breaking scale. In fact we will soon see that the top-color boson mass,  $M$ , remains fixed as  $N \rightarrow \infty$ , which means the vacuum expectation value that is responsible for the breaking must actually grow like  $\sqrt{N}$ .

Let  $\Lambda_{tc}$  be the scale at which the top-color interactions would become strong if top color remained unbroken, i.e., it is the analog of  $\Lambda_{QCD}$  for the ordinary strong interactions.  $\Lambda_{tc}$  is then independent [19] of  $N$  as  $N \rightarrow \infty$ . If top color is broken at scale  $M$ , we can analyze the theory in two limits [22]. First, if  $M \gg \Lambda_{tc}$ , we expect the low-energy theory to contain massless fermions which interact (ignoring the standard model interactions) only by the exchange of heavy, weakly coupled, top-quark gluons. In this limit, chiral symmetry is unbroken and  $\langle \Phi \rangle = 0$ . On the other hand, if  $M \ll \Lambda_{tc}$  the top-color interactions become strong and we may expect chiral symmetry to be broken in a manner similar to that in QCD. Here chiral symmetry is broken and  $\langle \Phi \rangle \propto I \neq 0$ . If the transition between these two regimes is continuous, i.e., if the chiral-symmetry breaking transition is of second order as  $g_{tc}^2(M)$  (the value of the top-color coupling at scale  $M$ ) is varied, then the mass-squared of the effective  $\Phi$  field goes continuously through zero at a critical coupling  $g_c^2$ .

Since  $\Lambda_{tc}$  is independent of  $N$  as  $N \rightarrow \infty$ ,  $M$  must also be independent of  $N$  as  $N \rightarrow \infty$ . For this to be consistent with the condition of criticality,  $g_{tc}^2(M) = g_c^2$ ,  $g_c^2$  must be proportional to  $1/N$ , i.e.

$$g_c^2 = \frac{\tilde{g}_c^2}{N}, \quad (3.2)$$

with  $\tilde{g}_c$  of  $\mathcal{O}(1)$ . This agrees with the NJL analysis in which  $\kappa_c = \pi/N$  in Eq. (2.5) is proportional to  $g_c^2$ .

If the transition is second-order, the effective low-energy theory for  $g_{tc}^2(M)$  close to  $g_c^2$  is one with a light scalar  $\Phi$  (with mass  $\ll M$ ) coupled to fermions, just as in the NJL analysis [22]. Therefore, while the analysis of top color presented in the previous section depends on both the large- $N$  and NJL approximations to the full top-color theory, the important properties may be expected to survive beyond these approximations so long as the chiral phase transition is sufficiently second-order [22].

Now consider the contribution of the isospin-violating U(1) interaction. In order to have a well-defined large- $N$  limit, we must insure that the vacuum polarization diagrams (see Fig. 2) have a finite large- $N$  limit. Hence, in analogy with Eq. (3.1), we must take a U(1) coupling

$$g_1 = \frac{\tilde{g}_1}{\sqrt{N}}, \quad (3.3)$$

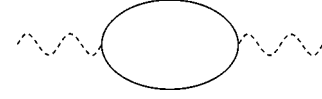


FIG. 2. Fermion-loop contribution to vacuum polarization correction to the U(1) gauge-boson propagator. The corresponding term in the  $\beta$  function for the coupling grows as  $N$ . To have a well-defined large- $N$  limit, therefore, the U(1) coupling must scale as  $N^{-1/2}$ .

and hold  $\tilde{g}_1$  fixed as  $N \rightarrow \infty$ . This immediately implies that  $\kappa_1$  in Eq. (2.1), is proportional to  $1/N$ —so that  $\kappa_1/\kappa_{tc}$  is independent of  $N$ , as promised above.

We can now generalize the analysis to the general effective field theory description, beyond the NJL approximation. To leading order in  $1/N$ , the contribution of the U(1) interaction to the effective potential for  $\Phi$  comes from planar diagrams involving one fermion loop and one U(1) gauge-boson exchange (Fig. 3). Regardless of the specific charges chosen, these contributions are  $\mathcal{O}(1)$ .

For the U(1) interaction to “tilt” the vacuum and break  $SU(2)_V$ , the contribution of the U(1) coupling to the mass-squared of the  $\Phi$  field must compete with the leading contribution from topcolor. Therefore, the contribution of the top-color interactions to the mass-squared of  $\Phi$  must be adjusted to be  $\mathcal{O}(1)$ . That is, the top-color chiral phase transition must be a second order transition to subleading order<sup>2</sup> in  $N$  and

$$\frac{\Delta g_{tc}^2(M)}{g_c^2} = \frac{g_{tc}^2(M) - g_c^2}{g_c^2} = \mathcal{O}\left(\frac{1}{N}\right); \quad (3.4)$$

i.e.  $\tilde{g}_{tc}^2(M)$  must be “tuned” to equal  $\tilde{g}_c^2$  to  $\mathcal{O}(1/N)$ .

These considerations have an immediate phenomenological consequence. Treating  $\Phi = (\phi_t \phi_b)$  as a pair of Higgs fields, we see that the difference in the mass-squared of  $\phi_t$  and  $\phi_b$ ,

$$\frac{|m_{\phi_t}^2 - m_{\phi_b}^2|}{M^2} = \mathcal{O}\left(\frac{1}{N}\right), \quad (3.5)$$

is subleading<sup>3</sup> in  $N$  [9,26]. Independent of the NJL approximation, therefore, we expect that the mass of the  $\phi_b$  doublet cannot be significantly larger than the weak scale unless  $M$  is taken to be much larger than the weak scale. This extra light  $\phi_b$  doublet could give rise to dangerous effects in  $B$ - $\bar{B}$  mixing and  $B$ -meson decays [9,10].

Note that this argument does not depend on any of the details that lead to Eq. (2.1). We need not assume that the top color and U(1) gauge boson masses are equal, or even of the same order of magnitude.

<sup>2</sup>It is of note that at next-to-leading order in  $N$  there are contributions which may make the phase transition weakly first order [23,24] due to the Coleman-Weinberg mechanism [25].

<sup>3</sup>This can be seen in Eq. (2.5), for example, because  $m_{\phi_t}^2 - m_{\phi_b}^2 \propto |\kappa_t - \kappa_b| = \mathcal{O}(1/N)$ .

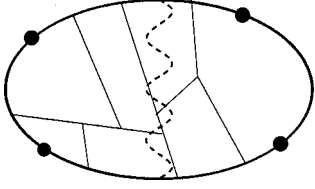


FIG. 3. A typical contribution to the effective potential for  $\Phi$  to leading order in  $1/N$  and lowest order in the (symmetry-violating)  $U(1)$  couplings. The  $U(1)$  gauge boson propagator is shown in dashed lines.

#### IV. BEYOND THE NJL APPROXIMATION: $\Delta\rho$

The NJL approximation treats only the terms in Eq. (2.1) which couple left-handed and right-handed currents. Consider the effects of the top-color interactions in Eq. (2.1) coupling *pairs* of left-handed (LL) or right-handed (RR) currents, which are not included in the NJL approximation. The LL terms may be Fierz transformed to color-singlet form

$$\left(\bar{\psi}_L \gamma^\mu \frac{\lambda^a}{2} \psi_L\right)^2 \rightarrow \frac{1}{2} (\bar{\psi}_L \gamma^\mu \psi_L)_y^x (\bar{\psi}_L \gamma_\mu \psi_L)_x^y - \frac{1}{2N} (\bar{\psi}_L \gamma^\mu \psi_L) (\bar{\psi}_L \gamma_\mu \psi_L). \quad (4.1)$$

Here  $x$  and  $y$  are  $SU(2)_L$  (flavor) indices and an analogous expression holds for the terms involving a product of right-handed currents with  $L \leftrightarrow R$ .

Treating the operator in Eq. (4.1) as a perturbation, its contribution in the low-energy effective scalar theory may be computed to leading-order as shown in Fig. 4. As in QCD [27,28], to leading-order in  $N$ , the ‘‘vacuum insertion’’ approximation may be used to evaluate this contribution. The left-handed and right-handed flavor currents can then be matched individually to the corresponding flavor currents in the low-energy scalar theory. Since  $\Phi$  transforms as a  $(2, \bar{2})$  under  $SU(2)_L \times SU(2)_R$ , we find

$$\begin{aligned} (\bar{\psi}_L \gamma^\mu \psi_L)_y^x &\rightarrow -i(\Phi \vec{\partial}^\mu \Phi^\dagger)_y^x + \mathcal{O}\left(\frac{\Phi \Phi^\dagger}{M^2}, \frac{\partial^2}{M^2}\right), \\ (\bar{\psi}_R \gamma^\mu \psi_R)_y^x &\rightarrow -i(\Phi^\dagger \vec{\partial}^\mu \Phi)_y^x + \mathcal{O}\left(\frac{\Phi \Phi^\dagger}{M^2}, \frac{\partial^2}{M^2}\right). \end{aligned} \quad (4.2)$$

The fermion currents above contain both  $SU(2)_{L,R}$  and  $U(1)_{L,R}$  pieces. However, in the *linear* sigma model, the lowest-order term in the effective Lagrangian which distinguishes between the  $SU(2)$  triplet and singlet pieces of  $\Phi$  is of dimension six. Therefore the relation of Eq. (4.2) holds for both parts up to corrections of  $\mathcal{O}(1/M^2)$ .

In the leading vacuum-insertion approximation we may then immediately write down the effective interaction arising from the left-handed and right-handed operators of the form of Eq. (4.1):

$$\delta\mathcal{L}_{LL} = + \frac{2\pi\kappa_{tc}}{M^2} \text{Tr}(\Phi \vec{\partial}^\mu \Phi^\dagger)(\Phi \vec{\partial}_\mu \Phi^\dagger),$$

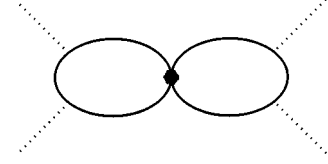


FIG. 4. Leading contribution of product of left-handed currents in Eq. (2.1) to the low-energy effective scalar theory. Fermion lines are solid, scalar ( $\Phi$ ) lines are dotted, and the left-handed four-fermion operator is represented by the solid circle in the center.

$$\delta\mathcal{L}_{RR} = + \frac{2\pi\kappa_{tc}}{M^2} \text{Tr}(\Phi^\dagger \vec{\partial}^\mu \Phi)(\Phi^\dagger \vec{\partial}_\mu \Phi), \quad (4.3)$$

plus corrections of order  $M^{-4}$  and  $1/N$ . Note that these interactions are  $SU(2)_L \times SU(2)_R$  invariant. This is as it should be since the top-color interactions are themselves  $SU(2)_L \times SU(2)_R$  invariant.

The vacuum expectation value of the field is

$$\langle \Phi \rangle = \begin{pmatrix} \frac{f_t}{\sqrt{2}} & 0 \\ 0 & 0 \end{pmatrix}, \quad (4.4)$$

which breaks  $SU(2)_L \times SU(2)_R \rightarrow U(1)_{em}$ . Note that the potential ‘‘custodial’’  $SU(2)_V$  [29] symmetry is broken by the alignment of the vacuum [12–15]. Because of the usual *accidental* symmetry, the terms of dimension four or less in the effective Lagrangian *cannot* give rise to a deviation of the rho parameter,

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W}, \quad (4.5)$$

from one. The leading contribution arises from the operator in Eq. (4.3). Promoting the partial derivatives to gauge covariant derivatives, we may calculate the gauge-boson masses and find the contribution to  $\rho - 1$

$$\Delta\rho = \frac{2\pi e^2 \kappa_{tc}}{\sin^2 \theta_W \cos^2 \theta_W} \frac{f_t^4}{M_Z^2 M^2}. \quad (4.6)$$

Note that this contribution to  $\Delta\rho$  is of leading-order in  $N$  and results from the LL and RR top-color interactions not included in the NJL approximation. To leading order in  $N$ , this result agrees with the calculation given in [11].

The calculation of  $\Delta\rho$  relies on treating the LL and RR portions of the top-color interactions as perturbations to the NJL model. However, even beyond this approximation we expect the effective Lagrangian for the composite  $\Phi$  will contain terms of the form shown in Eq. (4.3). We therefore generally expect contributions of this order of magnitude.<sup>4</sup>

The corrections discussed above are of leading order in  $1/N$ , but are suppressed by  $1/M^2$ . There are also corrections

<sup>4</sup>This may be viewed as a special case of the constraints on composite Higgs models discussed in [30].

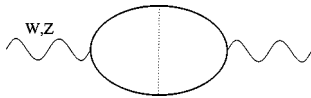


FIG. 5. Example of a potentially large two-loop contribution to  $\Delta\rho$  arising from exchange of light composite scalars. Fermions (the top and bottom) are represented by solid lines and scalars by the dotted line. These contributions are subleading in  $1/N$ , but are not suppressed by  $1/M^2$ .

coming from the exchange of the light scalars in  $\Phi$  (such as that shown in Fig. 5). These contributions are subleading in  $1/N$  but are *not* suppressed by  $1/M^2$ . They are also potentially large because the couplings to the top quark are of order  $m_t/f_t$ , approximately four times *larger* than the corresponding contributions from a standard model Higgs boson. It is amusing to note that if these particles were light, this contribution could be *negative* [31–34]. However, phenomenological constraints from  $Z \rightarrow b\bar{b}$  [35] require that these particles be relatively heavy and these contributions are probably suppressed.

Finally, we comment on the situation in models [36,26] which do *not* contain a U(1) tilting interaction. Instead, the strong topcolor group is arranged to couple to the left-handed top-quark–bottom-quark doublet (and, in “top-color II” [26], the charm-strange doublet) and the right-handed top quark, but not to the right-handed bottom quark. The large- $N$  analysis of the first section then no longer applies—however, anomaly cancellation implies that the number of *flavors* to which top color couples must scale with the number of colors. This situation is reminiscent of the large- $N$  and large- $N_f$  analysis of [37]. Such a theory will have a large number of scalars and contributions of the form shown in Fig. 5 would no longer be suppressed by  $1/N$ . Furthermore, as the top-color interactions themselves violate custodial SU(2), there

will still generically be contributions analogous to Eq. (4.3) which yield contributions to  $\Delta\rho$  of the same order of magnitude [30].

Top-color and top-color-assisted technicolor [1,2] provide examples of dynamical electroweak symmetry breaking which include top-quark condensation [3–8], thereby naturally incorporating a heavy top quark. In this paper we have discussed the roles of the Nambu–Jona-Lasinio and large- $N$  approximations used in phenomenological analyses and have discussed the dynamical behavior of top-color theories beyond these approximations. We have shown that, in models with a U(1) tilting interaction, in order to provide for top-quark condensation but not bottom-quark condensation the top-color coupling must be adjusted to equal the critical value for chiral symmetry breaking up to  $\mathcal{O}(1/N)$ . A consequence of these considerations is that the potentially dangerous “bottom-pions” [9,10] are naturally light. We have also considered contributions beyond the NJL approximation and shown that the other portions of the top-color interactions, in particular products of pairs of left-handed currents or right-handed currents, give rise to contributions to  $\rho - 1$ . These contributions, which have been estimated previously [11] by direct computation, are of leading-order in  $N$  and are the result of vacuum-alignment.

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