SU(3) breaking and hidden local symmetry

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We study the various existing implementations of SU(3) breaking in the hidden local symmetry model for the low energy hadronic sector following a mechanism originally proposed by Bando, Kugo and Yamawaki (BKY). We pay particular attention to Hermiticity and current conservation. Following this, we present a new method for including symmetry breaking effects which preserves the BKY mass relation among vector mesons. Symmetry breaking (SB) necessarily requires a transformation of the pseudoscalar fields, which, following BKY, we refer to as field renormalization. We examine the consequences of propagating this through all Lagrangian terms including the anomalous ones. We thus explore the consequences of these various SB schemes for both charged and neutral pseudoscalar decay constants as measured in weak and anomalous decays respectively. [S0556-2821(98)05219-9]

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I. INTRODUCTION

In a recent paper [1], it has been shown that the pion form factor is described in a perfectly coherent way by the hidden local symmetry (HLS) model proposed in Ref. [2]. Noticeably, it has been shown that a $\gamma \pi^+ \pi^-$ contact term is preferred in fits with a coupling strength c given by

$$c = 1 - \frac{f_{\rho\gamma} g_{\rho\pi\pi}}{m_\rho^2} \tag{1}$$

as predicted by the HLS model, where $f_{\rho\gamma}$ and $g_{\rho\pi\pi}$ are the usual coupling constants to respectively the photon and a pion pair. In addition to providing a nice description of the $e^+ e^- \rightarrow \pi^+ \pi^-$ data, it was also shown that the resulting phase of $F_\pi(s)$ accounted for the predicted $\pi\pi$ phase shift [3] up to about 1 GeV/c, without further constraint. Moreover, the values of $F_\pi(4m_\pi^2)$ and for the p-wave scattering length were found to agree perfectly with chiral perturbation theory (ChPT) predictions (see for instance Ref. [4]). This gives a hint that the HLS model could successfully describe other scattering data and that its extension to the anomalous sector [5] could describe radiative decays of light flavor mesons, with a very small number of free parameters.

In the sector explored by Ref. [1], one does not expect effects of the SU(3) symmetry breaking produced by the large mass difference between the s quark and the light u and d quarks. Other sectors such as the kaon form factors or most radiative decays are surely more sensitive to this. Therefore, a study of symmetry breaking within the HLS model is an *a priori* condition toward a full study of its relevance to low energy particle physics in sectors mixing vector and pseudoscalar mesons explicitly. In order to produce this breaking, Bando, Kugo and Yamawaki (BKY) [6]

proposed a way which leaves the u/d sector of pseudoscalar mesons unchanged while sharply breaking the s sector. We will refer to this (outlined in Sec. III) as the BKY mechanism.

Much attention to this SU(3) symmetry breaking has focused on its consequences for the anomalous meson sector [7–9] including proposed symmetry breaking variants [10]. The aim of this paper is to study the consequences of the original BKY mechanism [6] in both the anomalous and non-anomalous sectors. We use the full pseudoscalar field matrix (i.e. including the isoscalar sector), in order to examine the Hermitian version of the original BKY Lagrangian [6], as well as one proposed by Bramon, Grau and Pancheri (BGP) [8] and one we introduce here. Our scheme allows one to recover interesting properties of both the BKY and BGP schemes, namely the mass relation among vector mesons of Ref. [6], and the current structure and conservation properties obtained with the BGP scheme. In this scheme the current coupling a vector meson to a pseudoscalar pair, PP', has a divergence proportional to $m_P^2 - m_{P'}^2$, which vanishes for massless P and P'. We shall refer to this throughout as current conservation, because we only consider the case of massless pseudoscalar mesons. Then in the physical case of massive pseudoscalar mesons current conservation is broken in the appropriate way, i.e. only by terms proportional to $m_P^2 - m_{P'}^2$.

As recognized by BKY, their symmetry breaking mechanism leads to a redefinition (we shall call this renormalization) of the pseudoscalar fields, which has to be propagated to all Lagrangian contributions. Focusing on the anomalous (Wess-Zumino-Witten) WZW terms [11,12], we show that the symmetry breaking in the non-anomalous HLS Lagrangian, produces in this way a new breaking of the anomalous terms and we illustrate why it does not exhaust all breaking effects (for example, these do not include loop effects [13]). This is of relevance for the physics of η/η' mesons, which has recently received much interest from various points of view [14–22].

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The paper is organized as follows, in Sec. II we briefly review the basics of the HLS model. Section III is devoted to an analysis of three ''natural'' variants of the original BKY mechanism for SU(3) symmetry breaking. We show that, even when made Hermitian, the BKY scheme does not separately conserve all currents occurring in the interaction Lagrangian, in the sense given above, while the unbroken HLS Lagrangian does. The variant proposed by BGP [8] does, but gives the vector meson masses the standard Gell-Mann–Okubo formula. We propose another variant which allows one to obtain the phenomenologically successful BKY mass formula ($m_\phi m_\omega = m_{K*}^2$) and conservation of all currents. We illustrate how the BKY mechanism, together with a departure from ideal $\omega - \phi$ mixing, generates a mass difference $m_\omega - m_\rho$ which goes to zero with the symmetry breaking parameter. In Sec. IV, we examine the consequences of the pseudoscalar field renormalization implied by the BKY mechanism, in (re-)deriving the decay constant f_K and we show how box and triangle anomalies are affected. Most lengthy expressions are left to the Appendix.

II. HIDDEN LOCAL SYMMETRY

We shall examine the low energy sector, including the octet and singlet pseudoscalars within the context of the HLS model. Here we present a brief account of the HLS [2,5,6] model. The HLS model allows us to produce a theory with

vector mesons as the gauge bosons of a hidden local symmetry. These then become massive due to the spontaneous breaking of a chiral $U(3)_L \otimes U(3)_R$ global symmetry. Let us consider the chiral Lagrangian [23]

$$\mathcal{L}_{\text{chiral}} = \frac{1}{4} \, \text{Tr}[\partial_\mu F \partial^\mu F^\dagger], \tag{2}$$

where $F(x) = f_P U(x)$ in normal notation and f_P is a constant with dimensions of mass. In practice, one identifies this parameter with the pion decay constant $f_P = f_\pi = 93$ MeV. This exhibits the chiral $U(3)_L \otimes U(3)_R$ symmetry under $U \to g_L U g_R^\dagger$. We can write this in exponential form and expand

$$F(x) = f_P e^{2iP(x)/f_P} = f_P + 2iP(x) - 2P^2(x)/f_P + \cdots \tag{3}$$

therefore, substituting into Eq. (2) we see the vacuum corresponds to $P = 0$, $U = 1$. That is, F has a non-zero vacuum expectation value which spontaneously breaks the $U(3)_L \otimes U(3)_R$ symmetry [24]. The massless Goldstone bosons contained in P, then correspond to the perturbations about the vacuum and we can think of expansions in this field given by the Hermitian matrix $P = P^a T^a$ where the SU(3) generators are normalized such that $\text{Tr}[T^a T^b] = \delta^{ab}/2$. Thus, for the pseudoscalars one has

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \pi_8 + \frac{1}{\sqrt{3}} \eta_0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \pi_8 + \frac{1}{\sqrt{3}} \eta_0 & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \pi_8 + \frac{1}{\sqrt{3}} \eta_0 \end{pmatrix}, \tag{4}$$

where we have included the singlet field η_0 assuming nonet symmetry.[1]

However, in addition to the global chiral symmetry, QCD possesses a local symmetry. The HLS scheme includes such a symmetry in Eq. (2) in the following way. Let

$$U(x) \equiv \xi_L^\dagger(x) \xi_R(x) \tag{5}$$

where the ξ fields undergo a local transformation, $h(x)$, which does not affect the chiral field $U(x)$. In addition to

pseudoscalar fields, $P(x)$, the ξ fields also possess a scalar constituent $S(x)$, and are thus characterized by

$$\xi_{R,L}(x) = e^{iS(x)/f_S} e^{\pm iP(x)/f_P}, \quad \xi_{R,L}(x) \to h(x) \xi_{R,L}(x) g_{L,R}^\dagger. \tag{6}$$

As can be seen from Eq. (5), $\mathcal{L}_{\text{chiral}}$ is obviously invariant under this local transformation. From now on, as per usual [2], we remove $S(x)$ and thus $\xi_L^\dagger = \xi_R = \xi$. We may rewrite $\mathcal{L}_{\text{chiral}}$ explicitly in terms of the ξ components

$$\mathcal{L}_{\text{chiral}} = -\frac{f_P^2}{4} \, \text{Tr}[(\partial_\mu \xi_L \xi_L^\dagger - \partial_\mu \xi_R \xi_R^\dagger)]^2. \tag{7}$$

The Lagrangian can be gauged for both electromagnetism and the hidden local symmetry by changing to covariant derivatives

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[1] One could allow for departure from nonet symmetry by affecting the η_0 field by a multiplying parameter x to be fixed by the data. Moreover, we will not address here the problem of the renormalization scale dependence associated with the singlet pseudoscalar field (see [21,22] for instance).

$$D_\mu \xi_{L,R} = \partial_\mu \xi_{L,R} - ig\mathcal{V}_\mu \xi_{L,R} + ie\xi_{L,R}A_\mu Q \qquad (8)$$

where A_μ is the photon four-vector and $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ the charge matrix. The vector field, $\mathcal{V} = \mathcal{V}^a T^a$, transforming locally as $\mathcal{V}_\mu \to h(x)\mathcal{V}_\mu h^\dagger(x) + ih(x)\partial_\mu h^\dagger(x)/g$, is given by

$$\mathcal{V} = \frac{1}{\sqrt{2}} \begin{pmatrix} (\rho^0 + \omega)/\sqrt{2} & \rho^+ & K^{*+} \\ \rho^- & (-\rho^0 + \omega)/\sqrt{2} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}. \qquad (9)$$

In Eq. (9) ω and ϕ correspond to the ideally mixed states. The HLS Lagrangian is then given by $\mathcal{L}_{\mathrm{HLS}} = \mathcal{L}_A + a\mathcal{L}_V$ where

$$\mathcal{L}_A = -\frac{f_P^2}{4} \mathrm{Tr}[D_\mu \xi_L \xi_L^\dagger - D_\mu \xi_R \xi_R^\dagger]^2 \equiv -\frac{f_P^2}{4} \mathrm{Tr}[L - R]^2$$

$$\mathcal{L}_V = -\frac{f_P^2}{4} \mathrm{Tr}[D_\mu \xi_L \xi_L^\dagger + D_\mu \xi_R \xi_R^\dagger]^2 \equiv -\frac{f_P^2}{4} \mathrm{Tr}[L + R]^2 \qquad (10)$$

and a is a parameter which is not fixed by the theory. However, setting $a = 2$ allows one to recover the usual expression for vector meson dominance (VMD) [2] and moreover, there is some experimental evidence [1] that a is slightly (but significantly) greater than 2. For this reason we shall keep track of a in the following expressions.

The full HLS Lagrangian is somewhat lengthy, so we leave it to the Appendix, where it is given by Eq. (A1). The photon and vector mesons acquire Lagrangian masses through an analogue of the Higgs-Kibble mechanism; we will refer to these masses as HK masses.[2] The photon, though, is seen to be massless once the vector meson corrections to the vacuum polarization are included, thus preserving EM gauge invariance (for a fuller discussion of this point see, for example, Ref. [26]). One should also notice that the singlet field η_0 does not appear in the SU(3) symmetric HLS Lagrangian.

III. FLAVOR SYMMETRY BREAKING

To account for deviations from SU(3) flavor symmetry in the low energy sector Bando, Kugo and Yamawaki (BKY) [6] introduced symmetry breaking terms to the HLS La-

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[2]In light of this model for low energy QCD it is interesting to note the electric-magnetic duality where the elementary electric degrees of freedom become strongly coupled, leading to confinement. The magnetic degrees of freedom, which are in the Higgs phase, can be described as composites of the electric ones. These magnetic particles typically include massless gauge bosons associated with a new magnetic gauge symmetry not present in the fundamental electric theory [25].

grangian, as $(3,3^*) + (3^*,3)$ representations. However, there is no unique way to do this, though naturally one has to recover the unbroken case smoothly when the symmetry breaking parameter goes to zero. The initial BKY symmetry breaking scheme was recognized as being non-Hermitian by BGP who proposed a variant which restores Hermiticity [8]. We shall now discuss each variant of the original BKY symmetry breaking scheme [6] in detail and present a new one.

A. The BKY scheme

The BKY scheme introduces the symmetry breaking term $\epsilon = \mathrm{diag}(0,0,c)$ through

$$\mathcal{L}_{A(V)}^{\mathrm{BKY}} = -\frac{1}{4} f_P^2 \mathrm{Tr}[(D_\mu \xi_L \xi_L^\dagger + D_\mu \xi_L \epsilon_{A(V)} \xi_R^\dagger)$$

$$\mp (D_\mu \xi_R \xi_R^\dagger + D_\mu \xi_R \epsilon_{A(V)} \xi_L^\dagger)]^2 \qquad (11)$$

where the subscripts A and V respectively correspond to the $-$ and $+$ signs in the RHS of this expression. The relevant components are, defining $X_{A,V} = (1 + \epsilon_{A,V})$,

$$\mathcal{L}_A^{\mathrm{BKY}} = \mathrm{Tr}[(\partial P X_A)^2 - i(g\mathcal{V}(P\epsilon_A + \epsilon_A P)$$

$$- e(PA - AP + PA\epsilon_A + A\epsilon_A P))\partial P X_A$$

$$+ i(g(P\epsilon_A + \epsilon_A P)\mathcal{V}$$

$$- e(AP - PA + P\epsilon_A A + \epsilon_A AP))X_A \partial P]$$

$$\mathcal{L}_V^{\mathrm{BKY}} = \mathrm{Tr}[f_P^2(g\mathcal{V}X_V - eAX_V)^2$$

$$+ 2i(g\mathcal{V} - eA)X_V \partial P(1 - \epsilon_V)P$$

$$+ i(eA - g\mathcal{V})X_V(\partial PP + P\partial P)X_V] \qquad (12)$$

where we have assumed the appropriate contractions over the Lorentz indices of $\mathcal{V}_\mu$, A_μ and ∂_μ. Equation (12) can easily be made Hermitian through the redefinition

$$\mathcal{L}_{\mathrm{BKY}} \to \frac{1}{2}(\mathcal{L}_{\mathrm{BKY}} + \mathcal{L}_{\mathrm{BKY}}^\dagger) \qquad (13)$$

where one recovers smoothly the unbroken Lagrangian in the limit $X_{A(V)} \to 1$, as desired. This Hermitian version of the BKY Lagrangian is given in Eq. (A2). One should note here that the BKY implementation of flavor symmetry breaking produces an interplay of the singlet η_0 field which is absent in the unbroken Lagrangian. From Eq. (A2) we see the BKY relation for the vector meson masses

$$\frac{m_{K*}}{m_\omega} = \frac{m_\phi}{m_{K*}} = \sqrt{1 + c_V}. \qquad (14)$$

This is very well fulfilled by the Breit-Wigner (BW) masses of the corresponding vector mesons [27] for $c_V \sim 0.3$. Whether it should also be true for the HK masses is presently

an open question.[3] One should also note that no mass splitting is generated among the ρ and ω mesons.

In QCD the divergence of a general vector current $J_\mu = \bar{a}\gamma_\mu b$ is proportional to the quark mass difference ($m_a - m_b$). In ChPT [31] the squares of the pseudoscalar masses are proportional to linear combinations of the quark masses, and hence to maintain this connection current divergences should be of the form ($M_A^2 - M_B^2$) and so should vanish in the absence of pseudoscalar meson mass terms. Therefore it is reasonable to use current conservation (as defined above) to constrain parameters and symmetry breaking mechanisms in the interaction Lagrangian.[4] Equation (A2) then leads first to the condition

$$c_A = a c_V \qquad (15)$$

where $a = 2$ reproduces VMD [2]. Usually, it is simply *assumed* that $c_A = c_V$ [6,7]. Examining the kaon form-factors at $s = 0$ we find

$$F_{K^+}(0) = 1 + c_A, \quad F_{K^0}(0) = 0. \qquad (16)$$

It is thus clear that field renormalization is required and BKY [6] remarked that the appropriate field renormalization is

$$P_R = (1 + \epsilon_A)^{1/2} P (1 + \epsilon_A)^{1/2}. \qquad (17)$$

Indeed, in addition to normalizing the kaon charge [Eq. (16)] the pseudoscalar kinetic term is restored to its canonical form

$$\mathcal{L}^{\text{kinetic}} = \partial K_R^- \partial K_R^+ + \partial K_R^0 \partial \bar{K}_R^0 + \partial \pi_R^+ \partial \pi_R^-$$
$$+ \frac{1}{2}(\partial \pi_R^0 \partial \pi_R^0 + \partial \pi_R^8 \partial \pi_R^8 + \partial \eta_R^0 \partial \eta_R^0), \quad (18)$$

where the subscript R, stands for ''renormalized.'' Unfortunately, this does not quite restore current conservation in the K^* interactions terms as is clear from Eq. (A3), which expresses $\mathcal{L}^{\text{int}}(K^*, K, \pi^8, \eta_0)$, due to terms quadratic in the

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[3]The vector meson masses reported by the Particle Data Group (PDG) are quite generally obtained from parametrizations assuming the (Breit-Wigner) form $s - m_V^2 - i m_V \Gamma_V(s)$ for the vector meson propagators in fitting expressions. In order to get an (approximate) estimate of the HK masses, one should rather use propagators written like $s - m_V^2 - \Pi_V(s)$, where $\Pi_V(s)$ is the vector meson vacuum polarization. Although there is some hint [28], that the meson masses defined in this way could be significantly different from the usual (BW) masses, other studies predict negligible contributions from the real part of pseudoscalar meson loops to vector meson masses, apart from $\rho \to 2\pi \to \rho$ [29]. This is further complicated by the model dependence of traditional mass extractions [30] and we shall not discuss this matter any further here.

[4]From a phenomenological point of view, this assumption ensures that the coupling of a vector meson to two pseudoscalars can be generally written $\epsilon_\mu^\lambda \cdot (p_1 - p_2)^\mu$, with the massive vector particle having three polarization states (denoted by λ), as usual. The assumption about current conservation prevents to rather have $A\epsilon_\mu^\lambda \cdot (p_1 - p_2)^\mu + B\epsilon_\mu^\lambda \cdot (p_1 + p_2)^\mu$.

symmetry breaking parameters. There are two cases, where current conservation (in the sense defined just above) can be restored, as in the unbroken Lagrangian. The first one is the unphysical case where $\eta_0 = \sqrt{2}\pi^8$, the other case is if $a = 1$, the Georgi vector limit [32]. Pure VMD supposes $a = 2$, while existing data prefer a slightly larger value [1] $a \simeq 2.4$, inconsistent with $a = 1$ anyway. Stated otherwise, the original BKY scheme, even if modified in order to restore Hermiticity, does not completely maintain current conservation under physically acceptable conditions.

B. The BGP scheme

Having noted the non-Hermiticity of the original BKY Lagrangian, BGP [8] proposed the following variant to the original BKY scheme:

$$\mathcal{L}_{A,V}^{\text{BGP}} = -\frac{f_P^2}{4} \text{Tr}[(D_\mu \xi_L \xi_L^\dagger \mp D_\mu \xi_R \xi_R^\dagger)^2$$
$$\times (1 + \xi_L \epsilon_{A,V} \xi_R^\dagger + \xi_R \epsilon_{A,V} \xi_L^\dagger)] \qquad (19)$$

yielding upon the relevant weak field expansion

$$\mathcal{L}_A^{\text{BGP}} = \text{Tr}[(\partial P \partial P - ie(\partial P(AP - PA)$$
$$+ (AP - PA)\partial P))(1 + 2\epsilon_A)] \qquad (20)$$

$$\mathcal{L}_V^{\text{BGP}} = \text{Tr}[(i/2\{[\partial P, P], (g\mathcal{V} - eA)\}$$
$$+ e g f_P^2 \{\mathcal{V}, A\} + f_P^2 g^2 \mathcal{V}^2)(1 + 2\epsilon_V)]. \qquad (21)$$

The full BGP Lagrangian is given by Eq. (A4). A check of the K^* interaction terms finds current conservation guaranteed, but as there is no connection between c_A and c_V, they remain independent parameters, at this stage. One should note that the BGP variant does not invoke the interplay of the singlet field η_0, as opposed to the original BKY scheme which does.

The other results of the BGP symmetry breaking scheme were alluded to in general by BKY for any Lagrangian lacking ϵ^2 terms, and the following relations are easily recognized as linear truncations of the BKY results. The vector meson masses are given by the Gell-Mann–Okubo formula

$$m_{K*}^2 - m_\omega^2 = m_\phi^2 - m_{K*}^2 = c_V a f_P^2 g^2 \qquad (22)$$

which is less phenomenologically successful than the BKY relation and $m_\omega = m_\rho$. Using the numerical values from PDG [27], this relation implies $c_V = 0.3 \sim 0.4$.

It is clear from the expression of the pseudoscalar kinetic energy term that the BKY prescription for field renormalization cannot change it to the canonical form. In order to get this, one has to perform another change of fields. As per usual, we find $\pi_R = \pi$ and $K_R = K/\sqrt{1 + c_A}$, where the subscript R stands for ''renormalized fields,'' while for the isoscalars, we have

$$\pi^8 = \frac{1}{3}\left[\frac{2}{\sqrt{1+2c_A}}+1\right]\pi_R^8 + \frac{\sqrt{2}}{3}\left[1-\frac{1}{\sqrt{1+2c_A}}\right]\eta_R^0$$

$$\eta^0 = \frac{\sqrt{2}}{3}\left[1-\frac{1}{\sqrt{1+2c_A}}\right]\pi_R^8 + \frac{1}{3}\left[\frac{1}{\sqrt{1+2c_A}}+2\right]\eta_R^0 \tag{23}$$

which has clearly a smooth limit for $c_A \to 0$. Therefore, the BGP prescription, even if slightly more complicated to renormalize than the BKY scheme, allows one to recover all expectations. We have already commented on the mass formula, which cannot be presently considered as fully conclusive.

C. An alternate scheme

We have shown that the BKY scheme (once made Hermitian) fails to preserve current conservation for the (π^8,η_0) sector, whilst the BGP scheme, though ensuring current conservation, seems less successful in reproducing the observed accepted vector meson mass splitting [27]. We therefore introduce breaking in the HLS Lagrangian in such a way that the desirable features of both previous studies are reproduced, namely the BKY mass formula and current conservation in all interactions. We generalize Eq. (10) through

$$\mathcal{L}_{A,V} = -\frac{f_P^2}{4}\,\text{Tr}[(L\mp R)(1+(\xi_L\epsilon_{A,V}\xi_R^\dagger + \xi_R\epsilon_{A,V}\xi_L^\dagger)/2)]^2 \tag{24}$$

which has also a smooth unbroken limit. The terms we will be interested in are then given by

$$\mathcal{L}_{A,V} = -\frac{f_P^2}{4}\,\text{Tr}[(L\mp R)X_{A,V}(L\mp R)X_{A,V}] \tag{25}$$

and hence

$$\mathcal{L}_A = \text{Tr}[\partial P X_A \partial P X_A + 2ie(PA-AP)X_A\partial P X_A] \tag{26}$$

$$\mathcal{L}_V = \text{Tr}[f_P^2((g\mathcal{V}-eA)X_V)^2$$
$$+ i(g\mathcal{V}-eA)X_V(\partial P P - P\partial P)X_V]. \tag{27}$$

It is obvious from these expressions that the field renormalization prescription of BKY [6] is relevant in this new scheme. The kinetic pseudoscalar term is renormalized by the same procedure as for BKY. The full expression for the corresponding Lagrangian is given in Eq. (A5). We also see the quadratic BKY relation between the ω, K^* and ϕ masses of Eq. (14). What is more, like BGP, current conservation is

explicitly guaranteed. As for the BGP scheme, in contrast with that of BKY, the pseudoscalar singlet field η_0, does not occur in the broken Lagrangian. This could be inferred by looking at Eqs. (26) and (27) for which symmetry breaking enters only through the combinations $X_{A,V}$ unlike Eq. (12). In this new scheme, as for BGP, c_A and c_V remain unrelated. As a final check we examine the kaon form factors and find Eq. (16) holds for general a.

D. The ρ-ω mass splitting

All symmetry breaking schemes outlined above, predict no mass splitting of the ρ and ω mesons (loop effects could be important here [28,29]). It should however be remarked that what have been called, up to now, ϕ and ω are ideally mixed states, where ω is purely non-strange and ϕ is purely strange. There is however, strong experimental evidence that, even if the mixing is close to ideal, it is not exactly ideal. Then the question arises as to whether a departure from ideal mixing can (or should) be accounted for at the level of the Lagrangian itself and if the BKY symmetry breaking mechanism is able to contribute to ρ-ω mass splitting. As ideal mixing is not a fundamental symmetry, this may not be actually considered as a symmetry breaking effect.

For ω and ϕ being considered as the ideally mixed states, we can define, the physical states[5] ω_P and ϕ_P, by

$$\omega_P = \omega\cos\delta + \phi\sin\delta, \quad \phi_P = \phi\cos\delta - \omega\sin\delta \tag{28}$$

where δ can be determined by the ratio of the measured coupling constants [16] $g_{\phi\pi^0\gamma}$ and $g_{\omega\pi^0\gamma}$ which is found to correspond to $\tan\delta$ and gives about 3.25 degrees [9,15,33]. The corresponding vector mixing angle θ_V is therefore of the order 32 degrees, slightly smaller than its ideal value.

If one performs this change of variables in the unbroken HLS Lagrangian [see Eq. (A1)], the mass term is strictly conserved, and only couplings of the physical ω_P and ϕ_P to pseudoscalar mesons are changed by terms of the order $\sin\delta \approx 6\times10^{-2}$ or higher. In the broken Lagrangians, the situation looks slightly different. The transformation generates a ρ-ω mass difference

$$(m_\rho^2 - m_\omega^2)/m_\rho^2 = c_V(2+c_V)\sin^2\delta \quad \text{BKY, new scheme}$$

$$= 2c_V\sin^2\delta \quad \text{BGP} \tag{29}$$

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[5]In order to stay consistent with the usual custom in the effective Lagrangian community, we shall use the ideal ϕ to be $+|s\bar{s}\rangle$, while another usual custom [15,27] prefers $-|s\bar{s}\rangle$, which allows one to get these ideally mixed states from the standard isovector singlet and octet states by a normal rotation matrix, without any change of sign.

while leaving the ϕ_P mass modified, with respect to the ideal ϕ, by a negligible amount (a factor of the order $\cos\delta$). If one considers likely values for c_V ($\simeq 0.3$), this generates a ρ-ω mass splitting of about 2 to 3 MeV.

However, the original mass term for vector mesons in the broken Lagrangians also generates a transition term from ω_P to ϕ_P:

$$-\omega_P\phi_P\sin 2\delta\times c_V(2+c_V) \quad \text{BKY, new scheme}$$

$$\times 2c_V \quad \text{BGP} \tag{30}$$

which adds $\omega_P-\phi_P$ direct transitions to the usual γ to vector meson direct transitions. It should be noted that such a term (which vanishes in the unbroken limiting case), is small (its coupling is of the order 1% of m_ρ^2), and probably inefficient because of the large mass difference between these mesons. Moreover, it comes supplementing already existing transition effects by means of the $K\bar{K}$ loop effects. Whether such a term could be experimentally visible is thus not obvious to answer. It is however interesting to see that a very small admixture of $s\bar{s}$ inside the ω is able to generate and explain a small mass splitting between the ρ and ω mesons by means of the BKY symmetry breaking mechanism, which vanishes with the symmetry breaking parameter.

IV. PSEUDOSCALAR DECAY CONSTANTS AND ANOMALIES

We are now in a position to determine the pseudoscalar decay constants, and examine some consequences of the pseudoscalar field renormalization implied by the BKY symmetry breaking mechanism. We have of course to distinguish the case of π^0, η and η', which proceed from the low energy anomalous Lagrangians [11,12], from $\pi^\pm$, $K^\pm$, K^0 and $\bar{K}^0$ mesons, which can be determined from the pseudoscalar meson coupling to an axial vector field.

A. Decay constants from non-anomalous sector

The charged pseudoscalar decay constants are measured in weak decays $P^\pm \to l^\pm + \nu_l$ and $P^\pm \to l^\pm + \nu_l\gamma$ [27]. Therefore to examine this in the HLS model we need to include axial vectors, $\mathcal{A}_\mu$, through [34]

$$D_\mu\xi_L=(\partial_\mu-ig\mathcal{V}_\mu+ig_A\mathcal{A}_\mu)\xi_L,$$

$$D_\mu\xi_R=(\partial_\mu-ig\mathcal{V}_\mu-ig_A\mathcal{A}_\mu)\xi_R. \tag{31}$$

The pseudoscalar decay constants defined through (note that unlike the mini-review of Suzuki in Ref. [27] we include $\sqrt{2}$)

$$\langle 0|\mathcal{A}_\mu|P(q)\rangle=i\sqrt{2}f_P q_\mu \tag{32}$$

are determined from the $\mathcal{A}^\mu\partial_\mu P$ interaction (set $g_A=1$), which for $\mathcal{L}^{\text{BKY}}$ and $\mathcal{L}^{\text{new}}$ is given by

$$\mathcal{L}^{\text{BKY,new}}_{\mathcal{A}\partial P}=-2f_P\text{Tr}[\mathcal{A}(1+\epsilon_A)\partial P(1+\epsilon_A)] \tag{33}$$

while for $\mathcal{L}^{\text{BGP}}$ one has

$$\mathcal{L}^{\text{BGP}}_{\mathcal{A}\partial P}=-f_P\text{Tr}[(\partial P\mathcal{A}+\mathcal{A}\partial P)(1+2\epsilon_A)]. \tag{34}$$

Constructing axial currents of the appropriate quark flavor one finds, for the renormalized pion and kaon fields in all three models,

$$\mathcal{L}_{\mathcal{A}\partial P}=-\sqrt{2}f_P[(\partial_\mu\pi_R^-+\partial_\mu\pi_R^+)\mathcal{A}^\mu(ud) $$
$$+\sqrt{1+c_A}(\partial_\mu K_R^++\partial_\mu K_R^-)\mathcal{A}^\mu(us)] \tag{35}$$

and hence

$$f_{\pi^+}=f_P, \quad f_K=(1+c_A)^{1/2}f_{\pi^+} \tag{36}$$

and we see that f_P is just the usual pion decay constant ~ 93 MeV. For the BKY scheme this gives a prediction for the kaon decay constant

$$f_K=(1+c_A)^{1/2}f_{\pi^+}=(1+2c_V)^{1/2}f_{\pi^+}\sim 1.26f_{\pi^+} \tag{37}$$

which is in very good agreement with experiment [27]. For both our new scheme and that of BGP c_A is a parameter to fit to data, thus using the experimental result $f_K/f_{\pi^+}=1.22$ we find $c_A=0.49$. It is likely that c_A can also be derived from scattering data like the kaon form factors or radiative decays.

B. The anomalous sector

The neutral decay constants, however, are measured in the anomalous processes $P^0\to\gamma\gamma$. Therefore we cannot obtain them in the previous manner but rather from analyzing the anomalous Lagrangians. We shall not consider explicit symmetry breaking terms in the anomalous action, but rather propagate the pseudoscalar field renormalization into the anomalous Lagrangian. As will be seen, this induces symmetry breaking effects in a way not previously reported. The field renormalization are given by Eq. (17) for the original BKY scheme and our proposed method, and by Eq. (23) for the BGP variant [8].

It has been claimed [7] that the singlet and octet decay constants can be determined by renormalizing the π^8 and η^0 fields with the (square root of) the coefficients in front of $(\partial\pi^8)^2$ and $(\partial\eta^0)^2$ in Eq. (A2), leaving aside the mixed term $\partial\pi^8\partial\eta^0$. Even if, numerically, the results could look interesting [7] compared with expectations [31,35], the procedure

seems questionable, as any reference to the triangle anomaly, which controls the two-photon decays of π^0, η and η', is missing.

A convenient form of the Wess-Zumino anomalous action [11] was constructed by Witten [12], and although this has been generalized to include vector mesons [5,9,10] we shall here consider only the soft limit in which they play no role. However, the field renormalization performed in the HLS Lagrangian has clearly to be propagated to all possible (anomalous) terms. The two relevant interactions are γPPP and $\gamma\gamma P$. For the first we have

$$\mathcal{L}_{\gamma PPP} = \frac{ieN_c}{3\pi^2 f_P^3} \varepsilon^{\mu\nu\alpha\beta} A_\mu \mathrm{Tr}[Q \partial_\nu P \partial_\alpha P \partial_\beta P] \quad (38)$$

where our f_P is half Witten's F_π [12]. Renormalizing the bare pseudoscalar fields through Eq. (17) or Eq. (23) we express Eq. (38) in terms of the physical pseudoscalar fields,

$$\mathcal{L}_{\gamma\pi^+\pi^- P^0} = -\frac{ieN_c}{12\pi^2 f_P^3} \varepsilon^{\mu\nu\alpha\beta} A_\mu$$

$$\times \left[\partial_\nu \pi_R^0 + \frac{1}{\sqrt{3}} \partial_\nu \pi_R^8 + \sqrt{\frac{2}{3}} \partial_\nu \eta_R^0 \right]$$

$$\times \partial_\alpha \pi_R^+ \partial_\beta \pi_R^- . \quad (39)$$

No symmetry breaking results from field renormalization in any of the three implementations of symmetry breaking for all $\gamma\pi^+\pi^- P^0$ vertices; things are, of course, different in other sectors. This is easily understood in terms of the underlying quark substructure for such a process in which the s quark responsible for symmetry breaking cannot contribute. More precisely, this follows from the fact that all variants of the BKY symmetry breaking, leave invariant the combination of π^8 and η^0 fields (or of η and η' fields) which corresponds to the $u\bar{u} + d\bar{d}$ field component. This is indeed a specific feature of all implementations of symmetry breaking discussed here.

The anomalous Lagrangian for the decay $P \to \gamma\gamma$, is given by [12]

$$\mathcal{L}_{\gamma\gamma P} = -\frac{N_c e^2}{16\pi^2 f_P} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta} \, \mathrm{Tr}[Q^2 P]. \quad (40)$$

In the original BKY scheme, as well as the new scheme, we find

$$\mathcal{L}_{\gamma\gamma P}^{\mathrm{BKY,new}} = -\frac{N_c e^2}{48\pi^2 f_P} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$$\times \left[\frac{\pi_R^0}{2} + \frac{3+5c_A}{6\sqrt{3}(1+c_A)} \pi_R^8 + \frac{6+5c_A}{3\sqrt{6}(1+c_A)} \sigma_R^0 \right], \quad (41)$$

using Eqs. (17), while using Eqs. (23), the BGP variant gives

$$\mathcal{L}_{\gamma\gamma P}^{\mathrm{BGP}} = -\frac{N_c e^2}{48\pi^2 f_P} \varepsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$$\times \left[\frac{\pi_R^0}{2} + \frac{5\sqrt{1+2c_A}-2}{6\sqrt{3}\sqrt{1+2c_A}} \pi_R^8 + \frac{5\sqrt{1+2c_A}+1}{3\sqrt{6}\sqrt{1+2c_A}} \eta_R^0 \right]. \quad (42)$$

Therefore the BKY mechanism for U(3) breaking leads to a new modification of the anomaly equations which leaves the usual box anomaly terms $\gamma\pi^-\pi^+(\pi^0/\eta/\eta')$ and the coupling $\gamma\gamma\pi^0$ unchanged, while changing only slightly the triangle anomaly equations for $\gamma\gamma(\eta/\eta')$ (see below for numerical estimates). This is in contrast with Kisselev and Petrov [21] who find a stronger breaking affecting both the anomalous triangle and box couplings of pseudoscalar mesons, keeping them structurally unchanged. Leutwyler on the other hand predicts a deep change in the structure of the triangle anomaly matrix element [22]. Feldman and Kroll [20] also break deeply the structure of the triangle anomaly.

Examining the full effect of the BKY symmetry breaking schemes requires a refitting of all data on the box anomaly in order to stay consistent with the HLS model and accurately test all its assumptions in the anomalous sector [5]. This work, which goes far beyond the aim of this paper, is presently under way [33].

Equation (39) clearly shows that the change of fields required by the BKY breaking mechanism does not exhaust the expected symmetry breaking effects as this would imply $f_0 = f_8 = f_\pi$. Therefore higher order effects have to be accounted for when using Eq. (39); they can formally be described by changing appropriately a factor of f_P to f_π, f_8 and f_0 depending on the (renormalized) field it multiplies (π_R^0, π_R^8, η_R^0), while the two remaining powers of f_P have to be changed to f_π. Consistency then implies the corresponding changes in the relations for the triangle anomaly couplings (41) and (42).

Then, the couplings occurring in the $\gamma\gamma P$ sector are to be affected by weighting factors, relative to the unbroken case, which result in

$$\frac{1}{f_\pi} \to \frac{1}{f_\pi}, \quad \frac{1}{f_8} \to \frac{1.22}{f_8}, \quad \frac{1}{f_0} \to \frac{0.94}{f_0}, \quad \text{BKY, new scheme}$$

$$\frac{1}{f_\pi} \to \frac{1}{f_\pi}, \quad \frac{1}{f_8} \to \frac{1.19}{f_8}, \quad \frac{1}{f_0} \to \frac{0.72}{f_0}, \quad \text{BGP} \quad (43)$$

using $c_A = 0.49$.

Using Eqs. (39) and (41) [or (42)], one can easily write down the matrix elements for $\gamma\pi^+\pi^-(\pi^0/\eta/\eta')$ and $\gamma\gamma(\pi^0/\eta/\eta')$ after introducing the physical η and η' fields through [13,17,19,27]

$$\eta \equiv \cos\theta\,\pi_R^8 - \sin\theta\,\eta_R^0, \quad \eta' \equiv \sin\theta\,\pi_R^8 + \cos\theta\,\eta_R^0, \quad (44)$$

which leaves the pseudoscalar kinetic energy term with its canonical form (no $\eta - \eta'$ mixing is introduced) for each variant of the BKY SU(3) breaking schemes. This breaks the anomaly set of equations in an original way, i.e. the box anomaly equations (involving $\gamma\pi^+\pi^-P^0$) are strictly unchanged, while triangle anomaly couplings undergo modest symmetry breaking effects under realistic conditions. This is indeed obtained by including the s quark symmetry breaking while keeping the u and d degenerate. Of course, all anomalous terms within the HLS approach [5], undergo breaking by this field renormalization.

The full physics consequences of this breaking mechanism is under consideration [33].

V. CONCLUSION

We have shown that a few variants of the SU(3) breaking mechanism proposed by BKY [6], allow one to maintain current conservation (in the sense defined in the Introduction, namely that currents are *strictly* conserved for massless pseudoscalar mesons) in all sectors of the broken HLS Lagrangian. The original BKY breaking scheme, once Hermitized, achieves this symmetry breaking with a single parameter, since $c_A = a c_V$, but current conservation in the full isoscalar sector implies the unrealistic condition that $a = 1$, in contradiction with VMD ($a = 2$) and data ($a = 2.4$). It remains to find places where possible departures from the usual assumption of the current conservation (as we defined it) can be tested, and which arises only from symmetry breaking effects in the HLS Lagrangian.

However, the new scheme we propose, as well as the BGP scheme, maintain this current conservation. They mainly differ from each other by the mass relation among vector mesons once symmetry breaking has occurred. The broken Lagrangians in both cases depend on two independent breaking parameters c_A and c_V. Data from e^+e^- annihilations could allow one to fix them with a consideration of radiative decays of light mesons to check their consistency, and thus the relevance of the BKY symmetry breaking mechanism. Moreover, some work is still needed in order to see whether the mass relation for vector mesons (Gell-Mann–Okubo versus Bando-Kugo-Yamawaki) has to be considered conclusive.

We have also shown that the field ''renormalization'' (or transformation) following from the BKY symmetry breaking mechanism modifies in a very mild way the anomalous WZW Lagrangian terms. This contrasts with several other proposed mechanisms.

A full phenomenological study of this symmetry breaking mechanism may allow us to answer the question of the experimental relevance of the HLS Lagrangian. The present hint, relying on several studies, is optimistic.

ACKNOWLEDGMENTS

APPENDIX: LAGRANGIAN EXPRESSIONS

Here we present expressions for the full Lagrangians. For all Lagrangians, especially the SU(3) broken ones, expressions are given in terms of bare fields in the sense of the BKY [6] pseudoscalar field renormalization.

Defining ($P \overset{\leftrightarrow}{\partial} P' \equiv P \partial P' - P' \partial P$) the original (unbroken) HLS Lagrangian can be expanded as follows:

$$\mathcal{L}_{\text{HLS}} = \frac{2}{3}ae^2f_P^2A^2 + \frac{af_P^2g^2}{2}(\rho^2 + \omega^2 + \phi^2) + af_P^2g^2(\rho^+\rho^- + K^{*+}K^{*-} + \bar{K}^{*0}K^{*0})$$

$$- aef_P^2g\left[\rho + \frac{\omega}{3} - \frac{\sqrt{2}}{3}\phi\right]\cdot A + \frac{i}{2}[ag\rho + e(2-a)A]\cdot[\pi^-\overset{\leftrightarrow}{\partial}\pi^+]$$

$$+ \frac{i}{4}[ag(\rho + \omega - \sqrt{2}\phi) + 2e(2-a)A]\cdot[K^-\overset{\leftrightarrow}{\partial}K^+] + \frac{iag}{4}[\rho - \omega + \sqrt{2}\phi]\cdot[K^0\overset{\leftrightarrow}{\partial}\bar{K}^0]$$

$$+ \frac{iag}{2\sqrt{2}}\rho^+[K^-\overset{\leftrightarrow}{\partial}K^0 + \sqrt{2}\pi^0\overset{\leftrightarrow}{\partial}\pi^-] + \frac{iag}{2\sqrt{2}}\rho^-[\bar{K}^0\overset{\leftrightarrow}{\partial}K^+ + \sqrt{2}\pi^+\overset{\leftrightarrow}{\partial}\pi^0]$$

$$+ \frac{iag}{4}K^{*0}[\bar{K}^0\overset{\leftrightarrow}{\partial}\pi^0 + \sqrt{2}\pi^+\overset{\leftrightarrow}{\partial}K^- + \sqrt{3}\pi^8\overset{\leftrightarrow}{\partial}\bar{K}^0] + \frac{iag}{4}\bar{K}^{*0}[\pi^0\overset{\leftrightarrow}{\partial}K^0 + \sqrt{2}K^+\overset{\leftrightarrow}{\partial}\pi^- + \sqrt{3}K^0\overset{\leftrightarrow}{\partial}\pi^8]$$

$$+ \frac{iag}{4}K^{*-}[K^+\overset{\leftrightarrow}{\partial}\pi^0 + \sqrt{2}K^0\overset{\leftrightarrow}{\partial}\pi^+ + \sqrt{3}K^+\overset{\leftrightarrow}{\partial}\pi^8] + \frac{iag}{4}K^{*+}[\pi^0\overset{\leftrightarrow}{\partial}K^- + \sqrt{2}\pi^-\overset{\leftrightarrow}{\partial}\bar{K}^0 + \sqrt{3}\pi^8\overset{\leftrightarrow}{\partial}K^-]. \quad (A1)$$

The BKY Lagrangian, SU(3) broken as prescribed in Ref. [6] is, after Hermitization,

$$\mathcal{L}_{\text{BKY}}^{\text{Herm}} = (1+c_A)\partial K^- \partial K^+ + (1+c_A)\partial K^0 \partial \bar{K}^0 + \partial \pi^+ \partial \pi^- + (1/2)\partial \pi^0 \partial \pi^0$$

$$+ \frac{1}{2}\left(1 + \frac{4}{3}c_A + \frac{2}{3}c_A^2\right)\partial \pi^8 \partial \pi^8 + \frac{1}{2}\left(1 + \frac{2}{3}c_A + \frac{1}{3}c_A^2\right)\partial \eta_0 \partial \eta_0 - \frac{\sqrt{2}}{3}c_A(2+c_A)\partial \eta_0 \partial \pi^8$$

$$+ ae^2 f_P^2 A^2\left[\frac{2}{3} + \frac{2c_V}{9} + \frac{c_V^2}{9}\right] + ag^2 f_P^2(\rho^+ \rho^- + (1+c_V)(K^{*-}K^{*+} + \bar{K}^{*0}K^{*0}))$$

$$+ \frac{1}{2}ag^2 f_P^2(\rho^{02} + \omega^2 + (1+c_V)^2 \phi^2) - aegf^2 A\left[\rho^0 + \frac{1}{3}\omega - \frac{\sqrt{2}}{3}(1+c_V)^2 \phi\right]$$

$$+ \frac{ie}{3}A[(3(1-a/2) + ac_V/2 + (3c_A - c_A^2)/2)K^- \overleftrightarrow{\partial} K^+ + (ac_V + c_A^2/2)K^0 \overleftrightarrow{\partial} \bar{K}^0]$$

$$+ ie(1-a/2)A\pi^- \overleftrightarrow{\partial} \pi^+ + \frac{ig}{4}\rho^0[(a(1-c_V) + c_A)(K^- \overleftrightarrow{\partial} K^+ + K^0 \overleftrightarrow{\partial} \bar{K}^0) + 2a\pi^- \overleftrightarrow{\partial} \pi^+]$$

$$+ \frac{ig}{2\sqrt{2}}\rho^+[(a(1-c_V) + c_A)K^- \overleftrightarrow{\partial} K^0 + \sqrt{2}a\pi^0 \overleftrightarrow{\partial} \pi^-] - \frac{ig}{2\sqrt{2}}\rho^-[(a(1-c_V) + c_A)K^+ \overleftrightarrow{\partial} \bar{K}^0 + \sqrt{2}a\pi^0 \overleftrightarrow{\partial} \pi^+]$$

$$+ \frac{ig}{4}\omega(a(1-c_V) + c_A)[K^- \overleftrightarrow{\partial} K^+ - K^0 \overleftrightarrow{\partial} \bar{K}^0] - \frac{ig}{2\sqrt{2}}\phi(a(1+c_V) + c_A + c_A^2)[K^- \overleftrightarrow{\partial} K^+ - K^0 \overleftrightarrow{\partial} \bar{K}^0]$$

$$+ \frac{ig}{4}K^{*0}[(a+c_A)(\bar{K}^0 \partial \pi^0 - \sqrt{2}K^- \partial \pi^+) - a(1+c_V)(\pi^0 \partial \bar{K}^0 - \sqrt{2}\pi^+ \partial K^-)]$$

$$- \frac{ig}{4}\bar{K}^{*0}[(a+c_A)(K^0 \partial \pi^0 - \sqrt{2}K^+ \partial \pi^-) - a(1+c_V)(\pi^0 \partial K^0 - \sqrt{2}\pi^- \partial K^+)]$$

$$+ \frac{ig}{4}K^{*-}[(a+c_A)(K^+ \partial \pi^0 + \sqrt{2}K^0 \partial \pi^+) - a(1+c_V)(\pi^0 \partial K^+ + \sqrt{2}\pi^+ \partial K^0)]$$

$$- \frac{ig}{4}K^{*+}[(a+c_A)(K^- \partial \pi^0 + \sqrt{2}\bar{K}^0 \partial \pi^-) - a(1+c_V)(\pi^0 \partial K^- + \sqrt{2}\pi^- \partial \bar{K}^0)] + \mathcal{L}^{\text{int}}(K^*, K, \pi^8, \eta_0) \tag{A2}$$

where

$$\mathcal{L}^{\text{int}}(K^*, K, \pi^8, \eta_0) = igK^{*0}\left[\frac{1}{4\sqrt{3}}\{(3a + 4c_A - ac_V)\pi^8 \partial \bar{K}^0 - (3(a+c_A) + 2(c_A^2 - ac_V^2))\bar{K}^0 \partial \pi^8\}\right.$$

$$\left. + \frac{1}{2\sqrt{6}}\{(c_A^2 - ac_V^2)\bar{K}^0 \partial \eta_0 - 2(c_A - ac_V)\eta_0 \partial \bar{K}^0\}\right]$$

$$- ig\bar{K}^{*0}\left[\frac{1}{4\sqrt{3}}\{(3a + 4c_A - ac_V)\pi^8 \partial K^0 - (3(a+c_A) + 2(c_A^2 - ac_V^2))K^0 \partial \pi^8\}\right.$$

$$\left. + \frac{1}{2\sqrt{6}}\{(c_A^2 - ac_V^2)K^0 \partial \eta_0 - 2(c_A - ac_V)\eta_0 \partial K^0\}\right]$$

$$+ igK^{*-}\left[\frac{1}{4\sqrt{3}}\{(3(a+c_A) + 2(c_A^2 - ac_V^2))K^+ \partial \pi^8 - (3a + 4c_A - ac_V)\pi^8 \partial K^+\}\right.$$

$$\left. + \frac{1}{2\sqrt{6}}\{2(c_A - ac_V)\eta_0 \partial K^+ - (c_A^2 - ac_V^2)K^+ \partial \eta_0\}\right]$$

$$- igK^{*+}\left[\frac{1}{4\sqrt{3}}\{(3(a+c_A) + 2(c_A^2 - ac_V^2))K^- \partial \pi^8 - (3a + 4c_A - ac_V)\pi^8 \partial K^-\}\right.$$

$$\left. + \frac{1}{2\sqrt{6}}\{(2(c_A - ac_V)\eta_0 \partial K^- - (c_A^2 - ac_V^2))K^- \partial \eta_0\}\right]. \tag{A3}$$

The HLS Lagrangian broken following the BGP prescription of Ref. [8] is given by

$$\mathcal{L}^{\text{BGP}}=\frac{1}{2}\left[(\partial\pi^0)^2+\left(1+\frac{4c_A}{3}\right)(\partial\pi^8)^2+\left(1+\frac{2c_A}{3}\right)(\partial\eta_0)^2-\frac{4\sqrt{2}c_A}{3}\partial\pi^8\partial\eta_0\right]$$

$$+(1+c_A)\partial K^-\partial K^++(1+c_A)\partial K^0\partial\bar{K}^0+\partial\pi^+\partial\pi^-$$

$$+\frac{2ae^2f_P^2}{3}(1+c_V/3)A^2-aegf_P^2A\left[\rho^0+\frac{\omega}{3}-(1+2c_V)\frac{\sqrt{2}}{3}\phi\right]$$

$$+ieA\left[\left(1-\frac{a}{2}+c_A-\frac{ac_V}{3}\right)K^-\overset{\leftrightarrow}{\partial}K^++\frac{ac_V}{3}K^0\overset{\leftrightarrow}{\partial}\bar{K}^0+\left(1-\frac{a}{2}\right)\pi^-\overset{\leftrightarrow}{\partial}\pi^+\right]$$

$$+ag^2f_P^2\left[\frac{1}{2}(\rho^{0^2}+\omega^2+(1+2c_V)\phi^2)+\rho^+\rho^-+(1+c_V)(K^{*+}K^{*-}+\bar{K}^{*0}K^{*0})\right]$$

$$+\frac{iag}{4}[\rho^0(K^-\overset{\leftrightarrow}{\partial}K^++K^0\overset{\leftrightarrow}{\partial}\bar{K}^0+2\pi^-\overset{\leftrightarrow}{\partial}\pi^+)+\omega(K^-\overset{\leftrightarrow}{\partial}K^+-K^0\overset{\leftrightarrow}{\partial}\bar{K}^0)]$$

$$+\frac{iag}{2\sqrt{2}}[\rho^+(K^-\overset{\leftrightarrow}{\partial}K^0+\sqrt{2}\pi^0\overset{\leftrightarrow}{\partial}\pi^-)-\rho^-(K^+\overset{\leftrightarrow}{\partial}\bar{K}^0+\sqrt{2}\pi^0\overset{\leftrightarrow}{\partial}\pi^+)]+\frac{iag}{2\sqrt{2}}(1+2c_V)\phi[K^+\overset{\leftrightarrow}{\partial}K^-+K^0\overset{\leftrightarrow}{\partial}\bar{K}^0]$$

$$+\frac{iag(1+c_V)}{4}K^{*0}[\bar{K}^0\overset{\leftrightarrow}{\partial}\pi^0+\sqrt{2}\pi^+\overset{\leftrightarrow}{\partial}K^-+\sqrt{3}\pi^8\overset{\leftrightarrow}{\partial}\bar{K}^0]+\frac{iag(1+c_V)}{4}\bar{K}^{*0}[\pi^0\overset{\leftrightarrow}{\partial}K^0+\sqrt{2}K^+\overset{\leftrightarrow}{\partial}\pi^-+\sqrt{3}K^0\overset{\leftrightarrow}{\partial}\pi^8]$$

$$+\frac{iag(1+c_V)}{4}K^{*-}[K^+\overset{\leftrightarrow}{\partial}\pi^0+\sqrt{2}K^0\overset{\leftrightarrow}{\partial}\pi^++\sqrt{3}K^+\overset{\leftrightarrow}{\partial}\pi^8]+\frac{iag(1+c_V)}{4}K^{*+}[\pi^0\overset{\leftrightarrow}{\partial}K^-+\sqrt{2}\pi^-\overset{\leftrightarrow}{\partial}\bar{K}^0+\sqrt{3}\pi^8\overset{\leftrightarrow}{\partial}K^-].$$

$$(A4)$$

The HLS Lagrangian broken as proposed following our new scheme, Eq. (24), is given by

$$\mathcal{L}_{\text{new}}=(1+c_A)\partial K^-\partial K^++(1+c_A)\partial K^0\partial\bar{K}^0+\partial\pi^+\partial\pi^-+(1/2)\partial\pi^0\partial\pi^0$$

$$+\frac{1}{2}\left[\left(1+\frac{4}{3}c_A+\frac{2}{3}c_A^2\right)\partial\pi^8\partial\pi^8+\left(1+\frac{2}{3}c_A+\frac{1}{3}c_A^2\right)\partial\eta_0\partial\eta_0\right]-\frac{\sqrt{2c_A}}{3}(2+c_A)\partial\eta_0\partial\pi^8$$

$$+ae^2f_P^2A^2\left[\frac{2}{3}+\frac{2c_V}{9}+\frac{c_V^2}{9}\right]-aegf_P^2A\left[\rho^0+\frac{1}{3}\omega-(1+c_V)^2\frac{\sqrt{2}}{3}\phi\right]$$

$$+ieA\left[(1-a/2+c_A-ac_V(2+c_V)/6)K^-\overset{\leftrightarrow}{\partial}K^++\frac{ac_V(2+c_V)}{6}K^0\overset{\leftrightarrow}{\partial}\bar{K}^0+(1-a/2)\pi^-\overset{\leftrightarrow}{\partial}\pi^+\right]$$

$$+\frac{1}{2}af_P^2g^2[\rho^{0^2}+\omega^2+(1+c_V)^2\phi^2+2\rho^+\rho^-+2(1+c_V)(K^{*+}K^{*-}+\bar{K}^{*0}K^{*0})]$$

$$+\frac{iag}{4}[\rho^0(K^-\overset{\leftrightarrow}{\partial}K^++K^0\overset{\leftrightarrow}{\partial}\bar{K}^0+2\pi^-\overset{\leftrightarrow}{\partial}\pi^+)+\omega(K^-\overset{\leftrightarrow}{\partial}K^+-K^0\overset{\leftrightarrow}{\partial}\bar{K}^0)]$$

$$+\frac{iag}{2\sqrt{2}}[\rho^+(K^-\overset{\leftrightarrow}{\partial}K^0+\sqrt{2}\pi^0\overset{\leftrightarrow}{\partial}\pi^-)-\rho^-(K^+\overset{\leftrightarrow}{\partial}\bar{K}^0+\sqrt{2}\pi^0\overset{\leftrightarrow}{\partial}\pi^+)]+\frac{iag(1+c_V)^2}{2\sqrt{2}}\phi[K^+\overset{\leftrightarrow}{\partial}K^-+K^0\overset{\leftrightarrow}{\partial}\bar{K}^0]$$

$$+\frac{iag(1+c_V)}{4}K^{*0}[\bar{K}^0\overset{\leftrightarrow}{\partial}\pi^0+\sqrt{2}\pi^+\overset{\leftrightarrow}{\partial}K^-+\sqrt{3}\pi^8\overset{\leftrightarrow}{\partial}\bar{K}^0]+\frac{iag(1+c_V)}{4}\bar{K}^{*0}[\pi^0\overset{\leftrightarrow}{\partial}K^0+\sqrt{2}K^+\overset{\leftrightarrow}{\partial}\pi^-+\sqrt{3}K^0\overset{\leftrightarrow}{\partial}\pi^8]$$

$$+\frac{iag(1+c_V)}{4}K^{*-}[K^+\overset{\leftrightarrow}{\partial}\pi^0+\sqrt{2}K^0\overset{\leftrightarrow}{\partial}\pi^++\sqrt{3}K^+\overset{\leftrightarrow}{\partial}\pi^8]+\frac{iag(1+c_V)}{4}K^{*+}[\pi^0\overset{\leftrightarrow}{\partial}K^-+\sqrt{2}\pi^-\overset{\leftrightarrow}{\partial}\bar{K}^0+\sqrt{3}\pi^8\overset{\leftrightarrow}{\partial}K^-].$$

$$(A5)$$

[1] M. Benayoun *et al.*, Eur. Phys. J. C **2**, 269 (1998).

[2] M. Bando, T. Kugo, S. Uehara, K. Yamawaki, and T. Yanagida, Phys. Rev. Lett. **54**, 1215 (1985); M. Bando, T. Kugo, and K. Yamawaki, Phys. Rep. **164**, 217 (1985).

[3] C. D. Froggat and J. L. Petersen, Nucl. Phys. **B129**, 89 (1977).

[4] M. Knecht, B. Moussallam, J. Stern, and N. H. Fuchs, Nucl. Phys. **B457**, 51 (1995).

[5] T. Fujiwara, T. Kugo, H. Terao, S. Uehara, and K. Yamawaki, Prog. Theor. Phys. **73**, 926 (1985).

[6] M. Bando, T. Kugo, and K. Yamawaki, Nucl. Phys. **B259**, 493 (1985).

[7] O. Hajuj, Z. Phys. C **60**, 357 (1993).

[8] A. Bramon, A. Grau, and G. Pancheri, Phys. Lett. B **344**, 240 (1995).

[9] M. Hashimoto, Phys. Lett. B **381**, 465 (1996); Phys. Rev. D **54**, 5611 (1996).

[10] M. Harada and J. Schechter, Phys. Rev. D **54**, 3394 (1996).

[11] J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).

[12] E. Witten, Nucl. Phys. **B223**, 422 (1983).

[13] J. F. Donoghue, B. R. Holstein, and Y. C. R. Lin, Phys. Rev. Lett. **55**, 2766 (1985).

[14] M. Benayoun *et al.*, Z. Phys. C **58**, 31 (1993).

[15] M. Benayoun, P. Leruste, L. Montanet, and J. L. Narjoux, Z. Phys. C **65**, 399 (1995).

[16] M. Benayoun, S. I. Eidelman, and V. N. Ivanchenko, Z. Phys. C **72**, 221 (1996).

[17] E. P. Venugopal and B. R. Holstein, Phys. Rev. D **57**, 4397 (1998).

[18] M. A. Ivanov and T. Mizutani, hep-ph/9710514; A. Bramon, R. Escribano, and M. D. Scadron, hep-ph/9711229; L. Burakovsky and T. Goldman, hep-ph/9802404.

[19] Th. Feldmann, P. Kroll, and B. Stech, hep-ph/9802409.

[20] Th. Feldmann and P. Kroll, hep-ph/9711231.

[21] A. V. Kisselev and V. A. Petrov, hep-ph/9803411.

[22] H. Leutwyler, Nucl. Phys. B (Proc. Suppl.) **64**, 223 (1998).

[23] S. Weinberg, Phys. Rev. **166**, 1568 (1968); S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2239 (1969); C. Callan, S. Coleman, J. Wess, and B. Zumino, *ibid.* **177**, 2247 (1969).

[24] M. Knecht and E. de Rafael, Phys. Lett. B **424**, 335 (1998).

[25] J. H. Schwarz and N. Seiberg, hep-th/9803179.

[26] H. B. O'Connell, B. C. Pearce, A. W. Thomas, and A. G. Williams, Prog. Part. Nucl. Phys. **39**, 201 (1997).

[27] R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[28] F. Klingl, N. Kaiser, and W. Weise, Z. Phys. A **356**, 193 (1996).

[29] P. Maris, C. D. Roberts, and S. Schmidt, Phys. Rev. C **57**, R2821 (1998); A. Bender, C. D. Roberts, and L. v. Smekal, Phys. Lett. B **380**, 7 (1996); K. L. Mitchell and P. C. Tandy, Phys. Rev. C **55**, 1477 (1997); L. C. L. Hollenberg, C. D. Roberts, and B. H. J. McKellar, *ibid.* **46**, 2057 (1992).

[30] S. Gardner and H. B. O'Connell, Phys. Rev. D **57**, 2716 (1998).

[31] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

[32] H. Georgi, Nucl. Phys. **B331**, 311 (1990).

[33] M. Benayoun *et al.* (in preparation).

[34] U. G. Meissner, Phys. Rep. **161**, 213 (1988).

[35] J. F. Donoghue and D. Wyler, Nucl. Phys. **B316**, 289 (1988).