

# Effective $CP$ -violating operators of the tau lepton and some of their phenomenologies

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The dimension-six  $CP$ -violating  $SU_L(2) \times U_Y(1)$  invariant operators involving the tau lepton are studied. The constraints from the available experimental data on tau dipole moments are derived. Under the current constraints, the induced  $CP$ -violating effects could possibly be observed in  $\tau \rightarrow 3\pi\nu_\tau$  at the future tau-charm factory. [S0556-2821(98)06017-2]

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## I. INTRODUCTION

The tau lepton is possibly a special probe of new physics in the leptonic sector due to the fact that it is the only lepton which is heavy enough to have hadronic decays and, as naively expected, heavier fermions are more sensitive to the new physics related to mass generations. In searching for the possible new physics associated with the tau lepton,  $CP$  violation is a particularly interesting probe. In the standard model (SM), the existence of a phase in the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [1] allows  $CP$  violation in the quark sector but not in the lepton sector. The origin of  $CP$  violation remains a mystery more than three decades after its discovery. In models beyond the SM, additional  $CP$  violation can appear rather naturally and non-CKM-type  $CP$  violation is necessary in order to account for the observed value of the baryon density to entropy ratio [2]. Therefore, the detection of any non-CKM-type  $CP$  violations, such as  $CP$  violating lepton interaction, will be an unequivocal signal of new physics, and may help illuminating the origin of  $CP$  violating and alleviating the difficulty in baryogenesis.

Experimental studies to date for  $CP$  violation in tau processes have been in production and not in decay processes. Since the bounds are relatively weak, there are plenty rooms for  $CP$ -violating new physics to be discovered in the tau sector. With the tau-charm factory, which will operate at an  $e^+e^-$  center-of-mass energy of around 4 GeV and a luminosity of  $L=10^{33} \text{ cm}^{-2} \text{ s}^{-1}$  with good  $\pi/K$  separation, tau properties will be measured to a very high precision. This will allow the tests of the SM and provide some information about new physics.

$CP$  violations in tau lepton decays have been investigated in Refs. [3–7]. Generally, the possible  $CP$  violating effects are expected to be larger and easier for detection in semileptonic tau decays than in production processes [8]. Analyses of new physics models yielding  $CP$ -violating effects in tau decays have been given in Ref. [6], where the  $CP$ -violating effects of multi-Higgs-doublet models are found to be possibly observable at the tau-charm factory. More recently, a systematical analysis for the potential of the tau-charm fac-

tory in probing the  $CP$  violation of the tau sector has been given in [9].

In this article, we give a model-independent study for the possible  $CP$ -violating effects associated with tau lepton by the use of the effective Lagrangian approach. The use of the effective Lagrangian approach in describing new physics is well motivated. The fact that no direct signal of new particles has been observed from collider experiments and the impressive success of the SM requires that the new physics preserves the SM structure around the SM energy scale and only very delicately improves it [10]. So it is likely that the only observable effects of new physics at energies not too far above the SM energy scale could be in the form of anomalous interactions which slightly affect the couplings of the SM particles. In this spirit, the residue effects of new physics can be expressed as non-standard terms in an effective Lagrangian with a form such as

$$\mathcal{L}_{eff} = \mathcal{L}_0 + \frac{1}{\Lambda^2} \sum_i C_i O_i + \mathcal{O}\left(\frac{1}{\Lambda^4}\right), \quad (1)$$

where  $\mathcal{L}_0$  is the SM Lagrangian,  $\Lambda$  is the new physics scale and  $O_i$  are  $CP$ -conserving or  $CP$ -violating  $SU_c(3) \times SU_L(2) \times U_Y(1)$  invariant dimension-six operators, and  $C_i$  are constants which represent the coupling strengths of  $O_i$ . The expansion in Eq. (1) was first discussed in Ref. [11] and further investigated in [12,13,14,15]. In this article, we focus on the  $CP$ -violating operators involving the tau lepton.

In Sec. II we list the possible dimension-six  $CP$ -violating  $SU_L(2) \times U_Y(1)$  invariant operators involving the tau lepton and give their expressions after electroweak gauge symmetry breaking. In Sec. III we give the induced  $CP$ -violating effective couplings  $W\nu\tau$ ,  $Z\tau\tau$ , and  $\gamma\tau\tau$  and classify the operators according to the interaction vertices. In Sec. IV we derive the bounds for the coupling strength from the available experimental data in tau dipole moments. In Sec. V we evaluate the possibility of observing the  $CP$ -violating effects of these operators in  $\tau \rightarrow 3\pi\nu_\tau$  at a future tau-charm factory. And finally in Sec. VI we present the summary.

## II. CP-VIOLATING OPERATORS INVOLVING THE TAU LEPTON

Here we assume that the new physics in the lepton sector resides in the interaction of third family to gauge bosons or Higgs boson. Therefore, the operators we are interested in are those containing third-family leptons coupling to gauge or Higgs bosons.

To restrict ourselves to the lowest order, we consider only tree diagrams and to the order of  $1/\Lambda^2$ . Therefore, only one vertex in a given diagram can contain anomalous couplings. Under these conditions, operators which are related by the field equations are not independent. As discussed in Ref. [14], to which we refer for the detail, the fermion and the Higgs boson equations of motion can be used but the equations of motion of the gauge bosons cannot when writing down the operators in Eq. (1). Also, we assume all the operators  $O_i$  to be Hermitian. Because of our assumption that the available energies are below the unitarity cuts of new-physics particles, no imaginary part can be generated by the new physics effect. Therefore the coefficients  $C_i$  in Eq. (1) are real.

The expressions of the CP-violating operators involving the third family leptons are parallel to their corresponding ones involving the third family quarks given in Ref. [15], but the number of independent operators is much less due to the absence of right-handed neutrino and the strong interactions. We follow the standard notation:  $L$  denotes the third family left-handed doublet leptons,  $\Phi$  is the Higgs doublet,  $W_{\mu\nu}$  and  $B_{\mu\nu}$  are the SU(2) and U(1) gauge boson field tensors in the appropriate matrix forms, and  $D_\mu$  denotes the appropriate covariant derivatives. For more details of the notation we refer to Ref. [14]. The possible operators are given by

$$O_{LW} = i[\bar{L}\gamma^\mu\tau^I D^\nu L - \overline{D^\nu L}\gamma^\mu\tau^I L]W_{\mu\nu}^I, \quad (2)$$

$$O_{LB} = i[\bar{L}\gamma^\mu D^\nu L - \overline{D^\nu L}\gamma^\mu L]B_{\mu\nu}, \quad (3)$$

$$O_{\tau B} = i[\bar{\tau}_R\gamma^\mu D^\nu \tau_R - \overline{D^\nu \tau_R}\gamma^\mu \tau_R]B_{\mu\nu}, \quad (4)$$

$$O_{\Phi L}^{(1)} = [\Phi^\dagger D_\mu \Phi + (D_\mu \Phi)^\dagger \Phi]\bar{L}\gamma^\mu L, \quad (5)$$

$$O_{\Phi L}^{(3)} = [\Phi^\dagger \tau^I D_\mu \Phi + (D_\mu \Phi)^\dagger \tau^I \Phi]\bar{L}\gamma^\mu \tau^I L, \quad (6)$$

$$O_{\Phi\tau} = [\Phi^\dagger D_\mu \Phi + (D_\mu \Phi)^\dagger \Phi]\bar{\tau}_R\gamma^\mu \tau_R, \quad (7)$$

$$O_{\tau 1} = i\left(\Phi^\dagger \Phi - \frac{v^2}{2}\right)[\bar{L}\tau_R \Phi - \Phi^\dagger \bar{\tau}_R L], \quad (8)$$

$$O_{D\tau} = i[(\bar{L}D_\mu \tau_R)D^\mu \Phi - (D^\mu \Phi)^\dagger (\overline{D_\mu \tau_R L})], \quad (9)$$

$$O_{\tau W\Phi} = i[(\bar{L}\sigma^{\mu\nu}\tau^I \tau_R)\Phi - \Phi^\dagger (\bar{\tau}_R \sigma^{\mu\nu}\tau^I L)]W_{\mu\nu}^I, \quad (10)$$

$$O_{\tau B\Phi} = i[(\bar{L}\sigma^{\mu\nu}\tau_R)\Phi - \Phi^\dagger (\bar{\tau}_R \sigma^{\mu\nu}L)]B_{\mu\nu}. \quad (11)$$

The expressions of these CP-violating operators after electroweak symmetry breaking in the unitary gauge are given by

$$\begin{aligned} O_{LW} = & \frac{i}{2}W_{\mu\nu}^3[\bar{\nu}_\tau\gamma^\mu P_L\partial^\nu\nu_\tau - \partial^\nu\bar{\nu}_\tau\gamma^\mu P_L\nu_\tau \\ & - \bar{\tau}\gamma^\mu P_L\partial^\nu\tau + \partial^\nu\bar{\tau}\gamma^\mu P_L\tau] \\ & + \frac{i}{\sqrt{2}}[W_{\mu\nu}^+( \bar{\nu}_\tau\gamma^\mu P_L\partial^\nu\tau - \partial^\nu\bar{\nu}_\tau\gamma^\mu P_L\tau) \\ & + W_{\mu\nu}^-(\bar{\tau}\gamma^\mu P_L\partial^\nu\nu_\tau - \partial^\nu\bar{\tau}\gamma^\mu P_L\nu_\tau)] \\ & + g_2\bar{L}\gamma^\mu[W_\mu, W_\nu]\partial^\nu L - g_2\partial^\nu\bar{L}\gamma^\mu[W_\mu, W_\nu]L \\ & + \frac{1}{2}g_2(\vec{W}_{\mu\nu}\cdot\vec{W}^\nu)\bar{L}\gamma^\mu L - g_1B^\nu\bar{L}\gamma^\mu W_{\mu\nu}L, \end{aligned} \quad (12)$$

$$\begin{aligned} O_{LB} = & iB_{\mu\nu}\left[\bar{L}\gamma^\mu\partial^\nu L - \partial^\nu\bar{L}\gamma^\mu L - 2i\bar{L}\gamma^\mu\right. \\ & \left.\times\left(g_2W^\nu - \frac{1}{2}g_1B^\nu\right)L\right], \end{aligned} \quad (13)$$

$$O_{\tau B} = i[\bar{\tau}_R\gamma^\mu\partial^\nu\tau_R - \partial^\nu\bar{\tau}_R\gamma^\mu\tau_R]B_{\mu\nu} - 2g_1\bar{\tau}_R\gamma^\mu\tau_R B_{\mu\nu}B^\nu, \quad (14)$$

$$O_{\Phi L}^{(1)} = (H+v)\partial_\mu H[\bar{\nu}_\tau\gamma^\mu P_L\nu_\tau + \bar{\tau}\gamma^\mu P_L\tau], \quad (15)$$

$$\begin{aligned} O_{\Phi L}^{(3)} = & -O_{\Phi L}^{(1)} + 2(H+v)\partial_\mu H\bar{\tau}\gamma^\mu P_L\tau \\ & - \frac{ig_2}{\sqrt{2}}(H+v)^2(W_\mu^+\bar{\nu}_\tau\gamma^\mu P_L\tau - W_\mu^-\bar{\tau}\gamma^\mu P_L\nu_\tau), \end{aligned} \quad (16)$$

$$O_{\Phi\tau} = (H+v)\partial_\mu H\bar{\tau}\gamma^\mu P_R\tau, \quad (17)$$

$$O_{\tau 1} = \frac{1}{2\sqrt{2}}H(H+v)(H+2v)\bar{\tau}i\gamma_5\tau, \quad (18)$$

$$\begin{aligned} O_{D\tau} = & i\frac{1}{2\sqrt{2}}\partial^\mu H[\bar{\tau}\partial_\mu\tau - (\partial_\mu\bar{\tau})\tau + \partial_\mu(\bar{\tau}\gamma_5\tau) + i2g_1B_\mu\bar{\tau}\tau] \\ & + \frac{1}{2\sqrt{2}}\frac{m_Z}{v}(H+v)Z^\mu[\partial_\mu(\bar{\tau}\tau) + \bar{\tau}\gamma_5\partial_\mu\tau \\ & - (\partial_\mu\bar{\tau})\gamma_5\tau + 2g_1B_\mu(\bar{\tau}i\gamma_5\tau)] + \frac{g_2}{2}(H+v) \\ & \times[W_\mu^+(\bar{\nu}_\tau P_R\partial_\mu\tau + ig_1B_\mu\bar{\nu}_\tau P_R\tau) \\ & + W_\mu^-(\partial_\mu\bar{\tau}P_L\nu_\tau - ig_1B_\mu\bar{\tau}P_L\nu_\tau)], \end{aligned} \quad (19)$$

TABLE I. The contribution status of dimension-six  $CP$ -violating operators to the tau couplings. The contribution of a  $CP$ -violating operator to a particular vertex is marked by  $\times$ .

	$W\nu\tau$	$Z\tau\tau$	$\gamma\tau\tau$	$H\tau\tau$
$O_{\Phi L}^{(3)}$	$\times$			$\times$
$O_{LW}$	$\times$	$\times$	$\times$	
$O_{D\tau}$	$\times$	$\times$		$\times$
$O_{\tau W\Phi}$	$\times$	$\times$	$\times$	
$O_{LB}$		$\times$	$\times$	
$O_{\tau B}$		$\times$	$\times$	
$O_{\tau B\Phi}$		$\times$	$\times$	
$O_{\Phi L}^{(1)}$				$\times$
$O_{\Phi\tau}$				$\times$
$O_{\tau 1}$				$\times$

$$\begin{aligned}
O_{\tau W\Phi} = & i \frac{1}{2} (H + v) \left[ W_{\mu\nu}^+ (\bar{\nu}_\tau \sigma^{\mu\nu} P_R \tau) - W_{\mu\nu}^- (\bar{\tau} \sigma^{\mu\nu} P_L \nu_\tau) \right. \\
& - \frac{1}{\sqrt{2}} W_{\mu\nu}^3 (\bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau) + i g_2 (W_\mu^+ W_\nu^3 - W_\mu^3 W_\nu^+) \\
& \times (\bar{\nu}_\tau \sigma^{\mu\nu} P_R \tau) + i g_2 (W_\mu^- W_\nu^3 - W_\mu^3 W_\nu^-) (\bar{\tau} \sigma^{\mu\nu} P_L \nu_\tau) \\
& \left. + i \frac{g_2}{\sqrt{2}} (W_\mu^+ W_\nu^- - W_\mu^- W_\nu^+) (\bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau) \right], \quad (20)
\end{aligned}$$

$$O_{\tau B\Phi} = \frac{i}{\sqrt{2}} (H + v) B_{\mu\nu} (\bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau), \quad (21)$$

where we use the convention  $Z_\mu = -\cos \theta_W W_\mu^3 + \sin \theta_W B_\mu$  and  $A_\mu = \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu$ . Note that most of the above operators clearly show the  $U_{em}(1)$  gauge invariance, while some of them do not manifest this invariance straightforwardly. We have checked that all the operators listed above give indeed a  $U_{em}(1)$  gauge invariant expression.

### III. EFFECTIVE VERTICES FOR THE GAUGE COUPLINGS OF TAU

The possibilities of contributions of the dimension-six  $CP$ -violating operators to some three-particle couplings are shown in Table I. According to their contribution to the three-particle vertices of charged and neutral current, we classify the operators as follows: class A,  $O_{\Phi L}^{(3)}$ , contributing to charged current; class B,  $O_{LW}$ ,  $O_{D\tau}$ , and  $O_{\tau W\Phi}$ , contributing to both charged and neutral currents; class C,  $O_{LB}$ ,  $O_{\tau B}$ , and  $O_{\tau B\Phi}$ , contributing to neutral currents; class D,  $O_{\Phi L}^{(1)}$ ,  $O_{\Phi\tau}$ , and  $O_{\tau 1}$ , no contribution to charged and neutral currents.

Since class D operators contribute only to the  $H\tau\tau$  coupling, they may not be probed at future colliders. We will not consider these operators here further. Both class B and class C operators affect neutral currents of the tau, and, as our analysis show, they will be strongly constrained by data from

the CERN  $e^+e^-$  collider LEP for the dipole moments of the tau. Since the constraints for the charged current of the tau from its leptonic decays are much weaker (see below), class A operator will not be strongly constrained and, as a result, will provide the best possibility for the observation of  $CP$ -violating effects in hadronic tau decays.

Collecting all the relevant terms we get the effective  $CP$ -violating couplings

$$\begin{aligned}
\mathcal{L}_{W\nu\tau} = & -i \frac{C_{\Phi L}^{(3)}}{\Lambda^2} \frac{g_2}{\sqrt{2}} v^2 W_\mu^+ (\bar{\nu}_\tau \gamma^\mu P_L \tau) \\
& -i \frac{C_{D\tau}}{\Lambda^2} \frac{v}{\sqrt{2}} \frac{g_2}{\sqrt{2}} W_\mu^+ \bar{\nu}_\tau P_R (i \partial^\mu \tau) \\
& +i \frac{C_{\tau W\Phi}}{\Lambda^2} \frac{v}{2} W_{\mu\nu}^+ (\bar{\nu}_\tau \sigma^{\mu\nu} P_R \tau) \\
& +i \frac{C_{LW}}{\Lambda^2} \frac{1}{\sqrt{2}} W_{\mu\nu}^+ [\bar{\nu}_\tau \gamma^\mu P_L (\partial^\nu \tau) - (\partial^\nu \bar{\nu}_\tau) \gamma^\mu P_L \tau], \quad (22)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{Z\tau\tau} = & i \left( \frac{C_{\tau W\Phi}}{\Lambda^2} \frac{c_W v}{2\sqrt{2}} + \frac{C_{\tau B\Phi}}{\Lambda^2} \frac{v}{\sqrt{2}} s_W \right) Z_{\mu\nu} (\bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau) \\
& +i \left( \frac{C_{LW}}{\Lambda^2} \frac{c_W}{2} + \frac{C_{LB}}{\Lambda^2} s_W \right) \\
& \times Z_{\mu\nu} (\bar{\tau} \gamma^\mu P_L \partial^\nu \tau - \partial^\nu \bar{\tau} \gamma^\mu P_L \tau) \\
& +i \frac{C_{\tau B}}{\Lambda^2} s_W Z_{\mu\nu} (\bar{\tau} \gamma^\mu P_R \partial^\nu \tau - \partial^\nu \bar{\tau} \gamma^\mu P_R \tau) \\
& -i \frac{m_Z}{2\sqrt{2}} \frac{C_{D\tau}}{\Lambda^2} Z^\mu [i (\bar{\tau} \gamma_5 \partial_\mu \tau - \partial_\mu \bar{\tau} \gamma_5 \tau) + i \partial_\mu (\bar{\tau} \tau)], \quad (23)
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{\gamma\tau\tau} = & i \left( \frac{C_{LB}}{\Lambda^2} c_W - \frac{C_{LW}}{\Lambda^2} \frac{s_W}{2} \right) A_{\mu\nu} (\bar{\tau} \gamma^\mu P_L \partial^\nu \tau - \partial^\nu \bar{\tau} \gamma^\mu P_L \tau) \\
& +i \frac{C_{\tau B}}{\Lambda^2} c_W A_{\mu\nu} (\bar{\tau} \gamma^\mu P_R \partial^\nu \tau - \partial^\nu \bar{\tau} \gamma^\mu P_R \tau) \\
& +i \left( \frac{C_{\tau B\Phi}}{\Lambda^2} c_W - \frac{C_{\tau W\Phi}}{\Lambda^2} \frac{s_W}{2} \right) \frac{v}{\sqrt{2}} A_{\mu\nu} (\bar{\tau} \sigma^{\mu\nu} \gamma_5 \tau), \quad (24)
\end{aligned}$$

where  $s_W \equiv \sin \theta_W$ ,  $c_W \equiv \cos \theta_W$ , and  $P_{L,R} \equiv (1 \mp \gamma_5)/2$ .

### IV. CURRENT CONSTRAINTS FROM EXPERIMENTAL DATA

#### A. Constraints from the measurement of the dipole moments

Including both the SM couplings and  $CP$ -violating new physics effects, we can write the  $V\tau\tau$  ( $V=Z, \gamma$ ) vertices, with both taus being on-shell, as

$$\Gamma_{V\tau\tau}^\mu = ie \left[ \gamma_\mu A_V - \gamma_\mu \gamma_5 B_V + \frac{C_V}{2m_\tau} k_\nu \sigma^{\mu\nu} \gamma_5 \right], \quad (25)$$

where  $k$  is the momentum of the vector boson. We have neglected the scalar and pseudoscalar couplings,  $k_\mu$  and  $k_\mu \gamma_5$ , since these terms give contributions proportional to the electron mass in  $e^+ e^- \rightarrow \tau^- \tau^+$ . We note that some of these neglected terms are needed to maintain the electromagnetic gauge invariance for the axial vector couplings in Eq. (25).  $A_V$  and  $B_V$  are the SM couplings. At the tree level, they are given by  $A_{Z,\gamma} = (-1 + 4s_W^2/4s_W c_W)$ ,  $-1$  and  $B_{Z,\gamma} = (-1/4s_W c_W)$ ,  $0$ .  $C_V$  arises from new physics given by

$$C_Z = -\frac{C_{LW}}{\Lambda^2} \frac{m_\tau^2}{e} c_W + \frac{C_{D\tau}}{\Lambda^2} \frac{m_\tau m_Z}{\sqrt{2}e} + \frac{C_{\tau W\Phi}}{\Lambda^2} \frac{vm_\tau}{e} \sqrt{2} c_W - \frac{C_{LB} - C_{\tau B}}{\Lambda^2} \frac{m_\tau^2}{e} 2s_W + \frac{C_{\tau B\Phi}}{\Lambda^2} \frac{vm_\tau}{e} 2\sqrt{2}s_W, \quad (26)$$

$$C_\gamma = -\frac{C_{LW}}{\Lambda^2} \frac{m_\tau^2}{e} s_W + \frac{C_{\tau W\Phi}}{\Lambda^2} \frac{vm_\tau}{e} \sqrt{2}s_W + \frac{C_{LB} - C_{\tau B}}{\Lambda^2} \frac{m_\tau^2}{e} 2c_W - \frac{C_{\tau B\Phi}}{\Lambda^2} \frac{vm_\tau}{e} 2\sqrt{2}c_W. \quad (27)$$

The electric and weak dipole moments are obtained by

$$d_\tau^{\gamma,Z} = \frac{e}{2m_\tau} C_{\gamma,Z} = 0.55 \times 10^{-14} C_{\gamma,Z} \quad (e \text{ cm}). \quad (28)$$

The  $CP$  violation introduced by the dipole moments can be searched in  $Z \rightarrow \tau^+ \tau^-$ . The dipole moments can be determined from the tau spin which can be measured from the tau decay products. No evidence of  $CP$  violation has been observed in  $Z \rightarrow \tau^+ \tau^-$  so far, which set strong limits on the dipole moments.

The limit on the weak dipole moment of tau lepton obtained at LEP is [16]

$$|\text{Re } d_\tau^Z| \leq 3.6 \times 10^{-18} \text{ e cm} \quad (95\% \text{ C.L.}). \quad (29)$$

Assuming the simple situation that cancellation among different operators does not take place, we get the bounds on the coupling strength:

$$\left| \frac{C_{LW}}{\Lambda^2} \right| < 6.8 \times 10^{-5} \text{ GeV}^{-2}, \quad (30)$$

$$\left| \frac{C_{D\tau}}{\Lambda^2} \right| < 1.7 \times 10^{-6} \text{ GeV}^{-2}, \quad (31)$$

$$\left| \frac{C_{\tau W\Phi}}{\Lambda^2} \right| < 3.5 \times 10^{-7} \text{ GeV}^{-2}, \quad (32)$$

$$\left| \frac{C_{LB}}{\Lambda^2} \right|, \left| \frac{C_{\tau B}}{\Lambda^2} \right| < 6.3 \times 10^{-5} \text{ GeV}^{-2}, \quad (33)$$

$$\left| \frac{C_{\tau B\Phi}}{\Lambda^2} \right| < 3.2 \times 10^{-7} \text{ GeV}^{-2}. \quad (34)$$

Compared with the constraints on the weak dipole moment, those on the electric dipole moment of tau lepton are weaker. The strongest constraint on  $d_\tau^\gamma$  has been derived from the  $Z \rightarrow \tau^+ \tau^-$  decay width, which is given by [17]

$$|d_\tau^\gamma| < 2.7 \times 10^{-17} \text{ e cm} \quad (95\% \text{ C.L.}), \quad (35)$$

which yield the following bounds on the coupling strength under the assumption that cancellation among different operators does not take place:

$$\left| \frac{C_{LW}}{\Lambda^2} \right| < 9.4 \times 10^{-4} \text{ GeV}^{-2}, \quad (36)$$

$$\left| \frac{C_{\tau W\Phi}}{\Lambda^2} \right| < 4.9 \times 10^{-6} \text{ GeV}^{-2}, \quad (37)$$

$$\left| \frac{C_{LB}}{\Lambda^2} \right|, \left| \frac{C_{\tau B}}{\Lambda^2} \right| < 2.6 \times 10^{-4} \text{ GeV}^{-2}, \quad (38)$$

$$\left| \frac{C_{\tau B\Phi}}{\Lambda^2} \right| < 1.3 \times 10^{-6} \text{ GeV}^{-2}. \quad (39)$$

## B. Constraints from the measurement of leptonic decays

The  $CP$ -violating contribution to the  $W\nu\tau$  vertex in the tau decay can be written in the momentum space as

$$\mathcal{L}_{W\nu\tau} = \frac{g_2}{\sqrt{2}} W_\mu^+ \bar{\nu}_\tau \left[ \gamma_\mu P_L (1 + ia) + k_\mu P_R \frac{1}{2m_\tau} (ib) + \frac{i}{2m_\tau} k^\nu \sigma_{\mu\nu} P_R (ic) \right] \tau, \quad (40)$$

where the form factors are given by

$$a = -\frac{C_{\Phi L}^{(3)}}{\Lambda^2} v^2 - \frac{C_{D\tau}}{\Lambda^2} \frac{vm_\tau}{2\sqrt{2}}, \quad (41)$$

$$b = -\frac{C_{D\tau}}{\Lambda^2} \frac{vm_\tau}{\sqrt{2}}, \quad (42)$$

$$c = \frac{C_{LW}}{\Lambda^2} \frac{2m_\tau^2}{g_2} - \frac{C_{D\tau}}{\Lambda^2} \frac{vm_\tau}{\sqrt{2}} - \frac{C_{\tau W\Phi}}{\Lambda^2} \frac{2\sqrt{2}vm_\tau}{g_2}. \quad (43)$$

The  $k_\mu$  term is negligible in leptonic tau decays. The theoretic prediction for branching fractions of the decay  $\tau^- \rightarrow l^- \bar{\nu}_l \nu_\tau$  ( $l = e^-, \mu^-$ ) are given by [18,19,20]

$$B_l = \frac{G_F^2 m_\tau^5}{192 \pi^3} \tau_\tau (1 - 8x - 12x^2 \ln x + 8x^3 - x^4) \times \left[ \left( 1 - \frac{\alpha(m_\tau)}{2\pi} \left( \pi^2 - \frac{25}{4} \right) \right) \left( 1 + \frac{3}{5} \frac{m_\tau^2}{m_W^2} - 2 \frac{m_l^2}{m_W^2} \right) \right] \times (1 + \Delta_l), \quad (44)$$

where  $\tau_\tau$  is the tau lifetime,  $x = m_l^2/m_\tau^2$  and  $\Delta_l = \tilde{\kappa}^2/10$  with  $\tilde{\kappa} = \sqrt{c^2 + 10a^2}$ . The world average values for  $B_l$  [21] constraint  $|\tilde{\kappa}| < 0.26$  at 95% C.L. [20]. Again assuming the simple situation that cancellation among different operators does not take place, this yields the upper bounds on coupling strengths of the operators

$$\left| \frac{C_{\Phi L}^{(3)}}{\Lambda^2} \right| < 1.4 \times 10^{-6} \text{ GeV}^{-2}, \quad (45)$$

$$\left| \frac{C_{LW}}{\Lambda^2} \right| < 2.4 \times 10^{-2} \text{ GeV}^{-2}, \quad (46)$$

$$\left| \frac{C_{D\tau}}{\Lambda^2} \right| < 4.4 \times 10^{-4} \text{ GeV}^{-2}, \quad (47)$$

$$\left| \frac{C_{\tau W\Phi}}{\Lambda^2} \right| < 1.2 \times 10^{-4} \text{ GeV}^{-2}. \quad (48)$$

These bounds on  $C_{LW}$ ,  $C_{D\tau}$ , and  $C_{\tau W\Phi}$  derived from charged current are much weaker than those derived from neutral currents given in Eqs. (30)–(34).

We summarize that the strongest bounds presently available on the seven operators in classes A, B, and C are given in Eqs. (30)–(34) and (45).

## V. $CP$ -VIOLATING EFFECTS IN $\tau \rightarrow 3\pi\nu_\tau$

As pointed out in Sec. I, there are various methods in searching for the  $CP$  violations of the tau lepton. Here we focus on the three-charged-pion decay  $\tau^\pm \rightarrow \pi^\pm + \pi^\mp + \pi^\pm + \nu_\tau$ , which has been argued to be a promising process for detecting  $CP$  violation [6,7]. This decay is dominated by the contributions of two overlapping resonances,  $a_1(1260)$  and  $\pi'(1300)$ , and has been extensively studied [22,23]. In our analyses we follow Ref. [23] for the phenomenological parameterization of the form factors.

Including the contributions of possible new physics, the matrix element for the parton-level process  $\tau^-(p, \sigma) \rightarrow \bar{u} + d + \nu_\tau(k, -)$ , where the momenta and helicities for  $\tau$  and  $\nu_\tau$  are indicated, is given by

$$M = \sqrt{2} G_F \left[ (1 + \chi) \bar{u}(k, -) \gamma_\mu P_L u(p, \sigma) \bar{d} \gamma^\mu (1 - \gamma_5) u + \eta \bar{u}(k, -) P_R u(p, \sigma) \bar{d} (1 + \gamma_5) u + \zeta \bar{u}(k, -) \frac{P_\mu}{m_\tau} P_R u(p, \sigma) \bar{d} \gamma^\mu (1 - \gamma_5) u \right]. \quad (49)$$

The form factors  $\chi$ ,  $\eta$ , and  $\zeta$ , which are from new physics, are given by

$$\chi = i(T_1 + T_2 + T_3), \quad (50)$$

$$\eta = -i \frac{m_u + m_d}{m_\tau} (T_2 + T_3), \quad (51)$$

$$\zeta = -i2(T_2 + T_3 + T_4), \quad (52)$$

where

$$T_1 = -\frac{C_{\Phi L}^{(3)}}{\Lambda^2} v^2, \quad (53)$$

$$T_2 = \frac{C_{\tau W\Phi}}{\Lambda^2} \frac{\sqrt{2} v m_\tau}{g_2}, \quad (54)$$

$$T_3 = -\frac{C_{LW}}{\Lambda^2} \frac{m_\tau^2}{g_2}, \quad (55)$$

$$T_4 = \frac{C_{D\tau}}{\Lambda^2} \frac{v m_\tau}{2\sqrt{2}} \quad (56)$$

Here  $m_u$  and  $m_d$  are the current masses of the  $u$  and  $d$  quarks. Under the constraints derived in Sec. IV, we have

$$|T_1| < 8.47 \times 10^{-2}, \quad (57)$$

$$|T_2| < 3.65 \times 10^{-4}, \quad (58)$$

$$|T_3| < 3.67 \times 10^{-4}, \quad (59)$$

$$|T_4| < 2.65 \times 10^{-4}. \quad (60)$$

Here we see that the bound on term  $T_1$  is much weaker than those on the other terms. In the following we only present the detailed analyses for term  $T_1$ , i.e., the effects of operator  $O_{\Phi L}^{(3)}$ . Then the matrix element for the decay  $\tau^- \rightarrow (3\pi)^- \nu_\tau$  can be written in the form

$$M = \sqrt{2} G_F (1 + \chi) \bar{u}(k, -) \gamma^\mu P_L u(p, \sigma) J_\mu, \quad (61)$$

where  $J_\mu$  is the vector hadronic matrix element given by [23]

$$\begin{aligned}
 J_\mu &= \langle (3\pi)^- | \bar{d} \gamma_\mu (1 - \gamma_5) u | 0 \rangle, \\
 &= N \left\{ \frac{2\sqrt{2}}{3} T^{\mu\nu} [(q_2 - q_3)_\nu F_1(q^2, s_1) \right. \\
 &\quad + (q_1 - q_3)_\nu F_2(q^2, s_2)] \\
 &\quad + q^\mu C_{\pi'} [s_1(s_2 - s_3) F_3(q^2, s_1) \\
 &\quad \left. + s_2(s_1 - s_3) F_4(q^2, s_2)] \right\}. \quad (62)
 \end{aligned}$$

Here  $q_1$  and  $q_2$  are the momenta of two identical  $\pi^-$ 's,  $q_3$  is the momentum of the  $\pi^+$ ,  $q$  is the momentum of the  $(3\pi)^-$  system, and  $T^{\mu\nu} = g^{\mu\nu} - q^\mu q^\nu / q^2$ .  $F_i$  are the form factors [23], with  $F_3$  and  $F_4$  being related by Bose symmetry under  $q_1 \leftrightarrow q_2$ . The kinematic invariants  $s_i$  are defined by

$$\begin{aligned}
 s_1 &= (q_2 + q_3)^2, \quad s_2 = (q_3 + q_1)^2, \\
 s_3 &= (q_1 + q_2)^2. \quad (63)
 \end{aligned}$$

The constants  $N$  and  $C_{\pi'}$  are obtained by [23]

$$N = \frac{\cos \theta_C}{f_\pi}, \quad (64)$$

$$C_{\pi'} = \frac{g_{\pi'\rho\pi} g_{\rho\pi\pi} f_{\pi'} f_\pi}{m_\rho^4 m_{\pi'}^2}, \quad (65)$$

where  $\theta_C$  is the Cabibbo angle, and  $g_{\pi'\rho\pi}$  and  $g_{\rho\pi\pi}$  are the strong coupling constants of  $\pi' - \rho - \pi$  and  $\rho - \pi - \pi$ , respectively. The values of the relevant parameters are given as [23]

$$\begin{aligned}
 \cos \theta_C &= 0.973, \quad m_\rho = 0.773 \text{ GeV}, \quad g_{\pi'\rho\pi} = 5.8, \\
 g_{\rho\pi\pi} &= 6.08, \quad f_\pi = 0.0933 \text{ GeV}. \quad (66)
 \end{aligned}$$

Note that for the  $\pi'$  decay constant,  $f_{\pi'}$ , the value of 0.02 GeV was used in Ref. [23]. As pointed out in Ref. [6], this value might be overestimated because the mixing between the chiral pion field and a massive pseudoscalar  $q\bar{q}$  bound state should be considered. Taking into account such mixing effects,  $f_{\pi'}$  was re-estimated in the chiral Lagrangian framework and was found to be  $(1-5) \times 10^{-3}$  GeV [6]. In our calculation, we use the most conservative value of  $1 \times 10^{-3}$  GeV and will comment on the effect of the larger  $f_{\pi'}$  later.

This matrix element can be casted into the form [6]

$$\begin{aligned}
 M &= \sqrt{2} G_F (1 + \chi) \left[ \sum_\lambda L_{\sigma\lambda} H_\lambda \right. \\
 &\quad \left. + \left( 1 - \frac{m_{\pi'}^2}{(m_u + m_d) m_\tau} \frac{\chi}{1 + \chi} \right) L_{\sigma s} H_s \right], \quad (67)
 \end{aligned}$$

where  $L_{\sigma\lambda}$  and  $L_{\sigma s}$  are the leptonic amplitudes, and  $H_\lambda$  ( $\lambda = 0, \pm$ ) and  $H_s$  are those involving hadrons. They are given by

$$L_{\sigma+} = 0, \quad (68)$$

$$L_{\sigma 0} = \frac{m_\tau}{\sqrt{q^2}} \sqrt{m_\tau^2 - q^2} \delta_{\sigma+}, \quad (69)$$

$$L_{\sigma-} = \sqrt{2} \sqrt{m_\tau^2 - q^2} \delta_{\sigma-}, \quad (70)$$

$$L_{\sigma s} = \sqrt{m_\tau^2 - q^2} \delta_{\sigma+}, \quad (71)$$

$$\begin{aligned}
 H_\lambda &= -\frac{2\sqrt{2}}{3} N \epsilon_\mu(q, \lambda) [(q_2 - q_3)^\mu F_1(q^2, s_1) \\
 &\quad + (q_1 - q_3)^\mu F_2(q^2, s_2)], \quad (72)
 \end{aligned}$$

$$\begin{aligned}
 H_s &= N m_\tau C_{\pi'} [s_1(s_2 - s_3) F_3(q^2, s_1) \\
 &\quad + s_2(s_1 - s_3) F_4(q^2, s_2)], \quad (73)
 \end{aligned}$$

where  $\epsilon(q, \lambda)$  is the polarization vector of the axial vector meson  $a_1$ .

The amplitude of  $\tau^+ \rightarrow (3\pi)^+ \bar{\nu}_\tau$  can be obtained from that of  $\tau^- \rightarrow (3\pi)^- \nu_\tau$  by the substitutions [6]:  $\chi \rightarrow \chi^*$ ,  $L_{\sigma\lambda} \rightarrow (-1)^\lambda L_{-\sigma, -\lambda}$  and  $L_{\sigma s} \rightarrow L_{-\sigma s}$ .

Following Ref. [6], we define two coordinate systems  $(x, y, z)$  and  $(x^*, y^*, z^*)$  in the  $(3\pi)^\pm$  rest frame. Both systems have a common  $y$  axis which is chosen along the  $\vec{k} \times \vec{q}_3$  direction. In the  $(x, y, z)$  system, the  $z$  axis is along the direction of  $\vec{k}$ , and the momentum  $\vec{q}_3$  is in the  $(z, x)$  plane with a positive- $x$  component. The  $(x^*, y^*, z^*)$  system is related to the  $(x, y, z)$  system by a rotation by  $\theta$  (the angle between  $\vec{k}$  and  $\vec{q}_3$ ) with respect to the common  $y$  axis, so that the  $z^*$  axis is along  $\vec{q}_3$ . In terms of the five variables,  $q^2$ ,  $s_1$ ,  $s_2$ ,  $\theta$  and  $\phi^*$ , where  $\phi^*$  is the azimuthal angle of  $\vec{q}_1$  in the  $(x^*, y^*, z^*)$  system, we denote the differential decay rates of  $\tau^- \rightarrow (3\pi)^- \nu_\tau$  and  $\tau^+ \rightarrow (3\pi)^+ \bar{\nu}_\tau$  by  $\bar{G}(q^2, s_1, s_2, \cos \theta, \phi^*)$  and  $G(q^2, s_1, s_2, \cos \theta, \phi^*)$ , respectively. Then a  $CP$ -conserving sum  $\Sigma$  and a  $CP$ -violating difference  $\Delta$  can be constructed [6]

$$\begin{aligned}
 \Sigma &= G(q^2, s_1, s_2, \cos \theta, \phi^*) + \bar{G}(q^2, s_1, s_2, \cos \theta, -\phi^*) \\
 &= \frac{G_F^2 m_\tau (1 - q^2/m_\tau^2)^2}{2^7 \pi^6} \frac{1}{q^2} |1 + \chi|^2 \\
 &\quad \times \left\{ 2|H_-|^2 + \frac{m_\tau^2}{q^2} |H_0|^2 + |1 + \xi|^2 |H_s|^2 \right. \\
 &\quad \left. - 2 \frac{m_\tau}{\sqrt{q^2}} [1 + \text{Re}(\xi)] \text{Re}(H_0 H_s^*) \right\}, \quad (74)
 \end{aligned}$$

$$\begin{aligned}
 \Delta &= G(q^2, s_1, s_2, \cos \theta, \phi^*) - \bar{G}(q^2, s_1, s_2, \cos \theta, -\phi^*) \\
 &= -2 \frac{G_F^2 m_\tau (1 - q^2/m_\tau^2)^2}{2^7 \pi^6} \frac{1}{q^2} |1 + \chi|^2 \frac{m_\tau}{\sqrt{q^2}} \text{Im}(\xi) \text{Im}(H_0 H_s^*), \quad (75)
 \end{aligned}$$

where  $\xi$  is given by

$$\xi = -\frac{m_{\pi'}^2}{(m_u + m_d)m_\tau} \left( \frac{\chi}{1 + \chi} \right). \quad (76)$$

Choosing a weight function  $w(q^2, s_1, s_2, \cos \theta, \phi^*)$ , we can obtain a  $CP$ -violating observable  $\langle w\Delta \rangle$  which is obtained by integrating the quantity  $w\Delta$  over the allowed phase space. Following Ref. [6], we consider two types of  $CP$ -violating forward-backward asymmetries,  $A_{1FB}$  and  $A_{2FB}$ , which are the  $\nu_\tau$  ( $\bar{\nu}_\tau$ ) distribution with respect to the  $\pi^+$  ( $\pi^-$ ) direction in the  $(3\pi)^-$  [ $(3\pi)^+$ ] rest frame, with the respective weight functions  $\text{sgn}[\cos \theta]$  and  $\text{sgn}[s_2 - s_1] \cdot \text{sgn}[\cos \phi^*]$ . We also consider the optimal asymmetry,  $A_{opt}$ , which is defined with the weight function  $\Delta/\Sigma$ . The statistical significance can be determined by the quantity  $\varepsilon = \langle w\Delta \rangle / \sqrt{\langle \Sigma \rangle \cdot \langle w^2 \Sigma \rangle}$ . To observe this  $CP$ -violating observable at the  $2\sigma$  level, the required statistical significance should be  $\varepsilon \geq 2/\sqrt{N_\tau} \text{Br}$ . Hence the sensitivity to probes of the coupling is proportional to  $\sqrt{N_\tau}$ .

Under the current constraint in Eq. (45), i.e.,  $C_{\Phi L}^{(3)}/(\Lambda/\text{TeV})^2 \leq 1.4$ , the number of  $\tau$  required to observe the effect of operator  $O_{\Phi L}^{(3)}$  at the  $2\sigma$  level is found to be (for  $f_{\pi'} = 1 \times 10^{-3}$  GeV):

$$N_\tau \geq \begin{cases} 7.2 \times 10^6 & (\text{for } A_{1FB}) \\ 0.5 \times 10^6 & (\text{for } A_{2FB}) \\ 0.8 \times 10^5 & (\text{for } A_{opt}). \end{cases} \quad (77)$$

So, at the tau-charm factory which will produce  $1 \times 10^7$  tau leptons per year, it is possible to observe such  $CP$ -violating effects. If an effect is not seen at the  $2\sigma$  level, stronger constraints can be obtained for the coupling strength of the operator under consideration,

$$\frac{|C_{\Phi L}^{(3)}|}{(\Lambda/\text{TeV})^2} \leq \begin{cases} 1.18 & (\text{for } A_{1FB}) \\ 0.30 & (\text{for } A_{2FB}) \\ 0.12 & (\text{for } A_{opt}). \end{cases} \quad (78)$$

For the class B operators ( $O_{LW}$ ,  $O_{D\tau}$  and  $O_{\tau W\Phi}$ ), we do not present their detailed analyses here. But we can roughly estimate the number of  $\tau$  leptons required to observe their effects from our results for the operator  $O_{\Phi L}^{(3)}$ . As shown in Sec. IV, the upper bound on the coupling strength of a class B operator is  $10^{-2}$  lower than that of  $O_{\Phi L}^{(3)}$ . Since the number of  $\tau$  leptons required to observe the effects of an operator is proportional to  $1/C_i^2$ , the number of  $\tau$  leptons required to observe the effects of a class B operator should be increased by a factor  $10^4$  relative to that in Eq. (77). Hence, under the current constraints,  $10^9$  taus are needed in order to observe the effects of a class B operator even with the most sensitive probe by  $A_{opt}$ . So it is impossible to observe their effects in the  $3\pi$  mode of the tau decay at the tau-charm factory which is expected to produce  $10^7$  taus per year.

From Eqs. (65), (73), and (75), we see that the  $CP$ -violating difference  $\Delta$  is proportional to the  $\pi'$  decay con-

stant  $f_{\pi'}$  and thus the number of  $\tau$  leptons required is proportional to  $1/f_{\pi'}^2$ . In our calculation we used the most conservative value of  $1 \times 10^{-3}$  GeV. If we take the largest value of  $5 \times 10^{-3}$  GeV [6] for  $f_{\pi'}$ , the number of  $\tau$  leptons required to observe the effects of the operators as given in Eq. (77) will be lowered by a factor of 1/25.

To conclude this section, let us discuss briefly the present experimental situation. The  $\tau \rightarrow 3\pi\nu_\tau$  has been investigated by Argus [24] at Doris II and by OPAL [25], ALEPH [26] and, most recently, DELPHI [27] at LEP. The statistics in these experiments are several times to an order of magnitude smaller than that required by Eq. (77). Two models have been used by these experiments to fit their data: One is the model by Isgur, Morningstar, and Reader (IMR) [22] and the other by Kuhn and Santamaria (KS) [22]. The two models differ in their parametrization [28] although both assume axial vector, i.e.,  $a_1$ , dominance of the  $3\pi$  system. The model we used is close to that of IMR. All data were found to be consistent with the two models although the detailed fits reveal some disagreement between the experimental data and the models. For the OPAL Collaboration which has made a detailed fit, both models were found to overestimate the  $\rho$ -peak and underestimate the low value region of invariant mass of the  $\pi^+\pi^-$  system. In particular the IMR model was found to need a 14% non- $a_1$  contribution which was represented as a polynomial background. In the most recent DELPHI analysis, the Dalitz plot and the invariant mass of the  $3\pi$  system were analyzed and found to be in reasonably good agreement with both the IMR and KS models. However, it is also found that the two models do not give a good fit for the  $s_1$  and  $s_2$  distributions for  $s > 2.3$  GeV<sup>2</sup>, where  $s$  is the  $3\pi$  invariant mass squared.

## VI. SUMMARY

We studied the dimension-six  $CP$ -violating  $SU_L(2) \times U_Y(1)$  invariant operators involving the tau lepton, which could be generated by new physics at a higher energy scale. Under our criteria, there are totally 10 such operators which are classified into 4 classes. Since the class D operators, ( $O_{\Phi L}^{(1)}$ ,  $O_{\Phi\tau}$  and  $O_{\tau 1}$ ), only contribute to  $H\tau\tau$  coupling, they are not constrained by current data and it will be difficult to probe them at future colliders. Class B operators ( $O_{LW}$ ,  $O_{D\tau}$  and  $O_{\tau W\Phi}$ ) and Class C operators ( $O_{LB}$ ,  $O_{\tau B}$  and  $O_{\tau B\Phi}$ ) give anomalous neutral currents and are strongly constrained by LEP experimental data. Class A operator ( $O_{\Phi L}^{(3)}$ ) only contribute to charged current and so far is only loosely constrained. Although both class A and class B contribute to charged current and affect the tau hadronic decay,  $\tau \rightarrow 3\pi\nu_\tau$ , only class A operator will be possibly observable at future tau-charm factory. The current strong limits on the class B operators make their contributions to  $CP$  violation effect in  $\tau \rightarrow 3\pi\nu_\tau$  unobservable.

To conclude, our analyses show that the  $CP$ -violating new physics, subject to existing experimental limits, are possibly observable in hadronic tau decays at the future tau-charm factory. The effective operators (12)–(21) contain vertices of

more complicated structures than the 3-point vertices we have investigated in this article. Their effects, in particular of operators given in Eqs. (12), (13), (14), (16), and (21), will be investigated in future works.

Despite the fact that the decay  $\tau \rightarrow 3\pi + \nu_\tau$  is an ideal place to investigate the lepton  $CP$ -violation effect, the validity of the model parametrization is not completely clear as discussed at the end of Sec. V. In the future, a model independent parametrization of the process will be desirable. Therefore, we see more theoretical effort is needed in order

to reliably extract any lepton  $CP$ -violation effect which may exist.

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