Cosmological nucleosynthesis and active-sterile neutrino oscillations with small mass differences: The nonresonant case

D. P. Kirilova*

*Teoretisk Astrofysik Center, Juliane Maries Vej 30, DK-2100, Copenhagen
and Copenhagen University, Niels Bohr Institute, Blegdamsvej 17, DK-2100, Copenhagen, Denmark*

M. V. Chizhov[†]

Centre for Space Research and Technologies, Faculty of Physics, University of Sofia, 1164 Sofia, Bulgaria

We study the nonresonant oscillations between left-handed electron neutrinos ν_e and nonthermalized sterile neutrinos ν_s in the early Universe plasma. The case when ν_s do not thermalize until 2 MeV and the oscillations become effective after ν_e decoupling is discussed. As far as for this model the rates of the expansion of the Universe, neutrino oscillations, and neutrino interactions with the medium may be comparable, we have analyzed the kinetic equations for the neutrino density matrix, accounting *simultaneously* for these processes. The evolution of neutrino ensembles was described numerically by integrating the kinetic equations for the neutrino density matrix in *momentum* space for small mass differences $\delta m^2 \leqslant 10^{-7}$ eV^2. This approach allowed us to study precisely the evolution of the neutrino number densities, energy spectrum distortion, and the asymmetry between neutrinos and antineutrinos due to oscillations for each momentum mode. We have performed a complete numerical analysis for the full range of the oscillations parameters of the model of the influence of the nonequilibrium $\nu_e \leftrightarrow \nu_s$ oscillations on the primordial production of ^{4}He. The exact kinetic approach enabled us to calculate the effects of neutrino population depletion, the distortion of the neutrino spectrum, and the generation of neutrino-antineutrino asymmetry on the kinetics of neutron-to-proton transitions during the primordial nucleosynthesis epoch and correspondingly on cosmological ^{4}He production. It was shown that the neutrino population depletion and spectrum distortion play an important role. The asymmetry effect, in case the lepton asymmetry is accepted initially equal to the baryon one, is proved to be negligible for the discussed range of δm^2. Constant helium contours in the δm^2-ϑ plane were calculated. Thanks to the exact kinetic approach more precise cosmological constraints on the mixing parameters were obtained. [S0556-2821(98)04115-0]

PACS number(s): 14.60.Pq, 05.70.Ln, 26.35.+c

I. INTRODUCTION

The idea of Gamow, proposed in the 1930s and 1940s [1], about the production of elements through thermonuclear reactions in the hot ylem during the early stages of the Universe expansion, has been developed during the last 60 years into an elegant famous theory of cosmological nucleosynthesis, explaining quantitatively the primordial abundances of the light elements inferred from observational data [2]. Thanks to that good accordance between theoretical predictions and observational facts, we nowadays believe to understand well the physical conditions of the nucleosynthesis epoch. Still, the uncertainties of the primordial abundances values extracted from observations yet leave room for physics beyond the standard model.

In this article we present a modification of the standard model of cosmological nucleosynthesis (CN): *CN with neutrino oscillations*. Our aim is twofold: (1) to construct a modification of CN using a more precise kinetic approach to the problem of nonequilibrium neutrino oscillations and to illustrate the importance of such an exact approach, and (2) to determine the cosmologically allowed range for oscillation parameters from an accurate study of the oscillation effect on the primordial production of helium-4, thus helping to clarify the mixing patterns of neutrinos.

The theme of neutrino oscillations has been with us for almost 40 years, since the hypothesis for them was proposed by Pontecorvo [3]. They were studied experimentally and theoretically and their cosmological and astrophysical effects have been considered in numerous publications [4] as far as their study helps to go deeper into the secrets of neutrino physics and the neutrino mass pattern. Nowadays there are three main *experimental indications* that neutrinos oscillate: namely, the solar neutrino deficit [5] (an indirect indication), the atmospheric neutrino anomaly [6] (an indirect indication), and the Liquid Scintillation Neutrino Detector (LSND) experiment results [7] (a direct indication).

(a) *Solar neutrino deficit:* Already four experiments using different techniques have detected electron neutrinos from the Sun, at a level significantly lower than predicted on the basis of the standard solar model and the standard electroweak theory. Moreover, there exists incompatibility between Chlorine and Kamiokande experiments data, as well as problems for predicted berilium and borum neutrinos in the gallium experiments [5,8]. Recently, it was realized that by changing the solar model it is hardly possible to solve

*Permanent address: Institute of Astronomy, Bulgarian Academy of Sciences, blvd. Tsarigradsko Shosse 72, Sofia, Bulgaria. Email address: dani@libra.astro.acad.bg

[†]Email address: mih@phys.uni-sofia.bg

these problems [9]. Therefore, it is interesting to find a solution beyond the standard electroweak model. The only known natural solutions of that kind today are the energy-dependent Mikheyev-Smirnov-Wolfenstein (MSW) neutrino transitions in the Sun interior [10] and the ''just-so'' vacuum oscillations solutions, as well as the recently developed hybrid solutions of MSW transitions +vacuum oscillations type [11].

(b) *Atmospheric neutrino anomaly:* Three of the five underground experiments on atmospheric neutrinos have observed the disappearance of muon neutrinos [6]. This is in contradiction with the theoretically expected flux of muon neutrinos from primary cosmic rays interacting in the atmosphere. A successful oscillatory solution of that problem requires large mixing and δm^2 of the order of 10^{-2} eV^2.

(c) *The Los Alamos LSND experiment* claimed evidence for the oscillation of $\tilde{\nu}_\mu$ into $\tilde{\nu}_e$, with a maximal probability of the order of 0.45×10^{-2}. A complementary ν_μ into ν_e oscillation search, with completely different systematics and backgrounds, also shows a signal, which indicates the same favored region of oscillation parameters [7].

There exists yet another observational suggestion for massive neutrinos and oscillations: the *dark matter problem.* Present models of structure formation in the Universe indicate that the observed hierarchy of structures is reproduced best by an admixture of about 20% hot dark matter to cold dark matter [12]. Light neutrinos with mass in the eV range are the only particle dark matter candidates, which are actually known to exist and are the most plausible candidates provided by particle physics [13]. Actually, recent most popular hot plus cold dark matter models assume that two nearly degenerate massive neutrinos each with mass 2.4 eV play the role of the hot dark matter. This small mass value is now accessible only by oscillations.

However, in case we take seriously each of these experiments pointing to a neutrino anomaly and the neutrino oscillation solution to them, a fourth neutrino seems inevitable. In the case of only three species of light neutrinos with normal interactions and a see-saw hierarchy between the three masses, it is hardly possible to accommodate all present data simultaneously. Successful attempts to reconcile the LSND results with neutrino oscillation solutions to the solar and atmospheric neutrino problems usually contain some ''unnatural'' features, like a fourth ultralight sterile neutrino species or an inverted neutrino mass hierarchy [14]. However, an additional light (with mass less than 1 MeV) flavor neutrino is forbidden both from cosmological considerations and the experiments on Z decays at the CERN e^+e^- collider LEP [15]. Hence, it is reasonable to explore in more detail the possibility for an additional light *sterile* neutrino. In addition, grand unified theories (GUTs) $[SO(10),\ E_6,$ etc.] [16,17] and supersymmetry (SUSY) theories [18,19] predict the existence of a sterile neutrino. Moreover, recently models of singlet fermions, which explain the smallness of the sterile neutrino mass and its mixing with the usual neutrino, were proposed [19]. Therefore, it may be very useful to obtain more precise information about the cosmologically allowed range for the neutrino mixing parameters and thus present an additional independent test for the already discussed neutrino

puzzles. In addition, the very small values of mass differences, which can be explored by the oscillation cosmological effects (like the ones discussed in our model), are beyond the reach of present and near future experiments.

The present work is a step towards this: we suppose the existence of a sterile neutrino $[SU(2)$-singlet$]$ ν_s, and explore the cosmological effect of *nonresonant neutrino oscillations* $\nu_e \leftrightarrow \nu_s$ on the primordial nucleosynthesis, obtaining thus cosmological constraints on the neutrino mixing parameters. The nonresonant case in the early Universe medium corresponds to the resonant case in the Sun; therefore, the obtained information is also of interest for the MSW solution of the solar neutrino problem. We discuss the special case of *nonequilibrium oscillations* between weak interacting and sterile neutrinos for *small mass differences δm^2,* insofar as the case of large δm^2 is already sufficiently well studied [20–27]. Oscillations between active and sterile neutrinos, effective before neutrino freezing at 2 MeV, leading to ν_s thermalization before 2 MeV have been studied there. I.e., mainly the equilibrium oscillations were considered with rates of oscillations and neutrino weak interactions greater than the expansion rate. Here we discuss nonequilibrium oscillations between electron neutrinos ν_e and sterile neutrinos ν_s for the case when ν_s do not thermalize until ν_e decoupling at 2 MeV and oscillations become effective after ν_e decoupling. Such a kind of active-sterile neutrino oscillations in vacuum was first precisely studied in [28] using an accurate kinetic approach for the description of oscillating neutrinos, proposed in the pioneer work of Dolgov [29]. As far as the thermal background in the prenucleosynthesis epoch may strongly affect the propagation of neutrinos [31,32] an account of the neutrino interactions with the primeval plasma is obligatory [33,20,24]. The precise kinetic consideration of oscillations in a medium was provided in [34]. It was proved that in the case when the Universe expansion, the oscillations, and the neutrino interactions with the medium have comparable rates, their effects should be accounted for simultaneously, using exact kinetic equations for the neutrino density matrix. Moreover, for the nonequilibrium oscillations the energy distortion and asymmetry between neutrinos and antineutrinos may play a considerable role. As far as both neutrino collisions and active-sterile neutrino oscillations distort the initially equilibrium active neutrino momentum distribution, the momentum degree of freedom in the description of the neutrino must be accounted for. Therefore, for the case of nonequilibrium oscillations the evolution of neutrino ensembles should be studied using exact kinetic equations for the *density matrix of neutrinos in momentum space.* This approach allows an exact investigation of the different effects of neutrino oscillations [28,34,35], depletion of the neutrino number densities, the energy distortion, and the generation of asymmetry, for each separate momentum of the neutrino ensembles.

In the present work we expand and complete the original investigation [34] for the full parameter space of the nonequilibrium oscillations model for the nonresonant case. (The resonant case will be discussed in a following publication.) We have provided an exact kinetic analysis of the neutrino evolution by a numerical integration of the *kinetic*

equations for the neutrino density matrix for each momentum mode. The kinetic equations are coupled nonlinear and, therefore, an analytic solution is hardly possible in the general case of oscillations in a medium. We have numerically described the evolution of the neutrino ensembles from the ν_e freezing at 2 MeV until the formation of helium-4.

We have calculated the production of helium-4 in a detail model of primordial oscillations nucleosynthesis, accounting for the direct kinetic effects of oscillations on the neutron-to-proton transitions. The oscillations effect on CN has been considered by many authors [20–27,30,36–41]. However, mainly the excitation of an additional degree of freedom due to oscillations (i.e., an increase of the effective degrees of freedom g) and the corresponding increase of the Universe expansion rate $H \sim \sqrt{g}$, leading to an overproduction of helium-4, was discussed. The excluded regions for the neutrino mixing parameters were obtained from the requirement (based on the accordance between the light element abundances theoretically predicted and those extracted from observations) that the neutrino types should be less than 3.4: $N_\nu < 3.4$ [20–24]. A successful account for the electron neutrino depletion due to oscillations was first made in [20] and [24]. In the present work we have precisely calculated the influence of oscillations on primordially produced helium-4 using exact kinetic equations in momentum space for the neutron number density and the density matrix of neutrinos, instead of their particle densities. The accurate numerical analysis of the ocillations effect on helium production within a model of nucleosynthesis with oscillations allowed us to account precisely for the following important effects of neutrino oscillations: neutrino population depletion, distortion of the neutrino spectrum, and the generation of neutrino-antineutrino asymmetry. This enabled us to investigate the zone of very small neutrino mass differences up to 10^{-11} eV^2, which has not been reached before. As a result, we have obtained constant helium contours in the mass-difference–mixing-angle plane for the full range of the parameter values of our model. No matter which will be the preferred primordial helium value favored by future observations, it will be possible to obtain the excluded region of the mixing parameters using the results of this survey.

The paper is organized as follows. In Sec. II we present the model of nonequilibrium neutrino oscillations. In Sec. III an exact analysis of the neutrino evolution, using kinetic equations for the neutrino density matrix for each momentum mode, is provided. The main effects of nonequilibrium oscillations are revealed. In Sec. IV we investigate the $\nu_e \leftrightarrow \nu_s$ oscillations effect on the primordial production of helium using a numerical nucleosynthesis code. We discuss the influence of nonequilibrium neutrino oscillations, namely, electron neutrino depletion, neutrino spectrum distortion, and the generation of neutrino-antineutrino asymmetry, on the primordial yield of helium-4. The results and conclusions are presented in Sec. V.

II. NONEQUILIBRIUM NEUTRINO OSCILLATIONS: THE MODEL

The model of nonequilibrium oscillations between weak interacting electron neutrinos ν_e and sterile neutrinos ν_s for the case when ν_s do not thermalize until ν_e decoupling at 2 MeV and oscillations become effective after ν_e decoupling is described in detail in [34]. The main assumptions are the following: (1) *Singlet neutrinos decouple much earlier, i.e., at a considerably higher temperature than the active neutrinos do*: $T^F_{\nu_s} > T^F_{\nu_e}$. This is quite a natural assumption, as far as sterile neutrinos do not participate in ordinary weak interactions. In models predicting singlet neutrinos, the interactions of ν_s are mediated by gauge bosons with masses $M = \mathcal{O}(1 \text{ TeV})$ [16,18,42]. Therefore, in later epochs after their decoupling, their temperature and number densities are considerably less than those of the active neutrinos due to the subsequent annihilations and decays of particles that have additionally heated the nondecoupled ν_e in comparison with the already decoupled ν_s.

(2) *We consider oscillations between ν_s ($\nu_s \equiv \widetilde{\nu}_L$) and the active neutrinos, according to the Majorana and Dirac (MD) mixing scheme* [43] *with mixing present just in the electron sector* $\nu_i = \mathcal{U}_{il} \nu_l, \ l = e, s$:[1]

$$\nu_1 = c \nu_e + s \nu_s,$$

$$\nu_2 = -s \nu_e + c \nu_s,$$

where ν_s denotes the sterile electron antineutrino, $c = \cos(\vartheta)$, $s = \sin(\vartheta)$, and ϑ is the mixing angle in the electron sector; the mass eigenstates ν_1 and ν_2 are Majorana particles with masses correspondingly m_1 and m_2. We consider the nonresonant case $\delta m^2 = m_2^2 - m_1^2 > 0$, which corresponds in the small mixing angle limit to a sterile neutrino heavier than the active one.

In this model the element of nonequilibrium is introduced by the presence of a small singlet neutrino density at 2 MeV, $n_{\nu_s} \ll n_{\nu_e}$, when the oscillations between ν_s and ν_e become effective. In order to provide such a small singlet neutrino density the sterile neutrinos should have decoupled from the plasma sufficiently early in comparison to the active ones and should have not regained their thermal equilibrium until 2 MeV [44,28,34]. Therefore, as far as the oscillations into ν_e and the following noncoherent scattering off the background may lead to the thermalization of ν_s, two more assumptions are necessary for the nonequilibrium case to take place: (3) *Neutrino oscillations should become effective after the decoupling of the active neutrinos*, $\Gamma_{osc} \geq H$ *for* $T \leq 2$ MeV, *which is realizable for* $\delta m^2 \leq 1.3 \times 10^{-7}$ eV^2 [34]; (4) *Sterile neutrinos should not thermalize until* 2 MeV *when oscillations become effective; i.e., the production rate of ν_s must be smaller than the expansion rate*. The problem of sterile neutrino thermalization was discussed in the pioneer work of Manohar [44] and in more recent publications

---

[1] The transitions between different neutrino flavors were proved to have a negligible effect on the neutrino number densities and on primordial nucleosynthesis because of the very slight deviation from equilibrium in that case $T_f \sim T'_f$ (f is the flavor index) [29,30,41].

[20–24]. This assumption limits the allowed range of oscillation parameters for our model: $\sin^2(2\vartheta)\delta m^2 \lesssim 10^{-7}$ eV^2 [34].

We have assumed here that electron neutrinos decouple at 2 MeV. However, the neutrino decoupling process is more complicated. It has been discussed in the literature in detail [45]. Decoupling occurs when the neutrino weak interaction rate $\Gamma_w \sim E^2 n_\nu(E)$ becomes less than the expansion rate $H \sim \sqrt{g}T^2$. Really, for electron neutrinos this happens at about 2 MeV. Nevertheless, because of the fact that the weak interaction rate is greater at a higher energy, some thermal contact between neutrinos and high energy plasma remains after 2 MeV, especially for the high energy tail of the neutrino spectrum. In case these high energy neutrinos begin to oscillate before their decoupling, the account of this dependence of decoupling time on the neutrino momentum will be essential for our model. Otherwise, in case these neutrinos do not start oscillating before decoupling, there will be no harm considering them decoupled earlier, as far as they preserve their equilibrium distribution anyway due to their extremely small mass. In [34] we have checked that neutrinos from the high energy tail start to oscillate much later than they decouple for the range of oscillation parameters considered in our model. It can easily be understood from the fact that the oscillation rate decreases with energy $\Gamma_{\rm osc} \sim \delta m^2/E_\nu$ and, therefore, neutrinos with higher energies begin to oscillate later, namely, when $\Gamma_{\rm osc}$ exceeds the expansion rate $H \sim \sqrt{g}T^2$. Hence, the precise account for the momentum dependence of the decoupling does not change the results of our model but unnecessarily complicates the analysis and leads to an enormous increase of the calculation time. Therefore, in what follows we have assumed a fixed decoupling time instead of considering the real decoupling period—i.e., we have accepted that the electron neutrinos have completely decoupled at 2 MeV.

III. KINETICS OF NONEQUILIBRIUM NEUTRINO OSCILLATIONS

The exact kinetic analysis of the neutrino evolution, discussed in this section, though much more complicated, reveals some important features of nonequilibrium oscillations that cannot be caught otherwise. As far as for the nonequilibrium model discussed the rates of the expansion of the Universe, neutrino oscillations, and neutrino interactions with the medium may be comparable, we have used kinetic equations for neutrinos, accounting *simultaneously* for the participation of neutrinos in expansion, oscillations, and interactions with the medium. All possible reactions of neutrinos with the plasma were considered, namely, reactions of neutrinos with the electrons, neutrons and protons, neutrinos of other flavors, and the corresponding antiparticles, as well as self-interactions of electron neutrinos. These equations contain all effects due to first order on G_F medium-induced energy shifts, second order effects due to nonforward collisions, and the effects nonlinear on the neutrino density matrices like neutrino refraction effects in a medium of neutrinos. In the case of nonequilibrium oscillations the density matrix of neutrinos may considerably differ from its equilibrium form.[2] Then, for the correct analysis of nonequilibrium oscillations, it is important to work in terms of the density matrix of neutrinos in momentum space [29,28,34,35]. Therefore, we have provided a proper kinetic analysis of the neutrino evolution using kinetic equations for the *neutrino density matrix for each momentum mode*.

Hence, the kinetic equations for the density matrix of the nonequilibrium oscillating neutrinos in the primeval plasma of the Universe in the epoch previous to nucleosynthesis, i.e., consisting of photons, neutrinos, electrons, nucleons, and the corresponding antiparticles, have the form

$$\frac{\partial\rho(t)}{\partial t} = Hp\frac{\partial\rho(t)}{\partial p} + i[\mathcal{H}_0,\rho(t)] + i[\mathcal{H}_{int},\rho(t)] + \mathrm{O}(\mathcal{H}_{int}^2),$$

(1)

where p is the momentum of electron neutrinos and ρ is the density matrix of the massive Majorana neutrinos in momentum space.

The first term in the equation describes the effect of expansion, the second is responsible for oscillations, the third accounts for forward neutrino scattering off the medium, and the last one accounts for second order interaction effects of neutrinos with the medium. $\mathcal{H}_0$ is the free neutrino Hamiltonian,

$$\mathcal{H}_0 = \begin{pmatrix} \sqrt{p^2+m_1^2} & 0 \\ 0 & \sqrt{p^2+m_2^2} \end{pmatrix},$$

while $\mathcal{H}_{int} = \alpha V$ is the interaction Hamiltonian, where $\alpha_{ij} = U_{ie}^* U_{je}$, $V = G_F(+L - Q/M_W^2)$, and in the interaction basis plays the role of an induced squared mass for electron neutrinos:

$$\mathcal{H}_{int}^{LR} = \begin{pmatrix} V & 0 \\ 0 & 0 \end{pmatrix}.$$

Hence, V is the time-varying (due to the Universe cooling) effective potential, induced by the interactions of the neutrino with the medium through which it propagates. Since ν_s does not interact with the medium, it has no self-energy correction, i.e., $V_s = 0$.

The first "local" term in V accounts for charged- and neutral-current tree-level interactions of ν_e with medium protons, neutrons, electrons and positrons, and neutrinos and antineutrinos. It is proportional to the fermion asymmetry of the plasma, $L = \Sigma_f L_f$, which is usually taken to be of the order of the baryon one, i.e., 10^{-10} (i.e., $B-L$ conservation is assumed):

---

[2]When neutrinos are in equilibrium their density matrix has its equilibrium form, namely, $\rho_{ij} = \delta_{ij}\exp(\mu/T - E/T)$, so that one can work with particle densities instead of ρ. In an equilibrium background, the introduction of oscillations slightly shifts ρ from its diagonal form, due to the extreme smallness of the neutrino mass in comparison with the characteristic temperatures and to the fact that the equilibrium distribution of massless particles is not changed by the expansion [29].

$$L_f \sim \frac{N_f - N_{\bar{f}}}{N_\gamma} T^3 \sim \frac{N_B - N_{\bar{B}}}{N_\gamma} T^3 = \beta T^3.$$

The second "nonlocal" term in V arises as a W/Z propagator effect, $Q \sim E_\nu T^4$ [33,20]. For the conditions of the early Universe both terms must be accounted for because, although the second term is of the second power of G_F, the first term is proportional except to the first power of G_F, also to the small value of the fermion asymmetry. Moreover, the two terms have different temperature dependence and an interesting interplay between them during the cooling of the Universe is observed. At high temperature the nonlocal term dominates, while with cooling of the Universe in the process of expansion the local one becomes more important.

The last term in Eq. (1) describes the weak interactions of neutrinos with the medium. For example, for the weak reactions of neutrinos with electrons and positrons, $e^+ e^- \leftrightarrow \nu_i \tilde{\nu}_j$, $e^\pm \nu_j \rightarrow e'^\pm \nu'_i$, it has the form

$$\int d\Omega(\tilde{\nu}, e^+, e^-) \left[n_e - n_{e^+} \mathcal{A} \mathcal{A}^\dagger - \frac{1}{2} \{ \rho, \mathcal{A}^\dagger \bar{\rho} \mathcal{A} \}_+ \right]$$

$$+ \int d\Omega(e^-, \nu', e'^-) \left[n'_e - \mathcal{B} \rho' \mathcal{B}^\dagger - \frac{1}{2} \{ \mathcal{B}^\dagger \mathcal{B}, \rho \}_+ n_{e^-} \right]$$

$$+ \int d\Omega(e^+, \nu', e'^+) \left[n'_{e^+} \mathcal{C} \rho' \mathcal{C}^\dagger - \frac{1}{2} \{ \mathcal{C}^\dagger \mathcal{C}, \rho \}_+ n_{e^+} \right],$$

where n stands for the number density of the interacting particles,

$$d\Omega(i,j,k) = \frac{(2\pi)^4}{2E_\nu} \int \frac{d^3 p_i}{(2\pi)^3 2E_i} \frac{d^3 p_j}{(2\pi)^3 2E_j} \frac{d^3 p_k}{(2\pi)^3 2E_k}$$
$$\times \delta^4(p_\nu + p_i - p_j - p_k)$$

is a phase space factor, $\mathcal{A}$ is the amplitude of the process $e^+ e^- \rightarrow \nu_i \tilde{\nu}_j$, $\mathcal{B}$ is the amplitude of the process $e^- \nu_j \rightarrow e'^- \bar{\nu}'_i$, and $\mathcal{C}$ is the amplitude of the process $e^+ \nu_j \rightarrow e'^+ \nu'_i$. They are expressed through the known amplitudes $\mathcal{A}_e(e^+ e^- \rightarrow \nu_e \tilde{\nu}_e)$, $\mathcal{B}_e(e^- \nu_e \rightarrow e^- \nu_e)$, and $\mathcal{C}_e(e^+ \nu_e \rightarrow e^+ \nu_e)$:

$$\mathcal{A} = \alpha \mathcal{A}_e, \quad \mathcal{B} = \alpha \mathcal{B}_e, \quad \mathcal{C} = \alpha \mathcal{C}_e.$$

Analogues equations hold for the antineutrino density matrix, the only difference being in the sign of the lepton asymmetry: L_f is replaced by $-L_f$. Medium terms depend on the neutrino density, thus introducing a nonlinear feedback mechanism. Neutrino and antineutrino ensembles evolve differently as far as the background is not CP symmetric. Oscillations may change the neutrino-antineutrino asymmetry and it in turn affects oscillations. The evolution of neutrino and antineutrino ensembles is coupled and hence it must be considered simultaneously.

We have analyzed the evolution of the neutrino density matrix for the case when oscillations become noticeable after the electron neutrino decoupling, i.e., after 2 MeV. Then the last term in the kinetic equation can be neglected. So Eq. (1)

results in a set of coupled nonlinear integro-differential equations with time-dependent coefficients for the components of the density matrix of neutrinos. It is convenient instead of $\partial/\partial t$ to use $\partial/\partial \mu$, where $\mu^2 = \omega t$, $\omega = \sqrt{16 \pi^3 g / 45} \, (\Delta^2 / M_{Pl})$, and $\Delta = m_n - m_p$.

Then from Eq. (1) we obtain

$$\begin{pmatrix} \rho'_{11} \\ \rho'_{22} \\ \rho'_{12} \\ \rho'_{21} \end{pmatrix} = \begin{pmatrix} 0 & 0 & +iscV & -iscV \\ 0 & 0 & -iscV & +iscV \\ +iscV & -iscV & -iM & 0 \\ -iscV & +iscV & 0 & +iM \end{pmatrix} \begin{pmatrix} \rho_{11} \\ \rho_{22} \\ \rho_{12} \\ \rho_{21} \end{pmatrix},$$

(2)

where a prime denotes $\partial/\partial \mu$ and $M = \delta m^2 \mu / (\omega E_\nu) + (s^2 - c^2) V$.

An analytical solution is not possible without drastic assumptions and, therefore, we have numerically explored the problem[3] using the Simpson method for integration and the fourth order Runge-Kutta algorithm for the solution of the differential equations.

The neutrino kinetics down to 2 MeV does not differ from the standard case; i.e., electron neutrinos maintain their equilibrium distribution, while sterile neutrinos are absent. So the initial condition for the neutrino ensembles in the interaction basis can be assumed to be of the form

$$\varrho = n_\nu^{eq} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

where $n_\nu^{eq} = \exp(-E_\nu/T)/[1 + \exp(-E_\nu/T)]$.

We have analyzed the evolution of nonequilibrium oscillating neutrinos by numerically integrating the kinetic equation (2) for the period after the electron neutrino decoupling until the freeze-out of the neutron-proton ratio (n/p ratio), i.e., for the temperature interval $[0.3, 2.0]$ MeV. The oscillation parameters range studied is $\delta m^2 \in [10^{-11}, 10^{-7}]$ eV^2 and $\vartheta \in [0, \pi/4]$.

The distributions of electrons and positrons were taken to be the equilibrium ones. Really, because of the enormous rates of the electromagnetic reactions of these particles, the deviations from equilibrium are negligible. We have also neglected the distortion of the neutrino spectra due to residual interactions between the electromagnetic and neutrino components of the plasma after 2 MeV. This distortion was accurately studied in [41], where it was shown that the relative correction to the ν_e density is less than 1% and the effect on the primordial helium abundance is negligible.

The neutron and proton number densities, used in the kinetic equations for neutrinos, were substituted from the numerical calculations in the CN code accounting for neutrino oscillations. I.e., we have simultaneously solved the equations governing the evolution of neutrino ensembles and

---

[3]For the case of vacuum neutrino oscillations this equation was analytically solved and the evolution of the density matrix was given explicitly in Ref. [28].

those describing the evolution of the nucleons (see the next section). The baryon asymmetry β, parametrized as the ratio of the baryon number density to the photon number density, was taken to be 3×10^{-10}.

Three *main effects of neutrino nonequilibrium oscillations* were revealed and precisely studied, namely, electron neutrino depletion, neutrino energy spectrum distortion, and the generation of asymmetry between neutrinos and their antiparticles.

(a) Depletion of ν_e population due to oscillations: As far as oscillations become effective when the number densities of ν_e are much greater than those of ν_s, $N_{\nu_e} \gg N_{\nu_s}$, the oscillations tend to reestablish the statistical equilibrium between different oscillating species. As a result N_{ν_e} decreases in comparison to its standard equilibrium value due to oscillations in favor of sterile neutrinos.[4] The effect of depletion may be very strong (up to 50%) for relatively large δm^2 and maximal mixing. This result of our study is in accordance with other publications concerning the depletion of the electron neutrino population due to oscillations, like [23]; however, we have provided a more precise account for this effect due to the accurate kinetic approach used.

In Figs. 1 the evolution of neutrino number densities is plotted. In Fig. 1(a) the curves represent the evolution of the electron neutrino number density in the discussed model with a fixed mass difference $\delta m^2 = 10^{-8}$ eV^2 and for different mixings. The numerical analysis showed that for small mixing, $\sin^2(2\vartheta) < 0.01$, the results do not differ from the standard case; i.e., then oscillations may be neglected. In Fig. 1(b) the evolution of the electron neutrino number density is shown for a nearly maximum mixing, $\sin^2(2\vartheta) = 0.98$, and different squared mass differences. Our analysis has proved that for mass differences $\delta m^2 < 10^{-11}$ eV^2, the effect of oscillations is negligible for any ϑ.

In the case of oscillations effective after the neutrino freeze-out, electron neutrinos are not in thermal contact with the plasma and, therefore, the electron neutrino state, depleted due to oscillations into steriles, cannot be refilled by electron-positron annihilations. That irreversible depletion of the ν_e population exactly equals the increase of the ν_s one [see Fig. 1(c)]. The number of the effective degrees of freedom does not change due to oscillations in that case, as far as the electron neutrino together with the corresponding sterile one contribute to the energy density of the Universe as one neutrino unit, even in the case when the steriles are brought into chemical equilibrium with ν_e. This fact was first noted in [23].[5]

(b) Distortion of the energy distribution of neutrinos: The effect was first discussed in [29] for the case of flavor neu-

---

[4]Note that while neutrinos are in thermal equilibrium with the plasma no dilution of their number density is expected as far as it is kept the equilibrium one due to the annihilations of the medium electrons and positrons.

[5]Note the essential difference from the case of electron neutrinos in thermal equilibrium, when the oscillations into sterile neutrinos bring an additional degree of freedom into thermal contact.

trino oscillations. However, as far as the energy distortion for that case was shown to be negligible [29,41], it was not paid the necessary attention it deserved. The distortion of the neutrino spectrum was not discussed in publications concerning active-sterile neutrino oscillations, and was thought to be negligible. In [28] it was first shown that for the case of $\nu_e \leftrightarrow \nu_s$ vacuum oscillations this effect is considerable and may even exceed that of an additional neutrino species. In [34] we have discussed this effect for the general case of neutrino oscillations in a medium. The evolution of the distortion is the following: Different momentum neutrinos begin to oscillate at different temperatures and with different amplitudes. First the low energy part of the spectrum is distorted, and later on this distortion concerns neutrinos with higher and higher energies. This behavior is natural, as far as neutrino oscillations affect first low energy neutrinos, $\Gamma_{osc} \sim \delta m^2 / E_\nu$. Figures 2(a), 2(b), 2(c) and 2(d) give a snapshot of the evolution of the energy spectrum distortion of active neutrinos, $x^2 \rho_{LL}(x)$, where $x = E_\nu / T$, for maximal mixing, and $\delta m^2 = 10^{-8.5}$ eV^2, at different temperatures: $T = 1$ MeV (a), $T = 0.7$ MeV (b), $T = 0.5$ MeV (c), and $T = 0.3$ MeV (d). As can be seen from the figures, the distortion down to temperatures of 1 MeV is not significant as far as the oscillations are not very effective. However, for lower temperatures the distortion increases and at 0.5 MeV is strongly expressed. Its proper account is important for the correct determination of oscillations role in the kinetics of n-p transitions during the freeze-out of nucleons at about 0.3 MeV.

Our analysis has shown that the account for the nonequilibrium distribution by shifting the effective temperature and assuming the neutrino spectrum of equilibrium form, often used in the literature (see, for example, [26]), may give misleading results for the case $\delta m^2 < 10^{-7}$ eV^2. The effect cannot be absorbed merely in shifting the effective temperature and assuming equilibrium distributions. For larger neutrino mass differences oscillations are fast enough and the naive account is more acceptable, provided that ν_e have not decoupled.

(c) The generation of asymmetry between ν_e and their antiparticles: The problem of asymmetry generation in different contexts was discussed by several authors. The possibility of an asymmetry generation due to CP-violating flavor oscillations was first proposed in Ref. [38]. Later estimations of an asymmetry due to CP-violating MSW resonant oscillations were provided [39]. The problem of asymmetry was considered in connection with the exploration of the neutrino propagation in the early Universe CP-odd plasma also in [20–24] and this type of asymmetry was shown to be negligible. Recently it was realized in [40,27] that asymmetry can grow to considerable values for the case of great mass differences, $\delta m^2 \gtrsim 10^{-5}$ eV^2. The effect of asymmetry for small mass differences $\delta m^2 \lesssim 10^{-7}$ eV^2 on the primordial production of helium was also proved to be important for the case of resonant neutrino oscillations [34]. Our approach allows a precise description of the asymmetry evolution, as far as working with the *self-consistent kinetic equations for neutrinos in momentum space* enables us to calculate the behavior
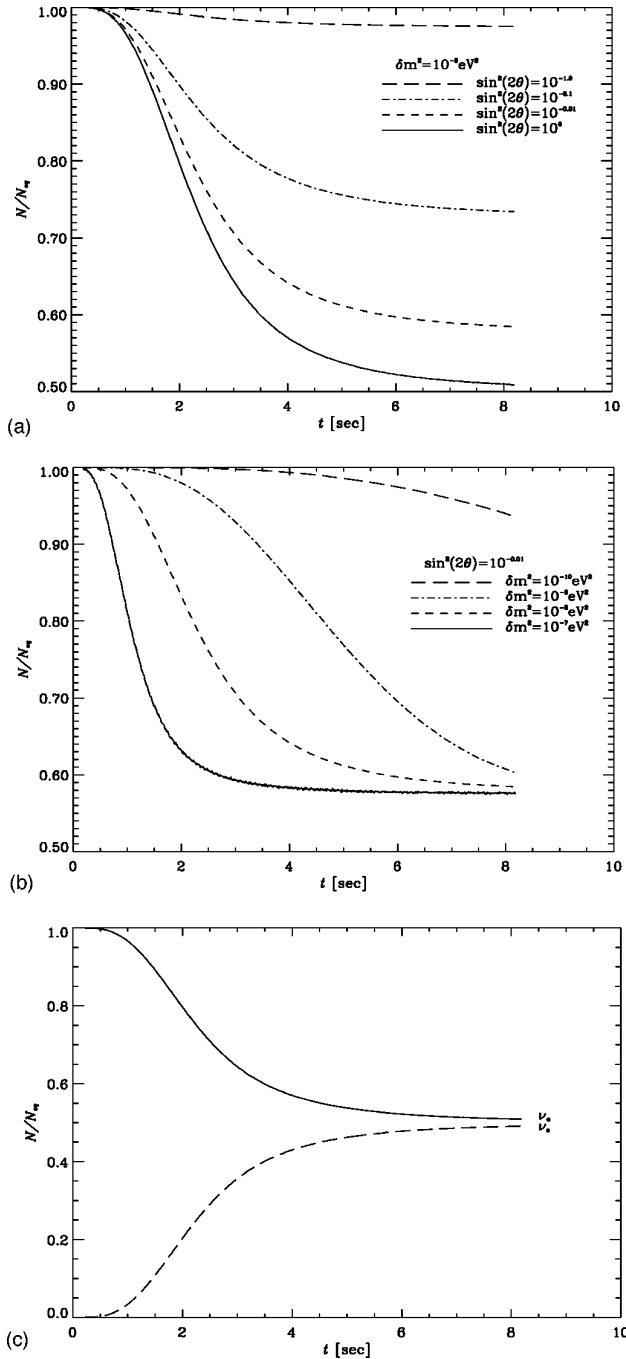
FIG. 1. The curves represent the calculated evolution of the electron neutrino number density in the discussed model of active-sterile neutrino oscillations with a mass difference $\delta m^2 = 10^{-8}$ eV^2 and different mixing, parametrized by $\sin^2(2\vartheta)$, namely, 1, $10^{-0.01}$, $10^{-0.1}$, and 0.1. (b) The curves show the evolution of the electron neutrino number density in the discussed model of nonresonant active-sterile neutrino oscillations for a nearly maximum mixing, $\sin^2(2\vartheta) = 0.98$, and different squared mass differences δm^2, namely, 10^{-7}, 10^{-8}, 10^{-9}, and 10^{-10} in eV^2. (c) The curves show the evolution of the electron neutrino number density (the solid curve) and the sterile neutrino number density (the dashed curve) in the case of nonresonant active-sterile neutrino oscillations for a maximal mixing and $\delta m^2 = 10^{-8}$ eV^2. The reduction of the active neutrino population is exactly counterbalanced by a corresponding increase in the sterile neutrino population.
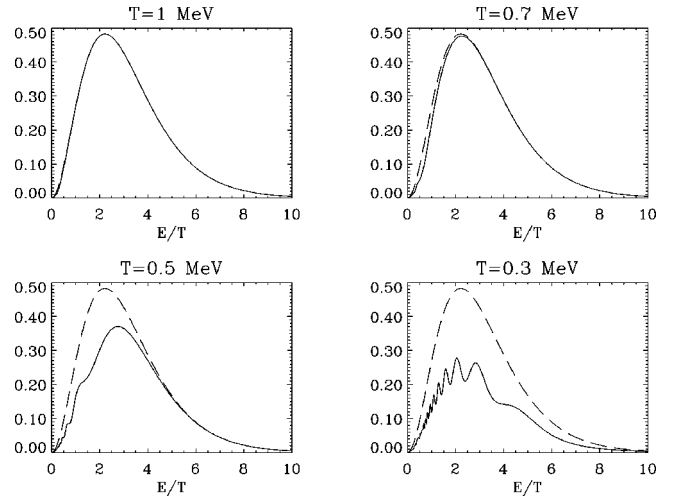

FIG. 2. The figures illustrate the evolution of the energy spectrum distortion of active neutrinos $x^2 \rho_{LL}(x)$, where $x = E_\nu/T$, for the case of nonresonant ν_e-ν_s oscillations with a maximal mixing and $\delta m^2 = 10^{-8.5}$ eV^2, at different temperatures: $T = 1$ MeV (a), $T = 0.7$ MeV (b), $T = 0.5$ MeV (c), and $T = 0.3$ MeV (d).

of the asymmetry at each momentum. This is important particularly when the distortion of the neutrino spectrum is considerable.

In the present work we have explored accurately the effect of the asymmetry in the nonresonant case for all mixing angles and for small mass differences $\delta m \lesssim 10^{-7}$ eV^2. Our analysis showed that when the lepton asymmetry is accepted initially as being equal to the baryon one [as is usually assumed for the popular $(L-B)$-conserving models], the effect of the asymmetry is small for the full parameters range of the model. And although the asymmetry is not wiped out by the coupled oscillations, as stated by some authors [21,24], *nonresonant* neutrino oscillations really *cannot generate* large neutrino-antineutrino asymmetry in the early Universe. This result is in accordance with the conclusions concerning asymmetry evolution in [22,24,34]. We have also checked that the neutrino asymmetry even in the case of an initial neutrino asymmetry by two orders of magnitude higher does not have a significant effect on the cosmologically produced ^{4}He. Therefore, for such small initial values of the lepton asymmetry, the neutrino asymmetry should be better neglected when calculating primordial element production for the sake of computational time. Mind, however, that for higher values of the initial asymmetry the effect could be significant, and should be studied in detail. The asymmetry evolution and its effect on ^{4}He production for unusually high initial values of the lepton asymmetry will be studied elsewhere [46].

In conclusion, our numerical analysis showed that the nonequilibrium oscillations can considerably deplete the number densities of electron neutrinos (antineutrinos) and distort their energy spectrum.

IV. NUCLEOSYNTHESIS WITH NONEQUILIBRIUM OSCILLATING NEUTRINOS

As an illustration of the importance of these effects, and hence of the proposed approach to the analysis of nonequi-

librium neutrino oscillations, it is useful to discuss their influence on the primordial production of ^{4}He. The effect of oscillations on other light elements like deuterium and lithium-7 is expected not to be considerable in the sense that it lies within the nowadays observational uncertainties for these elements. So in what follows we discuss in detail ^{4}He production in a modified CN model with nonresonant nonequilibrium neutrino oscillations. The effect of oscillations on helium-4 has been discussed in numerous publications [20–28,36,38,39]. A detail kinetic calculation of the primordial yield of helium for the case of the nonequilibrium oscillations in vacuum was made in [28] and the proper consideration accounting for neutrino forward scattering processes off the background particles was done in [34] for some neutrino mixing parameters.[6] In the present work the analysis of [34] is expanded and completed for the full space of the mixing parameters values.

The numerical analysis is performed working with exact kinetic equations for the nucleon number densities and neutrino density matrix in momentum space. This enabled us to analyze the direct influence of oscillations on the kinetics of neutron-to-proton transfers and to account precisely for the neutrino depletion, neutrino energy distortion, and the gen-

eration of asymmetry due to oscillations.

Primordial element abundances depend primarily on the neutron-to-proton ratio at the weak freeze-out [$(n/p)_f$ ratio] of the reactions interconverting neutrons and protons: $n + \nu_e \leftrightarrow p + e$ and $n + e^+ \leftrightarrow p + \tilde{\nu}_e$. The freeze-out occurs when due to the decrease of temperature with Universe expansion these weak interaction rates $\Gamma_w \sim E_\nu^2 n_\nu$ become comparable and less than the expansion rate $H \sim \sqrt{g}\, T^2$. Hence, the $(n/p)_f$ ratio depends on the effective relativistic degrees of freedom g (through the expansion rate) and the neutrino number densities and neutrino energy distribution (through the weak rates). Therefore, we calculate accurately the evolution of the neutron number density until its freeze-out. Further evolution is due to the neutron decays $n \rightarrow p + e + \tilde{\nu}_e$ that proceed until the effective synthesis of deuterium begins. As far as the expansion rate exceeds considerably the decay rate for the characteristic period before the freeze-out, decays are not essential. Therefore, we have accounted for them adiabatically.

The master equation, describing the evolution of the neutron number density in momentum space n_n for the case of oscillating neutrinos $\nu_e \leftrightarrow \nu_s$, reads

$$(\partial n_n/\partial t) = H p_n (\partial n_n/\partial p_n) + \int \mathrm{d}\Omega(e^-,p,\nu)|\mathcal{A}(e^- p \rightarrow \nu n)|^2 [n_e - n_p(1-\rho_{LL}) - n_n \rho_{LL}(1-n_{e^-})]$$

$$- \int \mathrm{d}\Omega(e^+,p,\tilde{\nu})|\mathcal{A}(e^+ n \rightarrow p \tilde{\nu})|^2 [n_{e^+} n_n(1-\bar{\rho}_{LL}) - n_p \bar{\rho}_{LL}(1-n_{e^+})]. \tag{3}$$

The first term on the right-hand side describes the effect of expansion while the next ones the processes $e^- + p \leftrightarrow n + \nu_e$ and $p + \tilde{\nu}_e \leftrightarrow e^+ + n$, directly influencing the nucleon density. It differs from the standard scenario one only by the substitution of ρ_{LL} and $\bar{\rho}_{LL}$ instead of $n_\nu^{eq} = [1 + \exp(E_\nu/T)]^{-1}$. The neutrino and antineutrino density matrices differ, $\bar{\rho}_{LL} \neq \rho_{LL}$, contrary to the standard model, as a result of the different reactions with the CP-odd plasma of the prenucleosynthesis epoch. We have accounted for final state Pauli blocking for neutrinos and electrons.

Particle number densities per unit volume are expressed as $N = (2\pi)^{-3} \int \mathrm{d}^3 p\; n(p)$. Performing the integration also on the right-hand side of the equation one gets the final equations for the time evolution of the neutron number density:

$$(\partial N_n/\partial t) = -3H N_n + G_F^2 \frac{g_V^2 + 3g_A^2}{\pi^3} T^5 \left\{ N_p \int_0^\infty [1 - \rho_{LL}(x)] \frac{\mathrm{e}^{-x-y}}{1 + \mathrm{e}^{-x-y}} f(x,y)\mathrm{d}x \right.$$

$$- N_n \int_0^\infty \rho_{LL}(x) \frac{1}{1 + \mathrm{e}^{-x-y}} f(x,y)\mathrm{d}x - N_n \int_{(1+\zeta)y}^\infty [1 - \bar{\rho}_{LL}(x)] \frac{\mathrm{e}^{-x+y}}{1 + \mathrm{e}^{-x+y}} f(x,-y)\mathrm{d}x$$

$$\left. + N_p \int_{(1+\zeta)y}^\infty \bar{\rho}_{LL}(x) \frac{1}{1 + \mathrm{e}^{-x+y}} f(x,-y)\mathrm{d}x \right\}, \tag{4}$$

---

[6]Calculations of helium production within the full big bang nucleosynthesis code with oscillations were provided also in [26]; however, there the momentum degree of freedom of neutrinos was not considered and a simplifying account of the nonequilibrium was used—by merely shifting the neutrino effective temperature and working in terms of equilibrium particle densities.
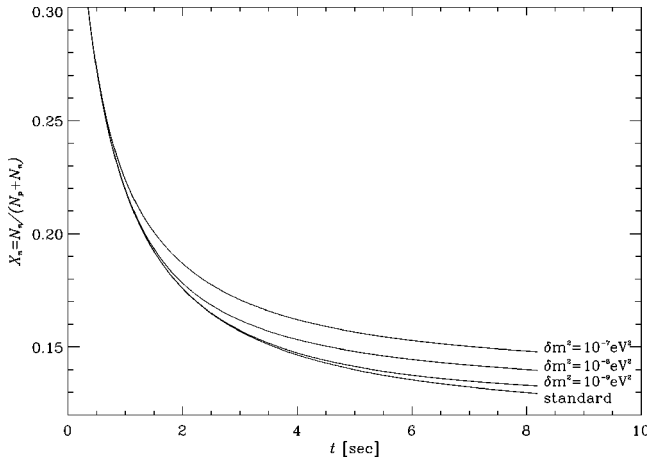
FIG. 3. The evolution of the neutron number density relative to nucleons, $X_n(t) = N_n(t)/(N_p + N_n)$, for the case of nonresonant oscillations with maximal mixing and different δm^2 is shown. For comparison the standard model curve is plotted also.

where $f(x,y) = x^2(x+y)\sqrt{(x+y)^2 + \zeta^2 y^2}$ and $y = (\Delta + m_e)/T$, $\zeta = m_e/\Delta$, $\Delta = m_n - m_p$.

The first term on the right-hand side describes the dilution effect of expansion; the next ones describe the weak processes, as pointed out above. We have numerically integrated this equation for the temperature range of interest, $T \in [0.3, 2.0]$ MeV, for the full range of oscillation parameters of our model. The value of $\rho_{LL}(x)$ at each integration step was taken from the simultaneously performed integration of the set of equations (2); i.e., the evolution of neutrino and the nucleons was followed self-consistently. The initial values at $T = 2$ MeV for the neutron, proton, and electron number densities are their equilibrium values. Although the electron mass is comparable with the temperature in the discussed temperature range, the deviation of the electron density from its equilibrium value is negligible due to the enormous rate of the reactions with the plasma photons [29]. The parameters values of the CN model, adopted in our calculations, are the following: the mean neutron lifetime is $\tau = 887$ sec, which corresponds to the present weighted average value [47]; the effective number of relativistic flavor types of neutrinos during the nucleosynthesis epoch, N_ν, is assumed to be equal to the standard value 3. This is a natural choice as far as it is in good agreement both with the CN arguments [48],[7] and with the precision measurements of the Z decay width at LEP [15].

V. RESULTS AND CONCLUSIONS

The results of the numerical integration are illustrated in Fig. 3. As can be seen from the figure kinetic effects (neutrino population depletion and distortion of the neutrino spectrum) due to oscillations play an important role and lead

---

[7]However, mind also the possibilities for a somewhat relaxation of that kind of bound in modifications of the CN model with decaying particles as in [49,50].
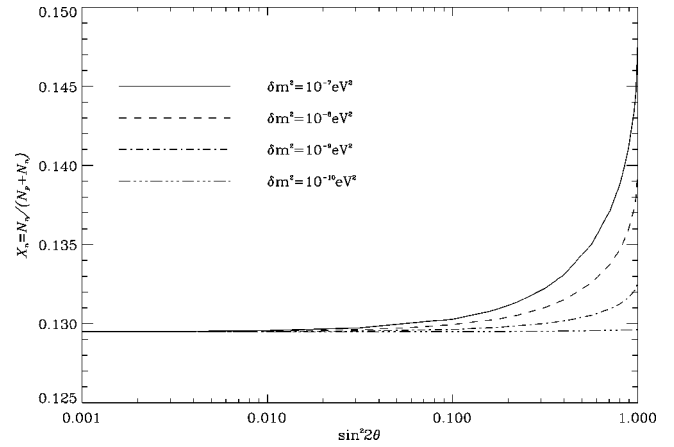

FIG. 4. The figure illustrates the dependence of the frozen neutron number density relative to nucleons, $X_n = N_n/(N_p + N_n)$, on the mixing angle for different δm^2.

to a considerable overproduction of helium.

Qualitatively the effect of oscillations on helium production can be described as follows.

The depletion of the electron neutrino number densities due to oscillations into sterile ones strongly affects the $n \leftrightarrow p$ reactions rates. It leads to an effective decrease in the processes rates, and hence to an increase of the freezing temperature of the n/p ratio and the corresponding overproduction of the primordially produced ^{4}He.

The effect of the distortion of the energy distribution of neutrinos has two aspects. On the one hand, an average decrease of the energy of active neutrinos leads to a decrease of the weak reactions rate, $\Gamma_w \sim E_\nu^2$, and subsequently to an increase in the freezing temperature and the produced helium. On the other hand, there exists an energy threshold for the reaction $\tilde{\nu}_e + p \rightarrow n + e^+$. And in the case when, due to oscillations, the energy of the relatively greater part of neutrinos becomes smaller than that threshold the n/p freezing ratio decreases, leading to a corresponding decrease of the primordially produced helium-4 [50]. The numerical analysis showed that the latter effect is less noticeable compared with the former ones.

The asymmetry calculations showed a slight predominance of neutrinos over antineutrinos, not leading, however, to a noticeable effect on the production of helium in case the lepton asymmetry is accepted initially equal to the baryon one. So the effect of asymmetry is proved to be negligible for all the discussed parameter range, i.e., for any ϑ and for $\delta m^2 \lesssim 10^{-7}$ eV^2. We have partially (not for the full range of model parameters) investigated the problem for the initial lepton asymmetry being higher than the baryon one. The preliminary results point out that even the lepton asymmetry, initially by two orders of magnitude higher, does not have a noticeable effect on the cosmologically produced ^{4}He. Lepton asymmetries higher than those, however, should be accounted for properly even in the *nonresonant* case.

Thus, the total result of nonequilibrium neutrino oscillations is an overproduction of helium in comparison to the standard value.

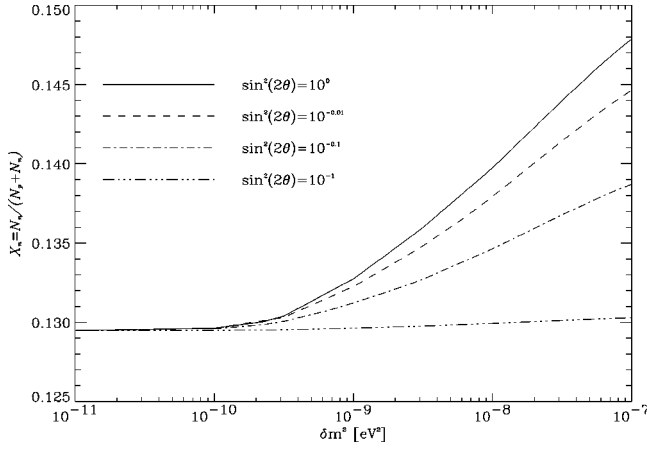In Fig. 4 the dependence of the frozen neutron number

FIG. 5. The figure illustrates the dependence of the frozen neutron number density relative to nucleons, $X_n = N_n/(N_p+N_n)$, on the mass difference for different mixing angles.

FIG. 7. On the δm^2-ϑ plane some of the constant helium contours calculated in the discussed model of cosmological nucleosynthesis with nonresonant neutrino oscillations are shown.

density relative to nucleons, $X_n = N_n/(N_p+N_n)$, on the mixing angle for different fixed δm^2 is illustrated. The dependence of the frozen neutron number density relative to nucleons, $X_n = N_n/(N_p+N_n)$ on the δm^2 for different fixed mixing angles is presented in Fig. 5. The effect of oscillations is maximal at maximal mixing for the nonresonant case of neutrino oscillations. As can be seen from the figures, it becomes almost negligible (less than 1%) for mixings as small as 0.1 for any δm^2 of the discussed range of our model. The value of the frozen n/p ratio is a smoothly increasing function of the mass difference. Our analysis shows that the effect of oscillations for δm^2 smaller than 10^{-10} eV^2 even for maximal mixing is smaller than 1%. The nonresonant oscillations with $\delta m^2 \lesssim 10^{-11}$ eV^2 do not have any observable effect on the primordial production of elements; i.e., the results coincide with the standard model values with great accuracy.
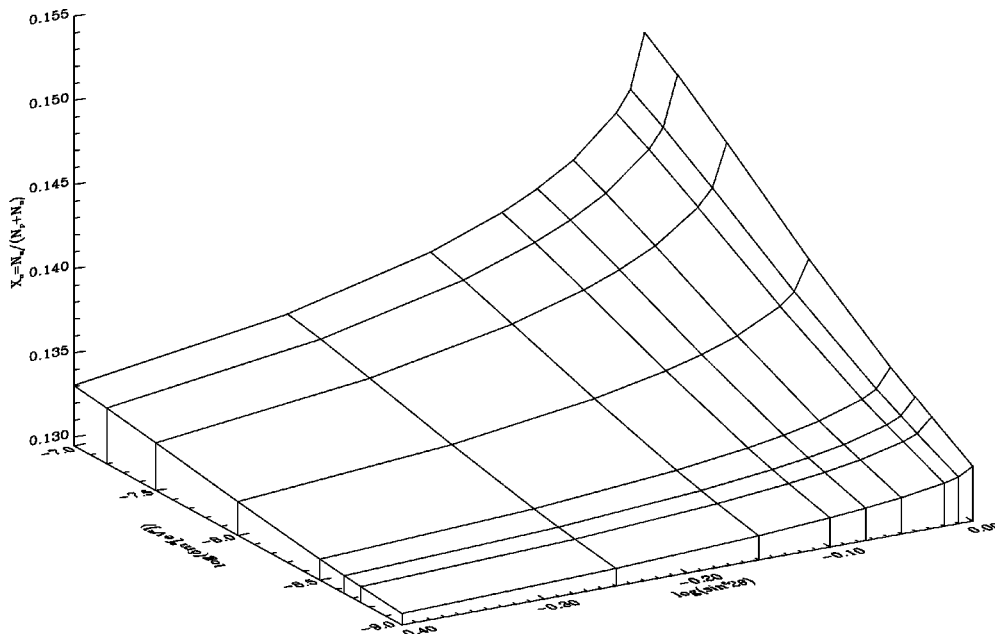
From the numerical integration for the full range of different oscillation parameters we have obtained the primordial helium yields $Y_p(\delta m^2,\vartheta)$, which is illustrated by the surface in Fig. 6. Some of the constant helium contours calculated in the discussed model of cosmological nucleosynthesis with nonresonant neutrino oscillations on the δm^2-ϑ plane are presented in Fig. 7.

On the basis of these results, requiring an agreement between the theoretically predicted and the observational values of helium, it is possible to obtain cosmological constraints on the neutrino mixing parameters. At present the primordial helium values extracted from observations differ considerably: for example, some authors believe that the systematic errors have already been reduced to about the same level as the statistical one and obtain the bounds for primordial helium, $Y_p(^4\text{He})=0.232\pm0.003$ [51], while others argue that underestimation of the systematic errors, such as

FIG. 6. The dependence of the primordially produced helium on the oscillation parameters is represented by the surface $Y_p(\delta m^2,\vartheta)$.
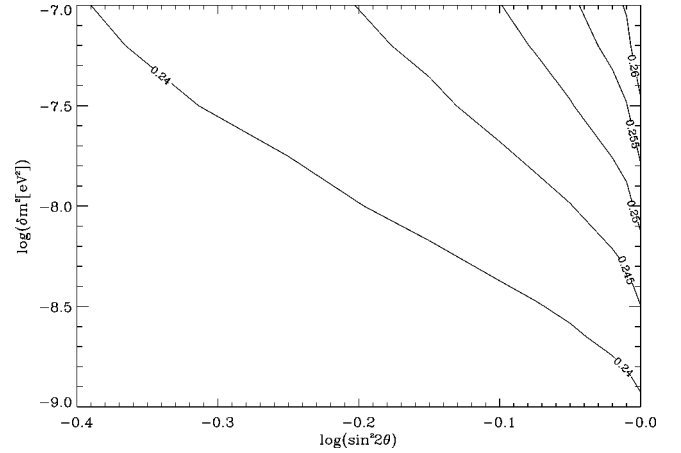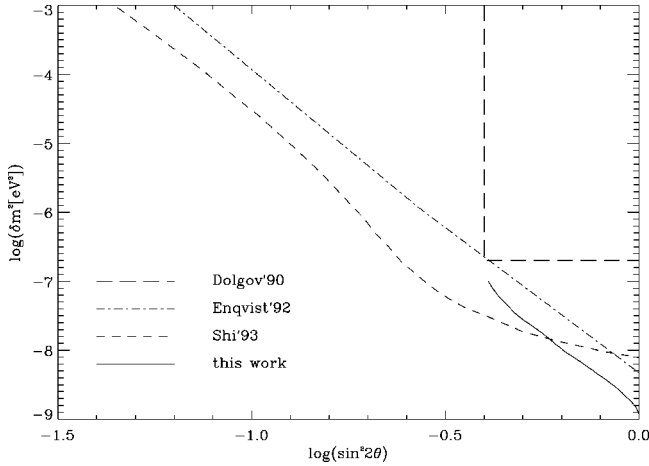
FIG. 8. The curves, corresponding to a helium abundance Y_p = 0.24, obtained in the present work and in previous works, analyzing the nonresonant active-sterile neutrino oscillations, are plotted on the δm^2-ϑ plane.

errors in helium emissivities, inadequaticies in the radiative transfer model used, corrections for underlying stellar absorption and fluorescent enhancement in the He I lines, and corrections for neutral helium, may be significant and their account may raise the upper bound on Y_p as high as 0.26 [52]. Thus in addition to the widely adopted "classical" bound $Y_p < 0.24$ [53] it is reasonable to have in mind the more "reliable" upper bound to the primordial helium abundance $Y_p < 0.25$ and even an extreme value as high as 0.26 [54]. Therefore, we considered it useful to provide precise calculations for helium contours up to 0.26. So whatever the primordial abundance of ^{4}He will be found to be in the future (within this extreme range) the results of our calculations may provide the corresponding bounds on mixing parameters of neutrino for the case of nonresonant active-sterile oscillations with small mass differences. Assuming the conventional observational bound on primordial ^{4}He $Y_p = 0.24$, the cosmologically excluded region for the oscillation parameters is shown on the plane $\sin^2(2\vartheta)$-δm^2 in Fig. 7. It is situated to the right of the $Y_p = 0.245$ curve, which gives 5% overproduction of helium in comparison with the standard value.

The curves, corresponding to a helium abundance Y_p = 0.24, obtained in the present work and in previous works, analyzing the nonresonant active-sterile neutrino oscillations, are plotted in Fig. 8. In [20] and [23] the authors estimated the effect of the excitement of an additional degree of freedom due to oscillations, and the corresponding increase of the Universe expansion rate, leading to an overproduction of helium-4. The excluded regions for the neutrino mixing parameters were obtained from the requirement that the neutrino types should be less than 3.4: $N_\nu < 3.4$. In these works the depletion effect was considered. The asymmetry was neglected and the distortion of the neutrino spectrum was not studied as far as the kinetic equations for neutrino mean number densities were considered. Our results are in good accordance with the estimations in [20] and the numerical analysis in [23], where a very successful account was made

for one of the discussed effects of nonequilibrium oscillations: the neutrino population depletion. The results of [26], as can be seen from Fig. 8, differ more both from the ones of the previously cited works and from our results. Probably the reason for that is the simplified account for nonequilibrium oscillations merely by shifting the effective neutrino temperature, not acceptable for a large range of model parameters.

As can be seen from the curves, for large mixing angles we exclude $\delta m^2 \geqslant 10^{-9}$ eV^2, which is almost an order of magnitude stronger constraint than that previously existing. This more stringent constraint obtained in our work for the region of great mixing angles and small mass differences is due to the more accurate kinetic approach we have used and to the precise account of neutrino depletion and energy distortion due to oscillations.

As far as we already have at our disposal some impressive indications for neutrino oscillations, it is interesting to compare our results also with the range of parameters which could eventually explain the observed neutrino anomalies:

The vacuum oscillation interpretation of the solar neutrino problem requires extremely small mass differences squared, less than and of the order of 10^{-10} eV^2. It is safely lower than the excluded region, obtained in our work and is, therefore, allowed from CN considerations. The MSW small mixing angle nonadiabatic solution (see, for example, Krastev, Liu, and Petcov in [11]) is out of the reach of our model. However, as we are in a good accordance with the results of active-sterile neutrino oscillation models with higher mass differences, it is obvious that a natural extrapolation of our excluded zone towards higher mass differences will rule out partially the possible solution range for large mixing angles.

Our pattern of neutrino mixing is compatible with models of degenerate neutrino masses of the order of 2.4 eV, necessary for the successful modeling of the structure formation of the Universe in hot plus cold dark matter models [12].

In connection with the increasing interest in problems of primordial deuterium in recent years it is relevant to comment on its production within the discussed modified CN model. According to our model oscillations change only the neutron-to-proton ratio. Neither the cosmic expansion rate nor the nucleon-to-photon ratio are influenced, which represent the most important parameters for the yields of deuterium. Similar is the situation for the primordial production of lithium-7 also. Therefore, the expected change in the yields of these light elements must not be considerable.[8] The preliminary estimations we made show that the expected increase in deuterium, for example, is safely within its observational uncertainties. As far as helium-4 is the most precisely calculated and measured light element, it is the

___

[8]However, their deviation from the standard values may be also due to the change in the neutron and proton number densities in comparison to the standard ones, which leads to some change of the nuclear reactions rates governing the synthesis and destruction of these elements. The exact account of this effect requires a detail numerical treatment of the problem, using the complete big bang nucleosynthesis code, which is beyond the aim of this work.

most reliable and convenient one for studying the allowed range of the oscillations parameters values. The uncertainties in the values of D and Li extracted from observations are much greater (by an order of magnitude) than the ones for helium-4. Therefore in that work we provide a precise numerical study of the oscillation effect upon helium-4, which allows us to obtain a reliable cosmological constraint on the oscillations parameters of the discussed model. However, in case the uncertainties in the other light elements could be considerably decreased in future, it will be interesting to perform a precise numerical calculation of their production within the modified CN model with active-sterile neutrino oscillations.

As a conclusion, we would like to outline the main achievements of this work: In a model of nonequilibrium nonresonant active-sterile oscillations, we studied the effect of oscillations on the evolution of the neutrino number densities, neutrino spectrum distortion, and neutrino-antineutrino asymmetry. We have used kinetic equations for the density matrix of neutrinos in *momentum* space, accounting *simultaneously* for expansion, oscillations, and interactions with the medium. This approach enabled us to describe precisely the behavior of neutrino ensembles in the early Universe in the period of interest for CN. A complete numerical analysis was provided for small mass differences $\delta m^2 \leqslant 10^{-7}$ eV^2 and all mixing angles. We have shown that the energy distortion may be significant, while the asymmetry in the case it is initially (i.e., before oscillations become effective) of the order of the baryon one, may be neglected.

Next, we have made a complete, precise survey of the influence of the discussed type of oscillations on the cosmological production of helium-4. We have calculated the evolution of the corresponding neutron-to-proton ratio from the time of the freeze-out of neutrinos at 2 MeV until the effective freeze-out of nucleons at 0.3 MeV for the full range of model parameters. As a result we have obtained the dependence $Y_p(\delta m^2, \vartheta)$ and constant helium contours on the δm^2-ϑ plane. Requiring an agreement between the observational and the theoretically predicted primordial helium abundances, we have calculated accurately the excluded regions for the neutrino mixing parameters, for different assumptions about the preferred primordial value of helium in the range 0.24–0.26.

[1] G. Gamow, Ohio. J. Sci **35**, 406 (1935); J. Wash. Acad. Sci. **32**, 353 (1942); Phys. Rev. **70**, 572 (1946).

[2] A. M. Boesgaard and G. Steigman, Annu. Rev. Astron. Astrophys. **23**, 319 (1985); R. A. Malaney and G. J. Mathews, Phys. Rep. **229**, 145 (1993); M. S. Smith, L. H. Kawano, and R. A. Malaney, Astrophys. J., Suppl. Ser. **85**, 219 (1993); B. Pagel, in *Nucleosynthesis and Chemical Evolution of Galaxies* (Cambridge University Press, Cambridge, England, 1997), Chap. 4; K. A. Olive, E. Skillman, and G. Steigman, Astrophys. J. **483**, 788 (1997); C. J. Hogan, in *Critical Dialogues in Cosmology*, edited by N. Turok (World Scientific, Singapore, 1997), p. 50; G. Steigman, in *Proceedings of the Conference on Critical Dialogues in Cosmology*, Princeton, NJ, 1996, edited by N. Turok (World Scientific, Singapore, 1997), p. 63; S. Sarkar, Rep. Prog. Phys. **59**, 1493 (1996), and the references therein.

[3] B. Pontecorvo, Sov. Phys. JETP **6**, 431 (1958); **7**, 172 (1958); **26**, 984 (1968).

[4] J. Ellis, in *Neutrino '96*, Proceedings of the 17 International Conference on Neutrino Physics and Astrophysics, Helsinki, Finland, 1996, edited by K. Enqvist *et al.* (World Scientific, Singapore, 1997), p. 541; A. D. Dolgov, Nucl. Phys. B (Proc. Suppl.) **48B**, 5 (1996); J. W. F. Valle, *ibid.* **48B**, 137 (1996): A. Yu. Smirnov, in *ICHEP '96*, Proceedings of the 28 International Conference on High Energy Physics Warsaw, Poland,

1996, edited by Z. Ajduk and A. Wroblewski (World Scientific, Singapore, 1997); hep-ph/9611465; P. Elmfors *et al.*, Nucl. Phys. **B503**, 3 (1997), and the references therein; K. Kainulainen, in *Neutrino 96* (this reference), 1996, p. 461, Singapore, and the references therein; N. Hata and P. Langacker, Phys. Rev. D **56**, 6107 (1997); K. Enqvist, in *New Physics with New Experiments*, edited by Z. Ajduk, S. Pokorski, and A. Wroblewski (World Scientific, Singapore, 1994), p. 380.

[5] Homestake Collaboration, K. Lande, in *Neutrino 96* [4], p. 25; R. Davis, Prog. Part. Nucl. Phys. **32**, 13 (1994); Homestake Collaboration, B. T. Cleveland *et al.*, Nucl. Phys. B (Proc. Suppl.) **38B**, 47 (1995); S. T. Petcov, *ibid.* **43B**, 12 (1995); J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989); J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **64**, 885 (1992); GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. B **388**, 384 (1996); GALLEX Collaboration, T. Kirsten, in *Neutrino 96* [4], p. 3; P. Anselmann *et al.*, Phys. Lett. B **327**, 377 (1994); **357**, 237 (1995); **342**, 440 (1995); SAGE Colloration, V. N. Gavrin, in *Neutrino 96* [4], p. 14; SAGE Collaboration, J. N. Abdurashitov *et al.*, Phys. Rev. Lett. **77**, 4708 (1996); Phys. Lett. B **328**, 234 (1994); Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **77**, 1683 (1996); Kamiokande Collaboration, Y. Suzuki, Nucl. Phys. **B38**, 54 (1995).

[6] Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Lett. B

335, 237 (1994); Kamiokande Collaboration, K. S. Hirata *et al.*, *ibid.* **205**, 416 (1988); **280**, 146 (1992); IBM Collaboration, D. Casper *et al.*, Phys. Rev. Lett. **66**, 2561 (1991); IBM Collaboration, R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992); Nucl. Phys. B (Proc. Suppl.) **38B**, 331 (1995); Soudan-2 Collaboration, W. W. M. Allison *et al.*, Phys. Lett. B **391**, 491 (1997).

[7] LSND Collaboration, C. Athanassopoulos *et al.*, Phys. Rev. Lett. **75**, 2650 (1995); **77**, 3082 (1996); Phys. Rev. C (to be published), nucl-ex/9706006; J. E. Hill, Phys. Rev. Lett. **75**, 2654 (1995).

[8] J. N. Bahcall, in *Proceedings of the 18th Texas Symposium on Relativistic Astrophysics*, Chicago, 1996, edited by A. Olinto, J. Frieman, and D. Schramm (World Scientific, Singapore, 1997).

[9] V. S. Berezinsky *et al.*, Phys. Lett. B **365**, 185 (1996); B. Ricci *et al.*, *ibid.* **407**, 155 (1997); J. N. Bahcall and M. H. Pinsonneault, Rev. Mod. Phys. **67**, 781 (1995); N. Hata and P. Langacker, Phys. Rev. D **50**, 632 (1994); J. N. Bahcall, Phys. Lett. B **338**, 276 (1994).

[10] S. P. Mikheyev and A. Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985); Nuovo Cimento C **9**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[11] S. T. Petcov, in *Neutrino 96* [4], p. 106; P. I. Krastev, Q. Y. Liu, and S. T. Petcov, Phys. Rev. D **54**, 7057 (1996); P. I. Krastev and S. T. Petcov, *ibid.* **53**, 1665 (1996); Q. Y. Liu and S. T. Petcov, *ibid.* **56**, 7392 (1997); P. I. Krastev and S. T. Petcov, Nucl. Phys. **B449**, 605 (1995); E. Calabresu *et al.*, Astropart. Phys. **4**, 159 (1995); Z. G. Berezhiani and A. Rossi, Phys. Rev. D **51**, 5229 (1995); A. Dar and G. Shaviv, Astrophys. J. **468**, 933 (1996); N. Hata and P. Langacker, Phys. Rev. D **52**, 420 (1995); **50**, 632 (1994); P. I. Krastev and A. Yu. Smirnov, Phys. Lett. B **338**, 282 (1994).

[12] J. R. Primack *et al.*, Phys. Rev. Lett. **74**, 2160 (1995); J. R. Primack, in *Critical Dialogues in Cosmology*, edited by N. Turok (World Scientific, Singapore, 1997), p. 535; D. G. Lee and R. N. Mohapatra, Phys. Lett. B **329**, 463 (1994); D. O. Caldwell and R. N. Mohapatra, Phys. Rev. D **50**, 3477 (1994); D. O. Caldwell and R. N. Mohapatra, *ibid.* **48**, 3259 (1993); J. A. Holtzman and J. R. Primack, Astrophys. J. **405**, 428 (1993); A. N. Taylor and M. Rowan-Robinson, Nature (London) **359**, 396 (1992); M. Davis *et al.*, *ibid.* **359**, 393 (1992); E. L. Wright *et al.*, Astrophys. J. Lett. **396**, L13 (1992).

[13] M. Drees, in *Neutrino 96* [4], p. 290, hep-ph/9703260.

[14] R. N. Mohapatra, hep-ph/9702229; P. Binetruy *et al.*, Nucl. Phys. **B496**, 3 (1997); N. Okada and O. Yasuda, Int. J. Mod. Phys. A **12**, 3669 (1997); Phys. Rev. D **54**, 4432 (1996); S. M. Bilenky, *ibid.* **54**, 1881 (1996); **54**, 4432 (1996), and the references therein; H. Minakata, hep-ph/9612259; H. Minakata, Phys. Rev. D **52**, 6630 (1995); H. Minakata, Phys. Lett. B **356**, 61 (1995); G. M. Fuller *et al.*, Phys. Rev. D **52**, 1288 (1995); E. Ma and J. Pantaleone, *ibid.* **52**, R3763 (1995); R. Foot and R. R. Volkas, *ibid.* **52**, 6595 (1995); Z. G. Berezhiani and R. N. Mohapatra, *ibid.* **52**, 6607 (1995); E. J. Chun *et al.*, Phys. Lett. B **357**, 608 (1995); S. T. Petcov and A. Yu. Smirnov, *ibid.* **322**, 109 (1994); A. S. Joshipura, Z. Phys. C **64**, 31 (1994); Note, however, that in case the zenith angle dependence reported by Kamioka is not a real effect, as proposed by the IBM Collaboration [R. Clark *et al.*, Phys. Rev. Lett. **79**,

2754 (1997); **79**, 345 (1997)] and the SuperKamiokande preliminary results, the atmospheric δm^2 solution may be compatible with the LSND solution; i.e., oscillations between the three flavor neutrinos could explain all the existing anomalies: G. Fogli, E. Lisi, and G. Scioscia, Phys. Rev. D **56**, 3081 (1997); C. Cardall and G. Fuller, *ibid.* **53**, 4421 (1996).

[15] V. F. Shvartsman, JETP Lett. **9**, 184 (1969); B. Adeva *et al.*, Phys. Lett. B **231**, 509 (1989); The LEP Collaborations, D. Abbaneo *et al.*, Report No. CERN-PPE/96-183, 1996.

[16] J. Hewett and T. Rizzo, Phys. Rep. **183**, 195 (1989).

[17] E. Ma and P. Roy, Phys. Rev. D **52**, 4780 (1995); E. Ma, Phys. Lett. B **380**, 286 (1996).

[18] J. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); R. Mohapatra and S. Pati, *ibid.* **11**, 2558 (1975).

[19] E. Chun, A. Joshipura, and A. Smirnov, Phys. Lett. B **357**, 371 (1995); Phys. Rev. D **54**, 4654 (1996).

[20] R. Barbieri and A. Dalgov, Phys. Lett. B **237**, 440 (1990).

[21] K. Kainulainen, Phys. Lett. B **244**, 191 (1990).

[22] R. Barbieri and A. Dalgov, Nucl. Phys. **B349**, 743 (1991).

[23] K. Enqvist, K. Kainulainen, and M. Thomson, Nucl. Phys. **B373**, 498 (1992).

[24] K. Enqvist, K. Kainulainen, and J. Maalampi, Nucl. Phys. **B349**, 754 (1991); Phys. Lett. B **249**, 531 (1990).

[25] J. M. Cline, Phys. Rev. Lett. **68**, 3137 (1992).

[26] X. Shi, D. N. Schramm, and B. D. Fields, Phys. Rev. D **48**, 2563 (1993).

[27] X. Shi, Phys. Rev. D **54**, 2753 (1996).

[28] D. P. Kirilova, Report No. JINR E2-88-301, 1988.

[29] A. D. Dolgov, Sov. J. Nucl. Phys. **33**, 700 (1981).

[30] P. G. Langacker *et al.*, Nucl. Phys. **B266**, 669 (1986); M. G. Savage *et al.*, Astrophys. J. **368**, 1 (1991).

[31] H. Weldon, Phys. Rev. D **26**, 2789 (1982); P. B. Pal and T. N. Pham, *ibid.* **40**, 259 (1989); J. F. Nieves, *ibid.* **40**, 866 (1989); N. P. Lausman and Ch. van Weert, Phys. Rep. **145**, 141 (1987).

[32] P. B. Pal and T. N. Pham, Phys. Rev. D **40**, 259 (1989); see also Langacker *et al.* [39]; L. Wolfenstein, Phys. Rev. D **20**, 2634 (1979); P. Langacker, J. Leveille, and J. Sheiman, *ibid.* **27**, 1228 (1983); V. Barger *et al.*, *ibid.* **22**, 2718 (1980).

[33] D. Nötzold and G. Raffelt, Nucl. Phys. **B307**, 924 (1988); R. A. Harris and L. Stodolsky, Phys. Lett. **78B**, 313 (1978); **116B**, 464 (1982); L. Stodolsky, Phys. Rev. D **36**, 2273 (1987).

[34] D. P. Kirilova and M. V. Chizhov, Phys. Lett. B **393**, 375 (1997); in *Neutrino 96* [4], p. 478.

[35] G. Sigl and G. Raffelt, Nucl. Phys. **B406**, 423 (1993).

[36] V. B. Semikoz and J. W. F. Valle, Nucl. Phys. **B425**, 651 (1994); **B485**, 545(E) (1997); **B425**, 65 (1994).

[37] D. Fargion and M. Shepkin, Phys. Lett. **146B**, 46 (1984); Stodolsky [33]; P. Langacker, Pennsylvania University, Report No. UPR 0401T 1989.

[38] M. Yu. Khlopov and S. T. Petcov, Phys. Lett. **99B**, 117 (1981); **100B**, 520(E) (1981).

[39] P. Langacker, S. Petcov, G. Steigman, and S. Toshev, Nucl. Phys. **B282**, 589 (1987); Wolfenstein [10]; Mikheyev and Smirnov [10]; T. K. Kuo and J. Panteleone, Rev. Mod. Phys. **61**, 937 (1989).

[40] R. Foot, M. J. Thomson, and R. R. Volkas, Phys. Rev. D **53**,

R5349 (1996); R. Foot and R. R. Volkas, *ibid.* **55**, 5147 (1997).

[41] A. D. Dolgov, S. H. Hansen, and D. V. Semikoz, Nucl. Phys. **B503**, 426 (1997); see also previous works on that theme and references therein: S. Hannestad and J. Madsen, Phys. Rev. D **52**, 1764 (1995); A. D. Dolgov and M. Fukugita, JETP Lett. **56**, 123 (1992); Phys. Rev. D **46**, 5378 (1992).

[42] J. A. Grifols and T. G. Masso, Nucl. Phys. **B31**, 244 (1990).

[43] S. M. Bilenky, Report No. JINR P2-83-441, 1983.

[44] A. Manohar, Phys. Lett. B **196**, 370 (1987); see also G. Fuller and R. Malaney, Phys. Rev. D **43**, 3136 (1991); E. K. Semikoz, Nucl. Phys. **B374**, 392 (1992).

[45] M. A. Herrera and S. Hacyan, Astrophys. J. **336**, 539 (1989); N. C. Rana and B. Mitra, Phys. Rev. D **44**, 393 (1991); T. P. Walker *et al.*, Astrophys. J. **376**, 51 (1991); S. Dodelson and M. S. Turner, Phys. Rev. D **46**, 3372 (1992); D. A. Dicus *et al.*, *ibid.* **26**, 2694 (1982); A. D. Dolgov and Y. B. Zeldovich, Rev. Mod. Phys. **53**, 1 (1989); Dolgov and Fukugita [41]; Hannestad and Madsen [41]; B. Fields, S. Dodelson, and M. S. Turner, Phys. Rev. D **47**, 4309 (1993); V. A. Kostelecký and S. Samuel, *ibid.* **52**, 3184 (1995).

[46] D. P. Kirilova and M. V. Chizhov, Report No. IC/98/61, 1998, hep-ph/9806441.

[47] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[48] Shvartsman [15]; G. Steigman *et al.*, Phys. Lett. **66B**, 202 (1977); G. Steigman *et al.*, Phys. Rev. Lett. **43**, 239 (1979); A. Dolgov and Ya. Zeldovich, Usp. Fiz. Nauk **130**, 559 (1980); Rev. Mod. Phys. **B503**, 1 (1981); J. Yang *et al.*, Astrophys. J. **281**, 493 (1984); T. Walker *et al.*, *ibid.* **376**, 51 (1991).

[49] N. Terasawa and K. Sato, Phys. Lett. B **185**, 412 (1987); S. Dodelson, G. Gyuk, and M. S. Turner, Phys. Rev. D **49**, 5068 (1994); M. Kawasaki, K. Kohri, and K. Sato, Phys. Lett. B **430**, 132 (1998); S. Hannestad, Phys. Rev. D **57**, 2213 (1998).

[50] A. Dolgov and D. Kirilova, Int. J. Mod. Phys. A **3**, 267 (1988).

[51] K. A. Olive and G. Steigman, Astrophys. J., Suppl. Ser. **97**, 49 (1995); E. Skillman *et al.*, Ann. (N.Y.) Acad. Sci. **688**, 739 (1993); B. Pagel *et al.*, Mon. Not. R. Astron. Soc. **255**, 325 (1992).

[52] D. Sasselov and D. Goldwirth, Astrophys. J. Lett. **444**, L5 (1995); K. Davidson and T. D. Kinman, Astrophys. J., Suppl. Ser. **58**, 321 (1985); N. Hata *et al.*, Phys. Rev. Lett. **75**, 3977 (1995); N. Hata and G. Steigman, Phys. Rev. D **55**, 540 (1997); G. M. Fuller and C. Y. Cardall, Nucl. Phys. B (Proc. Suppl.) **51B**, 71 (1996); Yu. I. Izotov *et al.*, Astrophys. J. **435**, 647 (1996).

[53] Walker *et al.* [45]; M. S. Smith, L. H. Kawano, and R. A. Malaney, Astrophys. J., Suppl. Ser. **85**, 219 (1993); P. J. Kernan and L. M. Krauss, Phys. Rev. Lett. **72**, 3309 (1994).

[54] P. J. Kernan and S. Sarkar, Phys. Rev. D **54**, 3681 (1996); L. M. Krauss and P. J. Kernan, Phys. Lett. B **347**, 347 (1995); C. J. Copi *et al.*, Science **267**, 192 (1995).