

Nonvanishing zero modes in the light-front current

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(Received 10 June 1998; published 9 September 1998)

We find that the zero mode ($q^+ = 0$ mode of a continuum theory) contribution is crucial to obtaining the correct values of the light-front current J^- in the Drell-Yan ($q^+ = 0$) frame. In the exactly solvable model of (1+1)-dimensional scalar field theory interacting with gauge fields, we quantify the zero-mode contribution and observe that the zero-mode effects are very large for the light meson form factors even though they are substantially reduced for the heavy meson cases.

[S0556-2821(98)50419-5]

PACS number(s): 11.15.Bt, 11.10.Kk, 11.40.-q

One of the distinguishing features in light-front quantization is the rational energy-momentum dispersion relation which gives a sign correlation between the light-front energy (P^-) and the light-front longitudinal momentum (P^+). In the old-fashioned time-ordered perturbation theory [1], this sign correlation allows one to remove the so-called ‘‘Z-graphs’’ such as the diagram of particle-antiparticle pair creation(annihilation) from(to) the vacuum. As an example, in the theory of scalar fields interacting with gauge fields [2,3], the covariant triangle diagram shown in Fig. 1(a) corresponds to only two light-front time-ordered diagrams shown in Figs. 1(b) and 1(c), while in the ordinary time-ordered perturbation theory, Fig. 1(a) would correspond to the six time-ordered diagrams including the ‘‘Z-graphs.’’ Furthermore, the Drell-Yan (or $q^+ = 0$) frame may even allow one to remove the diagram shown in Fig. 1(c) because of the same reasoning from the energy-momentum dispersion relation and the conservation of the light-front longitudinal momenta at the vertex of the gauge field and the two scalar fields.

Based on this idea, the Drell-Yan ($q^+ = 0$) frame is frequently used for the bound-state form factor calculations. Taking advantage of $q^+ = 0$ frame, one may need to consider only the valence diagram shown in Fig. 1(b), where the three-point scalar vertices should be replaced by the light-front bound-state wave function. Successful description of various hadron form factors can be found in the recent literatures [4–8] using the light-front quark model.

In this paper, however, we point out that even at $q^+ = 0$ frame, one should not overlook the possibility of non-zero contribution from the non-valence (pair creation or annihilation) diagram shown in Fig. 1(c). As we will show explicitly in the simple (1+1)-dimensional scalar field theory interacting with gauge fields, the current J^- is not immune to the zero-mode contribution shown in Fig. 1(c) at $q^+ = 0$. While the current J^+ does not have any zero-mode contribution from Fig. 1(c), the processes that involve more than one form factor, e.g., semileptonic decay processes, require the calculations of more components of the current other than J^+ in order to find all the necessary form factors in $q^+ = 0$ frame. For instance, in the analysis of the semileptonic decays between two pseudoscalar mesons, two form factors, $f_{\pm}(q^2)$, are involved and one has to use not only J^+ , but also J^- (or J^{\perp} in 3+1 dimensions) to obtain both form factors in $q^+ = 0$ frame. Thus, the zero-mode contribution is

crucial to obtaining the correct results of electroweak form factors. Only a brief exactly solvable model calculation is provided here. A full, detailed treatment of (3+1) dimensional semileptonic decay processes such as $K \rightarrow \pi, B \rightarrow \pi, B \rightarrow D$, etc., will be presented in a separate communication. We first describe the general formalism of the semileptonic decay form factors for non-zero momentum transfer in (1+1)-dimensions and then discuss the zero-mode problem in the limiting cases of the form factors as $q^+ \rightarrow 0$.

The semileptonic decay of a $Q_1 \bar{q}$ bound state into another $Q_2 \bar{q}$ bound state is governed by the weak current, viz.,

$$J^{\mu}(0) = \langle P_2 | \bar{Q}_2 \gamma^{\mu} Q_1 | P_1 \rangle = f_+(q^2)(P_1 + P_2)^{\mu} + f_-(q^2)(P_1 - P_2)^{\mu}, \quad (1)$$

where $P_2 = P_1 - q$ and the non-zero momentum transfer square $q^2 = q^+ q^-$ is time-like, i.e., $q^2 = [0, (M_1 - M_2)^2]$. One can easily obtain q^2 in terms of the fraction α as follows

$$q^2 = (1 - \alpha) \left(M_1^2 - \frac{M_2^2}{\alpha} \right), \quad (2)$$

where $\alpha = P_2^+ / P_1^+ = 1 - q^+ / P_1^+$. Accordingly, the two solutions for α are given by

$$\alpha_{\pm} = \frac{M_2}{M_1} \left[\frac{M_1^2 + M_2^2 - q^2}{2M_1 M_2} \pm \sqrt{\left(\frac{M_1^2 + M_2^2 - q^2}{2M_1 M_2} \right)^2 - 1} \right]. \quad (3)$$

The +(-) sign in Eq. (3) corresponds to the daughter meson recoiling in the positive (negative) z -direction relative to the parent meson. At zero recoil ($q^2 = q_{\max}^2$) and maximum recoil ($q^2 = 0$), α_{\pm} are given by

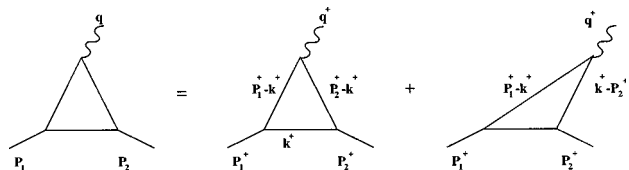


FIG. 1. Covariant triangle diagram (a) is represented as the sum of light-front triangle diagram (b) and the light-front pair-creation diagram (c).

TABLE I. Form factors of $f_{\pm}(0)$ obtained for different zero-momentum transfer limit, $q^+ = 0$ and $q^- = 0$. The notation of $\alpha_{p(m)}$ used in table are defined as $\alpha_p = 1 + \alpha_-(0)$ and $\alpha_m = 1 - \alpha_-(0)$, respectively.

Form factor	$q^+ = 0$	$q^- = 0$
$f_+(0)$	$I_1^+(\alpha_+(0))/2$	$\sum_{i=1}^2 I_i^-(\alpha_-(0))/2M_1^2$
$f_-(0)$	$[\sum_{i=1}^2 I_i^-(\alpha_+(0))/M_1^2 - \alpha_p I_1^+(\alpha_+(0))/2]/\alpha_m$	$\sum_{i=1}^2 [I_i^+(\alpha_-(0)) - \alpha_p I_i^-(\alpha_-(0))]/2M_1^2/\alpha_m$

$$\alpha_+(q_{\max}^2) = \alpha_-(q_{\max}^2) = \frac{M_2}{M_1},$$

$$\alpha_+(0) = 1, \quad \alpha_-(0) = \left(\frac{M_2}{M_1}\right)^2. \quad (4)$$

In order to obtain the form factors $f_{\pm}(q^2)$ which are independent of α_{\pm} , we can define

$$\langle P_2 | \bar{Q}_2 \gamma^{\mu} Q_1 | P_1 \rangle |_{\alpha=\alpha_{\pm}} \equiv 2P_1^+ H^+(\alpha_{\pm}) \quad \text{for } \mu = + \quad (5a)$$

$$\equiv 2 \left(\frac{M_1^2}{P_1^+} \right) H^-(\alpha_{\pm}) \quad \text{for } \mu = -, \quad (5b)$$

and obtain, from Eq. (1),

$$f_{\pm}(q^2) = \pm \frac{(1 \mp \alpha_-) H^+(\alpha_+) - (1 \mp \alpha_+) H^+(\alpha_-)}{\alpha_+ - \alpha_-} \quad \text{for } \mu = + \quad (6a)$$

$$= \pm \frac{(1 \mp \beta_-) H^-(\alpha_+) - (1 \mp \beta_+) H^-(\alpha_-)}{\beta_+ - \beta_-} \quad \text{for } \mu = -, \quad (6b)$$

where $\beta_{\pm} = \alpha_-(0)/\alpha_{\pm}$.

Now, the current $J^{\mu}(0)$ obtained from the covariant triangle diagram of Fig. 1(a) is given by

$$J^{\mu}(0) = \int d^2k \frac{1}{(P_1 - k)^2 - m_1^2 + i\epsilon} (P_1 + P_2 - 2k)^{\mu} \frac{1}{(P_2 - k)^2 - m_2^2 + i\epsilon} \frac{1}{k^2 - m_q^2 + i\epsilon}. \quad (7)$$

From this, we obtain for the “ \pm ”-components of the current $J^{\mu}(0)$ as

$$J^{\pm}(0) = -2\pi i (I_1^{\pm} + I_2^{\pm}), \quad (8)$$

where I_1^{\pm} and I_2^{\pm} corresponding to diagrams Figs. 1(b) and 1(c), respectively, are given by

$$I_1^+(\alpha) = \int_0^{\alpha} dx \frac{1 - 2x + \alpha}{x(1-x)(\alpha-x)(M_1^2 - m_1^2/(1-x) - m_q^2/x)(M_2^2/\alpha - m_2^2/(\alpha-x) - m_q^2/x)}, \quad (9a)$$

$$I_2^+(\alpha) = \int_{\alpha}^1 dx \frac{1 - 2x + \alpha}{x(1-x)(\alpha-x)(M_1^2 - m_1^2/(1-x) - m_q^2/x)(M_2^2/\alpha + m_1^2/(1-x) - m_2^2/(\alpha-x) - M_1^2)}, \quad (9b)$$

and

$$I_1^-(\alpha) = \int_0^{\alpha} dx \frac{M_1^2 + M_2^2/\alpha - 2m_q^2/x}{x(1-x)(\alpha-x)(M_1^2 - m_1^2/(1-x) - m_q^2/x)(M_2^2/\alpha - m_2^2/(\alpha-x) - m_q^2/x)}, \quad (10a)$$

$$I_2^-(\alpha) = \int_{\alpha}^1 dx \frac{M_2^2/\alpha - M_1^2 + 2m_q^2/(1-x)}{x(1-x)(\alpha-x)(M_1^2 - m_1^2/(1-x) - m_q^2/x)(M_2^2/\alpha + m_1^2/(1-x) - m_2^2/(\alpha-x) - M_1^2)}. \quad (10b)$$

TABLE II. Zero-mode (Z.M.) and non-valence (N.V.) contributions to the exact form factors of $f_{\pm}(0)$ for the semileptonic decays of $K(B) \rightarrow \pi$ and $B \rightarrow D$ in $(1+1)$ dimensions. We distinguished the zero-mode contribution at $q^+ = 0$ from the usual non-valence one at $q^- = 0$.

Frame	Ratio of $f_{\pm}^{\text{N.V.}(Z.M.)}(0)$ to $f_{\pm}^{\text{full}}(0)$	N.V.(Z.M.) factor	$K \rightarrow \pi$	$B \rightarrow \pi$	$B \rightarrow D$
$q^+ = 0$	$f_{+}^{\text{Z.M.}}(0)/f_{+}^{\text{full}}(0)$	None	0	0	0
	$f_{-}^{\text{Z.M.}}(0)/f_{-}^{\text{full}}(0)$	$\propto I_2^{\pm}(\alpha_+(0))$	6.9	0.03	0.1
$q^- = 0$	$f_{+}^{\text{N.V.}}(0)/f_{+}^{\text{full}}(0)$	$\propto I_2^{\pm}(\alpha_-(0))$	2.8	1.3	0.05
	$f_{-}^{\text{N.V.1.}}(0)/f_{-}^{\text{full}}(0)^{\text{a}}$	$\propto I_2^+(\alpha_-(0))$	3.8	3.8	0.6
	$f_{-}^{\text{N.V.2.}}(0)/f_{-}^{\text{full}}(0)^{\text{a}}$	$\propto I_2^-(\alpha_-(0))$	-11.1	-4.0	-1.1

^aWe show the separate contributions of the non-valence terms proportional to $I_2^+(\alpha_-(0))$ and $I_2^-(\alpha_-(0))$ to the exact form factor of $f_{-}(0)$ at $q^- = 0$.

Note that at zero momentum transfer limit, $q^2 = q^+ q^- \rightarrow 0$, the contributions of $I_2^{\pm}(\alpha)$ come from either $\lim_{q^+ \rightarrow 0} I_2^{\pm}(\alpha) = I_2^{\pm}(\alpha_+(0))$ or $\lim_{q^- \rightarrow 0} I_2^{\pm}(\alpha) = I_2^{\pm}(\alpha_-(0))$. It is crucial to note in $q^+ = 0$ frame that, while $I_2^+(\alpha_+(0))$ vanishes, $I_2^-(\alpha_+(0))$ does not vanish because the integrand has a singularity even though the region of integration shrinks to zero [9]. Its nonvanishing term is thus given by

$$I_2^-(\alpha_+(0)) = -\frac{2}{m_1^2 - m_2^2} \ln\left(\frac{m_2^2}{m_1^2}\right). \quad (11)$$

This nonvanishing term is ascribed to the term proportional to $k^- = P_1^- - m_1^2/(P_1^+ - k^+)$ in Eq. (10b), which prevents Eq. (10b) from vanishing in the limit, $\alpha \rightarrow 1$. This is precisely the contribution from ‘‘zero-mode’’ at $q^+ = 0$ frame. $I_2^-(\alpha_+(0))$ should be distinguished from the other nonvanishing pair-creation diagrams at $q^- = 0$ frame, i.e., $I_2^{\pm}(\alpha_-(0))$. Some relevant, but different applications of zero-modes were discussed in the literature [10].

In Table I, we summarized the form factors $f_{\pm}(0)$ obtained from both currents, J^+ and J^- , for different zero momentum transfer limit, i.e., $q^+ = 0$ or $q^- = 0$. As shown in Table I, the non-valence contributions, $I_2^{\pm}(\alpha_{\pm}(0))$, are separated from the valence contributions, $I_1^{\pm}(\alpha_{\pm}(0))$. Of special interest, we observed that the form factor $f_{-}(0)$ at $q^+ = 0$ is no longer free from the zero-mode, $I_2^-(\alpha_+(0))$.

To give some quantitative idea how much these non-valence contributions $I_2^{\pm}(\alpha_{\pm}(0))$ are for a few different decay processes, we performed model calculations for $K \rightarrow \pi$, $B \rightarrow \pi$, and $B \rightarrow D$ transitions in $(1+1)$ dimensions using rather widely used constituent quark masses, $m_{u(d)} = 0.25$ GeV, $m_c = 1.8$ GeV, and $m_b = 5.2$ GeV. Numerically, we first verified that the form factors, $f_{+}(0)$ and $f_{-}(0)$, obtained from the $q^+ = 0$ frame are in fact exactly the same with $f_{+}(0)$ and $f_{-}(0)$ obtained from the $q^- = 0$ frame, respectively, once the non-valence contributions (including zero-mode) are added. The non-valence contributions to the form factors of $f_{\pm}(0)$ at $q^- = 0$ are also shown in Table II. In Figs. 2a(b)–4a(b), the effects of pair-creation (non-valence) diagram to the exact form factors are shown

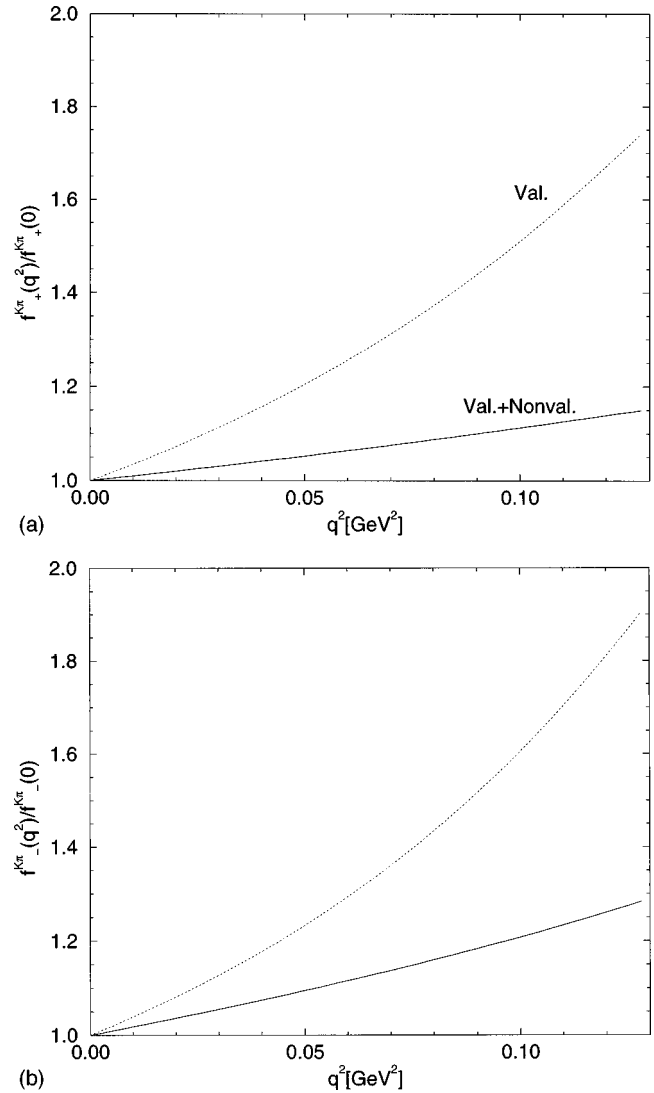
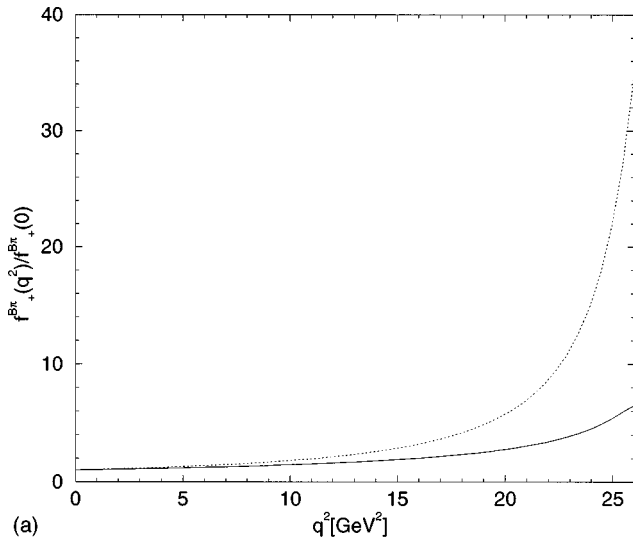
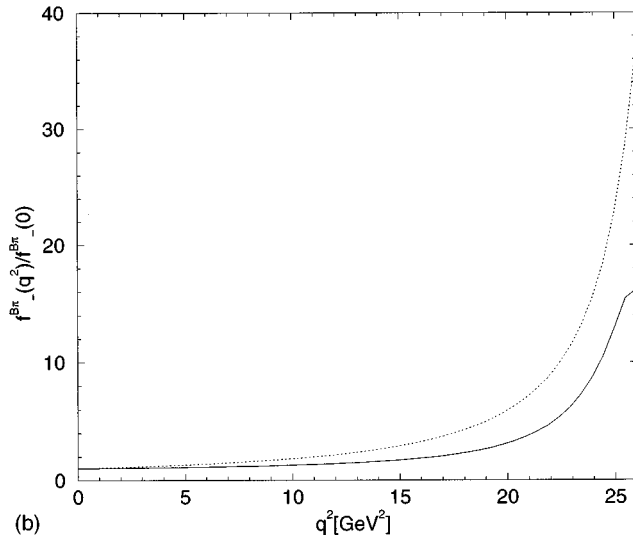


FIG. 2. (a) Normalized form factor of $f_{+}(q^2)$ for $K \rightarrow \pi$ in $(1+1)$ dimension. The solid line is the result from the valence plus non-valence contributions. The dotted line is the result from the valence contribution. (b) Normalized form factor of $f_{-}(q^2)$ for $K \rightarrow \pi$ in $(1+1)$ dimension. The same line code as in Fig. 2(a) is used.



(a)



(b)

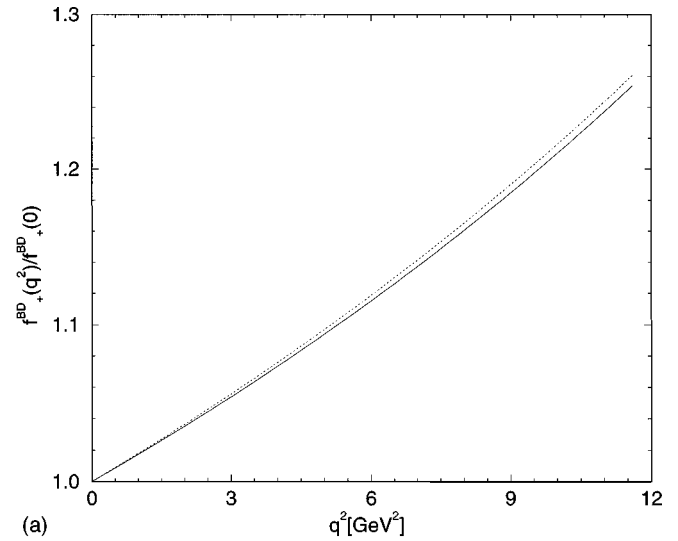
FIG. 3. (a) Normalized form factor of $f_+(q^2)$ for $B \rightarrow \pi$ in (1+1) dimension. The same line code as in Fig. 2(a) is used. (b) Normalized form factor of $f_-(q^2)$ for $B \rightarrow \pi$ in (1+1) dimension. The same line code as in Fig. 2(a) is used.

for the non-zero momentum transfer region for the above three decay processes. Especially, the zero-mode contributions $I_2^-(\alpha_+(0))$ to the exact solutions for the $f_-(0)$ at $q^+ = 0$, i.e., $f_-^{Z.M.}(0)/f_-^{\text{full}}(0)$, are estimated as 6.9 for $K \rightarrow \pi$, 0.03 for $B \rightarrow \pi$, and 0.12 for $B \rightarrow D$ decays. The zero-mode contributions on $f_-(0)$ at $q^+ = 0$ frame are drastically reduced from the light-to-light meson transition to the heavy-to-light and heavy-to-heavy ones. This qualitative feature of zero-mode effects on different initial and final states are expected to remain same even in (3+1) dimensional case, even though the actual quantitative values must be different from (1+1) dimensional case.

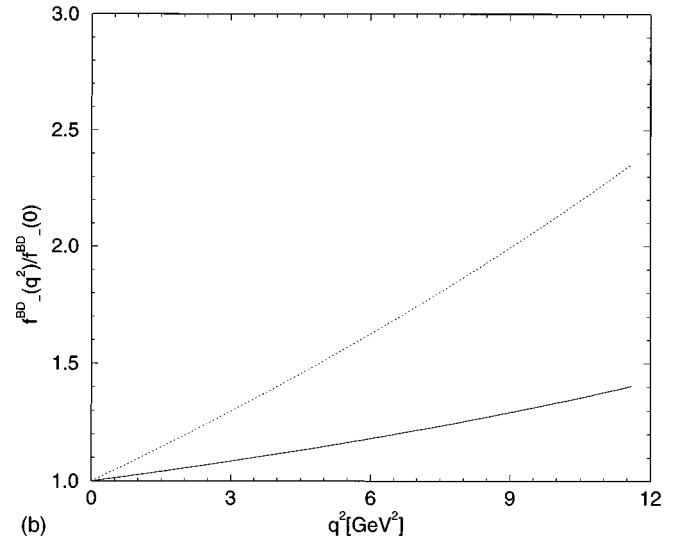
Furthermore, we have found the effect of zero mode to the EM form factor:

$$J^\mu(0) = (2P_1 - q)^\mu F_M(Q^2). \quad (12)$$

The EM form factor at $q^+ = 0$ using $J^-(0)$ current is obtained by



(a)



(b)

FIG. 4. (a) Normalized form factor of $f_+(q^2)$ for $B \rightarrow D$ in (1+1) dimension. The same line code as in Fig. 2(a) is used. (b) Normalized form factor of $f_-(q^2)$ for $B \rightarrow D$ in (1+1) dimension. The same line code as in Fig. 2(a) is used.

$$F_M(0) = N \left\{ \int_0^1 dx \frac{M^2 - m_q^2/x}{x(1-x)^2(M^2 - m_q^2/(1-x) - m_q^2/x)^2} + 1/m_q^2 \right\}, \quad (13)$$

where N is the normalization constant and the $1/m_q^2$ in Eq. (13) is the ‘‘zero-mode’’ term. Numerically, using the previous quark masses, the effects of zero-modes on the form factors of $F_\pi(0)$ and $F_B(0)$, i.e., $F_\pi^{Z.M.}(0)/F_\pi^{\text{full}}(0)$ and $F_B^{Z.M.}(0)/F_B^{\text{full}}(0)$, are estimated as 16.9 and 0.75, respectively. Again, the zero-mode contribution is drastically reduced for the heavy meson form factor. However, it gives a very large effect on the light meson form factors. The similar observation on the EM form factor was made in the Breit frame recently [11]. In (3+1) dimensions, however, we note

that the relation between the Breit frame and the Drell-Yan frame involves the transverse rotation in addition to the boost and therefore the results obtained from the Breit frame cannot be taken as the same with those obtained from the Drell-Yan frame or vice versa.

In conclusion, we investigated the zero-mode effects on the form factors of semileptonic decays as well as the electromagnetic transition in the exactly solvable model. Our main observation was the nonvanishing zero-mode contribution to the J^- current and our results are directly applicable to the real (3+1) dimensional calculations. The effect of zero-mode to the $f_-(0)$ form factor is especially important in the application for the physical semileptonic decays in the

Drell-Yan ($q^+ = 0$) frame. To the extent that the zero-modes have a significant contribution to some physical observables as shown in this work, one may even conjecture that the condensation of zero-modes could lead to the nontrivial realization of chiral symmetry breaking in the light-front quantization approach. The work along this line is in progress.

It is a pleasure to thank Stan Brodsky, Bernard Bakker, Matthias Burkardt, Tobias Frederico, Dae Sung Hwang and Carl Shakin for several informative discussions. This work was supported by the U.S. DOE under contracts DE-FG02-96ER 40947.

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