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No time machine construction in open $2+1$ gravity with timelike total energy-momentum

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It is shown that in $(2+1)$-dimensional gravity an open spacetime with timelike sources and total energy momentum cannot have a stable compactly generated Cauchy horizon. This constitutes a proof of a version of Kabat's conjecture and shows, in particular, that not only a Gott time machine cannot be formed from processes such as the decay of a single cosmic string as has been shown by Carroll *et al.*, but that, in a precise sense, a time machine cannot be constructed at all.
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I. INTRODUCTION AND OVERVIEW

Partly because of the possibility that topological defects such as cosmic strings may have been formed in the early universe, and also because of the fact that it had already been noted that some solutions in $2+1$ gravity corresponding to spinless particles do not have closed timelike curves (CTC) if the total energy momentum ($\mathcal{E}M$) is timelike [1,2], Gott's solution [3] has stimulated work discussing whether or not this spacetime is physically reasonable. The relation between cosmic strings and $2+1$ particles comes from the property that the spacetime of an infinitely long and stationary gauge cosmic string asymptotically tends to Minkowski spacetime with a deficit angle [4], and in the cases of interest the core is small enough that one can consider just Minkowski with a conical singularity (none of these properties holds for gauge but supermassive [5] or global strings [6]). Thus, Gott's solution approximates the spacetime of two infinitely long parallel gauge cosmic strings, but it can also be thought of as the spacetime of two (spinless) particles in $2+1$. The first objections to Gott's spacetime were due to the belief that it did not have an associated initial value problem, and to the fact that its total $\mathcal{E}M$ is timelike, in some similarity with tachyons [7]. Approximately at the same time, Cutler showed that in Gott's spacetime there are regions without CTC; in particular, in these regions there are achronal, edgeless, nonasymptotically null surfaces, so that it can be thought that the spacetime evolves from an initial data in any of these surfaces [8] (these surfaces must be suitably chosen, in this sense see also [9]). The apparent analogy with tachyons comes from the fact that parallel transport of vectors around a Gott pair is the same as for a tachyon, but this is not true for parallel transport of spinors [10], basically because a Gott pair satisfies the dominant energy condition (also the weak and strong ones) while a tachyon does not. Therefore, spatial $\mathcal{E}M$ must not necessarily be considered as unphysical (for more discussions on CTC in $2+1$ gravity and on the nontachyonic character of a Gott pair, see [11]). But then it remains intriguing that all known exact solutions describing spinless particles do not have CTC if their total $\mathcal{E}M$ is timelike [1–3]. Kabat has suggested that this is a general feature, specifically, that spacetimes with spinless particles and timelike total $\mathcal{E}M$ do not have CTC [12]. To this we should add that 't Hooft has shown that although a Gott pair can be produced from initial data with timelike $\mathcal{E}M$ momentum in a compact surface, a ''crunch'' will occur before the appearance of CTC [13].

Time machine constructions have been associated with compactly generated Cauchy horizons (CGCH) [14,15]. This is, on one side, because if for certain initial data on a surface $\mathcal{S}$ a domain of dependence without a Cauchy horizon is obtained, and changing the data in a compact region of $\mathcal{S}$ a Cauchy horizon appears, beyond which CTC exist, then it is compactly generated. On the other side, in certain points of a CGCH (the so called base points) strong causality is violated. In this work we will follow this approach and take the question of whether a time machine can be constructed in an $2+1$ open spacetime with timelike total $\mathcal{E}M$ as equivalent to asking whether such a spacetime can have a CGCH. The answer will be negative.

Note that working with a CGCH we get rid of a difficulty present in other formulations of Kabat's conjecture. This arises from the fact that it is not *a priori* obvious that, in a spacetime with CTC, a foliation in surfaces in which ''matter contributes positively'' exists, so that one can calculate the total $\mathcal{E}M$ via holonomy, without ''counting matter more than once.'' Specifically: we are interested in spacetimes arising from initial data, i.e., of the form $\mathcal{D}^+(\mathcal{S})$ where $\mathcal{S}$ is a simply connected, noncompact, closed, achronal and edgeless surface and its future domain of dependence (a stably causal region) is denoted by $\mathcal{D}^+(\mathcal{S})$. The dominant energy condition, i.e., that $T_{ab}t^a$ is a future directed timelike or null vector for all future directed timelike or null t^a, choosing t^a as the normal to $\mathcal{S}$, ensures that total $\mathcal{E}M$ is independent of time (a conserved quantity) and of the foliation. If there exists a Cauchy horizon, $\mathcal{H}^+(\mathcal{S})$, then the definition can be extended to the horizon if the matter ''crosses it,'' e.g., assuming that there are no lightlike sources; specifically, that $T_{ab}t^b$ is future directed and timelike for all future directed timelike t^a (DECa). In this work we will assume this energy condition but without requiring that $T_{ab}t^b$ is future directed (DECb), since in a CGCH the weak energy condition (WEC) is violated [15].

There are some previous results in connection with Kabat's conjecture: Seminara and Menotti have shown, assum-

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ing additional rotational symmetry and the WEC, that if there are no CTC at infinity then there are no CTC at all [16]; Headrick and Gott have shown that if a CTC is deformable to infinity, then the holonomy of the CTC itself cannot be timelike, except for a rotation of 2π [11]. Nevertheless, in a noncompact CGCH the WEC is violated and the first result is not related to time machine construction in the sense made precise above, while the total holonomy of a CTC is in principle not related to the total $\mathcal{E}M$, due to the problem mentioned in the preceding paragraph.

In Sec. II and Sec. III we will summarize and discuss some known results that are crucial in our proof, which will be given in Sec. IV.

II. COMPACTLY GENERATED CAUCHY HORIZONS

Since in Sec. IV we will analyze the dynamics of a CGCH in $2+1$, we need here to recall some properties of Cauchy horizons, obtained in $3+1$ gravity, but equally applicable to $2+1$ gravity. Let $\mathcal{S}$ be a (partial) Cauchy surface for $\mathcal{D}^+(\mathcal{S})$, an orientable, time orientable spacetime with a future Cauchy horizon $\mathcal{H}^+(\mathcal{S})$. Then the following is true.

(1) $\mathcal{H}^+(\mathcal{S})$ is compact if $\mathcal{S}$ is compact (see, for instance, [17,18]).

(2) $\mathcal{H}^+(\mathcal{S})$ is differentiable everywhere except in a set of zero measure. We will assume implicitly differentiability of the horizon each time it is needed. That is, we will assume, e.g., that the set of nondifferentiability is not dense (in this sense, see [19]).

(3) $\mathcal{H}^+(\mathcal{S})$ is generated by null geodesics that are complete in the past but may be incomplete in the future [17,18]. Let us denote them, generically, by $\beta(s,x):\mathcal{I}\times\mathcal{H}^+(\mathcal{S})\to\mathcal{H}^+(\mathcal{S})$, with $\mathcal{I}$ some interval of $\mathcal{R}$ and s some affine parameter and, unless otherwise stated, we always refer to generators directed to the past.

(4) $\mathcal{H}^+(\mathcal{S})$ is defined as compactly generated if all these geodesics enter some compact, connected region $\mathcal{K}$ and remain there forever. That is, for each $x\in\mathcal{H}^+(\mathcal{S})$, there exists s_0 such that $\beta(s,x)\in\mathcal{K}$ for $s\geqslant s_0$ [14,15].

(5) In a noncompact CGCH the WEC is violated, i.e., there exist points in $\mathcal{K}$ in which $T_{ab}k^ak^b<0$, with k^a the tangent to the generators [15].

(6) The base set, $\mathcal{B}\in\mathcal{H}^+(\mathcal{S})$, is defined as the set of terminal accumulation points of null generators. It can be seen that $\mathcal{B}$ is nonempty (this follows from the completeness of the generators in the past and the compactness of $\mathcal{K}$), that strong causality is violated in $\mathcal{B}$ (also from the completeness of the generators in the past), and that $\mathcal{B}$ is comprised by future and past inextendible null generators contained in $\mathcal{B}$, although not necessarily closed (''fountains'') [14] (as we will see in the last section, the last statement does not hold in $2+1$).

III. TOTAL MASS IN 2+1

From now on we will assume that the spacetime is open and that the total $\mathcal{E}M$ is timelike. Carroll *et al.* have shown that in such spacetimes, if they are composed of (spinless) particles, there cannot exist any subsystem with spatial $\mathcal{E}M$ [10]. In particular, a Gott time machine cannot be created out of the decay of a single cosmic string because there is not enough energy for that [20]. In principle this property is not obtained as a partial result in the version of Kabat's conjecture that we prove here, since a Gott pair satisfies the WEC and, indeed, it can explicitly be seen that it does not have a CGCH [8]. We mention it because a slight generalization is crucial in our proof. So, we need here to summarize the analysis given in [10].

Suppose, then, that the matter is composed by particles (assuming implicitly, in this way, the DECa). The total $\mathcal{E}M$ as defined by holonomies is constructed starting from a trivial loop in $\mathcal{S}$ (at constant but arbitrary time) and deforming it until it encircles all the particles. In the process, the corresponding holonomic operator describes a curve (let us call it γ) in the Lie group , which starts at the indentity (corresponding to the trivial loop) and finishes at the total $\mathcal{E}M$. We remark that (up to similarity transformations) the total $\mathcal{E}M$ does not depend on the way in which the deformation is carried out, although γ does not share this property, and is therefore not unique.

Coordinates for the double covering of $\mathcal{S}O(2,1)$, $\mathcal{S}U(1,1)$, can be chosen by decomposing every element in a rotation through angle θ followed by a boost of rapidity ζ and direction defined by the polar angle $(\psi+\theta)/2$. In these coordinates, the metric of $\mathcal{S}U(1,1)$ (naturally given by the structure constants) is

$$ds^2=-\frac{1}{4}\cosh\frac{\zeta}{2}d\theta^2+\frac{1}{4}d\zeta^2+\frac{1}{4}\sinh\frac{\zeta}{2}d\psi^2, \qquad (1)$$

which shows that $\mathcal{S}U(1,1)$ has the geometry of anti–de Sitter spacetime. A conformal diagram of (the universal covering of) this spacetime is shown in Fig. 1, with one dimension suppressed and $\xi\equiv 4\tan^{-1}(e^{\zeta/2})-\pi$. Systems with timelike (spacelike) total $\mathcal{E}M$ lie in region II (III).

Since we have assumed that the total $\mathcal{E}M$ is timelike, the corresponding holonomic operator is equivalent (through a similarity transformation) to a rotation through a certain angle θ_{total}, which is defined as the total mass. Since the topology of $\mathcal{S}$ is assumed to be $\mathcal{R}^2$ and (it is assumed that) the initial data are geodesically complete,

$$\theta_{total}=\int_\Sigma KdA\leqslant 2\pi, \qquad (2)$$

where K is the Gaussian curvature, the equality and inequality follow from the Gauss-Bonnet and Cohn-Vossen theorems, respectively.

At this point we remark that the matter need not be composed of particles (if it is not, the loop must encircle the support of T_{ab} or be deformed to infinity if this support is not compact) and that the same analysis holds if one just assumes DECa. This condition implies that γ is timelike and future directed, a condition that in turn implies that no subsystem can have spacelike $\mathcal{E}M$; since, if γ crosses from re-

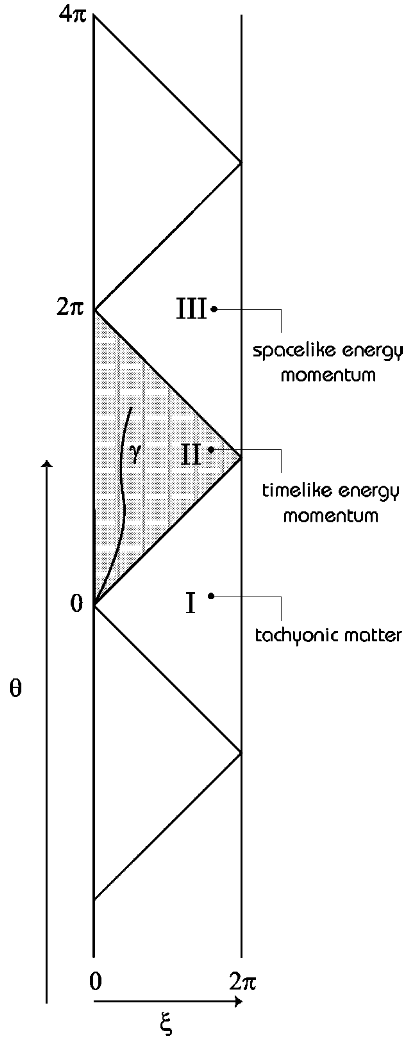

FIG. 1. A conformal diagram of the universal covering of $\mathcal{SO}(2,1)$.

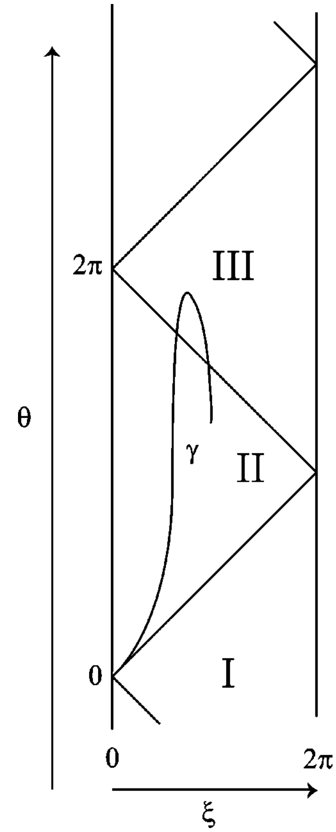

FIG. 2. When the WEC is violated γ is past directed, and can cross from region II to region III and turn back to region II, having thus timelike total $\mathcal{E}M$ and subsystems with spacelike $\mathcal{E}M$.

gion II to region III, it cannot return back to region II and lead to timelike total $\mathcal{E}M$, as we did assume (see [10] for further details).

We should also remember that Seminara and Menotti have explicitly shown (without using the causal structure of anti–de Sitter spacetime, or the Gauss-Bonnet and Cohn-Vossen theorems) similar properties [21]. Specifically, that in an open spacetime with timelike total $\mathcal{E}M$ satisfying DECa the mass increases as the loop encircles more and more matter and if, having reached the total mass of 2π, more matter is encircled then the total $\mathcal{E}M$ turns null or spacelike.

Now recall that there is already a ''standard'' formulation for asymptotic flatness in $2+1$, with analogues to the Arnowitt-Deser-Misner (ADM) [22] and Bondi masses [23]. In the Hamiltonian formulation it is required that asymptotically the spacetime approaches that of a (spinless) particle, i.e., Minkowski with a deficit angle: this angle defines the ''ADM'' mass. Note that for such a spacetime the total holonomy of a loop that is deformed to infinity is equivalent to a rotation through an angle which coincides with the deficit angle, so the ''ADM'' mass coincides with that defined via holonomies. It was emphasized in [22] that in $2+1$, in contrast to $3+1$, the total mass is not only bounded from below but also from above. This was noted in a Hamiltonian formulation, in which case the total mass must be strictly less than 2π and it was argued that it is not a feature arising from such formulation analyzing the limit of a particle's spacetime when the mass approaches and subsequently exceeds 2π, in which case the conical structure becomes cylindrical and subsequently geodesically incomplete. What we want to remark is that, on account of Eq. (2), and the discussion that preceded it, if geodesically completeness is assumed, then the total mass is effectively bounded, it must be $\leqslant 2\pi$. Let us also remark that not every compact system is asymptotically flat in the sense described in [23], since a spacetime with spacelike or null total $\mathcal{E}M$ is not, even if it has T_{ab} with compact support and thus curvature of compact support (the Weyl tensor vanishes identically in $2+1$).

Let us assume now that DECb holds. Then γ is timelike but not necessarily future directed and there are some subtleties that we need to discuss. Firstly, we shall assume that the total mass is non-negative. Therefore, near the identity γ can be chosen non past directed by simply choosing the point where the loop is initially expanded as one in which there is non negative mass. Now note that without assuming DECa the reasoning that led to the non existence of subsystems with spacelike $\mathcal{E}M$ is, in principle, no longer true (see Fig. 2). We shall overcome this difficulty by imposing that, once that γ has been chosen non past directed at the identity, it remains in region II.

IV. THE PROOF

Considering the previous sections, the proof of the version of Kabat's conjecture we give here reduces to showing that open $2+1$ spacetimes with a CGCH must have a subsystem with null or spacelike $\mathcal{EM}$, or timelike $\mathcal{EM}$ of mass zero or 2π. Imposing timelike total $\mathcal{EM}$ and the DECb, we will have arrived at a contradiction, except for the last two cases, which will turn out to be unstable, in an appropriate sense. Such a subsystem is, precisely, that encircled by the closed null geodesic whose existence we will now prove.

So, in $2+1$, not only strong but also stable causality is violated: the base set necessarily contains at least one closed null geodesic $\mathcal{C}$. The proof follows from the Poincaré-Bendixon-Schwartz theorem [24] applied to the dynamical system defined by the past directed null generators $\beta(s,x)$ in the two-dimensional compact manifold $\mathcal{K}$. We start by noting some properties of this dynamical system: (1)$\mathcal{K}$ is positively invariant (this results from the definition of $\mathcal{K}$); (2) the past directed generators $\beta(s,x)$ in $\mathcal{K}$ exist globally (they are complete in the past, and we are always referring to this direction in time); (3) from completeness of the generators in the past, there are no fixed points, i.e., there does not exist $x\in\mathcal{K}$ such that $\beta(s,x)=x\forall s$.

The first item allows us to think of our dynamical system as one in a compact manifold. The second item allows us to introduce what is usually called the ''ω limit set'' of a point $m\in\mathcal{K}$. This set is defined as the set of points $x\in\mathcal{K}$ such that the generator passing through m satisfies: for every open neighborhood $\mathcal{O}$ of x and every s_0 (in the domain in which they are defined) there exists $s>s_0$ such that $\beta(s,m)\in\mathcal{O}$. The ''ω limit set of $\mathcal{K}$'' is, similarly, defined as the union of the ω limit sets of all $y\in\mathcal{K}$. That is, the ω limit set of $\mathcal{K}$ is *by definition* the base set $\mathcal{B}$. With this in mind, we shall replace ω by $\mathcal{B}$.

The Poincaré-Bendixon-Schwartz theorem shows the following: let $\mathcal{K}$ be a compact, connected, orientable two dimensional manifold with $k^a\in T(\mathcal{K})$ and complete orbits, such that, for $m\in\mathcal{K}$, $\mathcal{B}(m)$ contains no fixed points (our dynamical system does satisfy these conditions). Then either $\mathcal{B}(m)=\mathcal{K}=\mathcal{T}^2$, or $\mathcal{B}(m)$ is a closed orbit $\mathcal{C}$, and $\beta(s,m)$ winds towards $\mathcal{C}$, where $\mathcal{T}^2$ is the torus, a manifold without boundary, excluded in our case. Thus we have a closed null geodesic $\mathcal{C}$.

In the proof of the Poincaré-Bendixon-Schwartz theorem the 2-dimensionality of $\mathcal{K}$ is crucial. Indeed, although not related to the failure of this theorem or the properties of CGCH in $2+1$ gravity, it has been emphasized that (in $3+1$) the base points are not, in general, made up by closed null geodesics (''fountains'') [25].

We will now show that the $\mathcal{EM}$ encircled by $\mathcal{C}$ is spatial, null or timelike of mass zero or 2π. For that purpose consider an arbitrary base point $p\in\mathcal{C}$, and an orthonormal base $\{v^a\}\in T_p$. Parallel transport of such a base around the loop $\mathcal{C}$ defines a new orthonormal base $\{V^a\}\in T_p$, related to the previous one by a proper Lorentz transformation $L:\{v^a\}\to\{V^a\}$, whose equivalence class defines the $\mathcal{EM}$ encircled by $\mathcal{C}$. In particular, let us take a base such that the tangent vector to $\mathcal{C}$ at p, k^a, belongs to $\{v^a\}$. $\mathcal{C}$ being a geodesic, k^a is parallel transported and, since it is a nonbroken geodesic, K^a (defined by $Lk^a=K^a$) is proportional to k^a, i.e., $K^a=\lambda k^a$. In other words, k^a is an eigenvector of L. It is straightforward to see that if L is timelike it has no null eigenvector, except when L is the identity, in which case the eigenvalue is obviously 1 and all the vectors are eigenvectors. When L is null it has exactly one null eigenvector, with eigenvalue 1 and when it is spatial it has two null eigenvectors, with eigenvalues $\lambda_1>1$ and $\lambda_2=1/\lambda_1$, so the first is attractive and the second repulsive. Then, the situation is different from $3+1$, because in that case L defines a map on the (past) sphere of null directions, and every orientation preserving map on the sphere has at least one fixed point, so that in $3+1$ every proper Lorentz transformation has at least one fixed null direction [26]. On the other hand, a homeomorphism on the circle which preserves orientation has a fixed point if its *rotation number* is zero [27]; in particular for rotations this number coincides with the angle of rotation, so one recovers that if L is timelike it has null eigenvectors if it is the identity.

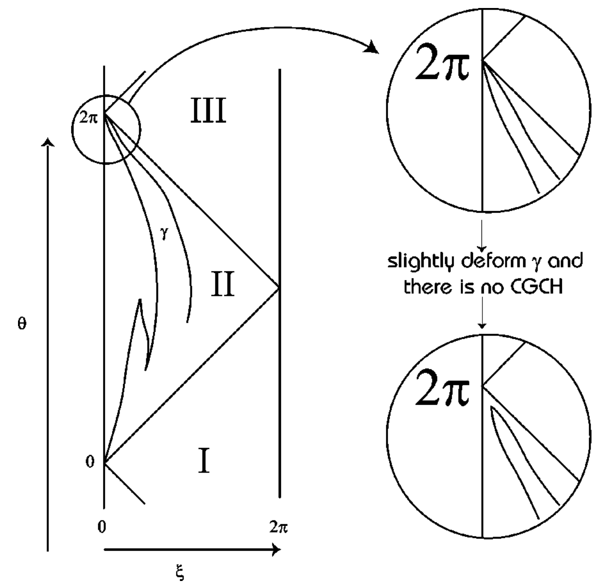

FIG. 3. This shows an example where a subsystem has 2π mass, but γ can be slightly deformed to rule out this possibility.

The point is that, applying the analysis of the previous paragraph to the closed null geodesic $\mathcal{C}$, we have shown that the $\mathcal{EM}$ it encircles is spatial, null or timelike of mass zero or 2π. Although it is not related to our proof, remember that the eigenvalue cannot be <1 because otherwise it can be shown that there would be CTC in $\mathcal{D}^+(\mathcal{S})$ [15] (see also Proposition 6.4.4 of [18]), a stably causal region. So, when the $\mathcal{EM}$ encircled by $\mathcal{C}$ is spacelike the corresponding eigenvalue is $\lambda=\lambda_2>1$.

Summarizing, if the total $\mathcal{EM}$ is timelike and DECb holds, there are no subsystems with null or spacelike $\mathcal{EM}$. Thereby, the $\mathcal{EM}$ encircled by $\mathcal{C}$ can only be timelike of mass zero or 2π. If we had supposed DECa we would have been able to discard the case of zero mass because $\mathcal{C}$ would encircle a simply connected flat (vacuum + the identical vanishing of the Weyl tensor) region and thus causally well behaved. However, since WEC is violated, zero mass does not necessarily correspond to vacuum. Nevertheless, both cases, subsystems with zero or 2π mass, are unstable, in the sense that in every neighborhood (with the Lie group manifold topology) of these points all timelike elements do not have fixed null directions (these two points belong to the boundary of region II). By slightly altering the distribution of masses (or γ, equivalently) there will be no subsystem with 0 or 2π mass; see Fig. 3.

V. FINAL REMARKS

Carroll *et al.* have shown, from energy considerations, that a Gott time machine cannot be constructed in $(2+1)$-dimensional open gravity with timelike sources and total energy momentum. In this paper we have shown that, in a precise sense, a time machine cannot be constructed at all, providing a proof of a suitable version of Kabat's conjecture.

Note that it makes sense to talk about total $\mathcal{E}M$ of a time machine: this quantity remains unaltered if the initial data is changed in a compact region of $\mathcal{S}$, since it is defined by a loop that encircles (in particular) such a region.

The proof is constructive in some aspects, e.g., it shows the existence of closed null geodesics in a CGCH, a property which is interesting in its own.

In a noncompact CGCH the WEC is violated, the known classical fields obey this condition but they do not when quantized (even in Minkowski), although an averaged version, the averaged null energy condition, has been proven to hold in some cases [28]. Therefore, it can be said that in order to create a time machine quantum matter is needed, and it is natural to ask whether the laws of physics allow CTC or a CGCH. There have been different and opposite conclusions to this question (see, for instance, [15,29]) and it seems reasonable to say that it will be difficult to have a complete answer within semiclassical gravity since not even the usual quantum field theory can be extended from $\mathcal{D}^+(\mathcal{S})$ to the base set of a CGCH [14].

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