Constraints on free parameters of the simplest bilepton gauge model from the neutral kaon system mass difference

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We consider the contributions of the exotic quarks and gauge bosons to the mass difference between the short- and the long-lived neutral kaon states in the $SU(3)_C \times SU(3)_L \times U(1)_N$ model. The lower bound $M_{Z'} \sim 14$ TeV is obtained for the extra neutral gauge boson Z'^0. Ranges for values of one of the exotic quark masses and quark mixing parameters are also presented.
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The $\Delta m = m_{K_L} - m_{K_S}$ mass difference between the long- and the short-lived kaon states was successfully used in a two-generation standard model to predict the charmed quark mass [1]. In the following years several authors studied the K^0-$\bar{K}^0$ system in order to find constraints on the parameters of new gauge theories such as gauge and scalar boson masses and mixing angles (see, for example, Refs. [2–4]). The idea behind this is that, since the c quark mass was predicted with good precision, a possible new contribution to Δm must be smaller than the result obtained from the two generations.

In this paper we go back to this subject in order to constrain the new neutral gauge boson mass and quark mixing parameters imposed by the $SU(3)_C \times SU(3)_L \times U(1)_N$ (3-3-1) model [5–7]. The 3-3-1 model is a gauge theory based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ semisimple symmetry group. It has the interesting feature that the anomaly cancellation does not happen within each generation, as in the standard model, but only when the three generations are considered together. Thus, the number of families must be a multiple of the color degrees of freedom and, as a consequence, the 3-3-1 model suggests a route towards the solution of the flavor question [5–8].

Let us summarize the most relevant points of the model. In the minimal particle content of Ref. [5] the left-handed quark fields transform under the $SU(3)_L$ group as the triplets

$$Q_{1L} = \begin{pmatrix} u_\varphi \\ d_{1\theta} \\ J_1 \end{pmatrix}_L \sim \left(\mathbf{3}, \tfrac{2}{3}\right), \quad Q_{\alpha L} = \begin{pmatrix} J_{\alpha\phi} \\ u_{\alpha\varphi} \\ d_{\alpha\theta} \end{pmatrix}_L \sim \left(\mathbf{3^*}, -\tfrac{1}{3}\right) \tag{1}$$

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$(\alpha = 2,3)$, where 2/3 and $-1/3$ are the $U(1)_N$ charges. Each left-handed quark field has its right-handed counterpart transforming as a singlet of the $SU(3)_L$ group. In order to avoid anomalies one of the quark families must transform in a different way with respect to the two others. In Ref. [6] the singled family is the third one, but this is not relevant here. The exotic quark J_1 carries 5/3 units of electric charge while J_2 and J_3 carry $-4/3$ each. In the gauge sector the single-charged ($V^\pm$) and the double-charged ($U^{\pm\pm}$) vector bileptons [9], together with a new neutral gauge boson Z'^0, complete the particle spectrum with the charged $W^\pm$ and the neutral Z^0 standard gauge bosons. At low energy the 3-3-1 model recovers the standard phenomenology [7,10].

An important property is that the bileptons can have a low energy scale. Low energy data constrain the vector bilepton masses to $M_X > 230$ GeV ($X \equiv V^+, U^{++}$) [11]. Some authors have used the running of the coupling constant to impose upper bounds on 3-3-1 gauge boson masses [12,13]. However, this procedure involves an arbitrary normalization of N [6,7]. Usually this is done like in the standard model, although it is not mandatory. Recently these upper bounds were reexamined and it was found that the $M_{Z'}$ mass does not have upper bound, differently from previous calculations, while $M_X < 3.5$ TeV [14].

The Δm mass difference has already been studied in the context of the 3-3-1 model at the tree level (Fig. 1a) in order to put a lower bound on the $M_{Z'}$ mass [15] and on the mixing parameters [13], in the last case, taking into account an upper bound on $M_{Z'}$. Here we apply the experimental lower limits and these reexamined upper bounds on the M_X gauge boson masses in order to obtain the impositions of Δm upon some of the 3-3-1 free parameters. Since the bileptons couple exotic to ordinary quarks, leading to additional contributions to the box diagram for the K^0-$\bar{K}^0$ transition (Fig. 1b), we combine the tree level with the box contribution.

The charged current interactions for the quarks are given in Ref. [5] and we can rewrite them as

(a)

(b)
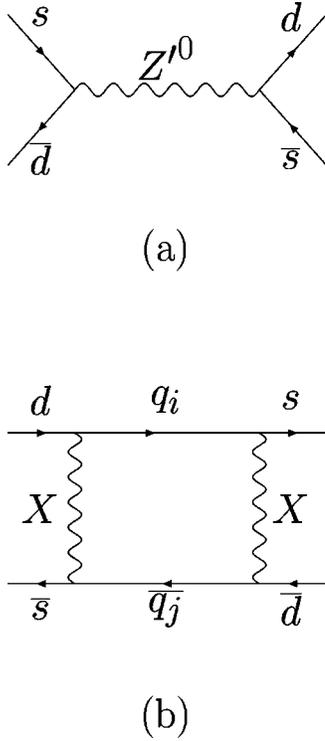
FIG. 1. K^0-$\bar{K}^0$ transition originating the $\Delta m = m_{K_L} - m_{K_S}$ mass difference at the tree level (a) and the box diagram (b) in the 3-3-1 model. The q's are all quarks coupled by X gauge bosons to d and s.

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}[\bar{U}\gamma^\mu(1-\gamma_5)V_{\mathrm{CKM}}DW_\mu^+ + \bar{U}\gamma^\mu(1-\gamma_5)\zeta\mathcal{J}V_\mu$$

$$+\bar{D}\gamma^\mu(1-\gamma_5)\xi\mathcal{J}U_\mu] + \mathrm{H.c.}, \qquad (2)$$

where

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \quad D = \begin{pmatrix} d \\ s \\ b \end{pmatrix}, \quad \mathcal{V}_\mu = \begin{pmatrix} V_\mu^+ \\ U_\mu^{--} \\ U_\mu^{--} \end{pmatrix}, \quad \mathcal{U}_\mu = \begin{pmatrix} U_\mu^{--} \\ V_\mu^+ \\ V_\mu^+ \end{pmatrix}, \qquad (3)$$

and $\mathcal{J} = \mathrm{diag}(J_1 \; J_2 \; J_3)$. The V_{CKM} is the usual Cabibbo-Kobayashi-Maskawa mixing matrix and ζ and ξ are mixing matrices containing the new unknown parameters due to the presence of the exotic quarks. Here, unlike Eq. (1), we are working with the mass eigenstates.

We are using the unitary gauge. The box diagram for the K^0-$\bar{K}^0$ transition is represented in Fig. 1b, where, in the standard model, $q_i = U_i$ $(i=1,2,3)$ and X is the W^- boson. In the 3-3-1 model, besides these, we have new contributions: for $i=1$, $q_1 = J_1$ and $X = U^{--}$, while for $i = \alpha = 2,3$, $q_\alpha = J_\alpha$ with $X = V^-$. We do not consider here contributions of the scalar bileptons. This would be very complicated because of the proliferation of unknown parameters as mixing angles and scalar boson masses [16]. However, because of the elu-

sivity of the scalar particles, we expect that the new contributions of the 3-3-1 model will be dominated by the gauge bosons.

Our calculation is standard [1]. Neglecting long-distance terms, we define $\mathcal{L}_X$—an effective Lagrangian corresponding to the X gauge boson exchange in the box diagram—as being the free quark amplitude $A_X(d\bar{s} \to \bar{d}s)$ and we evaluate the matrix element $\langle\bar{K}^0|-\mathcal{L}_X|K^0\rangle$. According to the MIT bag model we can write [4]

$$A_X = -0.7\left(\frac{G_F M_W^2}{\pi M_X}\right)^2 \mathcal{O}_{sd} \sum_{i,j} \Gamma_i \Gamma_j \frac{f(a_{q_i}^X) - f(a_{q_j}^X)}{a_{q_i}^X - a_{q_j}^X}, \qquad (4)$$

where the Γ_i are mixing parameters given in the standard model by $V_{is}^* V_{id}$ and in the 3-3-1 model by $\xi_{is}^* \xi_{id}$. From the momentum integration we have

$$f(x) = \frac{1}{1-x}\left(1 + \frac{x^2 \ln x}{1-x}\right). \qquad (5)$$

In Eq. (4), $a_{q_i}^X = (m_{q_i}/M_X)^2$ is the square ratio of the m_{q_i} quark and the M_X gauge boson masses. We have defined $\mathcal{O}_{sd} \equiv [\bar{v}_s \gamma_\mu (1-\gamma_5)u_d]^2$. The sum in Eq. (4) runs over all quarks coupled by the X gauge boson. The Δm_X mass difference due to the X gauge boson contribution is given by

$$\Delta m_X = -2\mathrm{Re}\langle\bar{K}^0|\mathcal{L}_X|K^0\rangle, \qquad (6)$$

with $\mathcal{L}_X \equiv A_X$. Taking into account the relation

$$\langle\bar{K}^0|\mathcal{O}_{sd}|K^0\rangle = \frac{4}{3}f_K^2 m_K, \qquad (7)$$

where $m_K = 498$ MeV and $f_K = 161$ MeV are the kaon meson mass and its decay constant, respectively, the total mass difference is

$$\Delta m = \frac{8\eta m_K}{3}\left(\frac{f_K M_W}{s_W M_X}\right)^2 \sum_{i,j} \Gamma_i \Gamma_j \frac{f(a_{q_i}^X) - f(a_{q_j}^X)}{a_{q_i}^X - a_{q_j}^X}, \qquad (8)$$

where $s_W = \sin\theta_W$, θ_W is the Weinberg weak mixing angle, and we have included the leading order QCD correction factor $\eta = 0.55$ [13,17].

Despite the uncertainties undergone by the evaluation of the effective Lagrangian $\mathcal{L}_X$, the application of this method to well-known processes tells us that the procedure is reliable [3].

In order to get physical results from Eq. (8) we perform some hypotheses on the free parameters. First, we notice that the standard model does not give the whole Δm mass difference. Considering only two quark generations we find $\Delta m_2 = 2 \times 10^{-15}$ GeV, while the experimental value is $\Delta m_{\mathrm{expt}} = 3.491 \times 10^{-15}$ GeV [18]. The top quark contribution is negligible because of the smallness of the corresponding
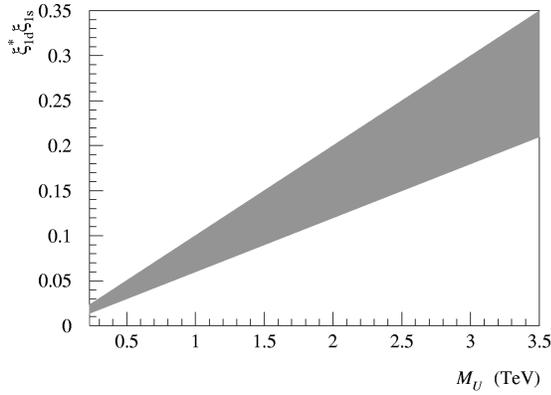
FIG. 2. Bounds on quark mixing parameters from the $m_{K_L} - m_{K_S}$ mass difference as a function of the double-charged bilepton mass. The dark region represents the allowed values for $\xi_{1d}^* \xi_{1s}$.

mixing parameters. Thus, if the 3-3-1 model is realized in nature, we can expect that its pure contribution to Δm is about 10^{-15} GeV.

Let us examine individual 3-3-1 contributions under the conservative hypothesis in which each one of them gives the total assumed 3-3-1 counterpart to Δm, i.e., 10^{-15} GeV. We begin analyzing the contribution of the U bilepton and the J_1 quark. For convenience we assume for the ξ mixing matrix the same parametrization advocated by the Particle Data Group for V_{CKM} [18]. In this case, since we choose all the mixing angles in the first quadrant, the parameter $\xi_{1d}^* \xi_{1s}$ is positive. We do not consider here CP violating phases in the mixing matrices. In Fig. 2 we plot this mixing parameter as function of M_U, according to Eq. (8), where we are using the bounds $M_U > 230$ GeV [11] and $M_U < 3500$ GeV [14]. Hence, we can see that $0.014 \lesssim \xi_{1d}^* \xi_{1s} \lesssim 0.35$. Physically this range for the mixing angles implies $m_{J_1} \lesssim M_U$. In order to apply the lower limit on the mixing parameter for obtaining a lower bound for $M_{Z'}$, we use the result of Ref. [15] for the

Z' contribution to Δm, but taking into account that we are assuming that the maximal contribution to Δm is $\sim 10^{-15}$ GeV (not the whole experimental value). We introduce also the leading order QCD correction factor $\eta = 0.55$. This leads to $M_{Z'} \sim 1.03 \times 10^3 [\text{Re}(V_{L\,11}^{D*} V_{L\,12}^{D})]^{1/2}$ TeV, where V_L^D, the mixing matrix relating the symmetry left-handed quark states carrying $-1/3$ units of electric charge (D_L') with the physical ones (D_L), is defined by $D_L' = V_L^D D_L$. Since only the J_1 quark carries $5/3$ units of electric charge, does it not mix. Therefore, the mixing parameter $V_{L\,11}^{D*} V_{L\,12}^{D}$ is the same as $\xi_{1s}^* \xi_{1d}$, whose lower bound gets $M_{Z'} \sim 14.4$ TeV.

A more rigorous analysis, considering the contributions of diagrams exchanging the single-charged V bilepton and J_2 and J_3 exotic quarks, is complicated since these two quarks can mix and the sign of the mixing parameters is not defined according to our parametrization of the mixing matrices and the choice of the mixing angles. However, it is not expected to make an appreciable difference in the results.

In this Brief Report we have studied the implications of the Δm mass difference on free parameters of the 3-3-1 model. However, differently from previous calculations [13,15], here we are taking into account the contribution of the box diagram exchanging the double-charged U^{--} bilepton. Our results differ from the previous ones because we have combined the analysis for the tree level and the box diagram. Therefore, if the 3-3-1 Higgs contribution to Δm is not important, we can assume the value we have estimated for $M_{Z'}$ as an approximate lower bound (i.e., $M_{Z'} \gtrsim 14$ TeV). We stress that this bound does not depend on the aforementioned arbitrary normalization of N. The crucial parameter for this value is the experimental lower bound on the mass of the U^{--} bilepton gauge boson.

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