Scalar glueball mass in Regge phenomenology

L. Burakovsky*
Theoretical Division, T-8, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
(Received 14 April 1998; published 29 July 1998)

We show that linear Regge trajectories for mesons and glueballs, and the cubic mass spectrum associated with them, determine a relation between the masses of the ρ meson and the scalar glueball, $M(0^{++})=3/\sqrt{2}M(\rho)$, which implies $M(0^{++})=1620\pm 10$ MeV. We also discuss relations between the masses of the scalar and tensor and 3^{--} glueballs, $M(2^{++})=\sqrt{2}M(0^{++})$, $M(3^{--})=2M(0^{++})$, which imply $M(2^{++})=2290\pm 15$ MeV, $M(3^{--})=3240\pm 20$ MeV. [S0556-2821(98)04317-3]

PACS number(s): 12.39.Mk, 12.40.Nn, 12.40.Yx, 14.40.Cs

The existence of a gluon self-coupling in QCD suggests that, in addition to the conventional $q\bar{q}$ states, there may be non-$q\bar{q}$ mesons: bound states including gluons (glueballs and $q\bar{q}g$ hybrids). However, the theoretical guidance on the properties of unusual states is often contradictory, and models that agree in the $q\bar{q}$ sector differ in their predictions about new states. Moreover, the abundance of $q\bar{q}$ meson states in the 1–2 GeV region and glueball-quarkonium mixing makes the identification of the would-be lightest non-$q\bar{q}$ mesons extremely difficult. To date, no glueball state has been firmly established yet.

Although the current situation with the identification of glueball states is rather complicated, some progress has been made recently in the 0^{++} scalar and 2^{++} tensor glueball sectors, where both experimental and QCD lattice simulation results seem to converge [1]. Recent lattice calculations predict the 0^{++} glueball mass to be 1600 ± 100 MeV [1–4]. Accordingly, there are two experimental candidates [5] $f_0(1500)$ and $f_0(1710)$ in this mass range which cannot both fit into the scalar meson nonet, and this may be considered as strong evidence for one of these states being a scalar glueball (and the other being dominantly $s\bar{s}$ scalar quarkonium).

It is known that in both lattice QCD [3,6–8] and pure Yang-Mills [9] calculations, the mass of the scalar glueball is determined by the dimensionless ratio

$$\gamma\equiv\frac{M(0^{++})}{\sqrt{\sigma}}, \tag{1}$$

where σ is the string tension, and the following values of γ have been claimed:

$$\gamma=\begin{cases} 3.95, & \text{Ref. [3]} \\ 4.77\pm 0.05, & \text{Ref. [6]}, \\ 3.88\pm 0.11, & \text{Ref. [8]}, \\ \approx 3.3, & \text{Ref. [9]}, \end{cases} \tag{2}$$

so that the data cluster around $\gamma\simeq 4$, with an uncertainty of $\sim 15\%$. The use of the value

$$\sqrt{\sigma}\approx 0.375 \text{ GeV}, \tag{3}$$

extracted from the light quark Regge phenomenology relation [10,11]

$$\alpha'=\frac{1}{8\sigma}\simeq 0.9 \text{ GeV}^{-2}, \tag{4}$$

in Eq. (1) with $\gamma\approx 4$ leads to the following estimate for the scalar glueball mass:

$$M(0^{++})\simeq 1500 \text{ MeV}. \tag{5}$$

Here we wish to suggest the following formula for the scalar glueball mass:

$$M(0^{++})=3\sqrt{2}\sqrt{\sigma}\approx 4.24\sqrt{\sigma}, \tag{6}$$

which, with $\sqrt{\sigma}$ given in Eq. (3), predicts

$$M(0^{++})\simeq 1600 \text{ GeV}. \tag{7}$$

A naive way to obtain the formula (6) is to follow the procedure of the minimization of the energy of a bound state of two massless gluons suggested by West [12]:

$$E=2p+\frac{9}{4}\sigma r-\frac{\alpha}{r}, \tag{8}$$

where p is the gluon momentum, α is the strong coupling constant, and 9/4 is a color factor in the long-range confining piece of the two-body Coulomb + linear potential. It follows from the uncertainty principle $pr\gtrsim 1$ that

$$E\geqslant\frac{2-\alpha}{r}+\frac{9}{4}\sigma r, \tag{9}$$

and minimizing the lower bound of Eq. (9) gives

$$E=3\sqrt{(2-\alpha)\sigma}\approx 3\sqrt{2}\sqrt{\sigma}. \tag{10}$$

We note that the procedure of West gives a reasonable result for, e.g., ordinary mesons: The analogue of Eq. (8) in this case is

$$E=2\sqrt{p^2+m^2}-2m+\sigma r-\frac{\alpha}{r}, \tag{11}$$

where m is the constituent quark mass, and minimizing the lower bound of the corresponding inequality following from Eq. (11) leads to the solution [in the nonrelativistic approximation $\sqrt{p^2+m^2}\approx m+p^2/(2m)$]

*Email address: BURAKOV@QMC.LANL.GOV

$$E \simeq 3\left(\frac{\sigma^2}{4m}\right)^{1/3}, \quad r \simeq \left(\frac{2}{m\sigma}\right)^{1/3}, \tag{12}$$

which, with σ given in Eq. (3) and $m \simeq 300$ MeV, gives

$$E \simeq 765 \text{ MeV}, \tag{13}$$

in agreement with the ρ meson mass of ~ 770 MeV and $r \simeq 0.7$ fm.

The way to derive Eq. (6) we suggest here is the use of the hadronic resonance spectrum. The idea of the spectral description of a strongly interacting gas which is a model for hot hadronic matter was suggested by Belenky and Landau [13] and consists in considering the unstable particles (resonances) on an equal footing with the stable ones in the thermodynamic quantities, by means of the resonance spectrum; e.g., the expression for pressure in such a resonance gas reads (in the Maxwell-Boltzmann approximation)

$$p = \sum_i g_i p(m_i) = \int_{M_l}^{M_h} dm\, \tau(m) p(m),$$

$$p(m) = \frac{T^2 m^2}{2\pi^2} K_2\left(\frac{m}{T}\right), \tag{14}$$

where M_l and M_h are the masses of the lightest and heaviest species, respectively, and g_i are particle degeneracies.

Phenomenological studies [14] have suggested that the cubic density of states, $\tau(m) \sim m^3$, for each isospin and hypercharge provides a good fit to the observed hadron spectrum. Let us demonstrate here that this cubic spectrum is intrinsically related to collinear Regge trajectories (for each isospin and hypercharge).

It is very easy to show that the asymptotic form of the mass spectrum of an individual Regge trajectory is cubic. Indeed, for a linear trajectory

$$\alpha(t) = \alpha(0) + \alpha' t, \tag{15}$$

the integer values of $\alpha(t)$ correspond to the states with integer spin, $J = \alpha(t_J)$, the masses squared of which are $m^2(J) = t_J$. Let J_0 be the spin of the ground state lying on the trajectory. Since a spin-J state has multiplicity $2J+1$, the number of states with spin $J_0 \leqslant J \leqslant \mathcal{J}$ for high enough $\mathcal{J}$ is

$$N(\mathcal{J}) = \sum_{J_0}^{\mathcal{J}} (2J+1) = (\mathcal{J}+1)^2 - J_0^2 \simeq \mathcal{J}^2 \simeq \alpha'^2 m^4(\mathcal{J}), \tag{16}$$

in view of Eq. (15), and therefore the asymptotic density of states per unit mass interval (the mass spectrum) is

$$\tau(m) = \frac{dN(m)}{dm} = 4\alpha'^2 m^3. \tag{17}$$

It is also clear that for a finite number of collinear trajectories, the resulting mass spectrum is

$$\tau(m) = 4N\alpha'^2 m^3, \tag{18}$$

where N is the number of trajectories, and does not depend on the numerical values of trajectory intercepts, as far as its asymptotic form $m \to \infty$ is concerned.

The result $\tau(m) \sim m^3$ is strongly supported by the thermodynamics of hadronic matter. Indeed, for $\tau(m) \sim m^3$, Eq. (14) and the corresponding expression for the energy density, ε, lead to the equation of state $p, \varepsilon \sim T^8$, $\varepsilon = 7p$ [14]. Bebie *et al.* [15] have calculated the ratio ε/p directly from Eq. (14) and the corresponding expression for ε with all known hadron resonances with the masses up to 2 GeV taken into account, and found that the curve ε/p first decreases very quickly and then saturates at the value $\varepsilon/p \simeq 7$, as read off from Fig. 1 of [15], which confirms both the cubic mass spectrum and thermodynamic relations of the type (14).

It turns out further that the cubic spectrum of the family of collinear Regge trajectories enables one to determine the mass of the state this family starts with. Before we dwell upon this point, let us make the following remark.

It is widely believed that pseudoscalar mesons are the Goldstone bosons of broken $SU(3) \times SU(3)$ chiral symmetry of QCD, and that they should be massless in the chirally symmetric phase. Therefore, it is not clear how well the resonance spectrum would be suitable for the description of the pseudoscalar mesons. Indeed, as we have tested in [16], this nonet is *not* described by the resonance spectrum. Moreover, pseudoscalar mesons are extrimely narrow (zero width) states to fit into a resonance description.

Thus, the resonance description should start with vector mesons, and the cubic spectrum of a linear trajectory enables one to determine the mass of the ρ meson, as follows.

We assume that the mass spectrum of hadronic (mesonic) matter is precisely cubic and given by Eqs. (17),(18). This is again strongly supported by thermodynamics: possible linear mass corrections to Eqs. (17),(18) would result in the corresponding T^6 corrections in the expressions for the pressure and energy density as calculated from relations of the type (14). The absence of such T^6 corrections has been established by Gerber and Leutwyler in chiral perturbation theory [17]. The same argument about the absence of the T^6 corrections led Shuryak to suggest the pion dispersion relation $E^2 = v^2 p^2 + m^2$, $v \leqslant 1$ [18] confirmed in a subsequent study by Pisarski and Tytgat [19].

Since the ρ meson has the lowest mass which the resonance description starts with, let us locate this state by normalizing the ρ trajectory to one state in the characteristic mass interval $(\sqrt{M^2(\rho) - 1/(2\alpha')}, \sqrt{M^2(\rho) + 1/(2\alpha')})$. With the cubic spectrum (17) of a linear trajectory, one has[1]

$$1 = 4\alpha'^2 \int_{\sqrt{M^2(\rho) - 1/(2\alpha')}}^{\sqrt{M^2(\rho) + 1/(2\alpha')}} m^3 dm = 2\alpha' M^2(\rho), \tag{19}$$

and therefore

$$M^2(\rho) = \frac{1}{2\alpha'}. \tag{20}$$

[1]Since the ρ trajectory starts with a spin-1 isospin-1 state (ρ), it corresponds to the spectrum $\tau(m) = 9 \times 4\alpha'^2 m^3$. There is therefore no difference in normalizing this trajectory to nine states or Eq. (17) to one state, in the vicinity of the ρ mass.

We note that Regge slope is known to have a weak flavor dependence in the light quark sector [11]: (in GeV^{-2}) $\alpha'_{n\bar{n}}=0.88$, $\alpha'_{s\bar{n}}=0.85$, $\alpha'_{s\bar{s}}=0.81$. With $\alpha'=0.85$ GeV^{-2}, as the average of the above three values, Eq. (20) gives

$$M(\rho)=767 \text{ MeV}, \tag{21}$$

in excellent agreement with the measured ρ meson mass of 768.5 ± 0.6 MeV [5].

Note that the result (20) is consistent with the Veneziano amplitude for $\pi^+\pi^-$ scattering [20],

$$A(s,t)=\beta\frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(1-\alpha(s)-\alpha(t))}, \tag{22}$$

where s,t are the Mandelstam variables, and $\alpha(t)$ is the linear ρ meson trajectory, for which the Adler consistency condition [the vanishing of $A(s,t)$ at $s=t=u=M^2(\pi)$] leads to

$$M^2(\rho)-M^2(\pi)=\frac{1}{2\alpha'}, \tag{23}$$

in agreement with Eq. (20), since $M(\pi)\ll M(\rho)$.[2]

It is now tempting to apply similar arguments to glueballs. The slope of glueball trajectories can be related to that of meson ones by the product of two color and mechanical, factors:

$$\alpha'_{glue}=\gamma_c\gamma_{mech}\alpha', \tag{24}$$

where

$$\gamma_c=\tfrac{4}{9} \tag{25}$$

and

$$\gamma_{mech}=\tfrac{1}{2}\,. \tag{26}$$

Equation (25) is easily understood by looking at the confining pieces of the two-body potentials in Eqs. (8),(11), and Eq. (4). The mechanical factor is easily understood, also, by noting that in the string model the glueball represents a closed string, while the ordinary meson an open string, and a spin-energy relation for a closed string has an extra factor of 1/2 as compared to that for an open string (e.g., without color, $J=\alpha' E^2$ for an open string and $J=1/2\alpha' E^2$ for a closed string, in the classical case) [22]. We note that the relation $\alpha'_{glue}=4/9\alpha'$, derived by Simonov via the vacuum correlators method [23], capitalizes an only the color factor, Eq. (25), but not the mechanical one, Eq. (26). As we see here, both factors should be taken into account in the correct formula for the glueball slope, which is

$$\alpha'_{glue}=\tfrac{2}{9}\,\alpha'\simeq0.2 \text{ GeV}^{-2}. \tag{27}$$

We now apply a procedure similar to that above for locating the ρ meson mass, in order to locate the lightest state which lies on glueball trajectories. It is firmly established that the lightest glueball state is the scalar glueball [12,24]. We therefore locate the scalar glueball mass as that for which its trajectory is normalized to one state in the mass interval $(M^2(0^{++})-1/(2\alpha'_{glue}),M^2(0^{++})+1/(2\alpha'_{glue}))$. Similar to Eq. (20), this procedure leads to

$$M^2(0^{++})=\frac{1}{2\alpha'_{glue}}, \tag{28}$$

which reduces, through Eqs. (20),(27), to

$$M(0^{++})=\frac{3}{\sqrt{2}}M(\rho). \tag{29}$$

We take $M(\rho)=764\pm5$ MeV, to accommodate the value given in [5] and results on the ρ^0 meson mass, both theoretical [25] and experimental [26], which concentrate around 760 MeV. With this $M(\rho)$, the above relation yields

$$M(0^{++})=1620\pm10 \text{ MeV}, \tag{30}$$

in excellent agreement with QCD lattice results 1600 ± 100 MeV [1–4]. It [and Eqs. (3),(6)] is also in agreement with the continuum limit lattice QCD result [27]

$$r_0M(0^{++})=4.33(5), \tag{31}$$

where $r_0^{-1}=0.372$ GeV [27], which leads to

$$M(0^{++})=1611\pm30 \text{ MeV}. \tag{32}$$

We note that, as follows from Eqs. (4),(27),(28),

$$M^2(0^{++})=18\sigma, \tag{33}$$

which is equivalent to Eq. (6).

Finally, we note that the mass spectrum can also establish a relation between the scalar and tensor glueball mass. Indeed, as discussed in [28], the mass splitting between S-wave spin-0 and spin-1 meson states (ρ and π or K and K^*) is well reproduced by the linear and cubic spectra of the corresponding multiplets and Regge trajectories, respectively, leading to the relations [the first of them is again Eq. (23))

$$M^2(\rho)-M^2(\pi)=M^2(K^*)-M^2(K)=\frac{1}{2\alpha'},$$

in good agreement with the data. A similar procedure in the glueball sector will lead to the relation between the masses of the spin-0 scalar and spin-2 tensor glueballs:

$$M^2(2^{++})-M^2(0^{++})=\frac{1}{2\alpha'_{glue}}, \tag{34}$$

which implies, through Eqs. (28),(29),

$$M(2^{++})=\sqrt{2}M(0^{++})=3M(\rho)=2290\pm15 \text{ MeV}, \tag{35}$$

in excellent agreement with QCD lattice simulations [4,30] which give 2390 ± 120 MeV for the tensor glueball mass and corresponding three experimental candidates in this mass region [5], $f_J(2220)$, $J=2$ or 4, $f_2(2300)$ and $f_2(2340)$, the first of which is seen in $J/\psi\to\gamma+X$ transitions but not in $\gamma\gamma$ production [5], while the remaining two are observed in the OZI rule-forbidden process $\pi p\to\phi\phi n$ [5], which favors the gluonium interpretation of all three states. It is also in-

[2]Note that the pion mass was expressed in terms of α' in [21]: $M(\pi)=0.13/\alpha'\approx140$ MeV.

teresting to note that, as follows from Eqs. (28),(34), $M^2(2^{++})=1/\alpha'_{glue}$, and therefore, the intercept of the tensor glueball trajectory determined by the relation

$$2=\alpha'_{glue}M^2(2^{++})+\alpha_{glue}(0)$$

is $\alpha_{glue}(0)=1$. This is in agreement with the fact widely accepted in the literature that the tensor glueball is the lowest resonance lying on the Pomeron trajectory with unit intercept. It is also interesting to note that if one takes the 3^{--} glueball as the Regge recurrence of the tensor glueball with the above mass value, one will obtain $M^2(3^{--})=2/\alpha'_{glue}$, and therefore $M(3^{--})=\sqrt{2}M(2^{++})=2M(0^{++})=3240\pm 20$ MeV, consistent with a naive scaling from the two-gluon 2^{++} glueball to the three-gluon 3^{--} case: $M(3g)\simeq 1.5M(2g)\simeq 3.3$ GeV, with $M(2g)\simeq 2.2$ GeV. Also, the original constituent gluon model predicts $M(1^{--})/M(2^{++})\simeq M(3^{--})/M(2^{++})\simeq 1.5$ [31], and QCD sum rules find the $3g$-glueball mass to be $\simeq 3.1$ GeV [32].

Again, as in a previous paper [28] where we discuss the linear mass spectrum of an individual hadronic multiplet first established in [29] and then applied to the problem of the correct $q\bar{q}$ assignments for problematic meson nonets in [16], and the cubic spectrum of a strongly interacting gas as a model for hot hadronic matter, we have demonstrated that a mass spectrum represents a powerful tool for hadron spectroscopy. We have shown that linear trajectories for mesons and glueballs, and the cubic mass spectrum associated with them, determine a relation between the masses of the lightest states lying on these trajectories, the ρ meson and the scalar glueball, respectively, which implies that the mass of the latter is in the vicinity of 1600 MeV. We have also established a relation between the scalar glueball mass and string tension [Eq. (33)] which is the subject of lattice QCD and pure Yang-Mills calculations, and discussed relations between the scalar and tensor and 3^{--} glueball masses.

Correspondence with L. P. Horwitz during the preparation of this work is very much appreciated.

[1] R. Landua, in ICHEP '96, Proceedings of the International Conference on High Energy Physics edited by Z. Ajduk and A. Wroblewski, Warsaw, Poland, 1996 (World Scientific, Singapore, 1997), and references therein.
[2] UKQCD Collaboration, G. S. Bali *et al.*, Phys. Lett. B **309**, 378 (1993).
[3] J. Sexton, A. Vaccarino, and D. Weingarten, Phys. Rev. Lett. **75**, 4563 (1995).
[4] C. J. Morningstar and M. Peardon, Phys. Rev. D **56**, 4043 (1997).
[5] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
[6] M. Teper, Phys. Lett. B **289**, 115 (1992); Nucl. Phys. B (Proc. Suppl.) **30**, 529 (1993).
[7] C. J. Hamer, M. Scheppeard, Zheng Weihong, and D. Schütte, Phys. Rev. D **54**, 2395 (1996).
[8] X.-Q. Luo and Q.-Z. Chen, Mod. Phys. Lett. A **11**, 2435 (1996).
[9] S. Samuel, Phys. Rev. D **55**, 4189 (1997).
[10] J. S. Kang and H. Schnitzer, Phys. Rev. D **12**, 841 (1975).
[11] J.-L. Basdevant and S. Boukraa, Z. Phys. C **28**, 413 (1985); Ann. Phys. (Paris) **10**, 475 (1985).
[12] G. B. West, Nucl. Phys. B (Proc. Suppl.) **54**, 353 (1997).
[13] S. Z. Belenky and L. D. Landau, Sov. Phys. Usp. **56**, 309 (1955); Nuovo Cimento Suppl. **3**, 15 (1956).
[14] E. V. Shuryak, Sov. J. Nucl. Phys. **16**, 220 (1973); L. Burakovsky and L. P. Howritz, Nucl. Phys. **A614**, 373 (1997).
[15] H. Bebie, P. Gerber, J. L. Goity, and H. Leutwyler, Nucl. Phys. **B378**, 95 (1992).
[16] L. Burakovsky and L. P. Horwitz, Nucl. Phys. **A609**, 585 (1996).
[17] P. Gerber and H. Leutwyler, Nucl. Phys. **B321**, 387 (1989).
[18] E. V. Shuryak, Phys. Rev. D **42**, 1764 (1990).
[19] R. D. Pisarski and M. Tytgat, Phys. Rev. D **54**, 2989 (1996); Phys. Rev. Lett. **78**, 3622 (1997); ''On the physics of a cool pion gas,'' hep-ph/9609414.
[20] C. Lovelace, Phys. Lett. **28B**, 264 (1968); J. A. Shapiro, Phys. Rev. **179**, 1345 (1969).
[21] L. A. P. Balazs, Phys. Rev. D **29**, 533 (1984).
[22] X. Artru, Nucl. Phys. **B85**, 442 (1975); Phys. Rep. **97**, 147 (1983).
[23] Yu. A. Simonov, Phys. Lett. B **249**, 514 (1990).
[24] G. B. West, Phys. Rev. Lett. **77**, 2622 (1996).
[25] S. Dubnicka and L. Martinovic, J. Phys. G **15**, 1349 (1989); A. Bernicha, G. Lopez Castro, and J. Pestieau, Phys. Rev. D **50**, 4454 (1994).
[26] L. Capraro *et al.*, Nucl. Phys. **B288**, 659 (1987); LEBC-EHS Collaboration, M. Aguilar-Benitez *et al.*, Z. Phys. C **50**, 405 (1991).
[27] C. Michael, ''Hadronic physics from the lattice,'' hep-ph/9710249; ''Hadron spectroscopy from the lattice,'' hep-ph/9710502.
[28] L. Burakovsky, ''Relativistic Statistical Mechanics and Particle Spectroscopy,'' presented at the First International Conference on Parametrized Relativistic Quantum Theory, PRQT '98, Houston, Texas, 1998, LANL Report No. LA-UR-98-1694; Found, Phys. (to be published); ''Hadron Spectroscopy in Regge Phenomenology,'' LANL Report No. LA-UR-98-1915, hep-ph/9805286.
[29] L. Burakovsky and L. P. Horwitz, Found. Phys. Lett. **9**, 561 (1996); **10**, 61 (1997).
[30] M. Peardon, Nucl. Phys. B (Proc. Suppl.) **63**, 22 (1998).
[31] W. S. Hou and A. Soni, Phys. Rev. Lett. **50**, 569 (1983); Phys. Rev. D **29**, 101 (1984); J. M. Cornwall and A. Soni, Phys. Lett. **120B**, 431 (1983).
[32] S. Narison, Nucl. Phys. **B509**, 312 (1998); Nucl. Phys. B (Proc. Suppl.) **64**, 210 (1998).