PHYSICAL REVIEW D, VOLUME 58, 055003

Pseudoscalar glueball mass: QCD versus lattice gauge theory prediction

Gregory Gabadadze*
Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08855-0849

(Received 24 November 1997; published 16 July 1998)

We study whether the pseudoscalar glueball mass in full QCD can differ from the prediction of quenched lattice calculations. Using properties of the correlator of the vacuum topological susceptibility we derive an expression for the upper bound on the QCD glueball mass. We show that the QCD pseudoscalar glueball is lighter than the pure Yang-Mills theory glueball studied in quenched lattice calculations. The mass difference between those two states is of the order of $1/N_c$. The value calculated for the 0^{-+} QCD glueball mass cannot be reconciled with any physical state observed so far in the corresponding channel. The glueball decay constant and its production rate in J/ψ radiative decays are calculated. The production rate is large enough to be studied experimentally. [S0556-2821(98)04315-X]

PACS number(s): 12.60.Jv, 11.55.Hx, 14.80.Ly

INTRODUCTION

Glueballs are one of the intriguing theoretical predictions of QCD [1]. The search for these composites is a long-standing problem of theory and experiment. We now have detailed experimental studies of the resonances in the mass region up to 2.3 GeV, as well as great progress in lattice QCD calculations. A number of interesting particles have been detected [2]. Some of them, the $\eta(1410,0^{-+})$, the $f_0(1500,0^{++})$, the $f_0(1710,0^{++})$ and the $\zeta(2230,2^{++})$ appear to have a rich gluon content. (For a recent discussion of the phenomenology of these composites and a full set of references see [3].) The glueball candidates can be compared to lattice QCD predictions.[1] These calculations argue in favor of the following hierarchy of glueball masses: the 0^{++} glueball is the lightest one with a mass about 1.5–1.7 GeV [9,10]; the 2^{++} state, having a mass 2–2.2 GeV [9], is the next one; finally, the 0^{-+} pseudoscalar glueball, being predicted in the lattice calculations to have a mass 2.3 ± 0.2 GeV [9], is the heaviest one. Various studies of pure Yang-Mills (YM) theory also support the picture outlined above. In Ref. [11] the theorem was proved that in pure gluodynamics the pseudoscalar glueball is heavier than the scalar one. Instanton calculations [12] also confirm this picture.

Notice that all the theoretical facts listed above are firmly established results of pure YM theory. One might wonder whether this picture is affected when quarks are also included in the theory. Recent analyses [10,13,3,14] of the scalar glueball candidates indicate an important mixing with the nearby $\bar{q}q$ resonances. This leads to a modification of the mass spectrum and the decay constants of the glueball states [3,14]. The mass shifts due to mixing are approximately 100 MeV or so, and the lattice predictions are in good agreement at this level with the experimental data for the 0^{++} and the 2^{++} channels [3,14].

However, the situation for the pseudoscalar channel is problematic. One can expect that, because of the axial anomaly, quarks are crucial for 0^{-+} channel physics [15]. As we mentioned above, the lightest 0^{-+} glueball predicted by the lattice calculations has mass about 2.3 GeV [9]. On the other hand, there is evidence that the $\eta(1410)$ can have a substantial gluonic component [3].[2] Hence, an important question is how the quark degrees of freedom may shift the glueball mass in the 0^{-+} channel and whether one can identify the QCD glueball with any observed state. We examine here these questions.

In low energy QCD the $\eta'(958)$ makes the dominant contribution to correlators in the 0^{-+} flavor singlet channel. It is well known that the mass and decays of the η' meson are strongly affected by the gluonic sector of the theory. The axial anomaly and instantons play a key role in generating the mass and decay constant of the η' [15] (see also the papers [5,20,21] and references therein). This suggests that the pure 0^{-+} glueball should be also affected by the singlet $\bar{q}q$ admixture in full QCD [22]. In what follows we will refer to this mixed glueball-$\bar{q}q$ state as a QCD glueball in distinction with the pure YM glueball and the physical η'.

We are going to derive an inequality between the mass of the QCD pseudoscalar glueball and the mass of the YM glueball as measured in quenched lattice calculations [9]. In the derivation we closely follow the argumentation of Witten [23] and Veneziano [24], but keep track of the finite N_c effects by using the method of QCD sum rules [4]. The decay constant and mass for the QCD pseudoscalar glueball will be determined. We show that the QCD glueball is lighter than the one of pure YM theory. Finally, we will look at J/ψ radiative decays and predict the production rate for the QCD glueball state.

*Email address: gabad@physics.rutgers.edu

[1]The earliest theoretical predictions were based on QCD sum rule [4] calculations [5–8].

[2]Note that the structure originally identified as $\eta(1440)$ is in fact two states, the $\eta(1410)$ and the $\eta(1490)$ [16–19]. The lighter $\eta(1410)$ seems to have a rich gluon content [3], while the heavier $\eta(1490)$ is dominantly $\bar{s}s$ state possibly with some admixture of glue [3].

I. QCD SUM RULES AND THE WITTEN-VENEZIANO RELATION

In this section we are going to study the properties of the correlator of the vacuum topological susceptibility. In Minkowski space-time this is defined as

$$\chi(q^2 \equiv -Q^2) = \left(\frac{g^2}{32\pi^2}\right)^2 i \int e^{iqx} \langle 0 | T G^a_{\mu\nu} \times \tilde{G}^a_{\mu\nu}(x) G^b_{\alpha\beta} \tilde{G}^b_{\alpha\beta}(0) | 0 \rangle d^4x, \tag{1}$$

where $\tilde{G}^a_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta} G^a_{\alpha\beta}$ and $(g^2/32\pi^2) G_{\mu\nu}\tilde{G}_{\mu\nu}$ is the renormalized composite operator with g being the running coupling constant.

This correlator has a different behavior in the vicinity of $Q^2=0$ depending on whether it is evaluated in full QCD or in pure Yang-Mills theory [23]. If light quarks are included in the theory, $\chi_{\rm QCD}(0)$ is proportional to the product of the quark masses and vanishes in the chiral limit. This fact is related to the absence of the theta angle in massless QCD. In general, the theta angle can always be rotated away by an appropriate chiral transformation of a massless fermionic field. On the other hand, there are no massless fermions in pure YM theory. As a result, the explicit theta dependence cannot be removed [23] and $\chi_{\rm YM}(0)$ turns out to be a nonzero number.

Let us consider the dispersion relation for the function $\chi(Q^2)/Q^2$:

$$\frac{\chi(0)-\chi(Q^2)}{Q^2} = \frac{1}{\pi}\int_0^\infty \frac{\rho(s)\,ds}{s(s+Q^2)} + \text{subtractions}. \tag{2}$$

Following the standard QCD sum rule approach [4], the spectral density for this correlator, ρ, can be decomposed into two parts. The first one consists of resonance (pole) contributions and the second one is determined by the perturbative expansion

$$\rho(s) = \rho^{poles}(s) + \tilde{\rho}(s/\mu^2)\,\theta(s - s_0(\mu)),$$

$$\rho^{poles}(s) \equiv \sum_n c_n \delta(s - m_n^2),$$

where the c_n's are resonance residues, m_n's are corresponding masses, and s_0 denotes the continuum threshold; $\tilde{\rho}$ is given by the perturbative expansion of the corresponding correlator. We are going to work in the next-to-leading order (NLO) of the perturbative expansion. In the case at hand, the leading contribution to $\tilde{\rho}$ is scale and scheme independent. However, the next-to-leading term depends on the renormalization scheme. In leading order $\tilde{\rho}$ is fixed by the diagram which contains only gluon propagators. This diagram is the same for QCD and pure Yang-Mills theory. Hence, in lowest order $\tilde{\rho}_{\rm QCD} = \tilde{\rho}_{\rm YM}$. However, this relation does not hold in the NLO. There are quark loop corrections to the gluon propagator in QCD. Hence, the result for $\tilde{\rho}$ in NLO QCD differs from the one of pure YM theory by quark loop contributions. In order to fix the scale or scheme ambiguity of the NLO corrections it is convenient to adopt the Brodsky-Lepage-Mackenzie (BLM) scale fixing procedure [25]. In the BLM scheme NLO quark loop insertions into the gluon propagator are summed up into the redefinition of the effective scale of the strong coupling constant. Hence, in the BLM scheme, the perturbative expansion for $\tilde{\rho}_{\rm QCD}$ in the next-to-leading order formally coincides with that for $\tilde{\rho}_{\rm YM}$. Keeping in mind that we have adopted the BLM scheme one can write down the following relation for the spectral densities in QCD and pure Yang-Mills theory[3] [7]:

$$\tilde{\rho}_{\rm QCD} = \tilde{\rho}_{\rm YM} = \left(\frac{g^2}{32\pi^2}\right)^2 \frac{2}{\pi}\left(1 + 5\frac{\alpha_s}{\pi}\right) s^2 \equiv a s^2. \tag{3}$$

Before we turn to the resonance part of the spectral density let us make an important comment. It deals with the definition of the running coupling constant in QCD and in YM theory. The expression for $\alpha_s(s/\mu^2)$ depends on the number of flavors N_f present in the theory. In pure YM theory $N_f = 0$ and the coupling constant of this theory differs from the one defined in full QCD. However, our goal is to stay maximally close to what is used in quenched lattice calculations. In those calculations only gluon degrees of freedom are taken into account. But this is not the whole story. Quenched lattice calculations effectively include some of the virtual quark effects through the formal substitution of the QCD running coupling with $N_f = 3$ instead of the coupling of YM theory with $N_f = 0$ (see Ref. [10] for this discussion). We are using this formal method through the paper. In particular, the theory to which we refer as pure YM is actually the theory with some of the virtual quark loops effectively included through the use of the QCD running coupling constant α_s instead of the pure YM running coupling. Thus, YM theory in our context refers to the model which has the full QCD coupling constant, but nevertheless, differs from true QCD by the absence of $\bar{q}q$ bound states and the absence of quark condensate effects. Let us stress again that these conventions differ from the ones normally used (adopted for example in Ref. [26]) and are motivated by the quenched lattice calculation procedure.

After this remark let us turn to the resonance part of the spectral density. This part for YM theory is assumed to be saturated by the pure glueball state G_0, and for QCD by the η' meson and QCD glueball state G. The expressions for the spectral densities are

$$\rho_{\rm YM}(s) = f^2_{G_0} m^4_{G_0} \delta(s - m^2_{G_0}) + \tilde{\rho}_{\rm YM}(s)\,\theta(s - s_0),$$

$$\rho_{\rm QCD}(s) = f^2_G m^4_G \delta(s - m^2_G) + f^2_{\eta'} m^4_{\eta'} \delta(s - m^2_{\eta'}) + \tilde{\rho}_{\rm QCD}(s)\,\theta(s - s_1), \tag{4}$$

[3] In this case the BLM scheme leads to the better convergence of the perturbation expansion. The NLO corrections are large in the minimal subtraction $\overline{\rm MS}$ scheme [7].

where s_0 and s_1 denote the continuum thresholds for YM theory and full QCD respectively. Other quantities in these equations are defined as follows:

$$\langle 0|\frac{g^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|G_0\rangle = f_{G_0}m^2_{G_0},$$

$$\langle 0|\frac{g^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|G\rangle = f_G m^2_G, \tag{5}$$

$$\langle 0|\frac{g^2}{32\pi^2}G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta'\rangle = f_{\eta'}m^2_{\eta'},$$

with m_G being the QCD glueball mass, m_{G_0} being the YM glueball mass, and f_G and f_{G_0} the corresponding decay constants.[4]

Before we turn to the application of the QCD sum rule method let us first compare the operator product expansions (OPE) for the quantity $B(Q^2)\equiv -\chi(Q^2)/Q^2$ in QCD and in YM theory. In leading order only the gluon fields contribute in both cases. The results of calculation of these OPE's can be found in Refs. [5,7,6]:

$$B^{\rm OPE}_{\rm YM}(Q^2)=\left(\frac{g^2}{32\pi^2}\right)^2 Q^2\left(\frac{2}{\pi^2}\log\frac{Q^2}{\mu^2_{\rm BLM}}+\frac{D_4}{Q^4}+\frac{D_6}{Q^6}\right)+\text{inst.},$$

$$B^{\rm OPE}_{\rm QCD}(Q^2)=\left(\frac{g^2}{32\pi^2}\right)^2 Q^2\left(\frac{2}{\pi^2}\log\frac{Q^2}{\mu^2_{\rm BLM}}+\frac{D'_4}{Q^4}+\frac{D'_6}{Q^6}\right)+\text{inst.}, \tag{6}$$

where

$$D_4=4\langle 0|G^a_{\mu\nu}G^a_{\mu\nu}|0\rangle,\quad D_6=8gf^{abc}\langle 0|G^a_{\mu\alpha}G^b_{\alpha\beta}G^c_{\beta\mu}|0\rangle,$$

$$D'_4=D_4+O(\alpha_s m_q\langle\bar{q}q\rangle),\quad D'_6=D_6+O(\alpha_s^2\langle\bar{q}q\rangle^2). \tag{7}$$

The instanton contributions in Eq. (6) are suppressed as Q^{-n}, where $n\simeq 12$ [5]. Since for practical calculations we use Eq. (3), the NLO perturbative corrections are not explicitly shown in Eq. (6) for brevity. Let us make some comments about the quantity $B^{\rm OPE}_{\rm QCD}$. As we mentioned above, the perturbative part of this correlator in the NLO is the same as that of $B^{\rm OPE}_{\rm YM}$. However, there are nonperturbative contributions in $B^{\rm OPE}_{\rm QCD}(Q^2)$ which do not appear in the expression for $B^{\rm OPE}_{\rm YM}(Q^2)$. Those are related to the quark condensate. The first such contribution modifies the $1/Q^4$ term in the OPE for $B^{\rm OPE}_{\rm QCD}$. The new contribution is proportional to $\alpha_s m_q\langle\bar{q}q\rangle$. The next nonperturbative correction related to the quark condensate appears at order $1/Q^6$ and yields a new term in addition to the operator O_6. The additional term is proportional to $\alpha_s^2\langle\bar{q}q\rangle^2$.

Now we can turn to the QCD sum rule analysis. It is useful to apply the Borel transformation [4] (with the Borel parameter denoted by M^2) to Eq. (2) with the phenomenological part on its left-hand side (LHS) and the operator product expansion (OPE) on its RHS. This leads to the following sum rules:

$$\int_0^\infty e^{-s/M^2}\frac{\rho^{poles}_{\rm YM}(s)}{s}ds=\int_0^{s_0}e^{-s/M^2}\frac{\tilde{\rho}_{\rm YM}(s)}{s}ds+\pi\left(\frac{g^2}{32\pi^2}\right)^2\left(D_4+\frac{D_6}{M^2}+\cdots\right)+\chi_{\rm YM}(0),$$

$$\int_0^\infty e^{-s/M^2}\frac{\rho^{poles}_{\rm QCD}(s)}{s}ds=\int_0^{s_1}e^{-s/M^2}\frac{\tilde{\rho}_{\rm QCD}(s)}{s}ds+\pi\left(\frac{g^2}{32\pi^2}\right)^2\left(D'_4+\frac{D'_6}{M^2}+\cdots\right).$$

Taking now the limit $M^2\to\infty$ we get[5]

$$-\chi_{\rm YM}(0)+f^2_{G_0}m^2_{G_0}=\frac{as_0^2}{2}+\pi\left(\frac{g^2}{32\pi^2}\right)^2 D_4, \tag{8}$$

$$f^2_G m^2_G+f^2_{\eta'}m^2_{\eta'}=\frac{as_1^2}{2}+\pi\left(\frac{g^2}{32\pi^2}\right)^2 D'_4. \tag{9}$$

We have mentioned already that our goal is to study how the QCD glueball mass differs from the one of quenched lattice gauge theory. However, Eqs. (8), (9) alone are not yet enough to determine whether that difference really exists. We need an additional relation. One way to get the new relation is to use the dispersion relation for the correlator of the topological susceptibility $\chi(Q^2)$ itself

$$\chi(Q^2)=\frac{1}{\pi}\int_0^\infty\frac{\rho(s)ds}{s+Q^2}+\text{subtractions}.$$

In order to get rid of the subtractions let us use again the Borel transformation. Applying this transformation to the correlator one gets the following relation:

$$\frac{1}{\pi}\int_0^\infty e^{-s/M^2}\rho^{poles}(s)ds=\frac{1}{\pi}\int_0^{s_0}e^{-s/M^2}\tilde{\rho}(s)ds-\left(\frac{g^2}{32\pi^2}\right)^2\left(D_6+O\left(\frac{1}{M^2}\right)\right),$$

[4]The gluonic operator $(g^2/32\pi^2)G\tilde{G}$ appearing in Eq. (5) has an anomalous dimension so that the constants f_{G_0}, f_G and $f_{\eta'}$ are not renormalization group invariant quantities [27]. For any of three f's one can construct the renormalization group invariant constant by means of a finite multiplicative renormalization of f's [28,29].

[5]In this way one derives the finite energy sum rules [30] modified by condensate contributions [31].

where we dropped the subscripts distinguishing QCD from YM theory. Taking the limit $M^2\to\infty$ we obtain the following relations:

$$f_{G_0}^2 m_{G_0}^4=\frac{as_0^3}{3}-\pi\left(\frac{g^2}{32\pi^2}\right)^2 D_6,$$

$$f_G^2 m_G^4+f_{\eta'}^2 m_{\eta'}^4=\frac{as_1^3}{3}-\pi\left(\frac{g^2}{32\pi^2}\right)^2 D_6'. \tag{10}$$

Having Eqs. (8)–(10), one can use them to find the relations between the quantities defined in YM theory and in full QCD. For that goal we are going to use the Witten-Veneziano arguments, but along with leading terms we keep corrections of order $1/N_c$. In the large N_c limit $f_G^2 m_G^2 \to f_{G_0}^2 m_{G_0}^2$, and in accordance with the Witten-Veneziano (WV) relation[6] [23, 24] $\chi_{\rm YM}(0)=-f_{\eta'}^2 m_{\eta'}^2$. As a result the left hand sides of Eqs. (8) and (9) are equal in the large N_c limit. On the other hand, the second terms on the RHS of Eqs. (8), (9) are also equal in that limit (difference between them is of order $1/N_c^2$). So, we come to the conclusion that $\lim_{N_c\to\infty}(s_1-s_0)=0$. The same result can be drawn from consideration of Eqs. (10) in the large N_c limit.

Since we are going to keep $1/N_c$ corrections as well as leading order terms let us introduce the following parametrization for the continuum thresholds

$$s_1=s_0+\frac{\delta}{N_c}, \tag{11}$$

where δ is an unknown quantity which is of order of the unity in the large N_c limit. In what follows we are going to solve the system of equations (8)–(11) keeping track of $1/N_c$ corrections. It is convenient to introduce the following notations

$$\tilde{D}_{4(6)}\equiv\pi\left(\frac{g^2}{32\pi^2}\right)^2 D_{4(6)}, \quad \tilde{D}'_{4(6)}\equiv\pi\left(\frac{g^2}{32\pi^2}\right)^2 D'_{4(6)}.$$

Based on the rules of the large N_c expansion one can see that

$$\tilde{D}'_4-\tilde{D}_4\propto O\left(m_q,\frac{1}{N_c^2}\right), \quad \tilde{D}'_6-\tilde{D}_6\propto O\left(\frac{\langle\bar{q}q\rangle^2}{N_c^4}\right).$$

Using the notations given above, and neglecting all contributions of order $1/N_c^2$ and higher, in the chiral limit one can rewrite the system of Eqs. (8)–(10) as follows:

$$-\chi_{\rm YM}(0)+f_{G_0}^2 m_{G_0}^2=\frac{as_0^2}{2}+\tilde{D}_4, \quad f_{G_0}^2 m_{G_0}^4=\frac{as_0^3}{3}-\tilde{D}_6, \tag{12}$$

$$f_G^2 m_G^2+f_{\eta'}^2 m_{\eta'}^2=\frac{as_1^2}{2}+\tilde{D}_4, \quad f_G^2 m_G^4+f_{\eta'}^2 m_{\eta'}^4=\frac{as_1^3}{3}-\tilde{D}_6. \tag{13}$$

Before we go further let us make some comments. Two equations given in (12) contain only two unknowns f_{G_0} and s_0 (assuming that the YM glueball mass m_{G_0} is known from lattice calculations [9]). Hence, those two equations can in general be solved and the values of f_{G_0} and s_0 can uniquely be determined. This is done in the next section. On the other hand, the two equations in (13) contain three unknowns, f_G and s_1 and m_G. So one cannot determine m_G uniquely. The only thing one can do is to calculate the decay constant f_G and mass m_G for chosen values of the continuum threshold s_1 which are dictated by previous analyses of the flavor singlet pseudoscalar channel. This calculation is also carried out below. Before we turn to the numerical simulations we can try to extract some analytic relations for the QCD glueball mass and decay constant studying the system of equations (12), (13). The relation between the continuum threshold parameters (11) allows one to set the equations (12), (13) as follows:

$$f_G^2 m_G^2+f_{\eta'}^2 m_{\eta'}^2=-\chi_{\rm YM}(0)+f_{G_0}^2 m_{G_0}^2+\frac{a\delta}{N_c}s_0, \tag{14}$$

$$f_G^2 m_G^4+f_{\eta'}^2 m_{\eta'}^4=f_{G_0}^2 m_{G_0}^4+\frac{a\delta}{N_c}s_0^2. \tag{15}$$

Now the key observation is that in pure YM theory there are no light meson states, thus the decay $G_0\to 3\pi$ does not occur in this theory. On the other hand, this decay should easily go in QCD. Hence, the continuum threshold for QCD s_1 should be less than or equal to the continuum threshold for YM theory[7]:

$$s_1\leqslant s_0 \quad \text{or} \quad \delta\leqslant 0. \tag{16}$$

Thus, up to order $1/N_c^2$ and in the chiral limit the following inequalities exist between the quantities defined in pure YM theory and in full QCD:

$$f_G^2 m_G^2+f_{\eta'}^2 m_{\eta'}^2\leqslant-\chi_{\rm YM}(0)+f_{G_0}^2 m_{G_0}^2, \tag{17}$$

$$f_G^2 m_G^4+f_{\eta'}^2 m_{\eta'}^4\leqslant f_{G_0}^2 m_{G_0}^4. \tag{18}$$

Let us point out that the inequality (17) yields the Witten-Veneziano relation in the limit of infinite N_c. Indeed, in the large N_c limit $f_G^2 m_G^2\to f_{G_0}^2 m_{G_0}^2$, $s_0\to s_1$. Hence, in that limit

---

[6]In our conventions χ is defined in Minkowski space-time and turns out to be a negative quantity. One can turn to Euclidean space-time in Eq. (1) making the substitutions $x_0\to -ix_4$ and $G\tilde{G}\to iG\tilde{G}$. The Euclidean quantity, which is used in the lattice calculations, is a positive number $\chi_{\rm YM}^{Eucl}(0)=-\chi_{\rm YM}^{Mink}(0)$.

[7]I am grateful to Glennys Farrar for bringing this line of arguments to my attention.

(17) is saturated and it turns into the Witten-Veneziano (WV) relation [23,24] $\chi_{\rm YM}(0)=-f^2_{\eta'}m^2_{\eta'}$.

Let us rewrite (18) in the following form:

$$m_G^4 \leqslant \left(\frac{f_{G_0}}{f_G}\right)^2 m_{G_0}^4-\left(\frac{f_{\eta'}}{f_G}\right)^2 m_{\eta'}^4. \qquad (19)$$

This inequality allows one to calculate the upper bound on the pseudoscalar glueball mass in QCD. The inequality shows that if the values for f_{G_0} and f_G are sufficiently close to each other, then the QCD glueball is lighter than the glueball of YM theory. In the next section, based on numerical studies, we demonstrate that this indeed is the case.

It is interesting to check the large N_c behavior of (19). Notice that, since the anomaly disappears in the limit when $N_c\to\infty$, there is no flavor singlet meson-glueball mixing term anymore in the effective Lagrangian [32] in that limit. One should therefore expect to have equal masses for the glueballs in QCD and pure YM theory when $N_c\to\infty$. Recalling that $f_{\eta'}\propto\sqrt{N_c}$, $m^2_{\eta'}\propto 1/N_c$, $f^2_{G_0}m^2_{G_0}\sim 1$ and substituting these into Eq. (19) we get

$$m_{G_0}^2-m_G^2\propto\frac{1}{N_c},$$

which is consistent with one's expectation.

II. SOME ESTIMATES AND PREDICTIONS

Let us now turn to numerical estimates. First of all let us list all the approximations we made deriving Eqs. (12)–(15). There are scheme dependent NLO perturbative corrections involved in the derivation. Besides that, we worked in the chiral limit neglecting u, d and, most importantly, s quark masses. Hence, the $\eta-\eta'$ mixing and all other nonsinglet pseudoscalar mesons are also neglected.[8] Below we show that the contributions of the $\eta(1295)$, the $\eta(1410)$ and the $\eta(1490)$ in the sum rules are rather small and can also be neglected.

If one knew all the numerical values for the quantities on the RHS of Eq. (19), one would be able to predict the upper bound on the QCD glueball mass. Unfortunately, f_{G_0} and f_G are not known. We can use however the sum rules derived in the previous section to calculate the value for f_{G_0}. Indeed, consider the equations given in (12). One can solve this system with f_{G_0} and s_0 treated as unknowns. The numerical solution for that system yields the following result: $f_{G_0}=(27\pm 3)$ MeV and $s_0=7.4\pm 0.5$ GeV^2.

In calculating the numbers given above we have used the following numerical values for[9] $f_{\eta'}=(29\pm 3)$ MeV, $m_{\eta'}=(957.77\pm 0.14)$ MeV, $m_{G_0}=(2.3\pm 0.2)$ GeV [9], $\alpha_s(2\ \text{GeV})=0.33\pm 0.05$. There are a number of estimates for the gluon condensate in the literature (see Refs. [4,33]). We take the world average value of these calculations $\langle(\alpha_s/\pi)G^2_{\mu\nu}\rangle=(2.5\pm 0.9)\ 10^{-2}$ GeV^4. The corresponding values for $\tilde{D}_4$ and $\tilde{D}_6$ are $\tilde{D}_4=(4.0\pm 1.7)10^{-4}$ GeV^4 and $\tilde{D}_6=(0.7\pm 0.3)10^{-4}$ GeV^6.

There are also several lattice estimates for the topological susceptibility $\chi_{\rm YM}(0)$ (for a recent review see [34]). For our estimates we take the result of Ref. [35], $\chi^{Eucl}_{\rm YM}(0)=(175\pm 5\ \text{MeV})^4$, which is also in good agreement with an earlier theoretical estimate[10] [24].

Now we can make the prediction for the matrix elements of the gluonic operator acting on the pseudoscalar pure YM glueball state with mass $m_{G_0}=(2.3\pm 0.2)$ GeV

$$\langle 0|g^2G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|G_0\rangle=(45\pm 9)\ \text{GeV}^3. \qquad (20)$$

The uncertainty in this result dominantly comes from the error bars associated with the value for the topological susceptibility and also with the value[11] of $f_{\eta'}$. The same matrix element for the η' meson state has the following numerical value $\langle 0|g^2G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta'\rangle=(8.4\pm 0.8)$ GeV^3. The values of the matrix element for the QCD glueball state G will also be given below. Notice that all the numerical results for the scale dependent quantities such as the decay constants f_{G_0}, f_G and the matrix element in Eq. (20) are to be taken at the normalization point approximately equal to the value of the glueball mass. Also, all the results presented above should be given 10–20 percent systematic error bars associated with the approximations and the method we have used.

As we mentioned already we checked whether the presence of the $\eta(1295)$, the $\eta(1410)$ and the $\eta(1490)$ mesons

[8] All these corrections are expected to be of order $O(m_{u,d,s}/\Lambda_{glueball})$, where $\Lambda_{glueball}\simeq 1.5-2$ GeV is an effective scale above which the existence of glueballs becomes important for hadron physics.

[9] Which corresponds to $F_0=(105\pm 12)$ MeV taken in conventional normalization for the singlet axial current.

[10] In order to study whether our results are sensitive to the numerical value of $\chi_{\rm YM}(0)$ we varied the value of this quantity from $(100\ \text{MeV})^4$ to $(190\ \text{MeV})^4$. Outside of the interval given by these numbers the system of equations does not have a solution. On the other hand, inside the interval the results are rather insensitive to the value of $\chi_{\rm YM}(0)$; varying $\chi_{\rm YM}(0)$, s_0 changes in the region 6.8–8 GeV and f_{G_0} varies in the interval 24–34 MeV.

[11] The only available lattice calculations for the matrix elements of the gluonic operators were presented in Ref. [36]. The predictions of Ref. [36] differ from the theoretical estimates in the case of 0^{++} channel by a factor of 2 or so [12]. In the pseudoscalar channel the deviation is more substantial; a factor of 7–10 difference occurs when one compares the results of the QCD sum rule calculations of the matrix element (20) (our result and the result of Ref. [37]) with the respective prediction of Ref. [36]. That discrepancy could be decreased by a factor of 2.7 if one uses the results of Ref. [36] with the updated value of the pseudoscalar glueball mass $m_{G_0}\simeq 2.3$ GeV [9] (instead of the old value $m_{G_0}\simeq 1.4$ GeV implied in Ref. [36]); however further theoretical and lattice studies are needed to clarify this issue completely.

TABLE I. Sum rule results.

s_1 GeV^2	f_G MeV	m_G GeV	s_1 GeV^2	f_G MeV	m_G GeV
7.4	29 ± 2	2.27 ± 0.04	5.0	21.5 ± 2.5	1.9 ± 0.05
7.0	28 ± 2	2.2 ± 0.04	4.5	20 ± 3	1.8 ± 0.1
6.5	26.5 ± 1.5	2.15 ± 0.05	4.0	18 ± 3.5	1.73 ± 0.12
6.0	25 ± 2	2.07 ± 0.05	3.5	16 ± 4	1.61 ± 0.14
5.5	23 ± 2	1.97 ± 0.05	3.0	13.5 ± 5.5	1.47 ± 0.20

could affect our results. We have included the contributions of these resonances in the sum rules used above. In order to determine the decay constants of these resonances we used the experimental data for the production rate of these states in J/ψ radiative decays [16,38]. The values of the decay constants [defined as in Eqs. (5)] are rather small numbers: $f_{\eta(1490)}=(7.5\pm 1.9)$ MeV, $f_{\eta(1410)}=(4.8\text{–}6.7)$ MeV and $f_{\eta(1295)}=(4.5\pm 1.0)$ MeV. Including these numbers into the sum rules one can see that the final results for the glueball mass and decay constants change unsubstantially.

Now let us turn to the calculation of the mass and decay constant of the QCD pseudoscalar glueball. Unfortunately, our method does not allow one to determine these values uniquely. The set of two equations given in (13) contains three unknowns, f_G, m_G and s_1. Below we present the results of numerical solutions of these equations for different values of the continuum threshold s_1. The values for s_1 are chosen between the upper bound determined as $s_1\leqslant s_0=7.4$ GeV^2 and the lower bound given by $s_1\geqslant 3$ GeV^2 (below this value the continuum threshold comes very close to the resonance mass square and applicability of the sum rule approach breaks down). The results are summarized in Table I.[12]

As we see from the table the QCD glueball is lighter than the glueball of YM theory. However, it is hard to identify the QCD glueball with the $\eta(1410)$. The very low value for the continuum threshold is needed in order to have QCD glueball mass at about 1.4 GeV.

Though we cannot determine uniquely the value for the continuum threshold from our consideration, one can use the estimate given in Ref. [26] $s_1\simeq 10m_\rho^2\simeq 6$ GeV^2. If this number is accepted, then in accordance with Table I the QCD glueball mass is $m_G=[2.07\pm 0.05\pm 0.3\ (\text{syst.})]$ GeV and decay constant $f_G=[25\pm 2\pm 4\ (\text{syst.})]$ MeV.

Having these numbers at hand one can predict the J/ψ decay width into a glueball state and photon. J/ψ radiative decays are very effective tools in studying the spectroscopy on light mesons. In the present case we are going to deal with the processes like $J/\psi\to R(0^{-+})\gamma$, where R stands for the resonance being considered. The theory of these decays was worked out in Ref. [39].[13] One can consider the ratio

$$r\equiv\frac{\Gamma(J/\psi\to G\gamma)}{\Gamma(J/\psi\to\eta'\gamma)}.$$

This ratio is independent of the J/ψ meson wave function and is completely defined by the properties of the pseudoscalar mesons produced in the decay. Assuming that the decay dominantly goes through the exchange of the intermediate gluons in the pseudoscalar state, the ratio r can be rewritten as follows [39]:

$$r=\frac{|\langle 0|g^2G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|G\rangle|^2}{|\langle 0|g^2G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|\eta'\rangle|^2}\left(\frac{m^2_{J/\psi}-m^2_G}{m^2_{J/\psi}-m^2_{\eta'}}\right)^3+O(\alpha_s^2).$$

Using the numerical results listed above in the table one can calculate the ratio r, matrix element $k\equiv\langle 0|g^2G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}|G\rangle$ and decay width $\Gamma(J/\psi\to G\gamma)$ for the QCD glueball state. These results are summarized in Table II.

The numerical values for the decay width of the J/ψ meson into the QCD glueball and photon are substantially large and this decay can be observed in recent experimental studies.

CONCLUSIONS

Let us summarize our results. We studied whether the pseudoscalar glueball mass in full QCD differs from the one determined in the quenched lattice calculations.

TABLE II. Some predictions.

f_G MeV	m_G GeV	k GeV^3	r ratio	Γ keV
29 ± 2	2.27 ± 0.04	47.2 ± 5.0	4.2 ± 1.1	1.57 ± 0.4
28 ± 2	2.2 ± 0.04	42.8 ± 4.7	4.2 ± 1.0	1.57 ± 0.37
25 ± 2	2.07 ± 0.05	33.8 ± 4.5	3.7 ± 1.0	1.38 ± 0.37
21.5 ± 2.5	1.9 ± 0.06	24.5 ± 4.6	2.8 ± 1.1	1.0 ± 0.4
18 ± 3.5	1.73 ± 0.12	17.0 ± 5.2	1.8 ± 1.2	0.67 ± 0.45
13.5 ± 5.5	1.47 ± 0.20	9.2 ± 7.5	0.75 ± 1.4	0.28 ± 0.53

[12]In Table I and also Table II below only the error bars associated with the method of numerical calculations are given.

[13]See also the approach developed in [40,3].

An inequality which sets the upper bound on the mass of the pseudoscalar glueball in QCD [Eq. (19)] is derived. In order to calculate that bound numerically one needs to know the decay constant of the QCD 0^{-+} glueball and also the mass and decay constant of the pure YM glueball.

The decay constants in this work are calculated using the QCD sum rule approach, while the value for the YM glueball mass is taken from lattice calculations. The value calculated for the decay constant of pure YM glueball is shown to be important for self-consistency checks of lattice results.

We found numerically that the mass of the QCD glueball is less than the mass of the glueball of pure YM theory. The values for the 0^{-+} QCD glueball mass and decay constant (for the phenomenologically preferred value of the continuum threshold parameter) are $m_G=[2.07\pm0.05 \pm 0.3\ (\mathrm{syst})]$ GeV and $f_G=[25\pm2\pm4\ (\mathrm{syst})]$ MeV. If these numbers are accepted, then there is no particle discovered so far which might be identified with the QCD pseudoscalar glueball. Further experimental searches in the 2 GeV region are needed. In this case the status of the $\eta(1410)$ is unclear.

In order to help resolve this question, we predict the production rate of the QCD glueball in the radiative decay of the J/ψ meson, $\Gamma(J/\psi\rightarrow G\gamma)$. For a 2.07 GeV glueball $\Gamma(J/\psi \rightarrow G\gamma)$ is about three or four times greater than for the η' meson. Thus, the prediction for the branching ratio for that process is large enough to be studied experimentally.

Note added. After this work was done we became aware of the paper [37] where the QCD sum rule method was used to calculate glueball masses and decay constants in scalar, pseudoscalar and tensor channels. The calculations in Ref. [37] are done (without referring to lattice results) using the optimization procedure with respect to Borel parameters and continuum thresholds. Our results for the mass and decay constant of the pseudoscalar glueball are in good agreement with the predictions of [37]. We are grateful to M. Schwetz for bringing Ref. [37] to our attention.

ACKNOWLEDGMENTS

I am grateful to Glennys R. Farrar for bringing this problem to my attention and for valuable discussions and suggestions. The author wishes to thank H. Neuberger and T. DeGrand for useful discussions.

[1] H. Fritzsch and M. Gell-Mann, in Proceedings of the XVI International Conference on High Energy Physics, Vol. 2, p. 135, Chicago, 1972.

[2] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).

[3] F. E. Close, G. R. Farrar, and Zh. Li, Phys. Rev. D **55**, 5749 (1997).

[4] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B147**, 385 (1979); **B147**, 448 (1979).

[5] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Lett. **86B**, 347 (1979).

[6] S. Narison, Z. Phys. C **26**, 209 (1984).

[7] A. L. Kataev, N. V. Krasnikov, and A. A. Pivovarov, Nucl. Phys. **B198**, 508 (1982); **B440**, 505E (1997).

[8] G. Grunberg, Phys. Rev. D **30**, 1570 (1984).

[9] UKQCD Collaboration, G. S. Bali *et al.*, Phys. Lett. B **309**, 378 (1993).

[10] H. Chen, J. Sexton, A. Vaccarino, and D. Weingarten, Nucl. Phys. B (Proc. Suppl.) **34**, 357 (1994); D. Weingarten, ''Evidence for the Observation of a Glueball,'' hep-ph/9607212.

[11] G. West, Phys. Rev. Lett. **77**, 2622 (1996).

[12] T. Schäfer and E. V. Shuryak, Phys. Rev. Lett. **75**, 1707 (1995).

[13] C. Amsler and F. E. Close, Phys. Lett. B **353**, 385 (1995); Phys. Rev. D **53**, 295 (1996).

[14] A. V. Anisovich, V. V. Anisovich, and A. V. Sarantsev, Z. Phys. A **359**, 173 (1997).

[15] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976).

[16] Mark III Collaboration, Z. Bai *et al.*, Phys. Rev. Lett. **65**, 2507 (1990).

[17] Mark III Collaboration, T. Bolton *et al.*, Phys. Rev. Lett. **69**, 1328 (1992).

[18] J. Augustin *et al.*, Phys. Rev. D **42**, 10 (1990).

[19] Crystal Barrel Collaboroation, C. Amsler *et al.*, Phys. Lett. B **358**, 389 (1995).

[20] J. Bartelski and S. Tatur, Phys. Lett. B **289**, 429 (1992).

[21] P. Ball, J.-M. Frère, and M. Tytgat, Phys. Lett. B **365**, 367 (1996).

[22] P. G. O. Freund and Y. Nambu, Phys. Rev. Lett. **34**, 1645 (1975).

[23] E. Witten, Nucl. Phys. **B156**, 269 (1979).

[24] G. Veneziano, Nucl. Phys. **B159**, 213 (1979).

[25] S. J. Brodsky, G. P. Lepage, and P. B. Mackenzie, Phys. Rev. D **28**, 228 (1983).

[26] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B191**, 301 (1981).

[27] G. M. Schore and G. Veneziano, Nucl. Phys. **B381**, 23 (1992).

[28] D. I. Kazakov, JETP Lett. **41**, 335 (1985).

[29] M. Glük and E. Reya, Z. Phys. C **39**, 569 (1988).

[30] K. G. Chetyrkin, N. V. Krasnikov, and A. N. Tavkhelidze, Phys. Lett. **76B**, 83 (1978); K. G. Chetyrkin and N. V. Krasnikov, Nucl. Phys. **B119**, 174 (1977).

[31] A. L. Kataev, N. V. Krasnikov, and A. A. Pivovarov, Phys. Lett. **123B**, 93 (1982); N. V. Krasnikov, A. A. Pivovarov, and N. N. Tavkhelidze, Z. Phys. C **19**, 301 (1983).

[32] P. Di Vecchia and G. Veneziano, Nucl. Phys. **B171**, 253 (1980); C. Rozenzweig, J. Schechter, and C. G. Trahern, Phys. Rev. D **21**, 3388 (1980); E. Witten, Ann. Phys. (N.Y.) **128**, 363 (1980).

[33] L. J. Reinders, H. Rubinstein, and S. Yazaki, Nucl. Phys. **B186**, 109 (1981); J. S. Bell and R. A. Bertlmann, *ibid.* **B177**, 218 (1981); K. J. Miller and M. G. Olsson, Phys. Rev. D **25**, 1247 (1982); **25**, 1253 (1982); G. Launer, S. Narison, and R. Tarrach, Z. Phys. C **26**, 433 (1984); R. A. Bertlmann *et al.*,

ibid. **39**, 231 (1988); C. A. Dominquez and J. Sola, *ibid.* **40**, 63 (1988); S. Narison, Phys. Lett. B **387**, 162 (1996).

[34] T. Schäfer and E. V. Shuryak, ''Instantons in QCD,'' hep-ph/9610451.

[35] J. Hoek, M. Teper, and J. Waterhouse, Phys. Lett. B **180**, 112 (1986); A. S. Kronfeld, M. L. Laursen, G. Schierholz, S. Schleiermacher, and U.-J. Wiese, Nucl. Phys. **B305**, 661 (1988); M. Teper, Phys. Lett. B **202**, 553 (1988); B. Alles, M. D'Elia, and A. Di Giacomo, Nucl. Phys. **B494**, 281 (1997).

[36] Y. Liang, K. Liu, B. Li, S. Dong, and K. Ishikawa, Phys. Lett. B **307**, 375 (1993).

[37] S. Narison, Nucl. Phys. B (Proc. Suppl.) **64**, 210 (1998).

[38] BES Collaboration, J. Z. Bai *et al.*, BIHEP-EP1 95-04, 1996.

[39] V. A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. **B165**, 55 (1980).

[40] M. B. Cakir and G. R. Farrar, Phys. Rev. D **50**, 3268 (1994).