

Scalar $B \rightarrow \pi$ and $D \rightarrow \pi$ form factors in QCD

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QCD sum rules on the light cone are derived for the sum $f^+ + f^-$ of the $B \rightarrow \pi$ and $D \rightarrow \pi$ form factors taking into account contributions up to twist four. Combining the results with the corresponding f^+ form factors calculated previously by the same method, we obtain the scalar form factors f^0 . Our sum rule predictions are compared with lattice results, current-algebra constraints, and quark-model calculations. Furthermore, we calculate decay distributions and the integrated width for the semileptonic decay $B \rightarrow \pi \bar{\tau} \nu_\tau$ which is sensitive to f^0 . Finally, the dependence of the sum rules on the heavy quark mass and the asymptotic scaling laws are discussed. [S0556-2821(98)02117-1]

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I. INTRODUCTION

The weak transition $B \rightarrow \pi$ plays an exceptional role in B physics, in particular at future B factories. The amplitude of this transition is given by the hadronic matrix element

$$\begin{aligned} \langle \pi(q) | \bar{u} \gamma_\mu b | B(p+q) \rangle \\ = 2f^+(p^2)q_\mu + (f^+(p^2) + f^-(p^2))p_\mu, \end{aligned} \quad (1)$$

where $p+q$ and q denote the initial and final state momenta, respectively, $\bar{u} \gamma_\mu b$ is the relevant weak vector current, and f^\pm are the two independent form factors. The form factor $f^+(p^2)$ was calculated in [1,2] using the technique of QCD sum rules on the light cone. Recently, one has also computed the perturbative QCD corrections to f^+ [3,4].

In the present paper we complete the calculation of the matrix element (1) by deriving the corresponding light-cone sum rule for the sum of form factors $f^+ + f^-$. This quantity turns out to be a pure higher-twist effect. The leading twist-2 contribution vanishes kinematically. From $f^+ + f^-$ and f^+ , one can construct the scalar form factor

$$\begin{aligned} f^0(p^2) = & \left(1 - \frac{p^2}{m_B^2 - m_\pi^2} \right) f^+(p^2) \\ & + \frac{p^2}{m_B^2 - m_\pi^2} (f^+(p^2) + f^-(p^2)), \end{aligned} \quad (2)$$

which determines the matrix element of the divergence of the weak vector current:

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$$\langle \pi(q) | \partial^\mu (\bar{u} \gamma_\mu b) | B(p+q) \rangle = (m_B^2 - m_\pi^2) f^0(p^2). \quad (3)$$

Using the results on f^+ and f^0 we predict the momentum transfer and lepton energy distributions as well as the width of the semileptonic decay $B \rightarrow \pi \bar{\tau} \nu_\tau$. As a by-product, we also obtain the analogous form factors of the $D \rightarrow \pi$ transition.

Furthermore, we investigate the heavy-mass dependence of heavy-to-light form factors. The asymptotic scaling laws are determined and found to differ at small and large momentum transfer. The origin of this difference is explained in detail. We also study the approach to the heavy-quark limit numerically and show that it is reached very slowly. Moreover, the behavior beyond the physical b -quark mass turns out to be very sensitive to the scale dependence of the pion wave functions.

The paper is organized as follows. In Sec. II, we derive the light-cone sum rule for $f^+ + f^-$ and compare it with the corresponding sum rule for f^+ . The numerical analysis of the new sum rule and the resulting prediction of the scalar form factor f^0 are presented in Sec. III. Section IV is devoted to the semileptonic decay $B \rightarrow \pi \bar{\tau} \nu_\tau$, and Sec. V to the heavy-mass dependence of the form factors. Our conclusions are summarized in Sec. VI.

II. LIGHT-CONE SUM RULE FOR $f^+ + f^-$

In order to obtain the QCD sum rule for the form factor combination $f^+ + f^-$ appearing in Eq. (1), we follow the method applied to f^+ and explained in detail in [1,2]. The main object of investigation is the vacuum-pion correlation function

$$\begin{aligned}
F_\mu(p, q) &= i \int d^4x \\
&\times e^{ipx} \langle \pi(q) | T \{ \bar{u}(x) \gamma_\mu b(x), \bar{b}(0) i \gamma_5 d(0) \} | 0 \rangle \\
&= F(p^2, (p+q)^2) q_\mu + \tilde{F}(p^2, (p+q)^2) p_\mu. \quad (4)
\end{aligned}$$

Insertion of a complete set of hadronic states with B -meson quantum numbers between the currents in Eq. (4) entails relations between the physical form factors f^+ and $f^+ + f^-$ and the invariant amplitudes F and \tilde{F} , respectively. More definitely, for \tilde{F} one finds

$$\begin{aligned}
\tilde{F}(p^2, (p+q)^2) &= \frac{m_b^2 f_B (f^+(p^2) + f^-(p^2))}{m_b (m_B^2 - (p+q)^2)} \\
&+ \int_{s_0^h}^{\infty} ds \frac{\tilde{\rho}^h(p^2, s)}{s - (p+q)^2}, \quad (5)
\end{aligned}$$

where the term proportional to $f^+ + f^-$ arises from the contribution of the ground state B meson, while the integral over the spectral density $\tilde{\rho}^h$ represents the contributions from excited resonances and continuum states above the threshold energy $\sqrt{s_0^h}$. In deriving this hadronic representation of \tilde{F} we have used the matrix element (1) and

$$\langle B | \bar{b} i \gamma_5 d | 0 \rangle = m_B^2 f_B / m_b, \quad (6)$$

f_B being the B meson decay constant.

In [1,2], the invariant amplitude F of the same correlation function (4) is calculated by expanding the T -product of the currents near the light-cone at $x^2=0$. The leading contribution to the operator product expansion (OPE) is obtained by contracting the b -quark fields in Eq. (4) and inserting the free b -quark propagator

$$\langle 0 | T \{ b(x) \bar{b}(0) \} | 0 \rangle = i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{\mathbf{k} + m_b}{k^2 - m_b^2}. \quad (7)$$

Substitution of Eq. (7) in Eq. (4) yields

$$\begin{aligned}
F_\mu(p, q) &= i \int \frac{d^4x d^4k}{(2\pi)^4 (m_b^2 - k^2)} \\
&\times e^{i(p-k)x} (m_b \langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_5 d(0) | 0 \rangle \\
&+ k^\nu \langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_\nu \gamma_5 d(0) | 0 \rangle). \quad (8)
\end{aligned}$$

This approximation is valid in the region of momenta ($p + q$)² $\ll m_b^2$ and

$$p^2 \leq m_b^2 - 2m_b \chi, \quad (9)$$

χ being a m_b -independent scale of order Λ_{QCD} . Since the pion is on-shell, $q^2 = m_\pi^2$ vanishes in the chiral limit adopted throughout this calculation. The above restrictions ensure that the b quark is sufficiently off-shell, and that the resonances in the $\bar{u}b$ channel are sufficiently far away.

The bilocal vacuum-to-pion matrix elements of light-quark fields encountered on the right-hand side (RHS) of Eq. (8) are expanded around $x^2=0$ leading to a series of contributions with increasing twist. The coefficient functions of this expansion can be parametrized by pion wave functions on the light cone [5–7]. Including terms up to order x^2 , the light-cone expansion of the first matrix element in Eq. (8) reads

$$\begin{aligned}
&\langle \pi(q) | \bar{u}(x) \gamma_\mu \gamma_5 d(0) | 0 \rangle \\
&= -i q_\mu f_\pi \int_0^1 du e^{iuqx} (\varphi_\pi(u) + x^2 g_1(u)) \\
&+ f_\pi \left(x_\mu - \frac{x^2 q_\mu}{qx} \right) \int_0^1 du e^{iuqx} g_2(u). \quad (10)
\end{aligned}$$

Here, φ_π is the leading twist 2 wave function, while g_1 and g_2 are twist 4 wave functions. Upon substitution of $\gamma_\mu \gamma_\nu = -i \sigma_{\mu\nu} + g_{\mu\nu}$, the second term in Eq. (8) is decomposed into the matrix elements

$$\langle \pi(q) | \bar{u}(x) i \gamma_5 d(0) | 0 \rangle = f_\pi \mu_\pi \int_0^1 du e^{iuqx} \varphi_p(u) \quad (11)$$

and

$$\begin{aligned}
&\langle \pi(q) | \bar{u}(x) \sigma_{\mu\nu} \gamma_5 d(0) | 0 \rangle \\
&= i (q_\mu x_\nu - q_\nu x_\mu) \frac{f_\pi \mu_\pi}{6} \int_0^1 du e^{iuqx} \varphi_\sigma(u), \quad (12)
\end{aligned}$$

with $\mu_\pi = m_\pi^2 / (m_u + m_d)$. In leading order, these matrix elements involve the twist 3 wave functions φ_p and φ_σ . It is worth pointing out that the path-ordered gluon operator

$$\Pi_G = P \exp \left\{ i g_s \int_0^1 d\alpha x_\mu A^\mu(\alpha x) \right\} \quad (13)$$

ensuring gauge invariance of the above matrix elements is unity in the light-cone gauge, $x_\mu A^\mu = 0$, assumed here. Therefore, the factor Π_G is not shown explicitly in Eqs. (10)–(12).

Substitution of Eqs. (10)–(12) in Eq. (8), integration over x and k , and collection of all terms proportional to p_μ yield the following expression for the invariant amplitude \tilde{F} :

$$\begin{aligned}
&\tilde{F}_{QCD}(p^2, (p+q)^2) \\
&= f_\pi \int_0^1 \frac{du}{m_b^2 - (p+uq)^2} \left\{ \mu_\pi \varphi_p(u) + \frac{\mu_\pi \varphi_\sigma(u)}{6u} \right. \\
&\times \left. \left(1 - \frac{m_b^2 - p^2}{m_b^2 - (p+uq)^2} \right) + \frac{2m_b g_2(u)}{m_b^2 - (p+uq)^2} \right\}. \quad (14)
\end{aligned}$$

The index QCD has been added to distinguish the above representation of the invariant function \tilde{F} in terms of quark and gluon degrees of freedom from the hadronic representa-

tion given in Eq. (5). Note that the twist 2 and 4 wave functions φ_π and g_1 , respectively, do not contribute to \tilde{F} . This is obvious from the definition (10). In general, the correlation function (4) also receives contributions from gluon emission by the b quark. This correction involves quark-antiquark-gluon wave functions as described in [1,2]. However, direct calculation shows that up to the twist 3 and 4 the three-particle correction vanishes in the invariant amplitude \tilde{F} . Hence, to twist 4 accuracy, the result for \tilde{F} turns out to be remarkably simple, at least when compared with the corresponding expression for the invariant amplitude F given in [1,2].

The equality of the two representations (5) and (14) of \tilde{F} implies a sum rule for $f^+ + f^-$ which, however, is only useful if one can remove the unknown contributions from the excited and continuum states. This is possible to a reasonable approximation by making use of quark-hadron duality. Following the standard procedure, the integral in Eq. (5) over the hadronic spectral function above the ground state is replaced by the corresponding integral over the imaginary part of \tilde{F}_{QCD} . Formally, one can substitute

$$\tilde{\rho}^h(p^2, s) \Theta(s - s_0^h) = \frac{1}{\pi} \text{Im} \tilde{F}_{QCD}(p^2, s) \Theta(s - s_0^B), \quad (15)$$

where s_0^B is an effective threshold parameter separating the duality interval of the ground state from the one of the higher states. With this approximation, it is straightforward to subtract the contribution of the excited and continuum states from the basic equation given by Eqs. (5) and (14). After performing the obligatory Borel transformation in $(p+q)^2$, one finally arrives at the sum rule

$$f_B(f^+(p^2) + f^-(p^2)) = \frac{m_b}{\pi m_B^2} \int_{m_b^2}^{s_0^B} \text{Im} \tilde{F}_{QCD}(p^2, s) \times \exp\left(\frac{m_B^2 - s}{M^2}\right) ds, \quad (16)$$

M^2 being the Borel mass parameter.

The remaining task is then to derive $\text{Im} \tilde{F}_{QCD}(p^2, s)$ from Eq. (14). This is explained below. Using $(p+uq)^2 = (1-u)p^2 + u(p+q)^2$ and changing variable from u to $s = (m_b^2 - p^2)/u + p^2$ one can rewrite Eq. (14) as follows:

$$\tilde{F}_{QCD}(p^2, (p+q)^2) = \sum_{i=1,2} \int_{m_b^2}^{\infty} ds \frac{\rho_i(p^2, s)}{(s - (p+q)^2)^i}, \quad (17)$$

where

$$\rho_1(p^2, s) = \frac{f_\pi \mu_\pi}{s - p^2} \left(\varphi_p(u) + \frac{\varphi_\sigma(u)}{6u} \right), \quad (18)$$

$$\rho_2(p^2, s) = \frac{f_\pi}{m_b^2 - p^2} \left(-\frac{\mu_\pi \varphi_\sigma(u)}{6u} (m_b^2 - p^2) + 2m_b g_2(u) \right). \quad (19)$$

The term $i=1$ in Eq. (17) already has the form of a dispersion integral in the variable $(p+q)^2$. In order to achieve this also for the term $i=2$ one has to perform a partial integration yielding in total:

$$\tilde{F}_{QCD}(p^2, (p+q)^2) = \int_{m_b^2}^{\infty} \frac{ds}{s - (p+q)^2} \left(\rho_1(p^2, s) + \frac{d\rho_2(p^2, s)}{ds} \right) - \int_{m_b^2}^{\infty} ds \frac{d}{ds} \left(\frac{\rho_2(p^2, s)}{s - (p+q)^2} \right). \quad (20)$$

Since the wave functions φ_σ and g_2 vanish at $u=0$ and $u=1$, that is $s=\infty$ and $s=m_b^2$, respectively, as can be seen from the explicit expressions given in the subsequent section, the second integral in Eq. (20) is zero. Hence, the imaginary part of \tilde{F}_{QCD} can be directly read off from the integrand of the first integral:

$$\frac{1}{\pi} \text{Im} \tilde{F}_{QCD}(p^2, s) = \rho_1(p^2, s) + \frac{d\rho_2(p^2, s)}{ds}. \quad (21)$$

Substitution of Eq. (21) in Eq. (16) yields

$$f_B(f^+(p^2) + f^-(p^2)) = \frac{m_b}{m_B^2} \int_{m_b^2}^{s_0^B} \left(\rho_1(p^2, s) + \frac{d\rho_2(p^2, s)}{ds} \right) \exp\left(\frac{m_B^2 - s}{M^2}\right) ds = \frac{m_b}{m_B^2} \left\{ \int_{m_b^2}^{s_0^B} \left(\rho_1(p^2, s) + \frac{\rho_2(p^2, s)}{M^2} \right) \exp\left(\frac{m_B^2 - s}{M^2}\right) ds + \rho_2(p^2, s_0^B) \exp\left(\frac{m_B^2 - s_0^B}{M^2}\right) \right\}. \quad (22)$$

In previous applications of QCD light-cone sum rules with the exception of the recent calculation of the $B \rightarrow \rho$ form factors in [8], surface terms similar to the last term on the RHS of Eq. (22) have been neglected. They originate from higher twist contributions to $\text{Im} \tilde{F}_{QCD}$ and play a minor role numerically. Nevertheless, in order to subtract the contributions from excited and continuum states in the duality approximation consistently we take these terms into account in the present calculation.

The final sum rule for $f^+ + f^-$ follows from Eqs. (18), (19) and (22) after returning to the variable u :

$$\begin{aligned}
& f_B(f^+(p^2) + f^-(p^2)) \\
&= \frac{f_\pi \mu_\pi m_b}{m_B^2} \exp\left(\frac{m_B^2}{M^2}\right) \left\{ \int_\Delta^1 \frac{du}{u} \exp\left(-\frac{m_b^2 - p^2(1-u)}{uM^2}\right) \right. \\
&\quad \times \left(\varphi_p(u) + \frac{\varphi_\sigma(u)}{6u} \left(1 - \frac{m_b^2 - p^2}{uM^2}\right) + \frac{2m_b g_2(u)}{\mu_\pi u M^2} \right) \\
&\quad \left. + \exp\left(-\frac{s_0^B}{M^2}\right) \left(-\frac{\varphi_\sigma(\Delta)}{6\Delta} + \frac{2m_b g_2(\Delta)}{\mu_\pi (m_b^2 - p^2)} \right) \right\} \quad (23)
\end{aligned}$$

with $\Delta = (m_b^2 - p^2)/(s_0^B - p^2)$. For comparison and later use, we also quote the analogous sum rule for f^+ obtained in [1,2]:

$$\begin{aligned}
f_B f^+(p^2) &= \frac{f_\pi m_b^2}{2m_B^2} \exp\left(\frac{m_B^2}{M^2}\right) \\
&\quad \times \left\{ \int_\Delta^1 \frac{du}{u} \exp\left(-\frac{m_b^2 - p^2(1-u)}{uM^2}\right) \right. \\
&\quad \times \left(\varphi_\pi(u) + \frac{\mu_\pi}{m_b} u \varphi_p(u) \right. \\
&\quad \left. + \frac{\varphi_\sigma(u)}{3} \left(1 + \frac{m_b^2 + p^2}{2uM^2}\right) \right] - \frac{4m_b^2 g_1(u)}{u^2 M^4} \\
&\quad \left. + \frac{2}{uM^2} \int_0^u g_2(v) dv \left(1 + \frac{m_b^2 + p^2}{uM^2}\right) \right\} \\
&\quad + t^+(s_0^B, p^2, M^2) + f_G^+(p^2, M^2) \Big\}. \quad (24)
\end{aligned}$$

Here, we have added the surface term t^+ which was neglected previously, and denoted the contribution from the quark-antiquark-gluon wave functions of twist 3 and 4 by f_G^+ . The explicit expressions for t^+ and f_G^+ can be found in the Appendix.

Since very recently, the perturbative $O(\alpha_s)$ correction to the leading twist 2 piece of the light-cone sum rule (24) for f^+ is also known [3,4]. However, the corresponding QCD corrections to the twist 3 term in Eq. (24) as well as in the sum rule (23) for $f^+ + f^-$ still remain to be calculated. Hence, for consistency, we will not include the $O(\alpha_s)$ effects in f^+ in the present analysis.

III. NUMERICAL RESULTS

For the numerical analysis of the new sum rule (23) we use the same input as in the evaluation of the sum rule (24) in [1,2]. From experiment we take $f_\pi = 132$ MeV and $m_B = 5.279$ GeV, whereas the parameters $m_b = 4.7 \pm 0.1$ GeV, $s_0^B = 35 \mp 2$ GeV², and $f_B = 140 \mp 30$ MeV are extracted from the QCD sum rule for the correlator of two $\bar{b}\gamma_5 u$ currents. For consistency, the $O(\alpha_s)$ correction is not included

in the latter two-point sum rule. This is reflected by the low value of f_B . Because of cancellations of QCD corrections in the ratios of Eq. (23) and f_B , respectively, Eq. (24) and f_B , the remaining corrections to the form factors themselves may in fact be small. This is precisely what happens in the case of f^+ as has been shown in [3,4].

Furthermore, the explicit expressions for the pion wave functions up to twist 4 are collected in [2]. Those entering the new sum rule (23) are given below for completeness:

$$\begin{aligned}
\varphi_\pi(u, \mu) &= 6u\bar{u} \left[1 + a_2(\mu) \frac{3}{2} [5(u-\bar{u})^2 - 1] \right. \\
&\quad \left. + a_4(\mu) \frac{15}{8} [21(u-\bar{u})^4 - 14(u-\bar{u})^2 + 1] \right], \quad (25)
\end{aligned}$$

$$\begin{aligned}
\varphi_p(u, \mu) &= 1 + B_2(\mu) \frac{1}{2} [3(u-\bar{u})^2 - 1] \\
&\quad + B_4(\mu) \frac{1}{8} [35(u-\bar{u})^4 - 30(u-\bar{u})^2 + 3], \quad (26)
\end{aligned}$$

$$\begin{aligned}
\varphi_\sigma(u, \mu) &= 6u\bar{u} \left[1 + C_2(\mu) \frac{3}{2} (5(u-\bar{u})^2 - 1) \right. \\
&\quad \left. + C_4(\mu) \frac{15}{8} (21(u-\bar{u})^4 - 14(u-\bar{u})^2 + 1) \right], \quad (27)
\end{aligned}$$

$$\begin{aligned}
g_1(u, \mu) &= \frac{5}{2} \delta^2(\mu) \bar{u}^2 u^2 + \frac{1}{2} \varepsilon(\mu) \delta^2(\mu) \left[\bar{u} u (2 + 13\bar{u} u) \right. \\
&\quad \left. + 10u^3 \ln u \left(2 - 3u + \frac{6}{5} u^2 \right) \right. \\
&\quad \left. + 10\bar{u}^3 \ln \bar{u} \left(2 - 3\bar{u} + \frac{6}{5} \bar{u}^2 \right) \right], \quad (28)
\end{aligned}$$

$$g_2(u, \mu) = \frac{10}{3} \delta^2(\mu) \bar{u} u (u - \bar{u}), \quad (29)$$

with $\bar{u} = 1 - u$ and μ being the renormalization scale. For a detailed discussion of the wave functions we refer the reader to the original literature [5–7]. Recent reviews and references can be found in [9,10]. Here, the specification of the various coefficients together with a few comments may suffice. The terms proportional to the coefficients a_i , B_i and C_i represent scale-dependent nonasymptotic corrections. They vanish logarithmically as $\mu \rightarrow \infty$.

In leading order, the value of the scale μ is ambiguous. As a reasonable choice we take $\mu_b = \sqrt{m_B^2 - m_b^2} = 2.4$ GeV. With this choice, the estimate in [2] gives $a_2(\mu_b) = 0.35$, $a_4(\mu_b) = 0.18$, $B_2(\mu_b) = 0.29$, $B_4(\mu_b) = 0.58$, $C_2(\mu_b) = 0.059$, $C_4(\mu_b) = 0.034$, $\delta^2(\mu_b) = 0.17$ GeV², and $\varepsilon(\mu_b) = 0.36$. Furthermore, the PCAC (partial conservation

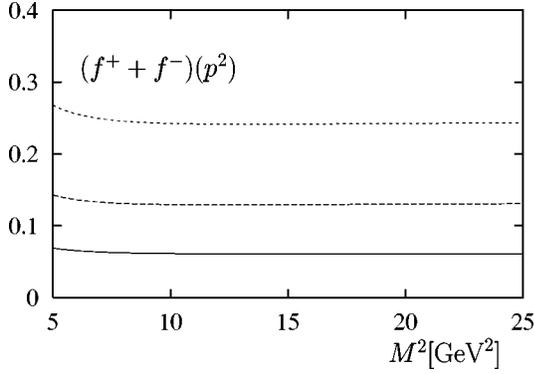


FIG. 1. Form factor $(f^+ + f^-)$ as a function of the Borel parameter at various values of the momentum transfer: $p^2=0$ (solid), $p^2=10$ GeV² (long-dashed) and $p^2=16$ GeV² (short-dashed).

of axial vector current) relation $f_{\pi}^2 \mu_{\pi}(\mu) = -2 \langle \bar{q}q \rangle(\mu)$ and $\langle \bar{q}q \rangle(\mu_b) = (-260 \pm 10 \text{ MeV})^3$ can be used to fix the final parameter appearing in Eqs. (23) and (24), namely $\mu_{\pi} = \mu_{\pi}(\mu_b) = 2.0 \pm 0.25$ GeV.

In the case of the sum rule (24) for f^+ the acceptable range of values of the Borel parameter M^2 was found to be $8 < M^2 < 12$ GeV² [2]. In the sum rule (23) for $f^+ + f^-$ we take the same interval after having checked that in this range of M^2 the twist 4 contribution does not exceed 10%, and that the excited and continuum states (15) do not contribute more than 30%. These are the usual conditions posed in sum rule applications.

Having specified the necessary numerical input, we are now ready to present quantitative results. In Fig. 1, the sum rule (23) is plotted as a function of Borel parameter M^2 . One can see that the variation is very moderate in the range $8 < M^2 < 12$ GeV², at least for $p^2 \leq 15$ GeV². This also applies to the sum rule (24). However, at $p^2 > 17$ GeV² the dependence on M^2 becomes strong, indicating that one is getting too close to the physical states in this channel. Figure 2 shows the momentum dependence of the form factors $f^+(p^2) + f^-(p^2)$, $f^+(p^2)$, and $f^0(p^2)$ for the central value $M^2 = 10$ GeV². Here, the scalar form factor f^0 is calculated from the other two form factors using the relation (2). In particular, at zero momentum transfer we predict

$$f^+(0) = f^0(0) = 0.30, \quad (30)$$

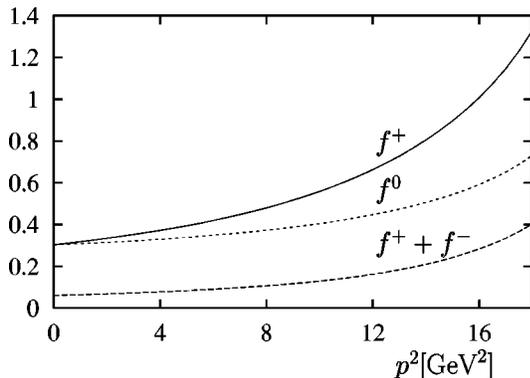


FIG. 2. $B \rightarrow \pi$ form factors obtained from light-cone sum rules.

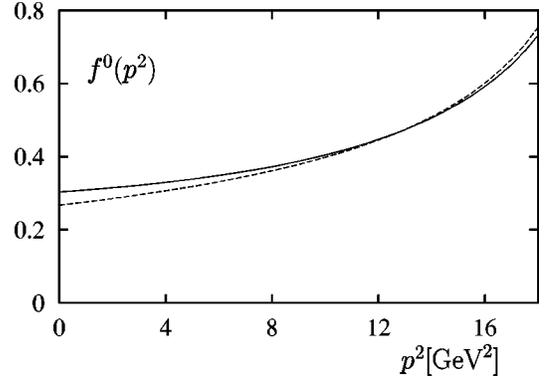


FIG. 3. Sensitivity of the form factor f^0 to the light-cone wave functions: nonasymptotic corrections included (solid) and purely asymptotic w.f. (dashed).

$$f^+(0) + f^-(0) = 0.06. \quad (31)$$

One of the greatest virtues of the sum rule approach is the possibility to estimate the theoretical uncertainties in the predictions, at least in principle. In practice, this task is not straightforward and, hence, the estimates should be considered with caution. The uncertainties in Eqs. (24) and (23) induced by the input parameters M^2 , m_b , s_0^B , f_B , and μ_{π} can be investigated by varying these parameters simultaneously in the sum rules for the form factors and for f_B . For instance, if the Borel mass is allowed to vary within the interval quoted above, f^0 is found to deviate by $\pm 3\%$ ($\pm 5\%$) at small (large) p^2 from the value obtained with the nominal choice $M^2 = 10$ GeV². The corresponding uncertainty in $f^+ + f^-$ can be inferred from Fig. 1. Furthermore, if m_b and s_0^B are varied in a correlated way within the ranges given at the beginning of this section such that one achieves maximum stability of the sum rule for f_B , f^0 changes by about $\pm 2\%$ relative to the value obtained with the nominal choice of parameters. This uncertainty is p^2 independent.

Another source of uncertainty is the precise shape of the pion wave functions. Keeping all other parameters fixed, we have studied the sensitivity of our results to the nonasymptotic terms in the twist 2 and 3 wave functions given in Eqs. (25)–(27). In Fig. 3 we compare the prediction on f^0 with the coefficients a_i , B_i , and C_i ($i=2,4$) as given earlier in this section to the result obtained by putting them to zero. The shifts are momentum dependent reaching -10% at $p^2 = 0$ and $+5\%$ at 15 GeV². The real uncertainty is certainly less than that.

In addition, there is some uncertainty due to the truncation of the light-cone expansion or, in other words, due to the neglect of terms with twist larger than 4. An upper limit on this uncertainty should be set by the size of the twist 4 contribution. The influence of the latter on the scalar form factor f^0 is displayed in Fig. 4. The twist 4 terms decrease f^0 by 2% at small p^2 and by 5% at large p^2 .

Finally, the present lack of knowledge of the perturbative QCD corrections to the twist 3 contributions in the sum rules (24) and (23) gives rise to uncertainties which will eventually be removed in near future. The scalar form factor f^0 is concerned in particular at large momentum transfer where

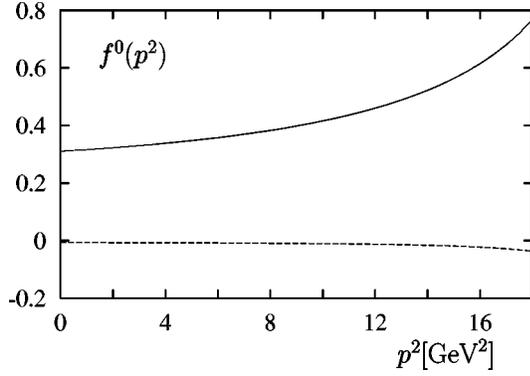


FIG. 4. $B \rightarrow \pi$ form factor f^0 : twist 2 and 3 contributions (solid), twist 4 contribution (dashed).

twist 3 should dominate as expected from the relation (2). Conversely, at small p^2 f^0 coincides with the form factor f^+ which receives the leading contribution from the twist 2 wave function φ_π . The $O(\alpha_s)$ corrections to this piece are known. They change the lowest order estimate (30) by only 10% [3]:

$$f^+(0) = f^0(0) = 0.27. \quad (32)$$

As already pointed out, this is due to a remarkable cancellation of the corrections to the sum rule (24) for $f^+ f_B$ and the sum rule for f_B in their ratio. Whether or not a similar cancellation takes place at the twist 3 level is questionable and can only be decided by direct calculation. For the time being the total uncertainty in f^0 is estimated to be about 20% at small p^2 and 30% at large p^2 .

The sum rules (23) and (24) for the $B \rightarrow \pi$ form factors are easily converted into sum rules for the corresponding $D \rightarrow \pi$ form factors. Formally, one only has to replace the flavor indices b by c , and B by D . Because of the relatively light charm mass, the region of validity of the sum rules covers only the low momentum region $0 \leq p^2 \leq 1.0 \text{ GeV}^2$ of the kinematically allowed range of p^2 . The values of the input parameters are $m_c = 1.3 \pm 0.1 \text{ GeV}$, $s_0^D = 6 \mp 1 \text{ GeV}^2$, and $f_D = 170 \pm 20 \text{ MeV}$. The scale μ is taken to be $\mu_c = \sqrt{m_D^2 - m_c^2} = 1.3 \text{ GeV}$, while the fiducial range of the Borel mass is $3 \text{ GeV}^2 < M^2 < 5 \text{ GeV}^2$. Correspondingly, the value of μ_π is lowered to $\mu_\pi(\mu_c) = 1.8 \pm 0.5 \text{ GeV}$. The numerical values of the nonasymptotic coefficients in the pion wave functions at the scale μ_c are given in [2,9]. There, one can also find further remarks on the above choice of parameters. Our numerical predictions for the $D \rightarrow \pi$ form factors are illustrated in Fig. 5 using the central values for the various input parameters. At zero momentum transfer, we estimate

$$f^+(0) = f^0(0) = 0.68, \quad (33)$$

$$f^+(0) + f^-(0) = 0.52. \quad (34)$$

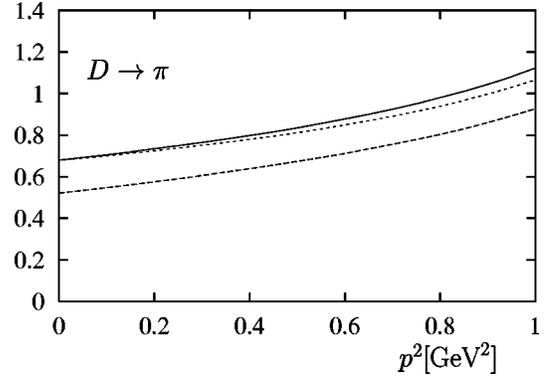


FIG. 5. $D \rightarrow \pi$ form factors obtained from light-cone sum rules: f^+ (solid), $f^+ + f^-$ (dashed) and f^0 (dotted).

IV. PREDICTIONS FOR $B \rightarrow \pi \bar{\nu} \nu_\tau$

The $B \rightarrow \pi$ form factors can be measured most directly in the weak semileptonic decays $B \rightarrow \pi \bar{l} \nu_l$ where $l = e, \mu$ or τ . The distribution of the momentum transfer squared in these decays is given by

$$\begin{aligned} \frac{d\Gamma}{dp^2} &= \frac{G_F^2 |V_{ub}|^2}{24\pi^3} \frac{(p^2 - m_l^2)^2 \sqrt{E_\pi^2 - m_\pi^2}}{p^4 m_B^2} \\ &\times \left\{ \left(1 + \frac{m_l^2}{2p^2} \right) m_B^2 (E_\pi^2 - m_\pi^2) [f^+(p^2)]^2 \right. \\ &\left. + \frac{3m_l^2}{8p^2} (m_B^2 - m_\pi^2)^2 [f^0(p^2)]^2 \right\} \quad (35) \end{aligned}$$

with $E_\pi = (m_B^2 + m_\pi^2 - p^2)/2m_B$ being the pion energy in the B rest frame. Another interesting observable is the distribution of the charged lepton energy E_l in the B rest frame:

$$\begin{aligned} \frac{d\Gamma}{dE_l} &= \frac{G_F^2 |V_{ub}|^2}{64\pi^3} \int_{p_{min}^2}^{p_{max}^2} dp^2 \left\{ \left[8 \frac{E_l}{m_B} (m_B^2 - m_\pi^2 + p^2) \right. \right. \\ &\left. \left. - 4(p^2 + 4E_l^2) + \frac{m_l^2}{m_B^2} (8m_B E_l - 3p^2 + 4m_\pi^2) - \frac{m_l^4}{m_B^2} \right] \right. \\ &\times [f^+(p^2)]^2 + \frac{2m_l^2}{m_B^2} [2m_B^2 + p^2 - 2m_\pi^2 \\ &\left. - 4m_B E_l + m_l^2] f^+(p^2) f^-(p^2) + \frac{m_l^2}{m_B^2} (p^2 - m_l^2) \right. \\ &\left. \times [f^-(p^2)]^2 \right\} \quad (36) \end{aligned}$$

with $p_{min}^2 = m_B(E_l \pm \sqrt{E_l^2 - m_l^2}) + O(m_\pi^2)$. Although the form factors are calculated in the chiral limit, otherwise the

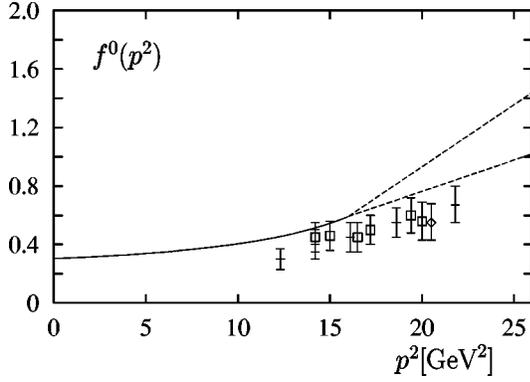


FIG. 6. The $B \rightarrow \pi$ form factor f^0 : direct sum rule estimate (solid) and linear extrapolations to the limit (38) (dashed). The lattice results are from [13].

finite pion mass is taken into account. In the case of light leptons $l = e, \mu$ the form factor f^0 or, equivalently, f^- plays a negligible role because of the smallness of the electron and muon masses. Hence, these decay modes can provide information only on the form factor f^+ . In contrast, the decay $B \rightarrow \pi \bar{\tau} \nu_\tau$ is also sensitive to the form factor f^0 . We therefore concentrate here on the latter case.

The sum rule results described in the preceding two sections allow to predict the decay spectra in the momentum region $0 \leq p^2 \leq 17 \text{ GeV}^2$. In order to include higher momentum transfers and to predict integrated widths one has to find another way to calculate the form factors up to the kinematical endpoint $p^2 = (m_B - m_\pi)^2 = 26.4 \text{ GeV}^2$. In [2], the single-pole approximation

$$f^+(p^2) = \frac{f_{B^*} g_{B^* B \pi}}{2m_{B^*} (1 - p^2/m_{B^*}^2)} \quad (37)$$

was used. Since the vector B ground state is only about 50 MeV heavier than the pseudoscalar B , the B^* pole is very near to the endpoint region. Consequently, at maximum p^2 the single-pole approximation can be expected to be very good. Moreover, the strong $B^* B \pi$ coupling which determines the normalization of the form factor at large p^2 can be calculated from the same correlation function (4) from which the sum rule (24) for f^+ at low to intermediate values of p^2 is derived. To this end one employs a double dispersion relation. The method and results are described in [2] and reviewed in [9]. Extrapolation of the single-pole model to smaller p^2 matches quite well with the direct estimate from the light-cone sum rule (24) at intermediate momentum transfer $p^2 = 15$ to 20 GeV^2 . This provides us with a consistent and complete theoretical prediction of f^+ .

Unfortunately, it is doubtful that a similar procedure can be applied to the scalar form factor f^0 , because the scalar B ground state is expected to be about 500 MeV heavier than the pseudoscalar B . Thus, the scalar B pole may be too distant from the kinematical endpoint of the $B \rightarrow \pi$ transition for

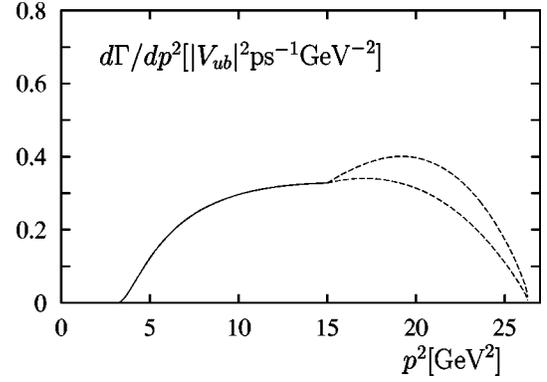


FIG. 7. Distribution of the momentum transfer squared in $B \rightarrow \pi \bar{\tau} \nu_\tau$. The two curves correspond to the two extrapolations of f^0 shown in Fig. 6.

the single-pole approximation to hold. Nearby nonresonant $B \pi$ states and excited scalar resonances may give comparable contributions.

Interestingly, there exists a model-independent constraint on the behavior of the form factor f^0 at large $p^2 \approx m_B^2$, i.e., near the kinematical endpoint. The constraint [11,12] is derived from a Callan-Treiman type relation obtained by combining current algebra and PCAC:

$$\lim_{p^2 \rightarrow m_B^2} f^0(p^2) = f_B / f_\pi. \quad (38)$$

In the following we make use of this bound in order to illustrate the sensitivity of the decay spectra in $B \rightarrow \pi \bar{\tau} \nu_\tau$ to the scalar form factor.

The form factor f^0 is extrapolated linearly from the value at $p^2 = 15 \text{ GeV}^2$ where the sum rules (23) and (24) still hold to the value at $p^2 \approx m_B^2$ dictated by Eq. (38). To be conservative we take $f_B = 150$ to 210 MeV in accordance with recent lattice data [13] and with QCD sum rule estimates (the latter including the perturbative correction, see, e.g., [3]). This is shown in Fig. 6 together with lattice estimates of f^0 .

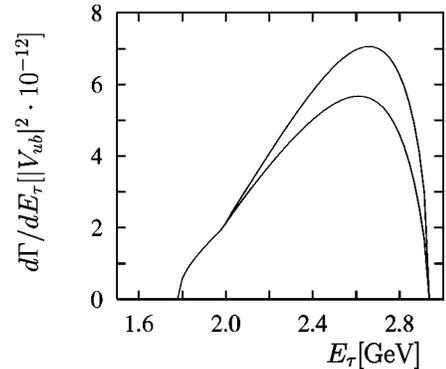


FIG. 8. Distribution of the τ -lepton energy in $B \rightarrow \pi \bar{\tau} \nu_\tau$. The two curves correspond to the two extrapolations of f^0 shown in Fig. 6.

Obviously, the lattice data favor the lower extrapolation. The distributions of p^2 and E_τ in $B \rightarrow \pi \bar{\tau} \nu_\tau$ resulting from the upper and lower bounds on f^0 are plotted in Fig. 7 and Fig. 8, respectively. This study demonstrates that measurements of these decay spectra at B factories should provide interesting information on the elusive form factor f^0 .

For the integrated partial width we predict

$$\Gamma(B^0 \rightarrow \pi^- \tau^+ \nu_\tau) = 5.7 \text{ to } 6.5 |V_{ub}|^2 \text{ ps}^{-1}, \quad (39)$$

where the range corresponds to the two extrapolations considered in Figs. 7 and 8. The theoretical uncertainties in the sum rule calculations discussed in Sec. III are not included. The latter drop out to a large extent in the ratio

$$\frac{\Gamma(B^0 \rightarrow \pi^- \tau^+ \nu_\tau)}{\Gamma(B^0 \rightarrow \pi^- e^+ \nu_e)} = 0.75 \text{ to } 0.85. \quad (40)$$

It should be noted that only the numerator is influenced by the scalar form factor.

The form factor f^0 also plays an important role in nonleptonic B decays where it enters the factorized two-body amplitudes for $B \rightarrow \pi h$. Depending on the mass of the meson h these decays probe f^0 in the range $m_\pi^2 < p^2 < m_\psi^2$. This is similar in D decays. However, there the form factor f^0 cannot be measured independently in semileptonic decays because only the electron and muon modes are kinematically accessible.

V. DEPENDENCE ON THE HEAVY QUARK MASS

The light-cone sum rules (23) and (24) offer the possibility to systematically investigate the dependence of heavy-to-light form factors on the heavy quark mass. Using the familiar scaling relations for mass parameters and decay constants, to wit

$$m_B = m_b + \bar{\Lambda}, \quad s_0^B = m_b^2 + 2m_b \omega_0, \quad M^2 = 2m_b \tau, \quad (41)$$

$$f_B = \hat{f}_B / \sqrt{m_b}, \quad (42)$$

where in the heavy quark limit $\bar{\Lambda}$, ω_0 , τ , \hat{f}_B are m_b -independent quantities, it is rather straightforward to expand the sum rules in inverse powers of m_b . At $p^2=0$, the leading terms are given by

$$\begin{aligned} f^+(0) &= m_b^{-3/2} \frac{f_\pi}{2\hat{f}_B} \exp\left(\frac{\bar{\Lambda}}{\tau}\right) \left\{ \int_0^{2\omega_0} d\rho \right. \\ &\quad \times \exp\left[-\frac{\rho}{2\tau}\right] \\ &\quad \times \left(-\rho \varphi'_\pi(1) + \mu_\pi \left[\varphi_\rho(1) - \frac{\rho}{12\tau} \varphi'_\sigma(1) \right] \right) \\ &\quad \left. - \frac{\mu_\pi \omega_0}{6} \exp\left(-\frac{\omega_0}{\tau}\right) \varphi'_\sigma(1) \right\} + O(m_b^{-5/2}), \quad (43) \end{aligned}$$

and

$$\begin{aligned} f^+(0) + f^-(0) &= m_b^{-3/2} \frac{f_\pi \mu_\pi}{\hat{f}_B} \exp\left(\frac{\bar{\Lambda}}{\tau}\right) \\ &\quad \times \left\{ \int_0^{2\omega_0} d\rho \exp\left[-\frac{\rho}{2\tau}\right] \left[\varphi_\rho(1) + \frac{\rho}{12\tau} \varphi'_\sigma(1) \right] \right. \\ &\quad \left. + \frac{\omega_0}{3} \exp\left(-\frac{\omega_0}{\tau}\right) \varphi'_\sigma(1) \right\} + O(m_b^{-5/2}), \quad (44) \end{aligned}$$

where $\varphi'_a(1)$ stands for the derivative $d\varphi_a/du$ at the endpoint $u=1$. It is important to note that whereas the twist 2 and 3 two-particle wave functions survive in the asymptotic limit (43) and (44), the higher-twist and three-particle components are suppressed by one additional power of m_b , and die out.

The asymptotic scaling laws derived above can be understood as follows. At $p^2=0$, the integration region $\Delta = m_b^2/s_0^B \leq u \leq 1$ in Eqs. (23) and (24) is rather narrow. In fact, it vanishes in the infinite mass limit as $1 - m_b^2/s_0^B \sim 2\omega_0/m_b$. In this limit, the asymptotic twist 2 and twist 3 wave functions behave like

$$\varphi_\pi \sim \varphi_\sigma \sim (1-u) \sim \omega_0/m_b \quad \text{and} \quad \varphi_\rho \sim 1. \quad (45)$$

Taking into account the extra factor $1/m_b$ multiplying φ_ρ , and noticing that the factors $1/m_b$ times bracket multiplying φ_σ in the sum rules approach unity at $m_b \rightarrow \infty$, one sees that the twist 2 and 3 terms lead to the same asymptotic scaling behavior. The latter is determined by a factor m_b^{-1} from the integrand, a factor m_b^{-1} from the integration region, and a factor $m_b^{1/2}$ from $1/f_B$. The fact that the light-cone sum rule predicts $f^+(0) \sim m_b^{-3/2}$ was first noticed in [14].

The situation changes drastically when the momentum transfer becomes large of order m_b^2 . To be definite, at the boundary $p^2 = m_b^2 - 2m_b \chi$ considered in Eq. (9), the integration region in Eqs. (23) and (24) is finite and independent of m_b . Therefore, the asymptotic scaling laws are simply determined by the factors in front of the duality integrals in the sum rules, that is $1/f_B \sim m_b^{1/2}$ in the case of f^+ and $1/m_b f_B \sim m_b^{-1/2}$ for $f^+ + f^-$. Explicitly, one obtains

$$\begin{aligned}
f^+(p^2 = m_b^2 - 2m_b\chi) \sim & m_b^{1/2} \frac{f_\pi}{2\hat{f}_B} \exp\left(\frac{\bar{\Lambda}}{\tau}\right) \left\{ \int_{\Delta}^1 \frac{du}{u} \exp\left[-\frac{\chi(1-u)}{\tau u}\right] \left[\varphi_\pi(u) + \frac{\mu_\pi}{6u\tau} \varphi_\sigma(u) - \frac{1}{u^2\tau^2} \left(g_1(u) - \int_0^u g_2(v) dv \right) \right] \right. \\
& + \frac{1}{\chi} \exp\left(-\frac{\omega_0}{\tau}\right) \left[\frac{\mu_\pi}{6} \varphi_\sigma(\Delta) - \frac{1}{\chi} \left(1 + \frac{\chi + \omega_0}{\tau} \right) \left(g_1(\Delta) - \int_0^\Delta g_2(v) dv \right) \right. \\
& \left. \left. + \frac{1}{\chi + \omega_0} \left(\frac{dg_1(\Delta)}{du} - g_2(\Delta) \right) \right] \right\} + O(m_b^{-1/2}), \tag{46}
\end{aligned}$$

and

$$\begin{aligned}
[f^+ + f^-](p^2 \sim m_b^2 - 2m_b\chi) \sim & m_b^{-1/2} \frac{f_\pi \mu_\pi}{\hat{f}_B} \exp\left(\frac{\bar{\Lambda}}{\tau}\right) \left\{ \int_{\Delta}^1 \frac{du}{u} \exp\left[-\frac{\chi(1-u)}{\tau u}\right] \left[\varphi_p(u) + \frac{\varphi_\sigma(u)}{6u} \left(1 - \frac{\chi}{u\tau} \right) + \frac{g_2(u)}{\mu_\pi u \tau} \right] \right. \\
& \left. + \frac{1}{\chi} \exp\left(-\frac{\omega_0}{\tau}\right) \left(-\frac{1}{6} (\chi + \omega_0) \varphi_\sigma(\Delta) + \frac{g_2(\Delta)}{\mu_\pi} \right) \right\} + O(m_b^{-3/2}), \tag{47}
\end{aligned}$$

where $\Delta = \chi/(\chi + \omega_0)$. In contrast to the heavy quark limit at $p^2 = 0$, here also twist 4 contributes asymptotically. However, the contributions from three-particle wave functions are still suppressed by an extra power of $1/m_b$.

Finally, using the relation (2) it is easy to check that the asymptotic scaling law of the form factor f^0 coincides with the expectation (43) for f^+ at small p^2 , and with Eq. (47) for $f^+ + f^-$ at large p^2 .

The above analysis shows that the light-cone expansion in terms of wave functions with increasing twist is consistent with the heavy mass expansion. The higher-twist contributions either scale with the same power of m_b as the leading-twist term, or they are suppressed by extra powers of m_b . The sum rules nicely reproduce the asymptotic dependence of the form factors f^\pm on the heavy quark mass as derived in [11,15] for small pion momentum in the rest frame of the B meson. In addition, they also allow to investigate the case of large pion momentum where neither heavy quark effective theory (HQET) nor the single-pole model can be trusted. Clearly, as $p^2 \rightarrow 0$ excited and continuum states are expected to become more and more important thus leading to a breakdown of the single-pole approximation. The change in the asymptotic mass dependence of the light-cone sum rules when going from large to small momentum transfers can be considered as a signal of this breakdown. Claims in the literature which differ from Eqs. (43) and (44) are often based on the pole model and therefore incorrect in our opinion. The above conclusions corroborate similar analyses carried out for the $B \rightarrow K^*$ [16] and $B \rightarrow \rho$ [8] transition form factors.

In order to clarify the relevance of the asymptotic scaling behavior in the mass range between m_c and m_b , we have studied the functional dependence of the sum rules (23) and (24) on the heavy quark mass m_Q numerically. The dependence of the parameters m_B , s_0^B and M^2 on m_Q is described approximately by the relations (41) using $\bar{\Lambda} = 0.6$ GeV, $\omega_0 = 1.4$ GeV, $\tau = 1.1$ GeV. Together with $m_Q = 4.7$ GeV this choice reproduces the central values of m_B etc. given at the beginning of Sec. III. In order to consistently include the

deviations of the decay constant f_B from the asymptotic scaling law (42) we have substituted f_B by the corresponding two-point sum rule using again the relations (41) analogously to the procedure described above. The logarithmic mass dependence of the wave functions and vacuum condensates through the scale $\mu_Q = \sqrt{2m_Q\bar{\Lambda}}$ is taken into account. Figure 9 shows the form factor $f^+(0)$ multiplied by the leading power $m_Q^{3/2}$ as a function of $1/m_Q$. Even at $m_Q > m_b$ there is still no sign that one is approaching the asymptotic limit. On the contrary, the mass dependence of the nonasymptotic terms in the pion wave function becomes more and more important, at least for an intermediate mass range. At $m_Q < m_b$, the integration region is still big enough to wash out these effects since the wave functions are normalized. In the region between m_c and m_b , the mass dependence can be fitted to the following quadratic polynomial in $1/m_Q$:

$$f^+(0) m_Q^{3/2} = 3.3 \text{ GeV}^{3/2} \left(1 - \frac{1.5 \text{ GeV}}{m_Q} + \frac{0.75 \text{ GeV}^2}{m_Q^2} \right), \tag{48}$$

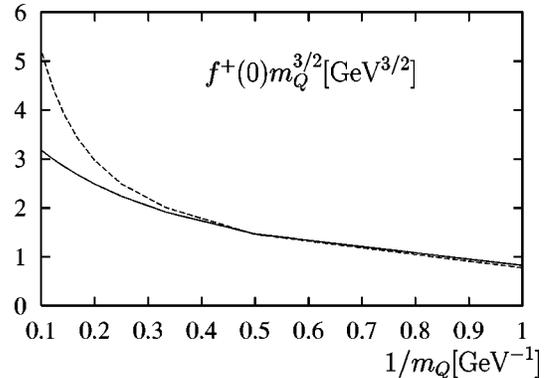


FIG. 9. Dependence of the form factor $f^+(0)$ on the heavy quark mass m_Q for purely asymptotic wave functions (solid), and the nonasymptotic corrections included at the scale $\mu_Q = \sqrt{2m_Q\bar{\Lambda}}$ (dashed).

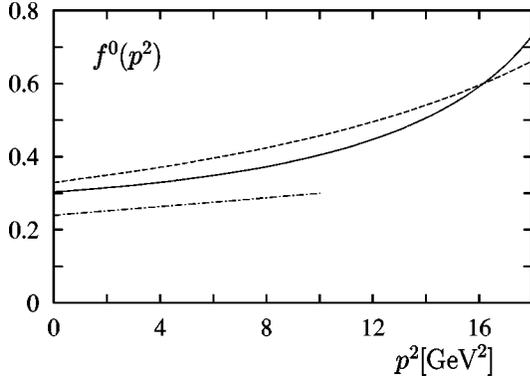


FIG. 10. The $B \rightarrow \pi$ form factor f^0 : light-cone sum rule (solid) in comparison to the quark model prediction from [17] (dashed) and the QCD sum rule result from [18] (dash-dotted).

indicating the existence of large $1/m_Q$ corrections in the physical mass range. Similar results have been obtained in the case of $B \rightarrow K^*$ [16] and $B \rightarrow \rho$ [8] form factors.

VI. CONCLUSION

In this paper, we have derived a new QCD sum rule for the combination $f^+(p^2) + f^-(p^2)$ of $B \rightarrow \pi$ (and $D \rightarrow \pi$) form factors. The light-cone approach used is designed for heavy-to-light transitions and incorporates the nonperturbative dynamics in terms of light-cone wave functions of the pion. The sum rule is essentially determined by the twist 3 $q\bar{q}$ wave functions. Terms involving the leading twist 2 wave function and the $q\bar{q}g$ wave function of twist 3 and 4 are absent, while the twist 4 $q\bar{q}$ wave functions give only small contributions. Higher-twist components are neglected. The sum rule is valid in the range of momentum transfers $0 \leq p^2 \leq 17 \text{ GeV}^2$.

Combining the new result on $f^+ + f^-$ with the corresponding calculation of f^+ in [2] we have been able to predict the scalar form factor f^0 . This prediction is compared with recent lattice results [13] in Fig. 6 and with quark model [17] and different sum rule [18] estimates in Fig. 10. Within the inherent uncertainties of both approaches there is agree-

ment with the lattice results. However, the latter tend to be systematically lower than our prediction. This could be an indication for the presence of perturbative QCD corrections which still need to be calculated. Our result on f^0 also agrees with the lattice-constrained parametrization of this form factor (the pole variant) discussed in [19]. The quark model estimate makes use of the single-pole approximation $f^0(p^2) = f^+(0)/(1 - m_0^2/p^2)$ with $m_0 = 6.0 \text{ GeV}$. In [12] it is suggested to use instead the relation (38) in order to normalize f^0 at maximum momentum transfer. Extrapolation to intermediate and small p^2 then leads to a result very similar to the one obtained in [17] and shown in the figure. Despite the rough agreement with our sum rule result we doubt the validity of the single-pole model at small p^2 for reasons explained in Sec. V. Finally, in the framework of the heavy-quark-effective theory one has derived a three-point sum rule for f^0 [18] giving a result in the region $0 < p^2 < 10 \text{ GeV}^2$ which is about 30% lower than the expectation from the light-cone sum rule. Concerning similar applications of the light-cone sum rules, one should also mention the study of the form factor f^- of the $B \rightarrow K$ transition in [20].

It would be very interesting to confront these predictions with experimental data, not only to test the theoretical methods of calculating form factors, but also since f^0 enters the factorized amplitudes for a class of nonleptonic two-body decays. Yet, direct measurements of f^0 are only feasible in the semileptonic decay $B^0 \rightarrow \pi^- \tau^+ \nu_\tau$. This mode may get in experimental reach at future B factories. We have presented the expected decay spectra and demonstrated the sensitivity to f^0 .

Last but not least, light-cone sum rules for heavy-to-light form factors such as f^0 provide very flexible tools to study the transition from small to large momentum transfers, and to relate the physics of D and B mesons. Moreover, they provide new insights in the heavy quark mass dependence of weak matrix elements.

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APPENDIX

Here, we give the explicit expressions for the surface term t^+ and the contribution f_G^+ from the quark-antiquark-gluon wave functions in the light-cone sum rule (24):

$$\begin{aligned}
 t^+(s_0^B, p^2, M^2) = & \exp\left(-\frac{s_0^B}{M^2}\right) \left\{ \frac{\mu_\pi(m_b^2 + p^2)}{6m_b(m_b^2 - p^2)} \varphi_\sigma(\Delta) - \frac{4m_b^2}{(m_b^2 - p^2)^2} \left(1 + \frac{s_0^B - p^2}{M^2}\right) g_1(\Delta) + \frac{4m_b^2}{(s_0^B - p^2)(m_b^2 - p^2)} \frac{dg_1(\Delta)}{du} \right. \\
 & \left. + \frac{2}{m_b^2 - p^2} \left[1 + \frac{m_b^2 + p^2}{m_b^2 - p^2} \left(1 + \frac{s_0^B - p^2}{M^2}\right) \right] \int_0^\Delta g_2(v) dv - \frac{2(m_b^2 + p^2)}{(m_b^2 - p^2)(s_0^B - p^2)} g_2(\Delta) \right\}, \quad (\text{A1})
 \end{aligned}$$

$$f_G^+(p^2, M^2) = - \int_0^1 u \, du \int \frac{\mathcal{D}\alpha_i \Theta(\alpha_1 + u\alpha_3 - \Delta)}{(\alpha_1 + u\alpha_3)^2} \exp\left(-\frac{m_b^2 - p^2(1 - \alpha_1 - u\alpha_3)}{(\alpha_1 + u\alpha_3)M^2}\right) \\ \times \left\{ \frac{2f_{3\pi}}{f_\pi m_b} \varphi_{3\pi}(\alpha_i) \left[1 - \frac{m_b^2 - p^2}{(\alpha_1 + u\alpha_3)M^2} \right] - \frac{1}{uM^2} [2\varphi_\perp(\alpha_i) - \varphi_\parallel(\alpha_i) + 2\tilde{\varphi}_\perp(\alpha_i) - \tilde{\varphi}_\parallel(\alpha_i)] \right\} \quad (\text{A2})$$

with $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. The definitions and functional forms of the twist 3 wave function $\varphi_{3\pi}$ and the twist 4 wave functions $\varphi_\perp, \varphi_\parallel, \tilde{\varphi}_\perp$ and $\tilde{\varphi}_\parallel$ can be found in [2,7].

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