Evidence for a scalar $\kappa(900)$ resonance in πK scattering

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Motivated by the $1/N_c$ expansion, we study a simple model in which the πK scattering amplitude is the sum of a *current-algebra* contact term and resonance pole exchanges. This phenomenological model is crossing symmetric and, when a putative light strange scalar meson κ is included, satisfies the unitarity bounds to well above 1 GeV. The model also features chiral dynamics, vector meson dominance and appropriate interference between the established $K_0^*(1430)$ resonance and its predicted background. We briefly discuss the physical significance of the results and directions for further work. [S0556-2821(98)00817-0]

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I. INTRODUCTION

In the present paper we will generalize to the case of πK scattering the recent treatment of $\pi\pi$ scattering given in [1–3]. There evidence was found to support the existence of a low-mass relatively broad scalar resonance, denoted $\sigma[m_\sigma = 560$ MeV, $\Gamma_\sigma = 370$ MeV, with pole position $s = (0.585 - 0.178i)$ GeV], in addition to the well-established scalar $f_0(980)$ resonance. A number of other authors have also found similar or related results in different models [4–14].

If one accepts a low-lying σ and notes the existence of the isovector scalar $a_0(980)$, as well as the $f_0(980)$, there would be three scalar resonances below 1 GeV. A great deal of discussion and controversy over the years has surrounded the issue of the nature of such very-low-mass scalars. The reason is that one expects the lowest-lying scalars in the quark model to be p-wave $q\bar{q}$ bound states and hence to have masses comparable to those of the axial and tensor mesons, already in the 1.2–1.6 GeV region (see for example [15]). As an example (see the discussion on p. 355 of [16] under the ''Note on Scalar Mesons'') one might form a conventional scalar nonet from the $f_0(1370)$, $a_0(1450)$, $K_0^*(1430)$ and $f_J(1710)$. If an assignment such as this is correct, it raises the question of why the three scalar candidates σ, $f_0(980)$ and $a_0(980)$ are so light, and whether a general organizing principle for their dynamics can be found. From this point of view it is extremely interesting to see if a light strange scalar resonance, to be denoted κ, emerges in the study of πK scattering. Evidence for such a resonance has been found by some authors—using a unitarized non-relativistic meson model [12] and using a method of interfering Breit-Wigner amplitudes with a repulsive background [17]—and disputed by others—using a unitarized quark model [18] and using a K-matrix analysis in a slightly higher energy region [19]. The existence of the κ would strengthen the point of view (see for example [20]) that there is a non-conventional scalar nonet lying below 1 GeV.

Of course another motivation for studying πK scattering using the approach of [1,2] is to test that approach itself in a context other than $\pi\pi$ scattering. According to experimental indications [21] the πK channel may be a particularly clean one for this purpose in that the effects of inelastic channels seem to be less important at moderate energies than for $\pi\pi$ scattering. Theoretically, too, the πK scattering seems cleaner in that its non-trivial quantum numbers reduce the number of nearby states which can mix with each other. This contrasts with the $\pi\pi$ isosinglet channel in which $(u\bar{u}+d\bar{d})$, $s\bar{s}$ and glueball states can *a priori* mix.

Perhaps it is useful to remark on the need to ''discover'' a light scalar meson by an analysis of the sort being undertaken here; why cannot one just rely on an inspection of the phase shifts obtained directly from experiment? In the case of the $\pi\pi$ isosinglet channel, the model of [1,2] for example shows that the light σ is on the broad side and does not dominate its own channel. Rather it is only one of three comparable and competing contributions. A similar situation is expected and will be seen to occur for the putative κ meson. Clearly, the reliability of such a prediction depends on how accurately the ''background'' of the κ can be modeled. In the present approach that job will be facilitated by using an effective chiral Lagrangian approach in which crossing symmetry is manifest. This ensures that important cross-channel contributions from resonances known to exist in a given energy region are included. Furthermore, by using the physical fields directly, we will not be limiting ourselves to any assumption about a particular kind of quark substructure for these fields. This is, on the one hand, an advantage, since it increases the generality of our analysis. On the other hand our demonstration of the need for a κ meson will not immediately answer the interesting question of what the quark substructure of light scalars is. In fact, we will not take a stand on this matter in the present paper and reserve our speculative notions for elsewhere [22].

This paper is organized as follows. In Sec. II there is a brief review of our approach as it was applied to the $\pi\pi$ scattering problem. This is used to motivate the specific approximations which we will make in the present case of πK scattering. Section III treats the very interesting $J=0$, $I=\frac{1}{2}$ channel. It is shown that postulating the existence of a light

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κ-type resonance enables us to satisfy the unitarity bound in this channel. In Sec. IV it is further shown that the existence of the κ also plays an important role in producing a background phase at the position of the $K_0^*(1430)$ resonance pole; this gives a shape for the $J=0$, $I=\frac{1}{2}$ partial wave amplitude in agreement with experiment. The $J=0$, $I=\frac{3}{2}$ channel, which apparently does not contain any exotic $I=\frac{3}{2}$ resonance poles, is studied in Sec. V. A brief summary and discussion are given in Sec. VI. For the reader's convenience, many technical details are compactly assembled in three Appendixes. Appendixes A, B and C are respectively devoted to scattering kinematics, the underlying chiral Lagrangian and the ''unregularized'' invariant amplitudes.

II. REVIEW OF THE MODEL

For the reader's convenience we will briefly review here the main features of [1,2] in which $\pi\pi$ scattering was discussed and indicate how they are expected to generalize to the πK case. For a fuller presentation of the ideas used, we refer the reader to [1,2].

The approach is inspired by the $1/N_c$ expansion [23] of QCD. It is desired to approximate the low energy (up to the roughly 1 GeV region) part of the leading, order of $1/N_c$, contribution to the meson-meson scattering amplitude. It seems to be an outstanding unsolved problem to obtain an analytic representation of even this leading contribution. However, certain of its features [23] are known. The amplitude should consist of tree diagrams—contact terms and resonance exchanges. Away from the poles (which contain divergences of the theory in leading order since the resonance widths go as $1/N_c$) the leading order amplitudes are purely real. Hence we restrict ourselves to comparing the real parts of our computed amplitudes with the real parts of the amplitudes deduced from experiment.

A crucial aspect is the regularization procedure at the s-channel poles. The guiding principle is to make the amplitude unitary in the neighborhood of the pole and the resulting regularization method used depends on the type of resonance under consideration. As illustrated in Sec. II of [2] this gives the Breit-Wigner prescription for a narrow isolated resonance, a Breit-Wigner prescription modified by a computed phase shift for a narrow resonance in a smoothly varying background and a slightly more general parameterization for the relatively broad light scalar resonance.

The crossing symmetric amplitude will, to ensure chiral symmetry which works very well near threshold, be computed from the chiral Lagrangian given in Appendix B (the same one used in [1,2]). The partial wave projections of interest will then be obtained according to Eq. (A8).

To see what happens in the case of the $\pi\pi$, $I=J=0$ partial wave amplitude let us start from the threshold and go up in energy. The threshold region is well explained by the so-called current algebra contact term. However, as shown in Fig. 1 of [2], this contact amplitude rises rapidly, already violating the unitarity bound at around 500 MeV. It is postulated that unitarity should be restored by nearby resonance contributions and this is called ''local cancellation.'' It is also seen in this figure that the introduction of the ρ-meson contribution markedly improves, but does not completely cure, the unitarity violation. However, this result makes the possibility of ''local cancellation'' seem plausible. A certain amount of experimentation, described in [1], showed that the remaining violation of the unitarity bound could be neatly cured by the introduction of a suitably parameterized light scalar σ meson. Figure 9 of [1] shows how such a σ meson, having a mass close to the energy where the unitarity bound is violated, kills two birds with one stone. At lower energies it boosts the ''current algebra'' result which is slightly too small when compared with the real part of the experimentally determined amplitude. At higher energies it falls rapidly to negative values to rescue unitarity. Furthermore, in the region of the $f_0(980)$ the real part of these contributions to the amplitude is brought to zero which yields a background phase of around 90°. In turn (see Sec. IV A of [2]), this leads to a Ramsauer-Townsend mechanism [25] which changes the $f_0(980)$ contribution to the cross-section from a peak to the experimentally observed dip. All in all a reasonable experimental fit for the isosinglet scalar amplitude is obtained up to about 1.2 GeV (see Fig. 4 of [2]). The great precision of the chiral perturbation theory [26] description of the amplitude very close to threshold has been slightly sacrificed to achieve an overall description over a considerably larger energy range.

Two additional points can be made. Investigation of the effect of the opening of the $\pi\pi\to K\bar{K}$ channel (Sec. V of [2]) showed that it made a relatively minor change in the qualitative treatment of $\pi\pi\to\pi\pi$ scattering up to about 1.2 GeV. Amusingly, the same mechanism for restoring unitarity which worked for $\pi\pi\to\pi\pi$ seemed also effective for the $\pi\pi\to K\bar{K}$, $I=J=0$ amplitude above the $K\bar{K}$ threshold. Second, it was noted [2,1] that there was a tendency for contributions from the exchange of the ''next group'' of resonances—the $f_2(1270)$, the $f_0(1300)$ and the $\rho(1450)$—to cancel among themselves. In any event they did not further improve the fit. Certainly, in order to carry this treatment still higher in energy it is necessary to treat the higher resonances more precisely. In the numerical treatment of [2,1], it was found that these effects of inelasticity and the higher resonances could all be absorbed in relatively minor adjustments of the three parameters used to describe the light scalar.

From this discussion, it seems that the appropriate model for an initial study of the generalization to the πK case would neglect the inelastic channels (here $\eta' K$ is apparently [21] the main first one) as well as resonances other than the vector mesons and the scalars which lie below 1 GeV. Since we are especially interested in the $J=0$, $I=\frac{1}{2}$ channel, we will make an important exception for the $K_0^*(1430)$ which has a direct pole in this channel. The $K_0^*(1430)$ seems to be a reasonable candidate for an ''ordinary'' p-wave $q\bar{q}$ scalar. The diagrams to be considered are shown in Fig. 1. Notice that a putative light scalar κ has been included. The main question is whether it is *needed* to satisfy the unitarity bound. Actually our treatment of the $I=\frac{1}{2}$ channel turns out to be conceptually similar to the experimental analysis of [21]. They parametrize the $I=\frac{1}{2}$, $J=0$ channel amplitude by an

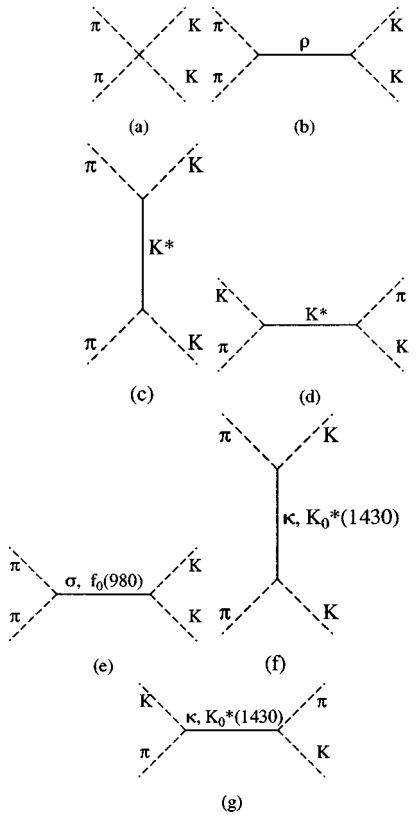

FIG. 1. Tree diagrams relevant for πK scattering in our model.

effective range background piece plus a modified Breit-Wigner term for the $K_0^*(1430)$. We work from our crossing symmetric invariant amplitude, and so in effect their background corresponds to the sum of all our diagrams, except for the $K_0^*(1430)$ pole terms in Fig. 1. Since their parameters for the $K_0^*(1430)$ are determined by this method, we choose to fit the $K_0^*(1430)$ and κ parameters simultaneously.

III. EVIDENCE FOR THE SCALAR $\kappa(900)$ IN THE $I=\frac{1}{2}$ CHANNEL

In this section we make an initial study of the $I=\frac{1}{2}$ and $J=0$ projection of the real part of the πK scattering amplitude $T_0^{1/2}$ defined in Eq. (A8). As in the $\pi\pi$ case we start with the well-known ''current algebra'' amplitude. This can be calculated from the second term of the Lagrangian (B7) together with Eq. (B10). If the vector mesons are not included in this chiral Lagrangian, then this is the same as using the more conventional chiral Lagrangian, including only pseudoscalars [24]:

$$\mathcal{L}_1 = -\frac{F_\pi^2}{8}\,\mathrm{Tr}(\partial_\mu U \partial_\mu U^\dagger) + \mathrm{Tr}[\mathcal{B}(U+U^\dagger)], \qquad (3.1)$$

in which $U=e^{2i\phi/F_\pi}$, with ϕ the 3×3 matrix of pseudoscalar fields and $F_\pi=132$ MeV the pion decay constant. $\mathcal{B}$ is a diagonal matrix (B_1,B_1,B_3) with $B_1=m_\pi^2F_\pi^2/8=B_2$ and $B_3=F_\pi^2(m_K^2-m_\pi^2/2)/4$. This is the dominant minimal symmetry breaking term for the pseudoscalar mesons. We shall

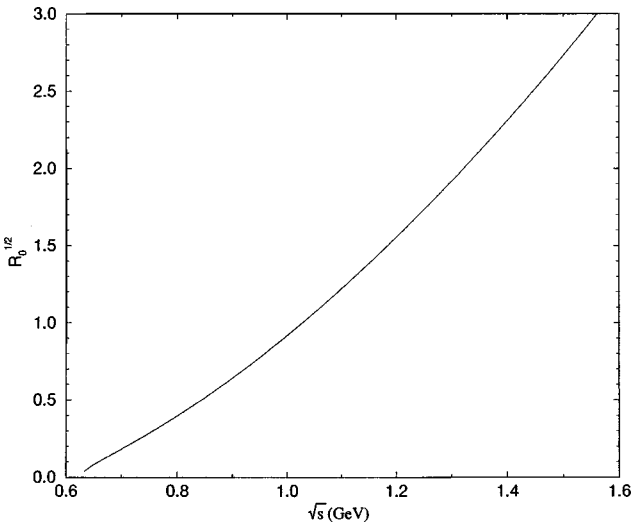

FIG. 2. Current algebra contribution to $R_0^{1/2}$.

choose $m_\pi=137$ MeV and $m_K=496$ MeV. Using Eq. (C1) together with Eq. (A7) gives the $I=\frac{1}{2}$ invariant amplitude

$$A_{CA}^{1/2}(s,t,u)=\frac{1}{2F_\pi^2}[2(s-u)+t], \qquad (3.2)$$

and we will refer to this as the *current algebra* result. Using Eq. (A8) we find the $J=0$ partial wave amplitude to be

$$R_0^{1/2}{}_{CA}=\frac{q}{8\pi F_\pi^2\sqrt{s}}[2(s-m_\pi^2-m_K^2)-3q^2], \qquad (3.3)$$

where the magnitude of the center of mass momentum $q(s)$ is given in Eq. (A9). The current algebra result is shown in Fig. 2, indicating a severe violation of the unitarity bound (A5) beyond approximately 900 MeV. This resembles the violation of the unitarity bound by the current algebra prediction in the $\pi\pi$ case. As in that case we will try to solve this problem by including resonance contributions to the scattering amplitude.

First consider the effect of the vector mesons. There are ρ and K^* exchanges and a direct K^* pole as illustrated in Figs. 1(b), 1(c) and 1(d). The relevant coupling constants are read off from the $\rho_\mu v_\mu$ piece in the first term of Eq. (B7). Symmetry breaking contributions are small [27] and will be neglected here. As an example, the invariant amplitude representing the two K^* diagrams is

$$A_{K^*}^{1/2}=\frac{3}{2}P(u,t,s)-\frac{1}{2}P(s,t,u), \qquad (3.4)$$

with

$$P(u,t,s)=\frac{g_{\rho\pi\pi}^2}{4m_{K^*}^2}\left[\frac{m_{K^*}^2(t-u)+(m_K^2-m_\pi^2)^2}{m_{K^*}^2-s-im_{K^*}\Gamma_{K^*}\theta(s-s_{th})}\right], \qquad (3.5)$$

where Γ_{K^*} is the K^* width, $s_{th}=(m_k+m_\pi)^2$, θ is the Heaviside step function and we take $m_\rho=0.77$ GeV, $g_{\rho\pi\pi}=8.56$

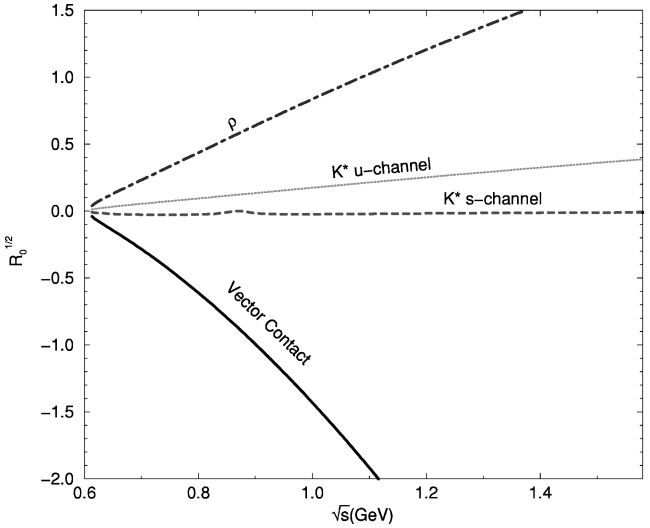

FIG. 3. Individual vector contributions to $R_0^{1/2}$.

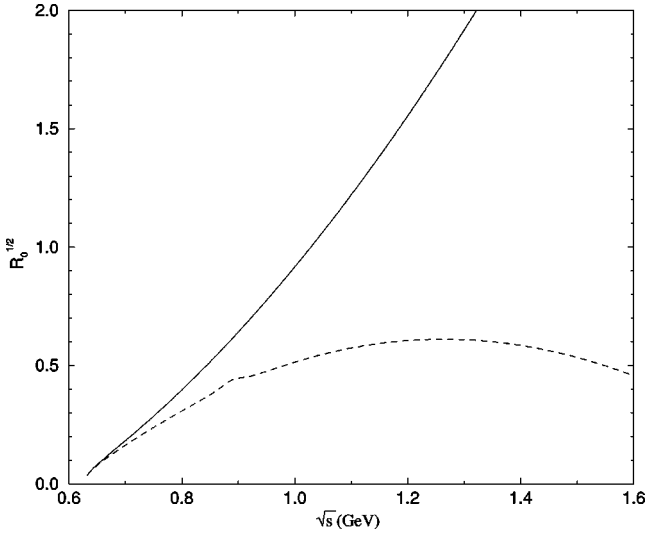

FIG. 4. Contribution of current algebra (solid line) and current algebra+vectors (dashed line) to $R_0^{1/2}$.

and $m_{K*}=0.89$ GeV. We have added a conventional width term in order to regularize the s-channel pole. We may more generally regard this regularization as the imposition of unitarity on the $J=0$, $I=\frac{1}{2}$ partial wave amplitude in the region near the K^* mass. Comparison with Eq. (A7) shows that this regularization formally maintains crossing symmetry. Actually our results are not very sensitive to the fine details of the regularization function.

The contributions associated with the vector mesons including the ρ exchange diagram, the K^* diagrams and a new contact term arising from the $v_\mu v_\mu$ piece in Eq. (B7) are plotted[1] in Fig. 3. As expected, the direct contribution due to the s-channel K^* pole upon projection into the scalar channel is almost zero. In fact it is the new contact term which is seen to play a crucial role in helping to restore unitarity. This term is negative and thus balances the positive current algebra piece. It arises as a consequence of casting the Lagrangian with vectors (B7) in a chirally invariant form. The effect of all the vector contributions, added to the current algebra piece is displayed in Fig. 4. It can be seen that, while individual terms violate the unitarity bound, the introduction of vectors has pulled the curve down so that it almost lies within the bound. A similar improvement, due to the inclusion of vectors, was observed in the analysis of the $\pi\pi$ scattering amplitude [1,2].

So far we have not used any unknown parameters, and so the current algebra and vector contributions are fixed. Actually the violation of unitarity is smaller than at the corresponding stage of the analogous $\pi\pi$ calculation and one might be inclined to stop at this point. However, in our framework, we should include other diagrams for resonances lying within the energy range of interest. There is the established $f_0(980)$ as well as the $\sigma(560)$ which should be included for self-consistency. Of course the role played by a possible strange scalar is of great interest. The relevant Feynman diagrams are shown in Figs. 1(e), 1(f) and 1(g). Another reason for inclusion of these resonances can be seen by looking ahead to the experimentally deduced form for $R_0^{1/2}$ (Fig. 7, below). The sharp dip near 1400 MeV could not be explained from the total current algebra plus vector amplitude of Fig. 4.

In order to compute the scalar exchange diagrams we need the following pieces of the scalar-pseudoscalar-pseudoscalar interaction Lagrangian given at the end of Appendix B:

$$\begin{aligned}\mathcal{L}_{scalars}=&-\sqrt{2}\,\gamma_{\sigma\pi\pi}(\sigma\partial_\mu\pi^+\partial_\mu\pi^-+\cdots)\\&-\frac{\gamma_{\sigma K\bar{K}}}{\sqrt{2}}(\sigma\partial_\mu K^+\partial_\mu K^-+\cdots)\\&-\sqrt{2}\,\gamma_{f_0\pi\pi}(f_0\partial_\mu\pi^+\partial_\mu\pi^-+\cdots)\\&-\frac{\gamma_{f_0K\bar{K}}}{\sqrt{2}}(f_0\partial_\mu K^+\partial_\mu K^-+\cdots)\\&-\gamma_{\kappa K\pi}(\kappa^0\partial_\mu K^-\partial_\mu\pi^++\cdots).\end{aligned}\tag{3.6}$$

For generality we are not assuming any model to relate these couplings to each other. Furthermore, as discussed in Appendix B, the derivative coupling is the one which would follow from a chiral invariant model. Also, the terms shown are the particular ones needed to compute the required π^+K^+ scattering amplitude in Eq. (A6). The coupling constants $\gamma_{\sigma\pi\pi}$, $\gamma_{f_0\pi\pi}$ and $\gamma_{f_0K\bar{K}}$ were estimated in [2]:

$$|\gamma_{\sigma\pi\pi}|=7.81\ \text{GeV}^{-1},\quad |\gamma_{f_0\pi\pi}|=2.43\ \text{GeV}^{-1},$$
$$|\gamma_{f_0K\bar{K}}|=10\ \text{GeV}^{-1}.\tag{3.7}$$

Of the needed σ and $f_0(980)$ coupling constants, only $\gamma_{\sigma K\bar{K}}$ was deduced using $SU(3)$ invariance in some way (which

[1]The bump in the s-channel K^* contribution arises because the amplitude is forced to *rise* to zero at the K^* mass by the spin 1 projection property of the K^* propagator.

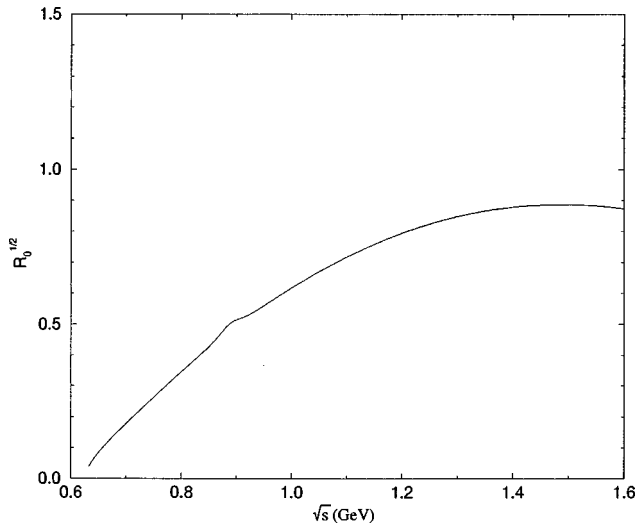

FIG. 5. Current algebra+vectors+σ+$f_0(980)$ contribution to $R_0^{1/2}$.

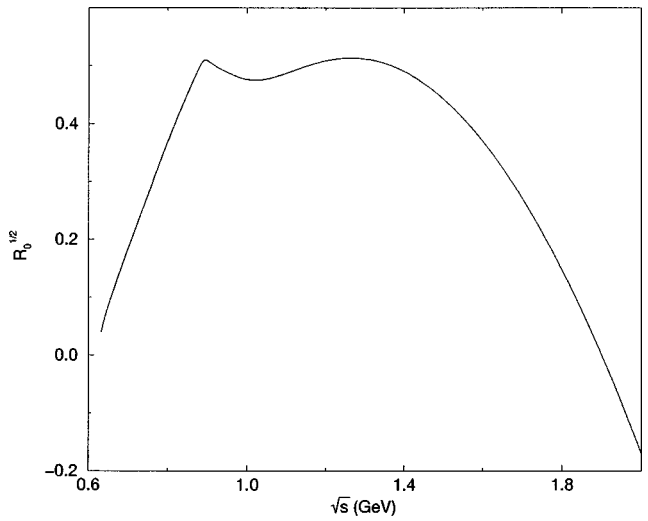

FIG. 6. Contribution of current algebra+vectors+σ+$f_0(980)$ +κ to $R_0^{1/2}$ for κ parameters quoted in Sec. III of text.

implies specializing to a given quark substructure for the scalars). In our final analysis we will thus, for generality, consider the effect of varying the magnitude and sign of $\gamma_{\sigma K\bar{K}}$. Because the $f_0(980)$ contribution is rather small, the relative sign of $\gamma_{f_0\pi\pi}$ and $\gamma_{f_0 K\bar{K}}$ is of less interest.

First, we take into account the σ-meson and the well-established $f_0(980)$. Using Eqs. (C4) and (A7) we find the σ contribution to the invariant amplitude to be

$$A_\sigma^{1/2}(s,t,u)=\frac{\gamma_{\sigma\pi\pi}\gamma_{\sigma K\bar{K}}}{4}\frac{(t-2m_\pi^2)(t-2m_K^2)}{m_\sigma^2-t}. \quad (3.8)$$

The $f_0(980)$ amplitude has an identical structure with $\sigma \to f_0$ everywhere. We shall take $m_\sigma=0.55$ GeV and $m_{f_0}=0.98$ GeV. For now we take $\gamma_{\sigma K\bar{K}}=\gamma_{\sigma\pi\pi}$ and $\gamma_{f_0 K\bar{K}}\gamma_{f_0\pi\pi}$ to be positive. Then a plot showing the effect of adding the projection of Eq. (3.8) into the scalar partial wave channel is given in Fig. 5. Both the σ and $f_0(980)$ contributions are positive, but that of the σ is roughly 3 times larger. It is clear that these contributions make the unitarity violation slightly worse.

Now let us consider the strange scalar κ contribution. Its regularized $I=\frac{1}{2}$ invariant amplitude is similarly found to be

$$A_\kappa^{1/2}(s,t,u)=\frac{\gamma_{\kappa K\pi}^2}{8}\left[\frac{3(s-m_\pi^2-m_K^2)^2}{m_\kappa^2-s-im_\kappa G_\kappa'\theta(s-s_{th})}-\frac{(u-m_\pi^2-m_K^2)^2}{m_\kappa^2-u-im_\kappa G_\kappa'\theta(u-s_{th})}\right]. \quad (3.9)$$

As for the K^*, this regularization is formally crossing symmetric (the u-channel regularization term will vanish in the physical region). We will treat m_κ, $\gamma_{\kappa K\pi}$ and G_κ' as independent parameters. Analogously to the treatment of the light broad $\sigma(560)$, we have introduced a possible deviation from the pure Breit-Wigner form by allowing G_κ' to be a free parameter. The first term in Eq. (3.9) is a direct channel pole and should be extremely important at energies around m_κ. Thus, as in the $\pi\pi$ case it may be used to cure the unitarity violation of the $J=0$ partial wave amplitude. Since the real part of a direct channel resonance contribution turns sharply negative just above the resonance energy and the graph in Fig. 5 rises above the positive unitarity bound at around 900 MeV, we are led to choose m_κ to lie roughly around this energy. With the additional illustrative choices $\gamma_{\kappa K\pi}=4.8$ GeV^{-1} and $G_\kappa'=280$ MeV we see from Fig. 6, which is a plot of $R_0^{1/2}$ including also the contribution of the $J=0$ partial wave projection of Eq. (3.9), that it is easy to achieve a fit in which the unitarity bound is roughly satisfied. The parameters chosen above will be seen in the next section to be close to those needed for a fit to the experimental data.

We obtain the deviation of our κ parametrization from a pure Breit-Wigner shape by noting that near the resonance the $J=0$ partial wave projection of Eq. (3.9) is

$$\frac{m_\kappa G_\kappa}{m_\kappa^2-s-im_\kappa G_\kappa'}, \quad (3.10)$$

where the perturbative width G_κ is given by

$$G_\kappa=\frac{3\gamma_{\kappa K\pi}^2 q(m_\kappa^2)}{64\pi m_\kappa^2}(m_\kappa^2-m_K^2-m_\pi^2)^2, \quad (3.11)$$

and $q(m_\kappa^2)$ is defined in Eq. (A9). Here $G_\kappa/G_\kappa'=1$ is the pure Breit-Wigner situation. The result $G_\kappa/G_\kappa'=0.13$ is similar to $G_\sigma/G_\sigma'=0.29$ which was previously obtained [1,2] for the σ. It seems that such deviations for the low mass scalars are a characteristic feature of our model. Ordinarily, when the resonance is a dominant feature by itself, the Breit-Wigner form may be regarded as equivalent to unitarity near the resonance. However, in our model, there are several different interfering contributions in the low mass region and all work together to keep the partial wave amplitude within the unitarity bound.

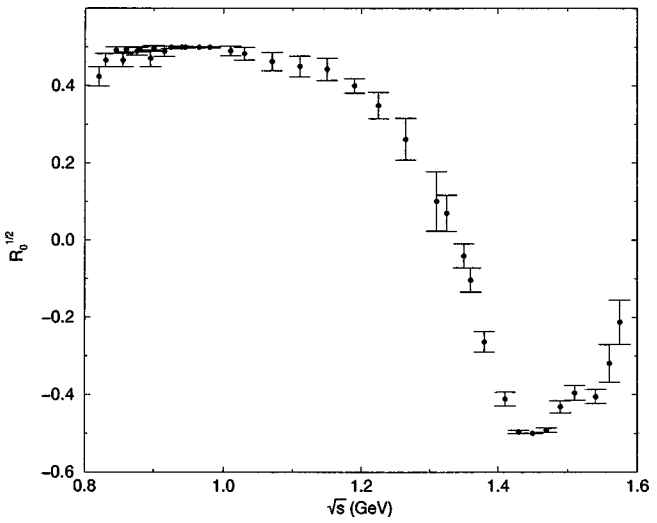

FIG. 7. Experimental data for $R_0^{1/2}$.

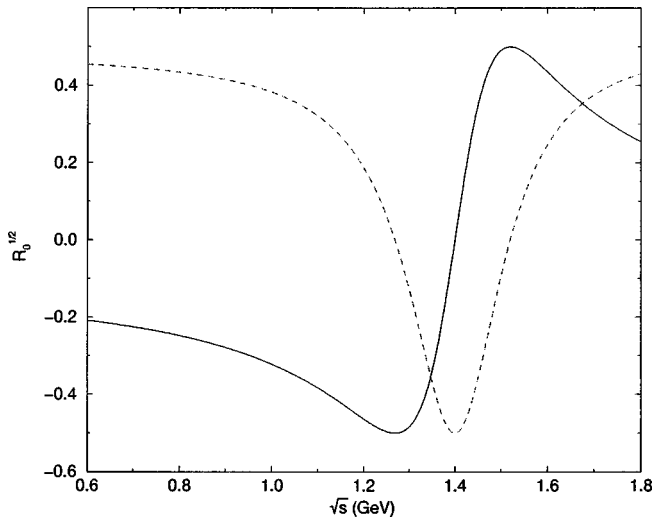

FIG. 8. Shape of $R_0^{1/2}$ derived from Eq. (4.1) for resonance ($m=1.4$ GeV and $\Gamma=0.25$ GeV) in the presence of a background. The plot shows two choices for the background phase: $\delta_{BG}=\pi/2$ (solid line) and $\delta_{BG}=\pi/4$ (dashed line).

IV. GLOBAL FIT TO DATA IN THE $J=0, I=\frac{1}{2}$ CHANNEL

The magnitude and phase of the experimental $I=\frac{1}{2}$ s-wave amplitude are given in Fig. 15 of Aston *et al.* [21], based on a high statistics study of the reaction $K^- p \to K^- \pi^+ n$. We have translated these to the real part $R_0^{1/2}(s)$, which is required for our approach, and show the results[2] in Fig. 7. It is clear that when one looks at the real part there is an interesting dip at around 1400 MeV. This is explained as the relatively narrow strange scalar resonance $K_0^*(1430)$, which is generally considered to be the best candidate for a p-wave $q\bar{q}$ state. From our point of view the most interesting question is whether our model including the κ meson provides the correct background structure to explain the overall shape of $R_0^{1/2}$ in this region. The role of the $K_0^*(1430)$ thus seems analogous to that of the $f_0(980)$ in the $I=J=0$ partial wave amplitude for $\pi\pi$ scattering.

In that case, as mentioned in Sec. I, the interplay between the narrow resonance with its background was introduced as a regularization of the direct channel resonance pole which is $\propto 1/(s-m_*^2)$. In the vicinity of the resonance, upon projection into the appropriate partial wave, one sets the amplitude equal to

$$\frac{e^{2i\delta} m_* \Gamma_*}{m_*^2 - s - i m_* \Gamma_*} + e^{i\delta} \sin\delta, \tag{4.1}$$

where m_* and Γ_* are the resonance mass and width, while δ is the background phase which is assumed to be constant in the neighborhood of the resonance. This form automatically makes the amplitude unitary in this region. We took our total calculated amplitude (which was crossing symmetric), without the $f_0(980)$ contribution, evaluated at the position of the resonance, to be the second term in Eq. (4.1); this allowed us to interpret the invariant amplitude (4.1) as being formally crossing symmetric.

It turns out that there is an interesting difference between the $\pi\pi$ and $K\pi$ situations. This can easily be seen by focusing on the real part of Eq. (4.1) which is

$$\frac{1}{2}\sin 2\delta + \frac{m_*\Gamma_*}{(m_*^2 - s)^2 + m_*^2\Gamma_*^2} \times [(m_*^2 - s)\cos 2\delta - m_*\Gamma_* \sin 2\delta]. \tag{4.2}$$

The shape of this curve depends on the value of δ. In the $\pi\pi$ case, the background naturally produced a phase $\delta \approx \pi/2$ at the position of the $f_0(980)$. This yields the shape indicated in Fig. 8 which just amounts to a sign reversal of the usual resonance function (in the absence of a background):—the Ramsauer-Townsend mechanism [25]. On the other hand, Fig. 6 shows that $R_0^{1/2}$ is almost $\frac{1}{2}$ at around 1400 MeV, so that we expect to have $\delta \approx \pi/4$. This gives the other shape shown in Fig. 8 which, in fact, basically agrees with the experimental $K\pi$ channel picture in Fig. 7.

Now let us consider the detailed application of this mechanism to $K\pi$ scattering. The contribution of the $K_0^*(1430)$ to the $I=\frac{1}{2}$ channel is structurally similar to that of the κ in Eq. (3.9). The real part of this contribution to the regularized invariant amplitude is

$$\begin{aligned}\mathrm{Re}[A_*^{1/2}(s,t,u)] &= \frac{\gamma_*^2}{8}\,\mathrm{Re}\left[e^{2i\delta\theta(s-s_{th})}\frac{3(s-m_\pi^2-m_K^2)^2}{m_*^2 - s - i m_* G'_* \theta(s-s_{th})}\right] \\ &\quad - \frac{\gamma_*^2}{8}\,\mathrm{Re}\left[e^{2i\delta\theta(u-s_{th})}\frac{(u-m_\pi^2-m_K^2)^2}{m_*^2 - u - i m_* G'_* \theta(u-s_{th})}\right].\end{aligned} \tag{4.3}$$

[2]Our error bars are based on propagating the errors in [21], assuming conservatively these in turn to be given by the experimental circles in Fig. 15 of [21].

TABLE I. Comparison of different fits in the $J=0$, $I=\frac{1}{2}$ channel, corresponding to different choices of $\gamma_{\sigma K\bar{K}}$.

Fitted parameter	$\gamma_{\sigma K\bar{K}}=\gamma_{\sigma\pi\pi}$	$\gamma_{\sigma K\bar{K}}=0$	$\gamma_{\sigma K\bar{K}}=-\gamma_{\sigma\pi\pi}$
m_κ	897 ± 2.1 MeV	951 ± 0.7 MeV	998 ± 1.1 MeV
G'_κ	322 ± 6.0 MeV	277 ± 10.6 MeV	195 ± 5.3 MeV
$\gamma_\kappa K_\pi$	5.0 ± 0.07 GeV^{-1}	4.32 ± 0.16 GeV^{-1}	4.04 ± 0.08 GeV^{-1}
m_*	1385 ± 3.3 MeV	1365 ± 2.5 MeV	1349 ± 2.1 MeV
G'_*	266 ± 9.5 MeV	201 ± 9.8 MeV	148 ± 5.6 MeV
γ_*	4.3 ± 2.1 GeV^{-1}	$3.7\pm .1$ GeV^{-1}	3.1 ± 0.05 GeV^{-1}
χ^2	4.0	9.0	25.7

Here we have denoted quantities associated with the $K_0^*(1430)$ by an asterisk subscript. In particular, m_* is now the mass of the $K_0^*(1430)$. The quantity γ_* is defined in terms of the $K_0^*(1430)$ partial width into $K\pi$ by

$$\Gamma(K_0^*(1430)\to K\pi)=\frac{3\gamma_*^2 q(m_*^2)(m_*^2-m_\pi^2-m_K^2)}{64\pi m_*^2}, \tag{4.4}$$

where $q(s)$ is defined in Eq. (A9). The background phase δ will not be considered an arbitrary parameter but shall be the constant quantity defined from

$$\frac{1}{2}\sin 2\delta=\tilde{R}_0^{1/2}(s=m_*^2), \tag{4.5}$$

where $\tilde{R}_0^{1/2}(s)$ is the real part of the partial wave amplitude previously computed as the sum of the crossing symmetric current algebra, vector, σ, $f_0(980)$ and κ pieces found in Sec. III. With these arrangements the total invariant amplitude is formally crossing symmetric. In order to see the connection with the unitary form (near the resonance) in Eqs. (4.1) and (4.2), we simply note that the second term in Eq. (4.3) is numerically dominated by the first term which contains a pole in the physical region. Finally, for the sake of generality, we shall consider G'_* to be a fitting parameter, not necessarily equal to $\Gamma(K_0^*(1430)\to K\pi)$. This allows for the possibility of some inelasticity.

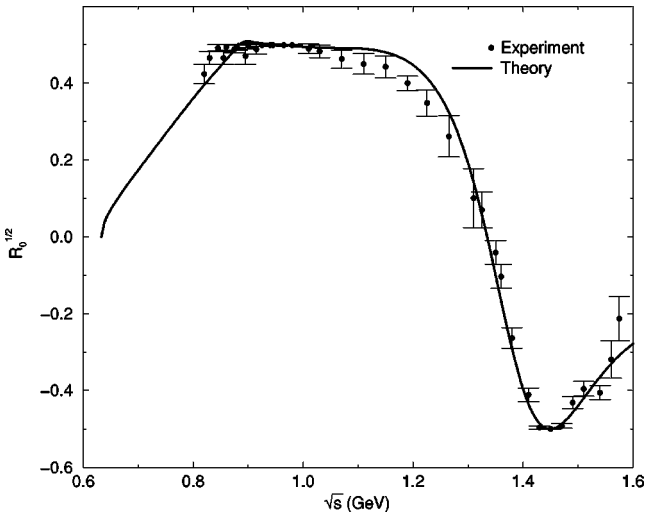

FIG. 9. Comparison of the theoretical prediction of $R_0^{1/2}$ with its experimental data (for the choice $\gamma_{\sigma K\bar{K}}=\gamma_{\sigma\pi\pi}$).

We notice that the mechanism shown in Eq. (4.1) implicitly demands a background which does not violate the unitarity bounds at the resonance mass m_*. This provides a justification for the existence of the κ meson, as we showed in the last section that it is needed to restore unitarity (compare Fig. 5 and Fig. 6).

We now continue with a more quantitative approach in order to extract the physical parameters of the κ meson and the $K_0^*(1430)$. We fit the theoretical amplitude, which consists[3] of the real part of the partial wave projection of Eq. (4.3) added to $\tilde{R}_0^{1/2}(s)$, defined above, to the experimental data displayed in Fig. 7. The parameters to be fit are the three quantities m_κ, $\gamma_{\kappa K\pi}$ and G'_κ for the κ [see Eq. (3.9)] and the corresponding quantities for the $K_0^*(1430)$, namely m_*, γ_* and G'_* [see Eq. (4.3)]. As discussed at the end of Sec. II, it seems reasonable to obtain the three $K_0^*(1430)$ parameters self-consistently from our model rather than taking them from [21]. The scalar meson coupling constants listed in Eq. (3.7) were used while, in light of its uncertainty, the calculation was performed for a range of values of $\gamma_{\sigma K\bar{K}}$. The fitting procedure made use of the MINUIT package and the fitted parameters, together with their χ^2 values, are shown in Table I. It is interesting to notice that the fitted parameters vary smoothly with $\gamma_{\sigma K\bar{K}}$. The actual comparison between experiment and the fitted amplitude, using the parameters from the first column in Table I, is shown in Fig. 9. The individual contributions due to the background and to the $K_0^*(1430)$ are shown in Fig. 10, indicating that the background does not violate the unitarity bound at $s=m_*^2$. The exact value of the phase found in this fit is $\sin 2\delta=0.937$. This agrees with the qualitative discussion regarding the background phase at the beginning of this section.

The partial decay width of $K_0^*(1430)$ can be calculated using Eq. (4.4). We find that $\Gamma(K_0^*(1430)\to\pi K)=238$ MeV and as a result (identifying G'_* as the total width) an estimate of the branching ratio of $K_0^*(1430)$ to decay to πK can be made:

[3]We also included the $f_0(1300)$ contribution, which is, however, very small.

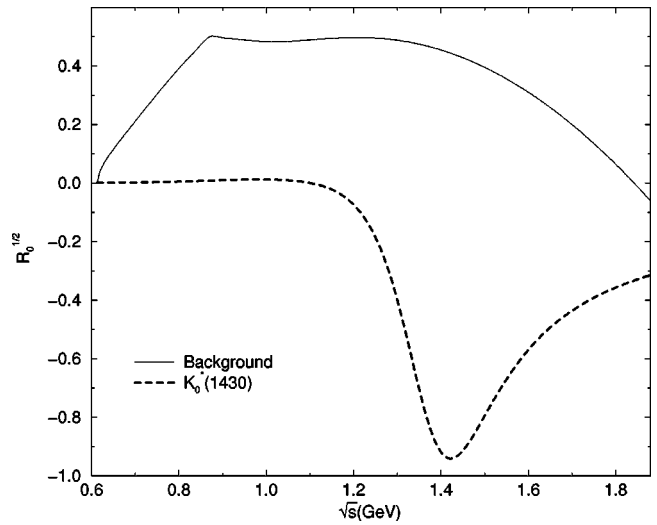

FIG. 10. Separate contributions of the background and $K_0^*(1430)$ to $R_0^{1/2}$ (for the choice $\gamma_{\sigma K\bar{K}}=\gamma_{\sigma\pi\pi}$).

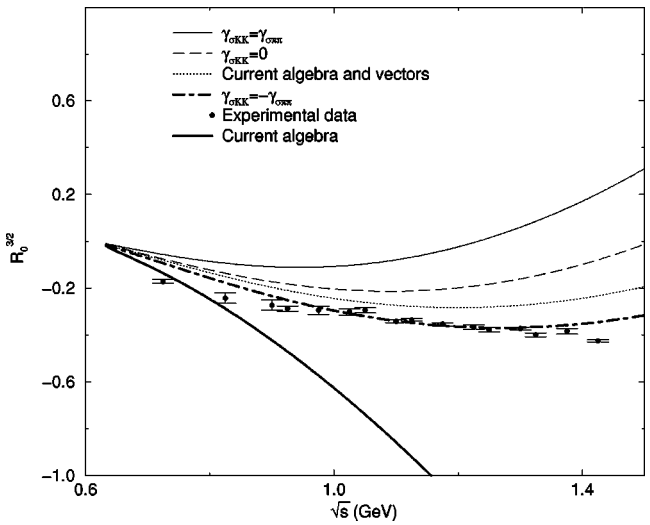

FIG. 11. Comparison of various predictions for $R_0^{3/2}$ with experiment.

$$\mathrm{B}[K_0^*(1430)\to\pi K]=\frac{\Gamma_{K_0^*(1430)}}{G'_*}=0.895. \qquad (4.6)$$

This quantity is comparable to the 0.93 obtained in [21]. Similarly, the (first column of Table I) mass and width we obtain—1385 MeV and 266 MeV—are in reasonable agreement with their [21] respective values:—1429 MeV and 287 MeV.

V. $J=0$, $I=\frac{3}{2}$ CHANNEL

It is interesting to compare with experiment the projection into the $J=0$, $I=\frac{3}{2}$ channel of the same invariant amplitude used for the last section. The structures of the invariant $I=\frac{3}{2}$ amplitudes may actually be read off from Eqs. (C1)–(C4) of Appendix C. Since there are no $I=\frac{3}{2}$ resonances in our model, there are no s-channel poles, and hence this calculation depends little on the details of the regularizations. As in the $I=\frac{1}{2}$ case, cancellations of individual contributions to the partial wave amplitude act to preserve the unitarity bound. The experimental points for the real part $R_0^{3/2}$ were translated from Fig. 12 of [28] and are displayed in our Fig. 11.

Figure 11 also shows various predictions from our model. First, we see that the current algebra prediction alone quite soon departs from the data points and begins to violate the unitarity bound at around 900 MeV. Inclusion of the ρ, K^* and contact contributions associated with the vector mesons can be seen to pull the curve up considerably so as to solve the unitarity problem and to give a much better fit to the data. This is very analogous to the situation in the $J=0$, $I=2$ partial wave for $\pi\pi$ scattering (see Fig. 4 of [1] and Fig. 2.10 of [29]). At this stage, the curve does not depend on any unknown parameters.

It turns out that the only additional important contribution to this channel comes from σ meson exchange. This will depend on the choice of the coupling constant $\gamma_{\sigma K\bar{K}}$ which was the important unknown parameter in the previous section. Figure 11 shows the results for the three choices of $\gamma_{\sigma K\bar{K}}$ given in Table I. The best choice for the $I=\frac{3}{2}$ amplitude is the case $\gamma_{\sigma K\bar{K}}=-\gamma_{\sigma\pi\pi}$ which unfortunately yields the fit with the highest χ^2 for the $I=\frac{1}{2}$ analysis. The small difference between the curve for $\gamma_{\sigma K\bar{K}}=0$ and the curve for the current algebra plus vector contribution measures the small impact of the other scalars. Actually the general trend of the data is reproduced for all values of $\gamma_{\sigma K\bar{K}}$ shown.

Since there are no large direct channel resonance contributions, the $I=\frac{3}{2}$ amplitude may be especially sensitive to exchanged resonances in the range above 1 GeV which we are currently neglecting. This is in contrast with the $I=\frac{1}{2}$ amplitude which contains fitting parameters that can absorb the effects of higher resonance exchanges. This was the case for the $\pi\pi$ scattering calculation also.

As we lower $\gamma_{\sigma K\bar{K}}$, we find fits with larger values of χ^2 that correspond to a κ that is heavier, narrower, and has a larger coupling constant, and to a $K_0^*(1430)$ that is lighter, narrower, and has a larger coupling constant.

VI. DISCUSSION

We have found that a large-N_c-motivated approximate treatment of πK scattering can give a crossing symmetric and unitary amplitude as a fit to the existing experimental data. A novel feature of this approach, which is analogous to that employed for $\pi\pi$ scattering in [1,2], is to start with the invariant perturbative amplitude which is manifestly crossing symmetric. This results in individual contributions dramatically violating the partial wave unitarity bounds. We rely on cancellations among these competing contributions to rescue unitarity. In our framework this suggests the existence of a light strange scalar resonance κ which has parameters mass $m_\kappa=897$ MeV and width $G_\kappa=322$ MeV. These give a pole position

$$(s_\kappa)^{1/2}=(0.911-0.158i)\ \ \mathrm{GeV}. \qquad (6.1)$$

We do not quote any error here since the main uncertainty in this analysis is clearly due to the theoretical model. It is noteworthy that these results are similar to those of [17] in which a different model was employed. In addition, the fit for the $K_0^*(1430)$ properties also obtained is similiar to that of the experimental analysis of [21]. Our work was simplified by directly making use of the analogous approximation seen to be reasonable in [2] for the $\pi\pi$ scattering case. Thus, as suggested by working to leading order in $1/N_c$, we compared the real part of the partial wave amplitude with experiment. Since elastic unitarity seems [21] to be a reasonable approximation until about the $K_0^*(1430)$ region for the $J=0$, $I=\frac{1}{2}$ partial wave amplitude, we can recover its imaginary part as

$$I_0^{1/2} \approx \frac{1}{2}[1 \pm \sqrt{(\eta_0^{1/2})^2 - 4(R_0^{1/2})^2}], \tag{6.2}$$

with $\eta_0^{1/2} \approx 1$ and an appropriate choice of sign. Of course the phase shift is recovered as $\tan(\delta_0^{1/2}) = I_0^{1/2}/R_0^{1/2}$.

As in the $\pi\pi$ treatment we neglected, for an initial analysis, the contributions of most resonances above 1 GeV. Specifically, we did not include diagrams with the radially excited vectors $\rho(1450)$ and $K^*(1420)$ or with the tensors $f_2(1270)$ and $K_2^*(1430)$. In a ''second generation'' treatment of this problem it would be desirable to fully investigate these aspects. It would be amusing to see if the complicated 1–2 GeV region is high enough so that the ''microscopic'' approach we are following merges with a kind of string picture [30].

If one accepts the existence of the $\kappa(900)$ and $\sigma(560)$, in addition to the $f_0(980)$ and $a_0(980)$, then there is a full set of candidates for a possibly unconventional (i.e. not of pure $q\bar{q}$ type) low mass scalar nonet. The nature of such a nonet is of great interest—see [31] for a recent discussion. A useful clue may arise from knowledge of the pattern of $0^+0^-0^-$ coupling constants defined in Eqs. (B11)–(B13). The numerical values obtained in our approach are given in Eq. (3.7) and Table I.

ACKNOWLEDGMENTS

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APPENDIX A: SCATTERING KINEMATICS

The partial wave scattering matrix for a channel like $\pi K \to \pi K$ can be written as

$$S = 1 + 2iT, \tag{A1}$$

where for simplicity the isospin and the angular momentum variables have not been indicated. The standard parameterization of the single-channel scattering amplitude is

$$S = \eta e^{2i\delta_{\pi K}}, \tag{A2}$$

where $\delta_{\pi K}$ is the phase shift and $0 < \eta \leqslant 1$ is the elasticity parameter. Evidently,

$$T_l^I(s) = \frac{\eta_l^I(s) e^{2i\delta^I_{\pi K;l}(s)} - 1}{2i}, \tag{A3}$$

where l and I label the angular momentum and isospin, respectively. The real and imaginary parts

$$R_l^I = \frac{\eta_l^I \sin(2\delta^I_{\pi K;l})}{2}, \quad I_l^I = \frac{1 - \eta_l^I \cos(2\delta^I_{\pi K;l})}{2} \tag{A4}$$

must satisfy the very important unitarity bounds

$$|R_l^I| \leqslant \frac{1}{2}, \quad 0 \leqslant I_l^I \leqslant 1. \tag{A5}$$

Now we relate the previous partial wave amplitudes to the $I=\frac{1}{2}$ and $I=\frac{3}{2}$ invariant amplitudes for the scattering process $\pi(p_1) + K(p_2) \to \pi(p_3) + K(p_4)$. This is simply achieved by first defining the $I=\frac{3}{2}$ amplitude via

$$A^{3/2}(s,t,u) = A(\pi^+(p_1)K^+(p_2) \to \pi^+(p_3)K^+(p_4)), \tag{A6}$$

where s, t and u are the Mandelstam variables. By crossing symmetry we have $A(\pi^+K^- \to \pi^+K^-) = A^{3/2}(u,t,s)$ which leads to

$$A^{1/2}(s,t,u) = \frac{3}{2}A^{3/2}(u,t,s) - \frac{1}{2}A^{3/2}(s,t,u). \tag{A7}$$

We then define the partial wave isospin amplitudes according to the formula

$$T_l^I(s) = \frac{\rho(s)}{2} \int_{-1}^{1} d\cos\theta P_l(\cos\,\theta) A^I(s,t,u), \tag{A8}$$

where θ is the scattering angle and

$$\rho(s) = \frac{q(s)}{8\pi\sqrt{s}} \equiv \frac{1}{16\pi s} \sqrt{[s-(m_\pi+m_K)^2][s-(m_\pi-m_K)^2]}. \tag{A9}$$

APPENDIX B: NOTATION AND LAGRANGIAN

Spontaneous chiral symmetry breaking plays a fundamental role at low energies and is often economically as well as successfully described by non-linear realizations. Associated with the standard chiral symmetry breaking pattern $SU(3)_{\rm L} \times SU(3)_{\rm R} \to SU(3)_{\rm V}$ we have an octet of pseudoscalar Nambu-Goldstone bosons ϕ. The latter are encoded in a 3×3 matrix U as follows:

$$U = \xi^2, \quad \xi = e^{i\phi/F_\pi}, \tag{B1}$$

where F_π is the pion decay constant. U transforms under a chiral transformation as

$$U \to U_{\rm L} U U_{\rm R}^\dagger, \tag{B2}$$

with $U_{L,R} \in U(3)_{L,R}$. While U transforms linearly under these transformations [see Eq. (B2)], ξ transforms non-linearly, i.e.

$$\xi \to U_L \xi K^\dagger(\phi, U_L, U_R) = K(\phi, U_L, U_R) \xi U_R^\dagger. \quad \text{(B3)}$$

The vector meson nonet ρ_μ may be formally introduced as a *gauge field* [32]. It transforms under chiral rotations as

$$\rho_\mu \to K \rho_\mu K^\dagger + \frac{i}{\tilde{g}} K \partial_\mu K^\dagger, \quad \text{(B4)}$$

where $\tilde{g}$ is the *gauge coupling constant*. (For an alternative approach see, for a review, Ref. [33].) It is convenient to define the following objects:

$$p_\mu = \frac{i}{2}(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi),$$

$$v_\mu = \frac{i}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi), \quad \text{(B5)}$$

which obey the transformation rules

$$p_\mu \to K p_\mu K^\dagger,$$

$$v_\mu \to K v_\mu K^\dagger + i K \partial_\mu K^\dagger. \quad \text{(B6)}$$

Using the above quantities we can construct the non-anomalous part of the chiral Lagrangian describing pseudoscalar and vector mesons:

$$\mathcal{L} = -\frac{1}{2} m_v^2 \, \mathrm{Tr}\left[\left(\rho_\mu - \frac{v_\mu}{\tilde{g}}\right)^2\right] - \frac{F_\pi^2}{2} \mathrm{Tr}[p_\mu p_\mu]$$

$$- \frac{1}{4} \mathrm{Tr}[F_{\mu\nu}(\rho) F_{\mu\nu}(\rho)], \quad \text{(B7)}$$

where $F_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - i\tilde{g}[\rho_\mu, \rho_\nu]$ is the vector meson *gauge field strength*. Chiral symmetry is explicitly broken in QCD by the presence of an explicit quark mass term $-\hat{m}\bar{q}\mathcal{M}q$, where $\hat{m} \equiv (m_u + m_d)/2$, and $\mathcal{M}$ is the dimensionless matrix:

$$\mathcal{M} = \begin{pmatrix} 1+y & & \\ & 1-y & \\ & & x \end{pmatrix}. \quad \text{(B8)}$$

Here x and y are the quark mass ratios:

$$x = \frac{m_s}{\hat{m}}, \quad y = \frac{1}{2}\left(\frac{m_d - m_u}{\hat{m}}\right). \quad \text{(B9)}$$

These quark masses induce a mass term for the pseudoscalar mesons which at the effective Lagrangian level is represented by the following term:

$$\mathcal{L}_{\phi \ \mathrm{mass}} = \delta' \ \mathrm{Tr}[\mathcal{M} U^\dagger + \mathcal{M}^\dagger U], \quad \text{(B10)}$$

where δ' is a real constant. A more general set of terms describing explicit chiral symmetry breaking in this framework is available in Refs. [27,34]. Scalar resonances, in the non-linear realization framework, interact with pseudoscalars with at least two derivatives. If we were to identify the scalars with a matter field nonet, i.e. which transforms under chiral transformations as $S \to K S K^\dagger$, a possible invariant interaction term is $\mathrm{Tr}[S p_\mu p_\mu]$. Since the quark content of the scalars is not yet firmly established and other possible terms may exist, we adopt here a more phenomenological approach by not relating the scalar couplings using SU(3) symmetry. For the present paper, the relevant interaction terms are

$$\mathcal{L}_\sigma = -\frac{\gamma_{\sigma\pi\pi}}{\sqrt{2}} \sigma \partial_\mu \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} - \frac{\gamma_{\sigma K\bar{K}}}{\sqrt{2}} \sigma (\partial_\mu K^+ \partial_\mu K^- + \cdots), \quad \text{(B11)}$$

$$\mathcal{L}_{f_0} = -\frac{\gamma_{f_0\pi\pi}}{\sqrt{2}} f_0 \partial_\mu \boldsymbol{\pi} \cdot \partial_\mu \boldsymbol{\pi} - \frac{\gamma_{f_0 K\bar{K}}}{\sqrt{2}} f_0 (\partial_\mu K^+ \partial_\mu K^- + \cdots), \quad \text{(B12)}$$

$$\mathcal{L}_\kappa = -\gamma_{\kappa K\pi} (\kappa^0 \partial_\mu K^- \partial_\mu \pi^+ + \cdots). \quad \text{(B13)}$$

Different models will relate the coupling constants in different ways. For example in the SU(3) limit, and if the scalars belong to the usual matter field nonet with no OZI violating interactions, we have $\gamma_{\sigma\pi\pi} = \gamma_{\sigma K\bar{K}} = \gamma_{f_0 K\bar{K}}/\sqrt{2} = \gamma_{\kappa K\pi}$ while $\gamma_{f_0\pi\pi} = 0$.

APPENDIX C: UNREGULARIZED AMPLITUDES

The current-algebra contribution to the $A^{3/2}(s,t,u)$ amplitude, obtained from Eqs. (B7) and (B10) is

$$A_{CA}^{3/2}(s,t,u) = \frac{t+u-s}{2F_\pi^2}. \quad \text{(C1)}$$

The vector meson contribution contains the following terms:

$$A_{vect}^{3/2}(s,t,u) = \frac{g_{\rho\pi\pi}^2}{4}\left[\frac{u-s}{m_\rho^2 - t} - \frac{m_{K*}^2(s-t) - (m_K^2 - m_\pi^2)^2}{(m_{K*}^2 - u) m_{K*}^2}\right]$$

$$+ \frac{g_{\rho\pi\pi}^2}{4 m_\rho^2}(2s - u - t), \quad \text{(C2)}$$

where $g_{\rho\pi\pi} = m_\rho^2/\tilde{g}F_\pi^2$ is the coupling of the vector to two pions, which is related to the width by $\Gamma(\rho \to 2\pi) = g_{\rho\pi\pi}^2 p_\pi^3 / 12\pi m_\rho^2$. The first and second terms correspond respectively to ρ^0 and K^* exchanges, while the third term represents the contact interaction $v_\mu v_\mu$ in Eq. (B7). The contribution of a strange scalar, denoted κ, is

$$A_\kappa^{3/2}(s,t,u) = \frac{\gamma_{\kappa K\pi}^2}{4} \frac{(u - m_\pi^2 - m_K^2)^2}{m_\kappa^2 - u}. \quad \text{(C3)}$$

Finally the σ exchange contribution is

$$A_\sigma^{3/2}(s,t,u)=\frac{\gamma_{\sigma\pi\pi}\gamma_{\sigma K\bar{K}}}{4}\frac{(2m_\pi^2-t)(2m_K^2-t)}{m_\sigma^2-t}. \quad \text{(C4)}$$

Note that Eq. (C3) can also be used to describe the contribution of the scalar resonance $K_0^*(1430)$, if we reidentify the coupling constant and the mass in the denominator. Similarly Eq. (C4) can be used for the f_0 exchange if we replace each subscript σ by a subscript f_0.

The $A^{1/2}$ amplitudes are obtained from these using Eq. (A7).

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