New explanation of the GAMS results on the $f_0(980)$ production in the reaction $\pi^- p \to \pi^0 \pi^0 n$

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The observed alteration of the S-wave $\pi^0\pi^0$ mass spectrum in the reaction $\pi^- p \to \pi^0\pi^0 n$ with increasing $-t$, i.e., the disappearance of a dip and the appearance of a peak in the region of the $f_0(980)$ resonance as $-t$ increases, is explained by the contribution of the $\pi^- p \to f_0(980) n$ reaction amplitude with the quantum numbers of the a_1 Regge pole in the t channel. It is very interesting that nontrivial evidence for the a_1 exchange mechanism in the reaction $\pi^- p \to \pi^0\pi^0 n$ follows for the first time from the experiment on an unpolarized target. The explanation of the GAMS results suggested by us is compared with that reported previously. Two ways of experimentally testing these explanations are pointed out. [S0556-2821(98)00317-8]

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I. INTRODUCTION

Recently, the GAMS Collaboration has continued the investigation of the reaction $\pi^- p \to \pi^0\pi^0 n$ at $P^{\pi^-}_{\rm lab} = 38$ GeV [1]. The goal of the new experiment is to study the t behavior of the S-wave $\pi^0\pi^0$ mass spectrum in the region of the $f_0(980)$ resonance (t is the square of the four-momentum transferred from the incoming π^- to the outgoing $\pi^0\pi^0$ system). The partial wave analysis performed in the range $0 < -t < 1$ GeV^2 gave a very interesting and unexpected result. The $f_0(980)$ resonance has been seen as a dip in the S-wave $\pi^0\pi^0$ mass spectrum for $-t < 0.2$ GeV^2 [see Fig. 1(a)], where the reaction $\pi^- p \to \pi^0\pi^0 n$ is dominated by the one-pion exchange mechanism,[1] whereas for $-t > 0.3$ GeV^2, it has been observed as a distinct peak [see Figs. 1(b)–1(f)]. This dip and peak behavior of the $f_0(980)$ has also been seen in the Brookhaven experiment on the reaction $\pi^- p \to \pi^0\pi^0 n$ at $P^{\pi^-}_{\rm lab} = 18$ GeV [16]. A partial wave analysis of these data is presently being undertaken [16].

In this work we show that the observed alteration of the S-wave $\pi^0\pi^0$ mass spectrum in the reaction $\pi^- p \to \pi^0\pi^0 n$ with increasing $-t$ can be explained by the contribution of the $\pi^- p \to f_0(980) n$ reaction amplitude with quantum numbers of the a_1 Regge pole in the t channel. So far this amplitude has been very poorly studied experimentally.

In fact, we suggest the following plausible scenario. At small $-t$, the reaction $\pi^- p \to (\pi^0\pi^0)_S n$ is dominated by the one-pion exchange mechanism, and the $f_0(980)$ resonance manifests itself in the $(\pi^0\pi^0)_S$ mass spectrum as a minimum [$(\pi\pi)_S$ denotes a $\pi\pi$ system with the orbital angular momentum $L=0$]. However, the one-pion exchange contribution decreases very rapidly with $-t$ (as is known, at least 85–90 % of the one-pion exchange cross section for the reactions $\pi N \to \pi\pi N$ originate from the region $-t < 0.2$ GeV^2). The most remarkable fact is that the reactions $\pi N \to (\pi\pi)_S N$ at high energies involve only two types of t-channel exchanges, namely, those with quantum numbers of the π and a_1 Regge poles. Thus, it is very probable that the reaction $\pi^- p \to (\pi^0\pi^0)_S n$ at large $-t$ is dominated by the a_1 exchange, and that the $f_0(980)$ resonance produced by this mechanism shows itself as a peak. Notice that a similar manifestation of the $f_0(980)$ resonance has been observed in many reactions not involving π exchange (i.e., in which the $\pi\pi$ interaction in the initial state is absent). For example, the $f_0(980)$ resonance has been seen as a clear peak in the two-pion mass spectra in the reaction $\pi^- p \to \pi^0\pi^0 n$ near threshold and for $-t$ from 0.33 to 0.83 GeV^2, where the one-pion exchange is small [17], in the reaction $K^- p \to \pi^+\pi^-(\Lambda,\Sigma)$ at 13 GeV [18], in the $J/\psi \to \phi\pi^+\pi^-$ [19] and $D_s^+ \to \pi^+\pi^+\pi^-$ [20] decays, in the reaction $\gamma\gamma \to \pi^0\pi^0$ [21], and also in the inclusive $\pi^+\pi^-$ production in γp, $\pi^\pm p$, $K^\pm p$ [22], and e^+e^- [23] collisions.

Our explanation of the GAMS results may be unambiguously verified experimentally in the reactions $\pi N \to \pi\pi N$ on polarized targets because this makes possible direct measurements of the interference between the π and a_1 exchange amplitudes. In a cross section summed over the nucleon polarizations, the contributions of these amplitudes are noncoherent and, generally speaking, they cannot be separated without additional assumptions. It is interesting to note in this connection that the GAMS Collaboration has probably become the first who succeeded in discovering a nontrivial evidence for the a_1 exchange mechanism in the reaction $\pi^- p \to (\pi^0\pi^0)_S n$ on an unpolarized target.[2]

In Sec. II, we perform a simultaneous description of the

[1] As is well known, such a manifestation of the $f_0(980)$ resonance, due to its strong destructive interference with the background, was observed in a large number of previous experiments on the reactions $\pi N \to \pi\pi N$ and $\pi N \to \pi\pi\Delta(1232)$, and according to their results, it has also been well established in the reaction $\pi\pi \to \pi\pi$ (see, for example, Refs. [2–11], and for reviews, Refs. [12–15]).

[2] As is known, the results of the measurements of the reactions $\pi^\pm N_\uparrow \to \pi^+\pi^- N$ on polarized targets are indicative of the a_1 exchange mechanism most definitely in the case of the $\rho^0(770)$ production [24–26]. However, in the $\pi\pi$ invariant mass region around 1 GeV, rather large experimental uncertainties in the available data present considerable problems for certain conclusions. Nevertheless, in a new analysis of the $\pi^- p_\uparrow \to \pi^+\pi^- n$ data at 17.2 GeV [6,25], which has been performed very recently in Ref. [27], one emphasizes that the a_1 exchange amplitude cannot be neglected especially around 1 and 1.5 GeV.

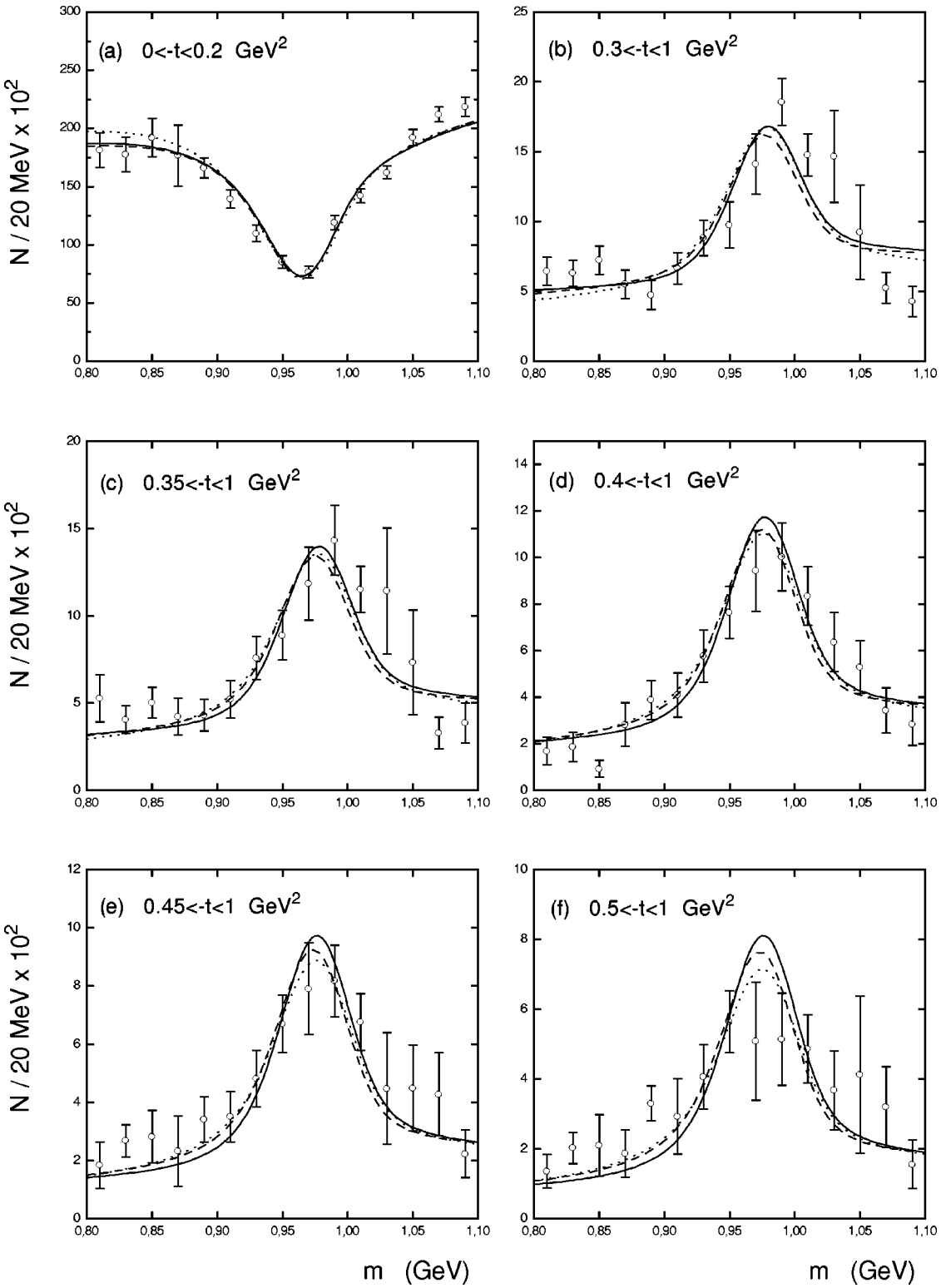

FIG. 1. The S-wave $\pi^0\pi^0$ mass spectra in the reaction $\pi^- p \to \pi^0\pi^0 n$ for six t intervals indicated in the figure. The data were obtained by the GAMS Collaboration [1]. The curves correspond to the fits using the π and a_1 exchange model which is described in detail in the text. The solid curves correspond to variant 1, the dotted curves to variant 2, and the dashed ones to variant 3.

GAMS data on the reaction $\pi^- p \to (\pi^0\pi^0)_S n$ [1] and the CERN-Munich data on the S-wave $\pi\pi$ scattering in an invariant mass region around 1 GeV [5]. We consider three simple parametrizations of the S-wave $\pi\pi \to \pi\pi$ reaction amplitude. As to the corresponding amplitude of the reaction $\pi^* \pi \to \pi\pi$ (where π^* denotes a Reggeized pion), it is constructed by using the t dependence factorization assumption which was extensively applied previously to obtain the $\pi\pi$ scattering data (see, for example, Refs. [5,7,8,27–29]). In parametrizing the $\pi^- p \to (\pi^0\pi^0)_S n$ reaction amplitude due

to the a_1 exchange, we use the above qualitative reason based on the observations of the $f_0(980)$ resonance in the reactions not involving π exchange. All considered parametrizations of the $\pi^- p \to (\pi^0\pi^0)_S n$ reaction amplitudes give similar results and, on the whole, quite reasonable fits to the GAMS data. In Sec. III, we compare our explanation of the GAMS data with that reported previously in Ref. [30] and point out two direct ways to test these explanations. The explanation of Ref. [30] differs crucially from ours in that it is based entirely on one-pion exchange or exchanges with these quantum numbers. Such a restriction, as we show, leads, in particular, to rather exotic predictions for the t distributions of the $\pi^- p \to (\pi^0\pi^0)_S n$ events. Our conclusions are briefly summarized in Sec. IV.

II. ALTERATION OF THE $(\pi^0\pi^0)_S$ MASS SPECTRUM IN THE $f_0(980)$ REGION IN THE REACTION $\pi^- p \to (\pi^0\pi^0)_S n$

We shall consider the reaction $\pi^- p \to (\pi^0\pi^0)_S n$ within the framework of the simplest Regge pole model and write the unpolarized differential distribution of the $\pi^- p \to (\pi^0\pi^0)_S n$ events at fixed $P^{\pi^-}_{\rm lab}$ in the following form:

$$\frac{d^2N}{dmdt} = \left| A_\pi \frac{\sqrt{-t}}{t-m_\pi^2} e^{\tilde{b}_\pi (t-m_\pi^2)/2} e^{-i\pi\alpha_\pi(t)/2} \sqrt{m/\rho_{\pi\pi}}\, T_{\pi^*\pi\to\pi\pi}(m,t) \right|^2 + |A_{a_1}(1+tC) e^{\tilde{b}_{a_1} t/2} i e^{-i\pi\alpha_{a_1}(t)/2} \sqrt{m}\, R_{a_1\pi\to\pi\pi}(m,t)|^2. \quad (1)$$

Here the first and second terms correspond to the π and a_1 Regge pole contributions, respectively (the π and a_1 exchanges do not interfere because, at high energies, they contribute to different helicity amplitudes), $\alpha_\pi(t) = \alpha'_\pi(t-m_\pi^2)$ and $\alpha_{a_1}(t) = \alpha_{a_1}(0) + \alpha'_{a_1} t$ are the trajectories of these poles, m is the invariant mass of the final $\pi\pi$ system, A_π and A_{a_1} are the normalization constants, $T_{\pi^*\pi\to\pi\pi}(m,t)$ and $R_{a_1\pi\to\pi\pi}(m,t)$ are the S-wave amplitudes for the subprocesses $\pi^{*+}\pi^- \to \pi^0\pi^0$ and $a_1^+\pi^- \to \pi^0\pi^0$, respectively, $\rho_{\pi\pi} = (1-4m_\pi^2/m^2)^{1/2}$, the slope $\tilde{b}_\pi = 2\alpha'_\pi \ln(P^{\pi^-}_{\rm lab}/1\ {\rm GeV}) + b_{\pi NN}$, i.e., it incorporates the slope of the Reggeized pion propagator and the slope of the $\pi^* NN$ residue taken in the exponential form, and the slope $\tilde{b}_{a_1}$ has a similar structure. According to the physical reasons which were discussed in the literature, the a_1 Regge pole amplitude has to have the so-called sense-nonsense wrong signature zero at $\alpha_{a_1}(t = t_0) = 0$, and hence, to be proportional to $\alpha_{a_1}(t)$ (see, for example, Refs. [31–33]). Thus, the factor $(1+tC)$ in the second term of Eq. (1) can be understood as the ratio $\alpha_{a_1}(t)/\alpha_{a_1}(0) = 1 + t\alpha'_{a_1}/\alpha_{a_1}(0)$. However, the value of $\alpha'_{a_1}/\alpha_{a_1}(0)$ is in fact unknown [32,33], and therefore, we consider C a free parameter. According to isotopic symmetry,

$$T_{\pi^*\pi\to\pi\pi}(m,t) = T_0^0(m,t) - T_0^2(m,t),$$

$$R_{a_1\pi\to\pi\pi}(m,t) = R_0^0(m,t) - R_0^2(m,t), \quad (2)$$

where $T_L^I(m,t)$ and $R_L^I(m,t)$ are the amplitudes with $L=0$ and isospin $I=0,2$ for the subprocesses $\pi^*\pi \to \pi\pi$ and $a_1\pi \to \pi\pi$, respectively; the amplitude $R_0^2(m,t)$ is assumed negligible. Now we suppose that the t dependences of the amplitudes $T_L^I(m,t)$ for the reaction $\pi^*\pi \to \pi\pi$ can be extracted in the form of overall exponential form factors. Thus we put

$$T_0^0(m,t) = e^{b_0^0(t-m_\pi^2)/2} T_0^0(m),$$

$$T_0^2(m,t) = e^{b_0^2(t-m_\pi^2)/2} T_0^2(m), \quad (3)$$

where the amplitudes $T_0^0(m)$ and $T_0^2(m)$ depend only on m and are determined by the on-mass-shell dynamics of the $\pi\pi$ scattering. This assumption about the t dependence factorization, together with the concrete shape of this dependence, was widely used as a simple working tool to obtain the $\pi\pi$ scattering data and gave results which were in close agreement with those of the more general Chew-Low extrapolation method [3–13,27–29].[3] Usually, the factorization assumption is applied to the $\pi N \to \pi\pi N$ one-pion exchange amplitudes in the region $0 < -t < (0.15-0.2)$ GeV^2

[3]For the pronounced solitary $\rho(770)$ and $f_2(1270)$ resonances produced in the reactions $\pi N \to \pi\pi N$ in the low-t region via the one-pion exchange, the factorization of the t and m dependences for the $\pi^*\pi \to \rho(770) \to \pi\pi$ and $\pi^*\pi \to f_2(1270) \to \pi\pi$ amplitudes is quite natural. However, in the S-wave case, the situation is more complicated. There are at least two strongly interfering contributions in the $L=I=0$ $\pi^*\pi \to \pi\pi$ channel at $m \approx 1$ GeV, namely, the narrow $f_0(980)$ resonance and the smooth large background which can be parametrized, for example, in terms of a broad elastic $\pi\pi$ resonance [34,35]. Even though the t dependence factorizes for each contribution, the whole $L=I=0$ $\pi^*\pi \to \pi\pi$ amplitude may possess this property only if the various contributions have rather close t dependence. In connection with the GAMS results, we discuss the $L=I=0$ $\pi^*\pi \to \pi\pi$ amplitude in the region of the $f_0(980)$ resonance beyond the t dependence factorization assumption at the end of this section and also in Sec. III.

[5,7,8,27,29]. We shall use Eq. (3) as ''a zeroth approximation'' (in the sense of a number of addition assumptions and new fitted parameters) for all $-t$ of interest from 0 to 1 GeV^2. Also we adopt a similar representation for $t<0$ for the amplitude $R_0^0(m,t)$ of the subprocess $a_1\pi\to\pi\pi$,

$$R_0^0(m,t)=e^{c_0^0 t/2}R_0^0(m). \tag{4}$$

Note that some smooth m dependence of the slopes b_0^0, b_0^2, and c_0^0 is not excluded. However, in the considered relatively narrow m range near the $f_0(980)$ resonance, $0.8<m<1.1$ GeV, we assume for simplicity that b_0^0, b_0^2, and c_0^0 are constant. From the fit to the data [1], the values of the overall slopes of the corresponding amplitudes, namely, $b_\pi^0=\tilde{b}_\pi+b_0^0$, $b_\pi^2=\tilde{b}_\pi+b_0^2$, and $b_{a_1}^0=\tilde{b}_{a_1}+c_0^0$ will be determined [see Eqs. (1)–(4)].

Let us now turn to the description of the model for the amplitudes $T_0^0(m)$, $T_0^2(m)$, and $R_0^0(m)$. On the mass shell of the reaction $\pi\pi\to\pi\pi$,

$$T_0^0(m)=(\eta_0^0 e^{2i\delta_0^0}-1)/2i, \quad T_0^2(m)=(\eta_0^2 e^{2i\delta_0^2}-1)/2i, \tag{5}$$

where δ_L^I and η_L^I are the phase shifts and elasticities which are functions of m. The data on the $L=0$, $I=2$ $\pi\pi$ channel in the region $2m_\pi<m<1.2$ GeV are described very well by $\eta_0^2=1$ and $\delta_0^2=-0.87q_\pi/(1+0.16q_\pi^2)$, where δ_0^2 is in radians if $q_\pi=m\rho_{\pi\pi}/2$ is taken in units of GeV (see, for example, Ref. [36]). At $m\approx1$ GeV, $\delta_0^2\approx-23°$. In the $L=I=0$ $\pi\pi$ channel, a very sharp rise of the phase δ_0^0 near the $K\bar{K}$ threshold [see Figs. 2(a) and 3(a)], together with a sharp drop of the elasticity η_0^0 just above the $K\bar{K}$ threshold [see Figs. 2(b) and 3(b)], is usually interpreted in term of the $f_0(980)$ resonance coupled to the $\pi\pi$ and $K\bar{K}$ channels [2–15,37]. However, in the $L=I=0$ $\pi\pi\to\pi\pi$ cross section this puzzling state shows itself not as a peak, but as a dip which occurs just below the $K\bar{K}$ threshold, and in fact, the cross section vanishes at a minimum point. Formally, this is because the phase δ_0^0 goes through 180°, but not though 90°, in the resonance region and $\eta_0^0=1$ with a good accuracy for $m<2m_K$. Note that the $I=2$ wave admixture shifts a minimum in the $L=0$ $\pi^+\pi^-\to\pi^0\pi^0$ reaction cross section approximately by 10 MeV to a lower mass region.[4] Let us write the amplitudes $T_0^0(m)$ and $R_0^0(m)$ as

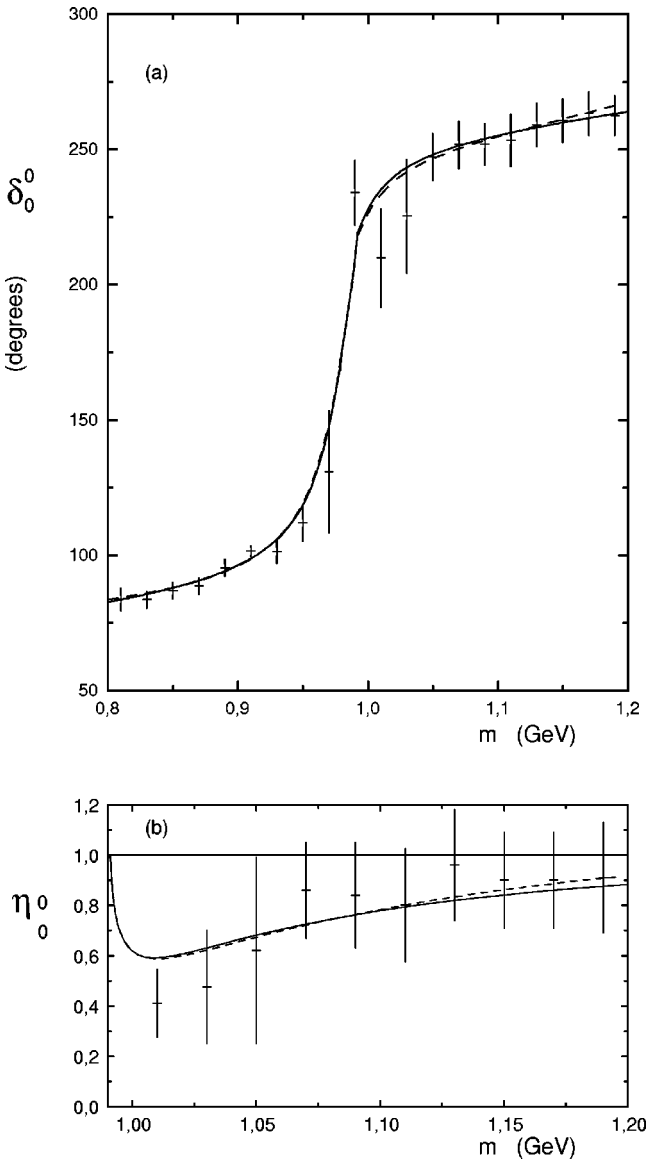

FIG. 2. The phase shift δ_0^0 (a) and the elasticity η_0^0 (b) pertaining to the $L=I=0$ $\pi\pi\to\pi\pi$ reaction amplitude $T_0^0(m)$ in the $f_0(980)$ region. The data are taken from Ref. [5]. The solid curves correspond to the fit for variant 1 and the dashed curves to that for variant 3.

$$T_0^0(m)=\frac{e^{2i\delta_B}-1}{2i}+e^{2i\delta_B}T^{\rm res}_{\pi\pi\to\pi\pi}(m),$$

$$R_0^0(m)=e^{i\delta_B}R^{\rm res}_{a_1\pi\to\pi\pi}(m), \tag{6}$$

where δ_B is the phase shift due to the smooth elastic background in the $\pi\pi$ channel, whereas $T^{\rm res}_{\pi\pi\to\pi\pi}(m)$ and $R^{\rm res}_{a_1\pi\to\pi\pi}(m)$ are the amplitudes due to the contributions of the mixed inelastic resonances. If we put $T^{\rm res}_{\pi\pi\to\pi\pi}(m)=(\eta_{\rm res}e^{2i\delta_{\rm res}}-1)/2i$, we find from Eqs. (5) and (6) that $\delta_0^0=\delta_B+\delta_{\rm res}$ and $\eta_0^0=\eta_{\rm res}$. To parametrize the resonance contributions we use the so-called propagator method [14,38,39]

[4]As seen from Fig. 1(a), the observed $\pi^-p\to(\pi^0\pi^0)_S n$ cross section does not vanish at a minimum but accounts for about 1/3 of the cross section at the side maxima. This is mainly because of a finite experimental $\pi^0\pi^0$ mass resolution which for the GAMS-2000 spectrometer has been characterized by a Gaussian distribution with the dispersion $\sigma_m\approx20$ MeV at $m\approx1$ GeV [1]. In the fit to the GAMS data, we certainly take into account this Gaussian smearing.

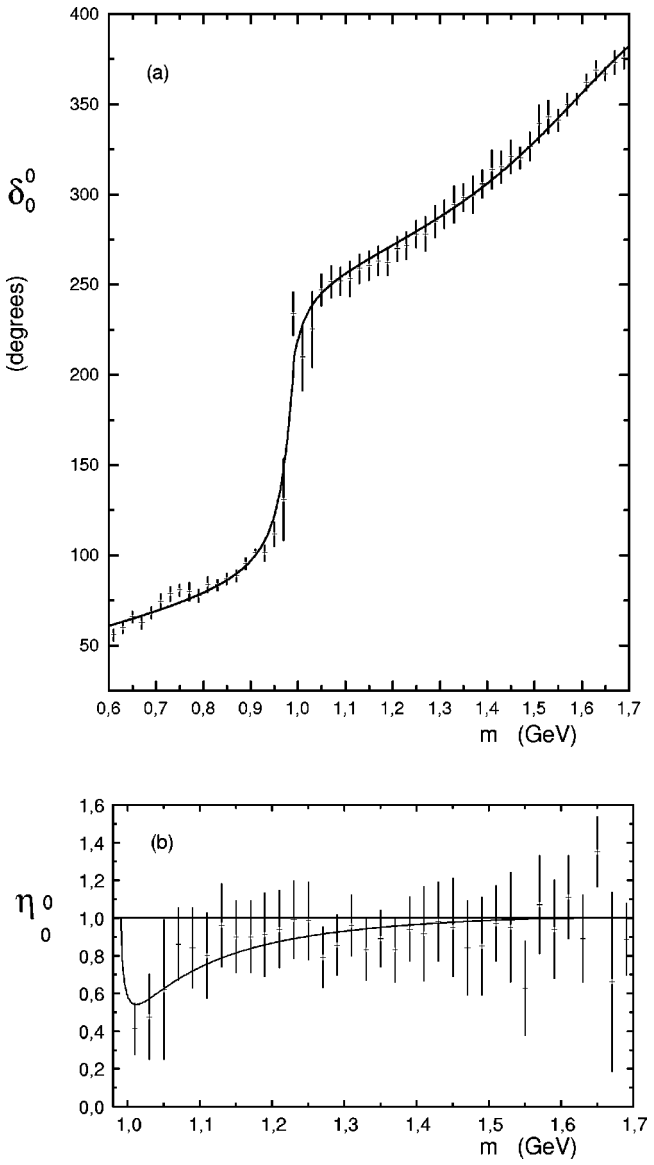

FIG. 3. The phase shift δ_0^0 (a) and the elasticity η_0^0 (b) pertaining to the $L=I=0$ $\pi\pi\to\pi\pi$ reaction amplitude. The data are taken from Ref. [5]. The curves correspond to the fit for variant 2.

and write the amplitude $\tilde{T}^{\rm res}_{ab\to cd}(m)$ for the process $ab\to cd$ in the following form (which satisfies the unitarity condition):

$$\tilde{T}^{\rm res}_{ab\to cd}(m)=\sum_{r,r'} g_{rab}G^{-1}_{rr'}(m)g_{r'cd}, \tag{7}$$

where the sum is evaluated over the resonances r,r' [$r(r')=r_1,r_2,\ldots$], $G_{rr'}(m)$ is the inverse propagator matrix for a resonance complex,

$$G_{rr'}(m)=\begin{pmatrix} D_{r_1}(m) & -\Pi_{r_1r_2}(m) & \ldots \\ -\Pi_{r_1r_2}(m) & D_{r_2}(m) & \ldots \\ \ldots & \ldots & \ldots \end{pmatrix}, \tag{8}$$

$$D_r(m)=m_r^2-m^2+\mathrm{Re}\,\Pi_r(m_r)-\Pi_r(m), \tag{9}$$

m_r and g_{rab}, $g_{r'cd}$ are, respectively, the masses and the coupling constants of the unmixed resonances. Since we are interested in a mass region around 1 GeV, we can restrict ourselves to the simplest case of resonances coupled only to the $\pi\pi$ and $K\bar{K}$ decay channels. We also imply that the resonance production occurs in $\pi\pi$ and $a_1\pi$ collisions (recall that the a_1 means here not a particle but a Reggeon). Then we can take, in Eq. (9),

$$\Pi_r(m)=\sum_{cd=\pi\pi,K\bar{K}} g^2_{rcd}\rho_{cd}\left(i+\frac{1}{\pi}\ln\frac{1-\rho_{cd}}{1+\rho_{cd}}\right) \tag{10}$$

and write the off-diagonal elements of the matrix $G_{rr'}(m)$ [see Eq. (8)], responsible for the resonance mixing, as

$$\Pi_{rr'}(m)=C_{rr'}+\sum_{cd=\pi\pi,K\bar{K}} g_{rcd}g_{r'cd}\rho_{cd}\left(i+\frac{1}{\pi}\ln\frac{1-\rho_{cd}}{1+\rho_{cd}}\right), \tag{11}$$

where $C_{rr'}$ are the mixing parameters, $\rho_{K\bar{K}}=(1-4m_K/m^2)^{1/2}$ for $m>2m_K$, and $\rho_{K\bar{K}}\to i|\rho_{K\bar{K}}|$ in the region $0<m<2m_K$. Here we neglect the K^+K^- and $K^0\bar{K}^0$ mass difference and put $m_K=(m_{K^+}+m_{K^0})/2$. Above the corresponding threshold, the partial decay width of the resonance r is $\Gamma_{rcd}(m)=g^2_{rcd}\rho_{cd}/m$. Using Eqs. (6) and (7) with due regard for the normalizations as defined in Eqs. (1)–(5), we finally obtain

$$T^{\rm res}_{\pi\pi\to\pi\pi}(m)=\rho_{\pi\pi}\tilde{T}^{\rm res}_{\pi\pi\to\pi\pi}(m),$$

$$R^{\rm res}_{a_1\pi\to\pi\pi}(m)=\sqrt{\rho_{\pi\pi}}\tilde{T}^{\rm res}_{a_1\pi\to\pi\pi}(m)/g_{r_1a_1\pi}, \tag{12}$$

where the second relation implies, in particular, that the coupling constant $g_{r_1a_1\pi}$ is taken up by the normalization constant A_{a_1} in Eq. (1).

Within the framework of the above model, we present the three simplest variants of the fit to the data [5] on δ_0^0 and η_0^0 in the $f_0(980)$ mass region. In variant 1, we assume that the amplitude $T_0^0(m)$ [see Eq. (6)] is dominated by a single resonance and a background, in variant 2 by two mixed resonances and a background, and in variant 3 by two mixed resonances.

Variant 1 yields the most economical and transparent parametrization. Using Eqs. (6)–(10) and (12), we find, in this case,

$$T_0^0(m)=\frac{e^{2i\delta_B}-1}{2i}+e^{2i\delta_B}\frac{m\Gamma_{f_0\pi\pi}(m)}{D_{f_0}(m)},$$

$$R_0^0(m)=e^{i\delta_B}\frac{\sqrt{m\Gamma_{f_0\pi\pi}(m)}}{D_{f_0}(m)}, \tag{13}$$

where f_0 is taken as a suitable notation for a single r_1 resonance and the background phase $\delta_B=a+mb$. The parametrization of $T_0^0(m)$ as given by Eq. (13) permits us to obtain

a good fit to the data on δ_0^0 and η_0^0 in the region $0.8<m<1.2$ GeV [see the solid curves in Figs. 2(a), 2(b)]. The corresponding parameters of the background and resonance are $\delta_B=35.5°+47°(m/\mathrm{GeV})$, $m_{f_0}=979$ MeV, $g^2_{f_0\pi\pi}=0.075$ GeV^2, and $g^2_{f_0K\bar{K}}=0.36$ GeV^2. Note that the above simple representation for $T_0^0(m)$ was also used for a similar purpose in a set of earlier analyses (see, for example, Refs. [3,9,35,40,41]). It is obvious that in this case a dip in the $L=I=0$ $\pi\pi\to\pi\pi$ reaction cross section in the $f_0(980)$ resonance region is due to the destructive interference between the resonance and the background whose contributions are near the S-wave unitarity limit.

Variant 2 allows a good fit to the data on δ_0^0 to be attained in the wider m interval from 0.6 to 1.7 GeV (see also Ref. [39]) and also turns out to be more flexible for the construction of the $\pi^-p\to(\pi^0\pi^0)_Sn$ reaction amplitude due to the a_1 exchange. In this case, using Eqs. (6)–(12), we have

$$T_0^0(m)=\frac{e^{2i\delta_B}-1}{2i}+e^{2i\delta_B}\rho_{\pi\pi}\frac{g_{r_1\pi\pi}[D_{r_2}(m)g_{r_1\pi\pi}+\Pi_{r_1r_2}(m)g_{r_2\pi\pi}]+g_{r_2\pi\pi}[D_{r_1}(m)g_{r_2\pi\pi}+\Pi_{r_1r_2}(m)g_{r_1\pi\pi}]}{D_{r_1}(m)D_{r_2}(m)-\Pi^2_{r_1r_2}(m)}, \quad (14)$$

$$R_0^0(m)=e^{i\delta_B}\sqrt{\rho_{\pi\pi}}\,\frac{[D_{r_2}(m)g_{r_1\pi\pi}+\Pi_{r_1r_2}(m)g_{r_2\pi\pi}]+(g_{r_2a_1\pi}/g_{r_1a_1\pi})[D_{r_1}(m)g_{r_2\pi\pi}+\Pi_{r_1r_2}(m)g_{r_1\pi\pi}]}{D_{r_1}(m)D_{r_2}(m)-\Pi^2_{r_1r_2}(m)}, \quad (15)$$

where $\delta_B=\rho_{\pi\pi}(a+mb)$. In the following, while referring to this variant, the lighter resonance r_1 will be denoted by f_0, and r_2 by σ. The curves shown in Figs. 3(a), 3(b) are the result of the fit to the data on δ_0^0 and η_0^0 using Eq. (14). These curves correspond to the following values of the parameters: $m_{f_0}=0.966$ GeV, $g^2_{f_0\pi\pi}=0.09$ GeV^2, $g^2_{f_0K\bar{K}}=0.36$, GeV^2, $m_\sigma=1.58$ GeV, $g^2_{\sigma\pi\pi}=0.73$ GeV^2, $g^2_{\sigma K\bar{K}}=0.002$ GeV^2, $C_{f_0\sigma}=\pm0.37$ GeV^2, and $\delta_B=\rho_{\pi\pi}[3°+50°(m/\mathrm{GeV})]$. Note that $C_{f_0\sigma}$ is defined up to a sign, but in so doing $C_{f_0\sigma}g_{f_0\pi\pi}g_{\sigma\pi\pi}>0$ and $g_{f_0\pi\pi}g_{\sigma\pi\pi}g_{f_0K\bar{K}}g_{\sigma K\bar{K}}<0$.

In variant 3, the amplitudes $T_0^0(m)$ and $R_0^0(m)$ are defined by Eqs. (14) and (15) with $\delta_B=0$. We consider this variant mainly to ease the following discussion of the results presented in Ref. [30] (see Sec. III). The fit to the data on δ_0^0 and η_0^0 in the region $0.8<m<1.2$ GeV with variant 3 gives $m_{r_1}=0.88$ GeV, $g^2_{r_1\pi\pi}=0.45$ GeV^2, $g^2_{r_1K\bar{K}}=0.57$ GeV^2, $m_{r_2}=1.23$ GeV, $g^2_{r_2\pi\pi}=0.74$ GeV^2, $g^2_{r_2K\bar{K}}=0.09$ GeV^2, $C_{r_1r_2}=\pm0.67$ GeV^2, $C_{r_1r_2}g_{r_1\pi\pi}g_{r_2\pi\pi}>0$, and $g_{r_1\pi\pi}g_{r_2\pi\pi}g_{r_1K\bar{K}}g_{r_2K\bar{K}}<0$ [see the dashed curves in Figs. 2(a), 2(b)].

Now we use the obtained parameters to describe the GAMS data on the $(\pi^0\pi^0)_S$ mass spectra in the reaction $\pi^-p\to(\pi^0\pi^0)_Sn$ which are shown in Figs. 1(a)–1(f). For each of the above variants we perform the fit to these data using Eq. (1) folded with a Gaussian mass distribution (see footnote 4) and integrated over t in six intervals indicated in Figs. 1(a)–1(f). For variant 1 we use Eqs. (2)–(4) and (13), and for variants 2 and 3 Eqs. (2)–(4), (14), and (15). As is seen from Figs. 1(a)–1(f), the observed alteration of the $(\pi^0\pi^0)_S$ mass spectrum in the $f_0(980)$ region with increasing $-t$ is satisfactorily reproduced in the three variants of the proposed π and a_1 exchange model. In variant 1, this takes place with $A^2_\pi=340\times10^2$ (number of events/GeV^2), $A^2_{a_1}=78.2$ (number of events/GeV^2), $C=-13.5$ GeV^{-2}, and the slopes $b^0_\pi=9.4$ GeV^{-2}, $b^2_\pi=5.3$ GeV^{-2}, and $b^0_{a_1}=5.4$ GeV^{-2} which are rather typical for similar reactions [see the solid curves in Figs. 1(a)–1(f)]. Note that the slope $b^2_\pi\approx5$ GeV^{-2} had been observed in the reaction $\pi^+p\to\pi^+\pi^+n$ at $P^{\pi^-}_{\mathrm{lab}}=12.5$ GeV for the $\pi^+\pi^+$ production in the invariant mass region from 0.75 to 1.25 GeV [29]. In variant 2, the fit to the GAMS data is characterized by the following values of the fitted parameters: $A^2_\pi=426\times10^2$ (number of events/GeV^2), $A^2_{a_1}=639$ (number of events/GeV^2), $C=-4.4$ GeV^{-2}, $b^0_\pi=12.4$ GeV^{-2}, $b^2_\pi=5.4$ GeV^{-2}, $b^0_{a_1}=5.8$ GeV^{-2}, and $(g_{\sigma a_1\pi}g_{\sigma\pi\pi})/(g_{f_0a_1\pi}g_{f_0\pi\pi})=0.16$ [see the dotted curves in Figs. 1(a)–1(f)]. In variant 3, the fit gives $A^2_\pi=355\times10^2$ (number of events/GeV^2), $A^2_{a_1}=91.8$ (number of events/GeV^2), $C=-13$ GeV^{-2}, $b^0_\pi=10.1$ GeV^{-2}, $b^2_\pi=5.2$ GeV^{-2}, $b^0_{a_1}=5.6$ GeV^{-2}, and $(g_{r_2a_1\pi}g_{r_2\pi\pi})/(g_{r_1a_1\pi}g_{r_1\pi\pi})=-0.863$ [see the dashed curves in Figs. 1(a)–1(f)]. Note that in this case the r_1 and r_2 resonances interfere destructively in the range $m_{r_1}<m<m_{r_2}$ in the $\pi^*\pi\to\pi\pi$ channel and constructively in the $a_1\pi\to\pi\pi$ channel.

Figure 4 shows the t distributions of the $\pi^-p\to(\pi^0\pi^0)_Sn$ events for three m regions 0.8−0.9 GeV, 0.9−1 GeV, and 1−1.1 GeV which we obtained for variant 1 using Eqs. (1)–(4) and (13). The figure illustrates how the one-pion exchange contribution falls and the a_1 exchange becomes dominant in the $f_0(980)$ region as $-t$ increases. Similar t distributions also take place for variants 2 and 3.

Up to now we have adhered to the t dependence factor-

ization assumption. However, it is easy to construct parametrizations which would permit one to move beyond the scope of this assumption. A simplest example is provided by variant 3 in which the amplitudes $T_0^0(m)$ and $R_0^0(m)$ are defined by Eqs. (14) and (15) with $\delta_B=0$. For example, for the $\pi^*\pi\to\pi\pi$ reaction amplitude $T_0^0(m,t)$, instead of Eq. (3) and Eq. (14) with $\delta_B=0$, one can write the more general expression

$$T_0^0(m,t)=\rho_{\pi\pi}\frac{g_{r_1\pi^*\pi}(t)[D_{r_2}(m)g_{r_1\pi\pi}+\Pi_{r_1r_2}(m)g_{r_2\pi\pi}]+g_{r_2\pi^*\pi}(t)[D_{r_1}(m)g_{r_2\pi\pi}+\Pi_{r_1r_2}(m)g_{r_1\pi\pi}]}{D_{r_1}(m)D_{r_2}(m)-\Pi^2_{r_1r_2}(m)}, \tag{16}$$

where the residues $g_{r_1\pi^*\pi}(t)$ and $g_{r_2\pi^*\pi}(t)$ characterizing r_1 and r_2 resonance production in $\pi^*\pi$ collisions, generally speaking, may be different functions of t [at $t=m_\pi^2$, $g_{r_{1,2}\pi^*\pi}(m_\pi^2)=g_{r_{1,2}\pi\pi}$]. Thus, if the t behaviors of these functions are appreciably different in a certain t region, then it is natural that the t dependence of the whole amplitude does not factorize in this region. However, we shall not exploit such a possibility, first, because it requires incorporating at least two additional fitted parameters (by one for every mechanism of the considered reaction), and secondly, because a certain version of the extremal violation of the factorization assumption has already been applied in Ref. [30] to explain the GAMS data within the framework of the pure one-pion exchange model. The results obtained in Ref. [30] are briefly discussed below.

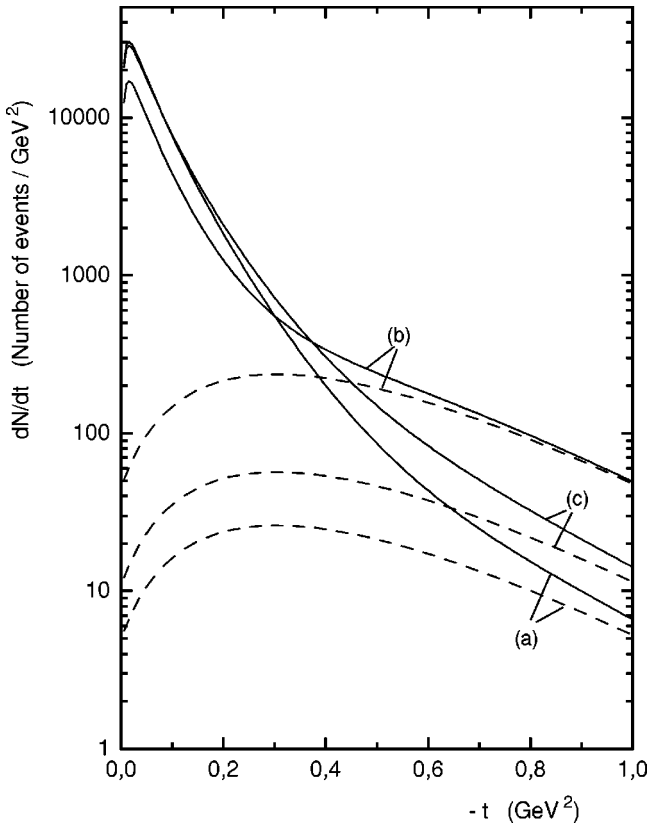

FIG. 4. The t distributions of the $\pi^-p\to(\pi^0\pi^0)_Sn$ events for three m intervals (a) 0.8−0.9 GeV, (b) 0.9−1 GeV, and (c) 1 −1.1 GeV corresponding to variant 1. The solid curves correspond to the sum of the π and a_1 exchange mechanisms and the dashed curves to the a_1 exchange contribution.

III. COMPARISON WITH THE PREVIOUS EXPLANATION

As already mentioned in the Introduction, the explanation of the GAMS data on the reaction $\pi^-p\to\pi^0\pi^0n$ [1] presented in Ref. [30] is based exclusively on the one-pion exchange model (this immediately follows from Eqs. (2), (5), (6), Fig. 3(a), and accompanying comments in Ref. [30][5]). As a consequence of such a restriction, this explanation leads to a strong violation of the t dependence factorization assumption. We can conveniently elucidate this assertion in terms of Eq. (16). Let us recall that the authors of Ref. [30] used the K matrix method to construct the $L=I=0$ $\pi^*\pi\to\pi\pi$ reaction amplitude, and that, in the 1 GeV region in the K matrix, two resonances coupled to the $\pi\pi$ and $K\bar{K}$ channels and some background terms were taken into account. However, the difference between the K matrix representation for the amplitude $T_0^0(m,t)$ obtained in Ref. [30] and Eq. (16) is unimportant for clearing up the question about the applicability of the pure one-pion exchange model for the description of the GAMS data.

Thus, if one takes into account only the one-pion exchange mechanism for the reaction $\pi^-p\to(\pi^0\pi^0)_Sn$ and uses the parametrization with two mixed resonances coupled to the $\pi\pi$ and $K\bar{K}$ channels for the $L=I=0$ $\pi^*\pi\to\pi\pi$ amplitude, then the observed alteration of the $(\pi^0\pi^0)_S$ mass spectrum can be understood only if the destructive interference between two resonances at $m\approx1$ GeV, which occurs in the low $-t$ region, is replaced by the constructive one with increasing $-t$. According to Eq. (16), this means a change of the interference type between the terms proportional to $g_{r_1\pi^*\pi}(t)$ and $g_{r_2\pi^*\pi}(t)$, which, in turn, is possible only if,

[5]It is worth noting that the comment after Eq. (8) in Ref. [30] about a flat term which can effectively describe the contribution of the a_1 exchange to the $\pi N\to(\pi\pi)_SN$ amplitude with the one-pion exchange quantum numbers from Eq. (5) or (6) in Ref. [30] is misleading. In fact, at high energies, the π and a_1 Regge amplitudes have different spin structures and in the unpolarized cross section their contributions are noncoherent as already emphasized above. So, the a_1 exchange has not been taken into account in Ref. [30] effectively.

as $-t$ increases, one of the residues, for example, $g_{r_1\pi^*\pi}(t)$, decreases in absolute value, vanishes at a certain value $t=t_0$, and then changes its sign. Also, this has to occur at least for $-t<0.3$ GeV^2. Hence, according to such an approach, the t dependence of the amplitude $T_0^0(m,t)$ must not factorize at $m\approx 1$ GeV even in the low $-t$ region. In Ref. [30], the following parametrization for the residues $g_{r_1\pi^*\pi}(t)$ and $g_{r_2\pi^*\pi}(t)$ was postulated:

$$g_{r_i\pi^*\pi}(t)=g_{r_i\pi\pi}[1+\xi_i(1-t/m_\pi^2)t/m_\pi^2], \quad i=1,2. \tag{17}$$

For the best fit $g_{r_1\pi\pi}=0.848$ GeV, $\xi_1=0.0565$, $g_{r_2\pi\pi}=0.884$ GeV, and $\xi_2=-0.0293$ [30]. As is seen, the residue $g_{r_1\pi^*\pi}(t)$ vanishes at $t\approx -0.0728$ GeV^2, and as $-t$ varies from 0 to 1 GeV^2, the functions $g^2_{r_1\pi^*\pi}(t)$ and $g^2_{r_2\pi^*\pi}(t)$ increase, respectively, by approximately factors of 22 000 and 6000. In order to compensate this enormous rise, the authors of Ref. [30] multiplied the $\pi^-p\to(\pi^0\pi^0)_Sn$ one-pion exchange amplitude by the overall form factor $F(t)=[(\Lambda-m_\pi^2)/(\Lambda-t)]^4$ with $\Lambda=0.1607$ GeV^2 which, however, they ascribed, for unknown reasons, to the nucleon vertex.[6] As a result, they obtained formally a very good description of the GAMS data on the $(\pi^0\pi^0)_S$ mass spectra. Recall that these spectra (dN/dm) correspond to the distribution $d^2N/dmdt$ integrated over t in the intervals indicated in Figs. 1(a)–1(f). Nevertheless, a detailed analysis shows that the model of Ref. [30] predicts rather exotic t distributions of the $\pi^-p\to(\pi^0\pi^0)_Sn$ events for $-t<0.2$ GeV^2. Figure 5 shows the unnormalized t distributions (dN/dt) for three m intervals $0.8<m<0.9$ GeV, $0.9<m<1$ GeV, and $1<m<1.1$ GeV which we obtained using the formulas from Ref. [30]. The most discouraging feature of the presented picture is a dip in dN/dt whose location depends on m. In fact, this is a straightforward consequence of a failure of the factorization for the amplitude $T_0^0(m,t)$. The t distribution for $0.8<m<0.9$ has a dip at $-t\approx 0.1$ GeV^2 and, as is seen from Fig. 5, changes very rapidly in the region $-t<0.2$ GeV^2. With increasing m, a dip in dN/dt moves to $t=0$. So, the t distribution for $0.9<m<1$ GeV has a dip at $-t\approx 0.072$ GeV^2. For the mass interval $1<m<1.1$ GeV which already belongs to the inelastic region of the reaction $\pi^*\pi\to\pi\pi$, a dip in dN/dt disappears. A comparison of the predictions for dN/dt shown in Figs. 4 and 5 shows that the choice between our explanation of the GAMS data and the explanation given by the authors of Ref. [30] can be easily realized experimentally. To do this, it is sufficient to have data on dN/dt in the region $-t<0.2$ GeV^2 for the m intervals $0.8<m<0.9$ GeV and $0.9<m<1$ GeV. So far, however, neither the GAMS Collaboration [1] nor the E852 Collaboration [16] have published data on the t distributions.

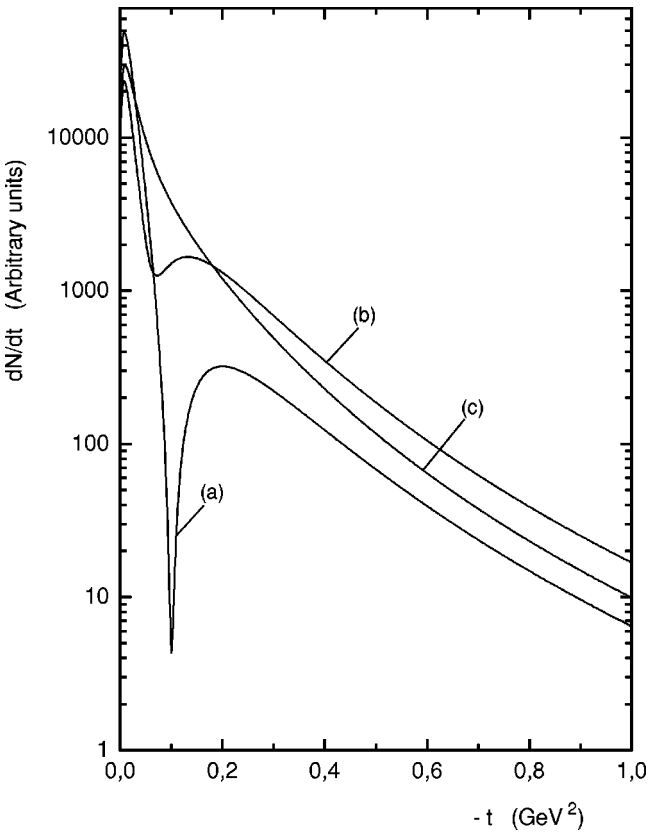

FIG. 5. The unnormalized t distributions for the reaction $\pi^-p\to(\pi^0\pi^0)_Sn$ for three m intervals (a) 0.8–0.9 GeV, (b) 0.9–1 GeV, and (c) 1–1.1 GeV corresponding to the pure one-pion exchange model used in Ref. [30] (see text).

Finally, let us emphasize that the best experimental test that we know of for the $\pi^-p\to(\pi\pi)_Sn$ reaction mechanisms are measurements on polarized targets, because they will permit the interference to be directly observed between the π and a_1 exchange amplitudes. As is known [24,25], in such experiments one can measure the triple distribution (in m, t, and ψ) which at fixed $P_{\rm lab}^{\pi^-}$ has the form

$$\frac{d^3N[\pi^-p_\uparrow\to(\pi\pi)_Sn]}{dmdtd\psi}=\frac{1}{2\pi}\frac{d^2N}{dmdt}+2P\cos\psi I(m,t), \tag{18}$$

where ψ is the angle between the normal to the reaction plane and the (transverse) proton polarization P. The first term in Eq. (18) corresponds to the distribution of events on an unpolarized target. It can be presented as $(d^2N/dmdt)/2\pi=|M^\pi_{+-}(m,t)|^2+|M^{a_1}_{++}(m,t)|^2$, where $M^\pi_{+-}(m,t)$ and $M^{a_1}_{++}(m,t)$ are the s-channel helicity ampli-

[6]Note that this leads to unsolvable difficulties. For example, if one describes the well-studied reaction $\pi^-p\to\rho^0n$ [6,42] using such a form factor in the π^*NN vertex it would be necessary to introduce a $\pi^*\pi\rho$ residue which increases with $-t$. In turn, this would lead to a rise of $d\sigma/dt$ for the process $\pi\pi\to\rho^0\rho^0$. It is evident that such a picture is incompatible with conventional ideas. Also, according to Eq. (17), we face a similar problem for the reaction $\pi\pi\to(\pi\pi)_S(\pi\pi)_S$. Furthermore, the above form factor would yield an abnormally sharp drop of the one-pion exchange contribution to the differential cross section of the charge exchange reaction $pn\to np$.

tudes with and without nucleon helicity flip, due to the π and a_1 exchange mechanisms, respectively. The second term in Eq. (18) describes the nucleon polarization effects. The function $I(m,t)$ in this term is stipulated by the interference between the π and a_1 exchange amplitudes and has the form $I(m,t)=\mathrm{Im}\{M^{\pi}_{+-}(m,t)[M^{a_1}_{++}(m,t)]^*\}$. In our model for the reaction $\pi^- p\to(\pi^0\pi^0)_S n$ the amplitude $\sqrt{2\pi}M^{\pi}_{+-}(m,t)$ [and, respectively, $\sqrt{2\pi}M^{a_1}_{++}(m,t)$] is given by the expression under the sign of modulus square in the first (second) term of Eq. (1). If one neglects the $I=2$ $\pi\pi$ S-wave contribution, then the phase of the product $M^{\pi}_{+-}(m,t)[M^{a_1}_{++}(m,t)]^*$ in the elastic region (i.e., for $m<2m_K$) would be completely defined by the Regge signature factors of the $M^{\pi}_{+-}(m,t)$ and $M^{a_1}_{++}(m,t)$ amplitudes. With these provisos in mind, one can easily write the function $I(m,t)$ in an explicit form for the three considered variants. For example, for the most simple variant 1, up to the sign

$$I(m,t)=\cos\{\pi[\alpha_\pi(t)-\alpha_{a_1}(t)]/2\}\times\frac{1}{2\pi}\left[A_\pi\frac{\sqrt{-t}}{t-m_\pi^2}e^{b^0_\pi(t-m_\pi^2)}A_{a_1}(1+tC)e^{b^0_{a_1}t}\right]\times\left\{\sin(\delta^0_0)\frac{\sqrt{m\Gamma_{f_0\pi\pi}(m)}}{|D_{f_0}(m)|}\right\}, \tag{19}$$

where, as seen, the t and m dependences factorize. It is natural that the pure one-pion exchange model [30] predicts $I(m,t)=0$.

IV. CONCLUSION

We have suggested a new explanation of the GAMS results on the $f_0(980)$ production in the reaction $\pi^- p\to\pi^0\pi^0 n$. A crucial role in our explanation is assigned to the amplitude with quantum numbers of the a_1 Regge pole in the t channel which is as of yet poorly studied. Moreover, we consistently used the standard assumption of the t dependence factorization. On the other hand, if one attempts to explain the GAMS data in the framework of the pure one-pion exchange model, as is done, for example, in Ref. [30], then this assumption must be rejected from the outset. To test the correctness of our explanation, the data on the t distributions of the $\pi^- p\to(\pi^0\pi^0)_S n$ events in the intervals $0.8<m<0.9$ GeV and $0.9<m<1$ GeV, and the measurements of the reaction $\pi^- p\to(\pi^0\pi^0)_S n$ on polarized targets, which can clearly demonstrate the presence of the a_1 exchange mechanism, are needed.

Recently we have shown [43] that new data on $d\sigma[\pi^- p\to a_0^0(980)n]/dt$ can be explained within the framework of the Regge pole model only if the reaction $\pi^- p\to a_0^0(980)n$ is dominated by the ρ_2 Regge pole whose partner by exchange degeneracy is the a_1 Regge pole. To all appearances, the time is right to study the pseudovector and pseudotensor Regge exchanges.

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