Charmed baryon strong coupling constants in a light-front quark model

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Light-front quark model spin-wave functions are employed to calculate the three independent couplings $g_{\Sigma_c\Lambda_c\pi}$, $f_{\Lambda_{c1}\Sigma_c\pi}$, and $f_{\Lambda^*_{c1}\Sigma_c\pi}$ of S wave to S wave and P wave to S wave one-pion transitions. It is found that $g_{\Sigma_c\Lambda_c\pi}=6.81\ \mathrm{GeV}^{-1}$, $f_{\Lambda_{c1}\Sigma_c\pi}=1.16$, and $f_{\Lambda^*_{c1}\Sigma_c\pi}=0.96\times10^{-4}\ \mathrm{MeV}^{-2}$. We also predict decay rates for specific strong transitions of charmed baryons. [S0556-2821(98)02917-8]

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In the heavy quark limit, the spin and parity of the heavy quark and light degrees of freedom are separately conserved in the hadron. In addition, strong and electromagnetic transitions among heavy baryon states are transitions solely of the light quark system. Therefore, heavy quark symmetry when supplemented by light flavor symmetries, such as SU(2) or SU(3) symmetry, relate these decays. Explicit relations between the various decay couplings of heavy baryons were derived in the constituent quark model [1,2]. S wave to S wave heavy baryon strong decays, for instance, are determined by a single coupling constant and two independent couplings are required to describe single-pion transitions from P-wave to S-wave states.

The coupling $g_{\Sigma_c\Lambda_c\pi}$ determines strong decays among charmed baryon ground states. Furthermore, single-pion transitions from the first excited states into the ground state are described in terms of two couplings $f_{\Lambda_{c1}\Sigma_c\pi}$ and $f_{\Lambda^*_{c1}\Sigma_c\pi}$. The Λ_{c1} and Λ^*_{c1} represent the two excited states discovered recently [3] with masses 2593 and 2625 MeV, respectively.

In a heavy baryon, a light diquark system with quantum numbers j^P couples with a heavy quark with $J^P_Q=1/2^+$ to form a doublet with $J^P=(j\pm1/2)$. Heavy quark symmetry allows us to write down a general form for the heavy baryon spin-wave functions (SWF) [1,4]:

$$\chi_{\alpha\beta\gamma}=(\phi_{\mu_1\cdots\mu_j})_{\alpha\beta}\psi_\gamma^{\mu_1,\ldots,\mu_j}(v). \tag{1}$$

Here, $v_\mu=P_\mu/M$ is the baryon four velocity, the spinor indices[1] α and β refer to the light quark system, and the index γ refers to the heavy quark. The number of the Lorentz indices μ_j is determined by the light diquark system quantum number j and is equal to 0, 1, and 2 for S-wave and P-wave baryon states. In the heavy quark limit, the $\chi_{\alpha\beta\gamma}$ satisfy the Bargmann-Wigner equation on the heavy quark index:

$$(\not{v})_\gamma^{\gamma'}\chi_{\alpha\beta\gamma'}=\chi_{\alpha\beta\gamma}. \tag{2}$$

[1]We have ignored the isospin indices which will be included in the transition amplitudes later on.

The light degrees of freedom spin-wave functions $(\phi_{\mu_1\cdots\mu_j})_{\alpha\beta}(v)$ are in general written in terms of the two bispinors $[\chi^0]_{\alpha\beta}$ and $[\chi^1_\mu]_{\alpha\beta}$. The matrix $[\chi^0]_{\alpha\beta}=[(\not{v}+1)\gamma_5 C]_{\alpha\beta}$ projects out a spin-0 object and is symmetric when interchanging α and β. However, $[\chi^1_\mu]_{\alpha\beta}=[(\not{v}+1)\gamma_{\perp\mu}C]_{\alpha\beta}$, which projects out a spin-1 object is antisymmetric. Here, C is the charge conjugation operator and $\gamma^\perp_\mu=\gamma_\mu-\not{v}v_\mu$. On the other hand, the "superfield" $\psi_\gamma^{\mu_1,\ldots,\mu_j}(v)$ stands for the two spin-wave functions corresponding to the two heavy quark symmetry degenerate states with spin $j-1/2$ and $j+1/2$. They are generally written in terms of the Dirac spinor u and the Rarita-Schwinger spinor u_μ. The S-wave heavy-baryon spin-wave functions are given by

$$(\phi^{\Lambda_Q})_{\alpha\beta}=(\chi^0)_{\alpha\beta}, \quad (\psi^{\Lambda_Q})_\gamma=u_\gamma \tag{3}$$

and

$$(\phi^{\mu\Sigma_Q})_{\alpha\beta}=(\chi^{1,\mu})_{\alpha\beta}, \quad (\psi^{\Sigma_Q}_\mu)_\gamma=\left\{\begin{matrix}\frac{1}{\sqrt{3}}\gamma^\perp_\mu\gamma_5 u\\ \\ u_\mu\end{matrix}\right\}_\gamma. \tag{4}$$

For P-wave heavy baryon states, we shall use the relative momentum $K=1/\sqrt{6}(p_1+p_2-2p_3)$, symmetric under the interchange of the constituent light quark momenta p_1 and p_2, to represent the orbital excitation. The Λ_{Q1} degenerate state spin-wave functions can be written as

$$(\phi^{\mu\Lambda_{Q1}})_{\alpha\beta}=(\chi^0K^\mu)_{\alpha\beta}, \quad (\psi^{\Lambda_{Q1}}_\mu)_\gamma=\left\{\begin{matrix}\frac{1}{\sqrt{3}}\gamma^\perp_\mu\gamma_5 u\\ \\ u_\mu\end{matrix}\right\}_\gamma. \tag{5}$$

A more detailed analysis with all heavy baryon P-wave spin wave functions was presented in Refs. [1,4].

In the heavy quark limit, we can write down the general form for single-pion transition amplitudes between heavy baryons

$$M_\pi = \langle B_{Q'}(P')|j_\pi(q)|B_Q(P)\rangle$$
$$= f_{B_Q B'_Q \pi} \overline{\psi}^{\nu_1,\ldots,\nu_{j_2}}(P')(t^\pi(q))^{\mu_1,\ldots,\mu_{j_1}}_{\nu_1,\ldots,\nu_{j_2}} \psi_{\mu_1,\ldots,\mu_{j_1}}(P), \tag{6}$$

with j_π being the strong current, $f_{B_Q B'_Q \pi}$ is the appropriate strong-coupling constant and the pion momentum $q = P - P'$. The light degrees of freedom transition tensors $(t^\pi(q))^{\mu_1,\ldots,\mu_{j_1}}_{\nu_1,\ldots,\nu_{j_2}}$ of rank (j_1+j_2), built from $g_{\perp\mu\nu} = g_{\mu\nu} - v_\mu v_\nu$ and the pion momentum, should have the correct parity and project out the appropriate partial wave amplitude.

The $\Sigma_c^{(*)} \to \Lambda_c \pi$, $\Lambda_{c1} \to \Sigma_c \pi$, and $\Lambda_{c1}^* \to \Sigma_c \pi$ covariant tensors $(t^\pi(q))^{\mu_1,\ldots,\mu_{j_1}}_{\nu_1,\ldots,\nu_{j_2}}$ are $q_{\perp\mu}$, $g_{\perp\mu\nu}$, and $q_{\perp\mu} q_{\perp\nu}$, with $q_{\perp\mu} = q_\mu - v \cdot q v_\mu$, corresponding to *P*-wave, *S*-wave, and *D*-wave transitions, respectively. Making use of the heavy baryon spin-wave functions given in Eqs. (3)–(5) the strong transition amplitudes, therefore, can be written as

$$\langle \Lambda(P',\lambda')|j_\pi(q)|\Sigma(P,\lambda)\rangle$$
$$= \frac{1}{\sqrt{3}} g_{\Sigma_c \Lambda_c \pi} I_1 \overline{u}(P',\lambda') \not{q}_\perp \gamma_5 u(P,\lambda), \tag{7}$$

$$\langle \Lambda(P',\lambda')|j_\pi(q)|\Sigma^*(P,\lambda)\rangle$$
$$= g_{\Sigma_c^* \Lambda_c \pi} I_1 \overline{u}(P',\lambda') q_{\perp\mu} u^\mu(P,\lambda), \tag{8}$$

$$\langle \Sigma(P',\lambda')|j_\pi(q)|\Lambda_{c1}(P,\lambda)\rangle$$
$$= f_{\Lambda_{c1}\Sigma_c \pi} I_3 \overline{u}(P',\lambda') u(P,\lambda), \tag{9}$$

and

$$\langle \Sigma(P',\lambda')|j_\pi(q)|\Lambda_{c1}^*(P,\lambda)\rangle$$
$$= \frac{1}{\sqrt{3}} f_{\Lambda_{c1}^* \Sigma_c \pi} I_3 \overline{u}(P',\lambda') \gamma_5 \not{q}_\perp u^\mu(P,\lambda) q_{\perp\mu}, \tag{10}$$

where λ (λ') is the helicity of the initial (final) spin-$\frac{1}{2}$ or spin-$\frac{3}{2}$ heavy baryon. The $I_1 \equiv I(6 \to 3^* + \pi)$ and $I_3 \equiv I(3^* \to 6 + \pi)$ are the appropriate group-theoretical flavor factors. In fact, these are the only amplitudes allowed by Lorentz invariance and parity conservation. As was discussed in [1], the *S*-wave coupling of Eq. (9) is different from the one introduced in the heavy hadron chiral perturbation theory (HHCPT) which is related to the scalar component of the axial vector current. The matrix elements, Eqs. (7)–(10), can be transformed into their equivalent effective chiral amplitudes [2,5–7] by replacing the pion momentum q_μ by $-\partial_\mu \pi$ with the spinors $u(p)$, $u_\nu(p)$, and $\overline{u}(p)$ being replaced by the corresponding heavy baryon fields. The couplings $g_{\Sigma_c \Lambda_c \pi}$, which is equal to $g_{\Sigma_c^* \Lambda_c \pi}$ in the heavy quark limit, $f_{\Lambda_{c1}\Sigma_c \pi}$, and $f_{\Lambda_{c1}^* \Sigma_c \pi}$ are related, respectively, to g_2, h_2, and h_8 defined in the HHCPT [2,5,8] such that $g_{\Sigma_c \Lambda_c \pi} = \sqrt{3} g_2/\sqrt{2} f_\pi$, $f_{\Lambda_{c1}\Sigma_c \pi} = (\sqrt{2} h_2/f_\pi) E_\pi$, and $f_{\Lambda_{c1}^* \Sigma_c \pi} = 6 h_8/\sqrt{5} f_\pi$ with $f_\pi = 0.093$ GeV.

The single-pion decay rates are calculated using the general formula

$$\Gamma = \frac{1}{2J_1+1} \quad \frac{|\vec{q}|}{8\pi M_{B_Q}^2} \sum_{\text{spins}} |M^\pi|^2, \tag{11}$$

with $|\vec{q}|$ being the pion momentum in the rest frame of the decaying baryon. Using Eqs. (7)–(10) and (11), we get

$$\Gamma(\Sigma_c \to \Lambda_c \pi) = \Gamma(\Sigma_c^* \to \Lambda_c \pi) = g^2_{\Sigma_c^{(*)}\Lambda\pi} I_1^2 \frac{|\vec{q}|^3}{6\pi} \frac{M_{\Lambda_c}}{M_{\Sigma_c^{(*)}}}, \tag{12}$$

$$\Gamma(\Lambda_{c1} \to \Sigma_c \pi) = f^2_{\Lambda_{c1}\Sigma\pi} I_3^2 \frac{|\vec{q}|}{4\pi} \frac{M_{\Sigma_c}}{M_{\Lambda_{c1}}}, \tag{13}$$

$$\Gamma(\Lambda_{c1}^* \to \Sigma_c \pi) = f^2_{\Lambda_{c1}^*\Sigma\pi} I_3^2 \frac{|\vec{q}|^5}{36\pi} \frac{M_{\Sigma_c}}{M_{\Lambda_{c1}^*}}. \tag{14}$$

Assuming that the width of Σ_c, Λ_{c1}, and Λ_{c1}^* are saturated by strong decay channels one can estimate the values of the three couplings using the experimental decay rates. Taking $\Gamma_{\Sigma_c^{*++} \to \Lambda_c^+ \pi^+} = 17.9^{+3.8}_{-3.2}$ MeV, $\Gamma_{\Sigma_c^{*0} \to \Lambda_c^+ \pi^-} = 13.0^{+3.7}_{-3.0}$ MeV reported by CLEO [3], Eq. (12) can be used to determine[2] the coupling $g_{\Sigma_c \Lambda_c \pi}$. One, therefore, respectively, gets

$$g_{\Sigma_c \Lambda_c \pi} = 8.03^{+1.97}_{-1.92} \ \text{GeV}^{-1} \tag{15}$$

and

$$g_{\Sigma_c \Lambda_c \pi} = 6.97^{+1.84}_{-1.74} \ \text{GeV}^{-1}. \tag{16}$$

These values, in return, give the analogous HHCPT coupling $g_2 = 0.61^{+0.15}_{-0.14}$ and $g_2 = 0.53^{+0.14}_{-0.13}$ defined in [2,5].

To estimate $f_{\Lambda_{c1}\Sigma\pi}$ we use the Particle Data Group [9] average value for $\Lambda_{c_1}(2593)$ width which is $\Gamma_{\Lambda_{c_1}(2593)} = 3.6^{+2.0}_{-1.3}$ MeV and Eq. (13) to obtain

$$f_{\Lambda_{c1}\Sigma\pi} = 1.11^{+0.31}_{-0.20}. \tag{17}$$

The corresponding HHCPT coupling constant h_2 is calculated to be $h_2 = 0.73^{+0.20}_{-0.13}$.

Finally, taking the upper bound on the $\Lambda_{c_1}^+(2625)$ width obtained by CLEO [3] ($\Gamma_{\Lambda_{c_1}^+(2625)} < 1.9$ MeV), Eq. (14) gives

$$f_{\Lambda_{c1}^*\Sigma\pi} = 1.66 \times 10^{-4} \ \text{MeV}^{-2}. \tag{18}$$

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[2]Numerical values for the masses will be taken from Table I of [8]. In this analysis, which is similar to those done in [2,5,8], we use the updated data reported by the Particle Data Group [9].

The value of the HHCPT D-wave coupling h_8 is determined to be $h_8 = 5.75\times 10^{-3}$ MeV^{-1}. The uncertainty in the values of the couplings is dominated by the experimental errors in the decay rates and in the baryons masses.

Theoretically, to calculate the three couplings one needs to evaluate the matrix elements of $j_\pi(q)$ in Eqs. (7), (9), and (10) at $\vec{\mathbf{q}}^2 = 0$ in an appropriate frame of reference. The light-front (LF) formalism [10] provides a consistent relativistic theory for composite systems with a fixed number of constituent. The other essential fact is that the Melosh rotation [11] is already included in the LF spinors which is important when calculating form factors. Therefore, we shall employ (LF) wave functions to describe the initial and final heavy baryons.

Without loss of generality, we choose to work in a Drell-Yan frame where the initial baryon momentum $P^\mu = (P^+, M^2/P^+, \mathbf{0}_\perp)$ and the pion momentum $q^\mu = (0, M^2 - M'^2 - \mathbf{q}_\perp^2/P^+, \mathbf{q}_\perp)$. With the aid of the light-front spinors and matrix elements of the appropriate γ matrices defined in the Appendix, which become even simpler since more elements will vanish in this frame, the three independent couplings are given by

$$g_{\Sigma_c\Lambda_c\pi} = -\frac{2\sqrt{3M_{\Lambda_c}M_{\Sigma_c}}}{(M_{\Sigma_c}^2 - M_{\Lambda_c}^2)}\langle \Lambda(P',\uparrow)|\hat{j}_\pi(0)|\Sigma(P,\uparrow)\rangle, \tag{19}$$

$$f_{\Lambda_{c1}\Sigma_c\pi} = \langle \Sigma(P',\uparrow)|\hat{j}_\pi(0)|\Lambda_{c1}(P,\uparrow)\rangle, \tag{20}$$

and

$$f_{\Lambda_{c1}^*\Sigma_c\pi} = \frac{3\sqrt{2}}{(M_{\Lambda_{c1}^*} - M_\Sigma)^2}\,\frac{M_{\Lambda_{c1}^*}^2}{(M_{\Lambda_{c1}^*}^2 - M_{\Sigma_c}^2)} \times \left\langle \Sigma(P',\uparrow)\middle|\hat{j}_\pi(0)\middle|\Lambda_{c1}^*\left(P,\frac{1}{2}\right)\right\rangle. \tag{21}$$

The LF matrix elements of the strong transition current $\hat{j}_\pi(q)$ between heavy baryon states are given by

$$\begin{aligned}\langle B'(P',\lambda')|\hat{j}_\pi(q)|B(P,\lambda)\rangle &= \int [dx_i][d^2\mathbf{p}_{\perp i}]\sum_{\lambda_i,\lambda_i'} \psi_{B'}^\dagger(x_i',\mathbf{p}_{\perp i}',\lambda_i';\lambda')\\ &\times\left(\sum_{j=1,2}\overline{u}(p_j',\lambda_j')\hat{j}_\pi(q)u(p_j,\lambda_j)\right)\psi_B(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda),\end{aligned} \tag{22}$$

where $\psi_B(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)$ and $\psi_{B'}^\dagger(x_i',\mathbf{p}_{\perp i}',\lambda_i';\lambda')$ are the initial and final heavy baryon wave functions explicitly given in Eq. (24) below. In the constituent quark model the pion is assumed to be emitted by each of the light quarks and the heavy quark is not affected. Therefore, the strong current is resolved into constituent quark transitions and its appropriate operator $\hat{j}_\pi(q)$ can be written as

$$(\hat{j}_\pi)^{\alpha\beta}_{\alpha'\beta'} = \frac{1}{2}[(\gamma_5)^\alpha_{\alpha'}\delta^\beta_{\beta'} + \delta^\alpha_{\alpha'}(\gamma_5)^\beta_{\beta'}]. \tag{23}$$

The most difficult part in calculating these form factors, however, is related to the choice of the form of initial and final baryon wave functions. One of the advantages of light-front (LF) formalism [10] is that all Fock-state wave functions $\Psi(x_i,p_{\perp i},\lambda_i;\lambda)$, with helicity λ and constituent transverse momenta $p_{\perp i}$, tend to vanish when the LF energy ϵ becomes infinitely large. This feature, is very much similar to the so-called ''valence'' constituent quark model where the dynamics are dominated by the valence quark structure.

In the LF formalism the total baryon spin-momentum distribution function can be written in the following general form:

$$\Psi(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda) = \chi(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)\,\psi(x_i,\mathbf{p}_{\perp i}). \tag{24}$$

Here, $\chi(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)$ and $\psi(x_i,\mathbf{p}_{\perp i})$ represent the spin and momentum distribution functions, respectively, and the longitudinal-momentum fraction

$$x_i = \frac{p_i^+}{P^+} \quad \text{with} \quad \sum_{i=1}^{3} x_i = 1. \tag{25}$$

These functions are normalized such that

$$\int [dx_i][d^2\mathbf{p}_{\perp i}]\sum_{\lambda_i} \psi_{B'}^{\lambda'\dagger}(x_i,\mathbf{p}_{\perp i};\lambda_i)\psi_B^\lambda(x_i,\mathbf{p}_{\perp i};\lambda_i) = \delta_{\lambda\lambda'}, \tag{26}$$

with

$$[dx_i] = \prod_i dx_i\,\delta\left(1-\sum_i x_i\right),$$

$$[d^2\mathbf{p}_{\perp i}] = \prod_i d^2\mathbf{p}_{\perp i}16\pi^3\delta^2\left(\sum_i \mathbf{p}_{\perp i}\right). \tag{27}$$

Assuming factorization of the longitudinal $\phi(x_i)$ and transverse momentum distribution functions, $\psi(x_i,\mathbf{p}_{\perp i})$ can be written as

$$\psi(x_i,\mathbf{p}_{\perp i}) = \phi(x_i)\exp\left[-\frac{\vec{\mathbf{k}}^2}{2\alpha_\rho^2} - \frac{\vec{\mathbf{K}}^2}{2\alpha_\lambda^2}\right]. \tag{28}$$

The transverse component of the momentum distribution are assumed to be harmonic-oscillator eigenfunctions with α_ρ and α_λ controlling the confinement of quarks in the heavy baryon. The momenta $\vec{\mathbf{k}}$ and $\vec{\mathbf{K}}$, corresponding to the nonrelativistic three-body momenta $\mathbf{k}_\rho$ and $\mathbf{k}_\lambda$, are the transverse component of the covariant vectors

$$k = \frac{1}{\sqrt{2}}(p_1 - p_2), \quad K = \frac{1}{\sqrt{6}}(p_1 + p_2 - 2p_3). \tag{29}$$

These harmonic-oscillator functions were used successfully in [12] to predict masses and decay rates of ground-state and excited charmed baryons. They were also employed to calculate baryon magnetic moments [13] and to calculate the Isgur-Wise function for Λ_Q semileptonic decay [14] in a relativistic quark model. The choice of the relative momenta k and K are also convenient for keeping track of the exchange symmetry for the light degrees of freedom spin-wave functions. They will be used later on to write down an explicit form for heavy baryon P-wave spin functions.

In the heavy quark limit, the heavy baryon longitudinal momentum distribution functions $\phi(x_i)$ are expected to have most of their strength in the neighborhood of the mean values $\overline{x_Q}=m_Q/M$. In the weak binding [15] or valence approximation [16] the longitudinal velocity of the constituent quarks are the same. One therefore expects that also for the light quarks the distribution is peaked fairly sharply around the equal velocity point $\overline{x_i}=m_i/M$ with $i=1$ and 2. Therefore, we can assume

$$\phi(x_i)=\prod_{i=1}^{3}\ \delta(x_i-\overline{x_i}). \tag{30}$$

In the equal velocity assumption [15,16] one may use the two projection operators $[\chi^0]_{\alpha\beta}$ and $[\chi^{1,\mu}]_{\alpha\beta}$, defined earlier, to write down the spin-dependent functions. The Λ_Q-like baryons spin-wave function $\chi_{\Lambda_Q}(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)$ must be antisymmetric when interchanging the light quark indices and is given by

$$\chi_{\Lambda_Q}(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)=\overline{u}^{\alpha_1}(p_1,\lambda_1)\overline{u}^{\alpha_2}(p_2,\lambda_2)\overline{u}^{\alpha_3}(p_3,\lambda_3)[\chi^0]_{\alpha_1\alpha_2}u_{\alpha_3}(P,\lambda), \tag{31}$$

here, the LF spinors $u^{\alpha_i}(p_i,\lambda_i)$ describe the constituent quarks with momentum p_i and helicity λ_i and $u^{\alpha}(P,\lambda)$ refers to the Λ_Q-like baryon with momentum P and helicity λ. $\chi_{\Lambda_Q}(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)$ can be rewritten in a more convenient form

$$\chi_{\Lambda_Q}(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)=\overline{u}(p_1,\lambda_1)[(\not{P}+M_\Lambda)\gamma_5]\nu(p_2,\lambda_2)\overline{u}(p_3,\lambda_3)u(P,\lambda). \tag{32}$$

For the Σ_Q-like baryon, the spin-wave functions are symmetric in the light quark indices and have the form

$$\chi_{\Sigma_Q}(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)=\overline{u}(p_1,\lambda_1)[(\not{P}+M_\Lambda)\gamma_\perp^\mu]\nu(p_2,\lambda_2)\times\overline{u}(p_3,\lambda_3)\gamma_{\perp\mu}\gamma_5u(P,\lambda), \tag{33}$$

The two relative momenta k and K can be used to specify the spin-wave functions for heavy baryon resonances. The excited states Λ_{Q1}, with $J^P=\frac{1}{2}^-$, and Λ^*_{Q1}, with $J^P=\frac{3}{2}^-$, have spin functions of the following forms:

$$\chi_{\Lambda_{QK1}}(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)=\overline{u}(p_1,\lambda_1)[(\not{P}+M_{\Lambda_{c1}})\gamma_5]\nu(p_2,\lambda_2)\times\overline{u}(p_3,\lambda_3)\not{K}\gamma_5u(P,\lambda), \tag{34}$$

and

$$\chi_{\Lambda^*_{QK1}}(x_i,\mathbf{p}_{\perp i},\lambda_i;\lambda)=\overline{u}(p_1,\lambda_1)[(\not{P}+M_{\Lambda^*_{c1}})\gamma_5]\nu(p_2,\lambda_2)\times\overline{u}(p_3,\lambda_3)K_\mu u^\mu(P,\lambda). \tag{35}$$

One can obtain the spin-wave functions for the corresponding antisymmetric excited states by replacing K with k. Explicit forms for the spinors $u(p,\lambda)$ and $u^\mu(p,\lambda)$ and antispinors $\nu(p,\lambda)$ in the LF formalism are given in the Appendix.

Since there are two free parameters in our model, namely, the oscillator couplings α_ρ and α_λ one, therefore, expects that the predictions made will depend mainly on these two parameters. The numerical values for the constituent quark masses are taken to be $m_u=m_d=0.33$ GeV, $m_c=1.51$ GeV and those for α_ρ and α_λ are $\alpha_\rho=0.40$ GeV/c and $\alpha_\lambda=0.52$ GeV/c. The same values for the oscillator couplings were chosen to fit the Λ baryon masses [14]. However, one would expect that these values might slightly change for the Ξ baryons since the constituent quarks are not the same as those in the Λ and Σ baryons. We shall postpone the study of the effect of these parameters for a future work since the sensitivity of the decay rates to the α values is such that a 10% increase results in about (5–8) % change in the calculated decay rates.

To evaluate the integrals in Eqs. (19),(20) we introduce the relative momentum variables

$$\boldsymbol{\zeta}_\perp=\frac{x_2\mathbf{p}_{\perp 1}-x_1\mathbf{p}_{\perp 2}}{x_1+x_2},\quad \boldsymbol{\eta}_\perp=(x_1+x_2)\mathbf{p}_{\perp 3}-x_3(\mathbf{p}_{\perp 1}+\mathbf{p}_{\perp 2}). \tag{36}$$

These variables have the crucial property of being spacelike four vectors because of the vanishing of the invariant + component $(\zeta^+=\eta^+=0)$. The momentum conservation relations are

$$x_iM=x_i'M', \tag{37}$$

and if the pion is emitted by quark number 1, we also have

$$\boldsymbol{\zeta}'_\perp=\boldsymbol{\zeta}_\perp-\frac{x_1}{x_1+x_2}\mathbf{q}_\perp \quad\text{and}\quad \boldsymbol{\eta}'_\perp=\boldsymbol{\eta}_\perp-x_3\mathbf{q}_\perp. \tag{38}$$

Using Eqs. (28), (30), and (32)–(35) the three charmed baryons strong couplings $g_{\Sigma_c\Lambda_c\pi}$, $f_{\Lambda_{c1}\Sigma_c\pi}$, and $f_{\Lambda^*_{c1}\Sigma_c\pi}$ are calculated to be

$$g_{\Sigma_c\Lambda_c\pi}=6.81\ \text{GeV}^{-1},\quad f_{\Lambda_{c1}\Sigma_c\pi}=1.16,$$
$$f_{\Lambda^*_{c1}\Sigma_c\pi}=0.96\times10^{-4}\ \text{MeV}^{-2}. \tag{39}$$

TABLE I. Decay rates for charmed baryon states.

$B_Q \to B'_Q \pi$	Γ (MeV)	$\Gamma_{\rm expt}$ (MeV)
P-wave transitions		
$\Sigma_c^{+} \to \Lambda_c \pi^0$	1.70	
$\Sigma_c^{0} \to \Lambda_c \pi^-$	1.57	
$\Sigma_c^{++} \to \Lambda_c \pi^+$	1.64	
$\Sigma_c^{*0} \to \Lambda_c \pi^-$	12.40	$13.0^{+3.7}_{-3.0}$
$\Sigma_c^{*++} \to \Lambda_c \pi^+$	12.84	$17.9^{+3.8}_{-3.2}$
$\Xi_c^{*0} \to \Xi_c^0 \pi^0$	0.72	<5.5
$\Xi_c^{*0} \to \Xi_c^+ \pi^-$	1.16	
$\Xi_c^{*+} \to \Xi_c^0 \pi^+$	1.12	<3.1
$\Xi_c^{*+} \to \Xi_c^+ \pi^0$	0.69	
S-wave transitions		
$\Lambda_{c1}(2593) \to \Sigma_c^0 \pi^+$	2.61	
$\Lambda_{c1}(2593) \to \Sigma_c^+ \pi^0$	1.73	$3.6^{+2.0}_{-1.3}$
$\Lambda_{c1}(2593) \to \Sigma_c^{++} \pi^-$	2.15	
$\Xi_{c1}^*(2815) \to \Xi_c^{*0} \pi^+$	4.84	$\Gamma_{\Xi_{c1}^*} < 2.4$
$\Xi_{c1}^*(2815) \to \Xi_c^{*+} \pi^0$	2.38	
D-wave transitions		
$\Lambda_{c1}^*(2625) \to \Sigma_c^0 \pi^+$	0.77	
$\Lambda_{c1}^*(2625) \to \Sigma_c^+ \pi^0$	0.69	$\Gamma_{\Lambda_{c1}^*} < 1.9$
$\Lambda_{c1}^*(2625) \to \Sigma_c^{++} \pi^-$	0.73	
$\Xi_{c1}^*(2815) \to \Xi_c^0 \pi^+$	0.30	
$\Xi_{c1}^*(2815) \to \Xi_c^+ \pi^0$	0.15	

These are in nice agreement with our earlier fit to the upgraded CLEO measurements for $\Gamma_{\Sigma_c^* \to \Lambda_c}$, $\Gamma_{\Lambda_{c_1}(2593) \to \Sigma_c}$, and $\Gamma_{\Lambda_{c_1}^*(2593) \to \Sigma_c}$ strong decay rates. The corresponding HHCPT couplings are determined using the values in Eq. (39);

$$g_2 = 0.52, \quad h_2 = 0.54, \quad h_8 = 3.33 \times 10^{-3} \ \ \text{MeV}^{-1}. \tag{40}$$

Having the three independent couplings in hand, we are now in a position to predict charmed baryons strong decay rates. Ground-state transitions are saturated by *P*-wave transitions which can be calculated using the value of $g_{\Sigma_c \Lambda_c \pi}$ and Eq. (12). On the other hand, transitions from the first excited states are *S*-wave or *D*-wave transitions and their decay rates are predicted using Eqs. (13) and (14), respectively. These decay rates are summarized in Table I as well as the experimental values presented in the updated version of the Particle Data Group [9].

From Table I, one notes that the strong width of Σ_c^* is about seven to eight times larger than the width of its spin-$\frac{1}{2}$ partner Σ_c. These values are within the range of the CLEO measurements. The Ξ_c^{*0} and Ξ_c^{*+} strong decay width are within the current upper bound obtained by CLEO.

The $\Lambda_{c1}(2593)$ decay width receives contributions from both its single-pion decay to Σ_c and from decaying to Λ_c via a two-pion transition. The two-pion contribution was computed in [6,7] with the result $\Gamma_{\Lambda_{c1}(2593) \to \Lambda_c \pi\pi} = 2.5$ MeV. Hence, the total decay rate is $\Gamma_{\Lambda_{c1}(2593)} = 6.49$ MeV which is still consistent with the CLEO result $\Gamma_{\Lambda_{c1}(2593)} = 3.6^{+2.0}_{-1.3}$ MeV. Actually, there is also a negligible *D*-wave single-pion contribution to the $\Lambda_{c1}(2593)$ width.

We also predict the *S*-wave branching ratios of $\Xi_{c1}(2815) \to \Xi_c^{*0} \pi^+$ to $\Xi_{c1}(2815) \to \Xi_c^{*+} \pi^0$ to be 67 and 33 %, respectively. The *S*-wave $\Xi_{c1}(2815)$ decay width receives an extra 2% contribution from *D*-wave modes giving a total width $\Gamma_{\Xi_{c1}(2815)} = 7.67$ MeV. This value is about three times higher than the upper bound obtained by CLEO $\Gamma_{\Xi_{c1}(2815)} < 2.4$ MeV.

Finally, the strong decay width of $\Lambda_{c1}^*(2625)$, the spin-$\frac{3}{2}$ partner of $\Lambda_{c1}(2593)$, is saturated by *D*-wave transitions to Σ_c and by two-pion decay to Λ_c. Adding the contribution from two-pion decay $\Gamma_{\Lambda_{c1}^*(2625) \to \Lambda_c \pi\pi} = 0.035$ MeV, calculated in [7], one gets $\Gamma_{\Lambda_{c1}^*(2625)} = 2.19$ MeV which is close to the upper limit obtained by CLEO $\Gamma_{\Lambda_{c1}^*(2625)} < 1.9$ MeV.

To summarize, we constructed light-front (LF) quark model functions with a factorized harmonic-oscillator transverse momentum component and a longitudinal component given by Dirac δ functions. The spin-wave functions are the LF generalization of the conventional constituent quark model spin-isospin functions. These bound-state distribution functions were used to calculate the strong couplings for $\Sigma_c \to \Lambda_c \pi$, $\Lambda_{c1} \to \Sigma_c \pi$, and $\Lambda_{c1}^* \to \Sigma_c \pi$ decay modes which correspond to *P*-wave, *S*-wave, and *D*-wave transitions, respectively. The LF quark model predictions for the numerical values of these couplings Eq. (39) are in good agreement with estimates obtained using the available experimental data Eqs. (15)–(18). Like other models, our results will mainly depend on the choice of the free parameters which are the harmonic-oscillator constants α_ρ and α_λ. The decay rates are also sensitive to the numerical values of the masses of the heavy baryon states and some of these masses have not been measured with high accuracy. We hope in the near future our results will be confirmed by the new experimental data.

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APPENDIX

In this appendix explicit forms for Dirac spinors $u(p,\lambda)$ and Rarita-Schwinger spinors $u^\mu(p,\lambda)$ in the light-front (LF) formalism are presented. Previously, the spin-$\frac{3}{2}$ wave functions have only been given in the canonical form [17]. We shall, also, give matrix elements of some useful γ matrices between LF spinors. The standard representation of γ matrices is used:

TABLE II. Spin-$\frac{3}{2}$ helicity eigenstates in the light-front formalism with $u^l(p,\lambda)=u^1(p,\lambda)-iu^2(p,\lambda)$ and $u^r(p,\lambda)=u^1(p,\lambda)+iu^2(p,\lambda)$.

$u^\mu(p,\lambda)$ λ	$u^+(p,\lambda)$	$u^-(p,\lambda)$	$u^r(p,\lambda)$	$u^l(p,\lambda)$
$\frac{3}{2}$	0	0	0	$u(p,\uparrow)$
$\frac{1}{2}$	$-\frac{1}{\sqrt{3}}\frac{p^+}{m}u(p,\uparrow)$	$\frac{1}{\sqrt{3}}\frac{p^-}{m}u(p,\uparrow)$	0	$\frac{1}{\sqrt{3}}u(p,\downarrow)$
$-\frac{1}{2}$	$-\frac{1}{\sqrt{3}}\frac{p^+}{m}u(p,\downarrow)$	$\frac{1}{\sqrt{3}}\frac{p^-}{m}u(p,\downarrow)$	$-\frac{1}{\sqrt{3}}u(p,\uparrow)$	0
$-\frac{3}{2}$	0	0	$-u(p,\downarrow)$	0

TABLE III. The $\overline{u}(p',\lambda')\Gamma u(p,\lambda)$, with $\Gamma=I$, γ^+, γ_5, and $\gamma^+\gamma_5$, matrix elements. They are in units of $\sqrt{p^+p'^+/mm'}$.

Γ	$\overline{u}(p',\uparrow)\Gamma u(p,\uparrow)$	$\overline{u}(p',\downarrow)\Gamma u(p,\downarrow)$	$\overline{u}(p',\downarrow)\Gamma u(p,\uparrow)$	$\overline{u}(p',\uparrow)\Gamma u(p,\downarrow)$
I	$\frac{1}{2}\frac{m'p^+ + mp'^+}{p^+p'^+}$	$\frac{1}{2}\frac{m'p^+ + mp^{+'}}{p^+p^{+'}}$	$-\frac{1}{2}\frac{p^+p'^r - p^{+'}p^r}{p^+p'^+}$	$\frac{1}{2}\frac{p^+p'^l - p'^+p^l}{p^+p'^+}$
γ^+	1	1	0	0
γ_5	$\frac{1}{2}\frac{m'p^+ - mp^{+'}}{p^+p^{+'}}$	$-\frac{1}{2}\frac{m'p^+ - mp'^+}{p^+p^{+'}}$	$-\frac{1}{2}\frac{p^+p'^r - p'^+p^r}{p^+p'^+}$	$-\frac{1}{2}\frac{p^+p'^l - p'^+p^l}{p^+p'^+}$
$\gamma^+\gamma^5$	1	-1	0	0

TABLE IV. The $\overline{u}(p',\lambda')\Gamma u^\mu(p,\lambda)$ matrix elements. The lower sign is for $\Gamma=\gamma^+$ and the upper sign is for $\Gamma=\gamma^+\gamma_5$. They are in units of $\sqrt{p^+p'^+/mm'}$.

	$\overline{u}(p',\uparrow)\Gamma u^+(p,\lambda)$ $\overline{u}(p',\downarrow)\Gamma u^+(p,\lambda)$	$\overline{u}(p',\uparrow)\Gamma u^-(p,\lambda)$ $\overline{u}(p',\downarrow)\Gamma u^-(p,\lambda)$	$\overline{u}(p',\uparrow)\Gamma u^r(p,\lambda)$ $\overline{u}(p',\downarrow)\Gamma u^r(p,\lambda)$	$\overline{u}(p',\uparrow)\Gamma u^l(p,\lambda)$ $\overline{u}(p',\downarrow)\Gamma u^l(p,\lambda)$
$\lambda=\frac{3}{2}$	0 0	0 0	0 0	$\pm\sqrt{2}$ 0
$\lambda=\frac{1}{2}$	$-\sqrt{\frac{2}{3}}\frac{p^+}{m}$ 0	$+\sqrt{\frac{2}{3}}\frac{p^-}{m}$ 0	0 0	0 $\mp\sqrt{\frac{2}{3}}$
$\lambda=-\frac{1}{2}$	0 $-\sqrt{\frac{2}{3}}\frac{p^+}{m}$	0 $+\sqrt{\frac{2}{3}}\frac{p^-}{m}$	$\pm\sqrt{\frac{2}{3}}$ 0	0 0
$\lambda=-\frac{3}{2}$	0 0	0 0	0 $\pm\sqrt{2}$	0 0

TABLE V. Same as Table IV but for $\overline{u}(p',\lambda')u^{\mu}(p,\lambda)$ matrix elements.

	$\overline{u}(p',\uparrow)u^{+}(p,\lambda)$ $\overline{u}(p',\downarrow)u^{+}(p,\lambda)$	$\overline{u}(p',\uparrow)u^{-}(p,\lambda)$ $\overline{u}(p',\downarrow)u^{-}(p,\lambda)$	$\overline{u}(p',\uparrow)u^{r}(p,\lambda)$ $\overline{u}(p',\downarrow)u^{r}(p,\lambda)$	$\overline{u}(p',\uparrow)u^{l}(p,\lambda)$ $\overline{u}(p',\downarrow)u^{l}(p,\lambda)$
$\lambda=\frac{3}{2}$	0	0	0	$\frac{1}{\sqrt{2}}\frac{m'p^{+}+mp'^{+}}{p^{+}p'^{+}}$
	0	0	0	$\frac{1}{\sqrt{2}}\frac{p^{r}p'^{+}-p'^{r}p^{+}}{p^{+}p'^{+}}$
$\lambda=\frac{1}{2}$	$-\frac{1}{\sqrt{6}}\frac{m'p^{+}+mp'^{+}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{m'p^{+}+mp'^{+}}{mp'^{+}}$	0	$\frac{1}{\sqrt{6}}\frac{p^{+}p'^{l}-p'^{+}p^{l}}{p^{+}p'^{+}}$
	$\frac{1}{\sqrt{6}}\frac{p^{+}p'^{r}-p'^{+}p^{r}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{p'^{+}p^{r}-p^{+}p'^{r}}{mp'^{+}}$	0	$\frac{1}{\sqrt{6}}\frac{m'p^{+}+mp'^{+}}{p^{+}p'^{+}}$
$\lambda=-\frac{1}{2}$	$\frac{1}{\sqrt{6}}\frac{p^{l}p'^{+}-p'^{l}p^{+}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{p'^{l}p^{+}-p^{l}p'^{+}}{mp'^{+}}$	$-\frac{1}{\sqrt{6}}\frac{m'p^{+}+mp'^{+}}{p^{+}p'^{+}}$	0
	$-\frac{1}{\sqrt{6}}\frac{m'p^{+}+mp'^{+}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{m'p^{+}+mp'^{+}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p^{+}p'^{r}-p'^{+}p^{r}}{p^{+}p'^{+}}$	0
$\lambda=-\frac{3}{2}$	0	0	$\frac{1}{\sqrt{2}}\frac{p'^{+}p^{l}-p^{+}p'^{l}}{p^{+}p'^{+}}$	0
	0	0	$-\frac{1}{\sqrt{2}}\frac{m'p^{+}+mp'^{+}}{p^{+}p'^{+}}$	0

$$\gamma_0=\begin{bmatrix} I & 0 \\ 0 & -I \end{bmatrix},\quad \gamma_i=\begin{bmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{bmatrix},\quad \gamma_5=\begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \tag{A1}$$

where σ^i being the usual Pauli matrices.

The spin-$\frac{1}{2}$ LF spinors $u_\lambda(p)$ with four momentum $p=(p^+,p^-,\mathbf{p}_\perp)$ and helicity $\lambda=(\uparrow or \downarrow)$ are given by [16,18]

$$u(p,\uparrow)=\frac{1}{2\sqrt{mp^+}}\begin{bmatrix} p^+ + m \\ p^r \\ p^+ - m \\ p^r \end{bmatrix},$$

$$u(p,\downarrow)=\frac{1}{2\sqrt{mp^+}}\begin{bmatrix} -p^l \\ p^+ + m \\ p^l \\ -(p^+ - m) \end{bmatrix}, \tag{A2}$$

here, we have defined $p^l=p_x-ip_y$ and $p^r=p_x+ip_y$. Similarly, the antispinors $\nu_\lambda(p)$ have the form

$$\nu(p,\uparrow)=\frac{1}{2\sqrt{mp^+}}\begin{bmatrix} -p^l \\ p^+ - m \\ p^l \\ -(p^+ + m) \end{bmatrix},$$

$$\nu(p,\downarrow)=\frac{1}{2\sqrt{mp^+}}\begin{bmatrix} p^+ - m \\ p^r \\ p^+ + m \\ p^r \end{bmatrix}. \tag{A3}$$

They are normalized such that

$$\overline{u}(p,\lambda)u(p,\lambda')=-\overline{\nu}(p,\lambda)\nu(p,\lambda')=\delta_{\lambda\lambda'}. \tag{A4}$$

The spin-$\frac{1}{2}$ projection operator is given by

$$\sum_{\lambda} u(p,\lambda)\overline{u}(p,\lambda)=\frac{(\not{p}+m)}{2m}. \tag{A5}$$

TABLE VI. Same as Table IV but for $\overline{u}(p',\lambda')\gamma_5 u^{\mu}(p,\lambda)$ matrix elements.

	$\overline{u}(p',\uparrow)\gamma_5 u^{+}(p,\lambda)$ $\overline{u}(p',\downarrow)\gamma_5 u^{+}(p,\lambda)$	$\overline{u}(p',\uparrow)\gamma_5 u^{-}(p,\lambda)$ $\overline{u}(p',\downarrow)\gamma_5 u^{-}(p,\lambda)$	$\overline{u}(p',\uparrow)\gamma_5 u^{r}(p,\lambda)$ $\overline{u}(p',\downarrow)\gamma_5 u^{r}(p,\lambda)$	$\overline{u}(p',\uparrow)\gamma_5 u^{l}(p,\lambda)$ $\overline{u}(p',\downarrow)\gamma_5 u^{l}(p,\lambda)$
$\lambda=\frac{3}{2}$	0	0	0	$\frac{1}{\sqrt{2}}\frac{m'p^{+}-mp'^{+}}{p^{+}p'^{+}}$
	0	0	0	$\frac{1}{\sqrt{2}}\frac{p^{r}p'^{+}-p'^{r}p^{+}}{p^{+}p'^{+}}$
$\lambda=\frac{1}{2}$	$-\frac{1}{\sqrt{6}}\frac{m'p^{+}-mp'^{+}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{m'p^{+}-mp'^{+}}{mp'^{+}}$	0	$-\frac{1}{\sqrt{6}}\frac{p^{+}p'^{l}-p'^{+}p^{l}}{p^{+}p'^{+}}$
	$\frac{1}{\sqrt{6}}\frac{p^{+}p'^{r}-p'^{+}p^{r}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{p'^{+}p^{r}-p^{+}p'^{r}}{mp'^{+}}$	0	$-\frac{1}{\sqrt{6}}\frac{m'p^{+}-mp'^{+}}{p^{+}p'^{+}}$
$\lambda=-\frac{1}{2}$	$-\frac{1}{\sqrt{6}}\frac{p^{l}p'^{+}-p'^{l}p^{+}}{mp'^{+}}$	$-\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{p'^{l}p^{+}-p^{l}p'^{+}}{mp'^{+}}$	$-\frac{1}{\sqrt{6}}\frac{m'p^{+}-mp'^{+}}{p^{+}p'^{+}}$	0
	$\frac{1}{\sqrt{6}}\frac{m'p^{+}-mp'^{+}}{mp'^{+}}$	$-\frac{1}{\sqrt{6}}\frac{p^{-}}{p^{+}}\frac{m'p^{+}-mp'^{+}}{mp'^{+}}$	$\frac{1}{\sqrt{6}}\frac{p'^{+}p^{r}}{p^{+}p'^{+}-p^{+}p'^{r}}$	0
$\lambda=-\frac{3}{2}$	0	0	$-\frac{1}{\sqrt{2}}\frac{p'^{+}p^{l}-p^{+}p'^{l}}{p^{+}p'^{+}}$	0
	0	0	$\frac{1}{\sqrt{2}}\frac{m'p^{+}-mp'^{+}}{p^{+}p'^{+}}$	0

These LF spinors are related to the canonical spinors by a Melosh transformations. The spin-$\frac{3}{2}$ helicity eigenstates $u^{\mu}(p,\lambda)$ are given in Table II which are normalized such that

$$\overline{u^{\mu}}(p,\lambda)u^{\mu}(p,\lambda')=-\delta_{\lambda\lambda'}. \tag{A6}$$

The spin-$\frac{3}{2}$ projection operator has the form

$$\sum_{\lambda} u^{\mu}(p,\lambda)\overline{u^{\nu}}(p,\lambda)=\frac{(\not p+m)}{2m}\left\{-g^{\mu\nu}+\frac{2}{3}v^{\mu}v^{\nu}+\frac{1}{3}\gamma^{\mu}\gamma^{\nu}+\frac{1}{3}(\gamma^{\mu}v^{\nu}-\gamma^{\nu}v^{\mu})\right\}. \tag{A7}$$

Table III contains matrix elements $\overline{u}(p',\lambda')\Gamma u(p,\lambda)$ with $(\Gamma=\{I,\gamma^{+},\gamma^{5}$ and $\gamma^{+}\gamma^{5}\})$. In Tables IV, V, and VI matrix elements $\overline{u}(p',\lambda')\Gamma u^{\mu}(p,\lambda)$ with $(\Gamma=\{I,\gamma^{+}\text{and}\gamma^{5}\})$, respectively, are presented.

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