Resonant and nonresonant $D^+ \to K^- \pi^+ l^+ \nu_l$ semileptonic decays

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We analyze the semileptonic decay $D^+ \to K^- \pi^+ l^+ \nu_l$ using an effective Lagrangian developed previously to describe the decays $D \to P l \nu_l$ and $D \to V l \nu_l$. Light vector mesons are included in the model which combines the heavy quark effective Lagrangian and chiral perturbation theory approach. The nonresonant and resonant contributions are compared. With no new parameters the model correctly reproduces the measured ratio $\Gamma_{\rm nres}/\Gamma_{\rm nres+res}$. We also present useful nonresonant decay distributions. Finally, a similar model, but with a modified current which satisfies the soft pion theorems at the expense of introducing another parameter, is analyzed and the results of the models are compared. [S0556-2821(98)02017-7]

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I. INTRODUCTION

The experimental result for the nonresonant semileptonic decay mode $D^+ \to K^- \pi^+ l^+ \nu_l$ is [1]

$$\frac{\Gamma[D^+ \to K^- \pi^+ \mu^+ \nu_\mu(\text{nonresonant})]}{\Gamma[D^+ \to K^- \pi^+ \mu^+ \nu_\mu]} = 0.083 \pm 0.029. \tag{1}$$

It is important to understand this result before making predictions for the other yet unmeasured $D \to P_1 P_2 l \nu_l$ decay modes. Furthermore, this could also be helpful in understanding the $D \to P l \nu_l$ [2,3] and $D \to V l \nu_l$ data [4,5].

Two main issues arise in the description of D meson semileptonic decays. The first potential problem is that the D mesons might not be heavy enough for heavy quark effective theory (HQET) [6] to be very accurate. The second possible problem is the application of chiral perturbation theory (CHPT) in $D \to P l \nu_l$ and $D \to P_1 P_2 l \nu_l$ decays, where the light pseudoscalar mesons can have quite large energies [6–9].

In order to investigate the semileptonic D_{l3} decays of D mesons we have developed a model [10] which accommodates the available experimental data using HQET and the CHPT description of the heavy and light meson sectors, respectively. The experimental data for the semileptonic D_{l3} decays are unfortunately not good enough at this time to empirically determine the q^2 dependence of the form factors. What is known experimentally are some branching ratios, based on measuring the relevant form factors at some kinematical point and *assuming* a pole-type behavior for all the form factors. The same assumption is also used in many theoretical calculations, for example in [8] and [11].

In our model [10] the *vertices* of the processes considered are assumed not to change appreciably from their value at the zero-recoil point, where they are predicted in the heavy quark limit. However, in our model [10] *the complete propagators for the heavy mesons* are used instead of the usual HQET propagators. Assuming that these modified Feynman rules can be approximately applied to the entire available q^2 region, one can naturally understand why some form factors do have a pole-type behavior and why others are mainly flat, which is also in agreement with the predictions of the QCD sum rules analysis [12]. Moreover, in the region where the heavy meson is nearly on-shell, where HQET is most reliable, the HQET prescription and our model [10] almost perfectly overlap, providing a simple and consistent picture. By calculating the decay widths of all the measured charmed meson D_{l3} semileptonic decays we found [10] that our model, which is a simple modification of orthodox HQET, worked well, providing confidence in extending it to the D_{l4} decays.

There are some previous theoretical calculations attempting to describe the $D \to K \pi l \nu$ D_{l4} semileptonic decays using HQET [9,13,14]. Reference [13] includes only the light pseudoscalars, while for the understanding of the experimental data one must also include the light vector mesons. The authors of Ref. [14] considered resonant and nonresonant contributions in the overlapping region and indicated that outside the resonant region both contributions could be the same order of magnitude. However, no predictions were made. In the present investigation we use our simple and instructive model [10], which was quite successful in describing the D_{l3} decays, to calculate the nonresonant contribution to the $D^+ \to K^- \pi^+ l^+ \nu_l$ semileptonic decay. The experimental ratio (1) is then found to be reproduced without introducing any new parameters, which is remarkable, considering the simplicity of the model.

Since the weak current in the model does not satisfy the soft pion limit exactly, we have also investigated a modified current which does have the exactly correct soft pion limit for comparison, even though the light mesons are not particularly soft in the D_{l4} decay. Of course, the strong Lagrangian automatically satisfies the soft pion constraint.

In Sec. II we briefly summarize the strong Lagrangian describing the heavy and light pseudoscalar and vector mesons, based on HQET and chiral symmetry. In Sec. III the

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two weak currents we consider are given: the usual one, whose form is based on HQET, and another, modified to exactly satisfy the soft pion theorems, albeit at the expense of additional parameters. In Sec. IV the expressions describing the D_{l4} semileptonic decays are given and used to calculate the resonant and nonresonant decay widths, as well as some distributions which might be useful in future analyses of experimental data. The results are presented in Sec. V. Finally, a brief summary and a few comments are given in Sec. VI.

II. HQET AND CHPT STRONG LAGRANGIAN

Our strong interaction Lagrangian [10], which incorporates both heavy meson $SU(2)$ spin symmetry [15,16] and $SU(3)_L \times SU(3)_R$ chiral symmetry, spontaneously broken to the diagonal $SU(3)_V$ [17], describes the heavy and light pseudoscalar and vector mesons. A similar Lagrangian, but without the light vector octet, was first introduced by Wise [6], Burdman and Donoghue [9], and Yan *et al.* [18]. It was then generalized to include the light vector mesons in [7,19,20].

The light meson sector of the strong Langangian is

$$\mathcal{L}_{\text{light}} = -\frac{f^2}{2}\{\text{tr}(\mathcal{A}_\mu \mathcal{A}^\mu) + 2\,\text{tr}[(\mathcal{V}_\mu - \hat{\rho}_\mu)^2]\} + \frac{1}{2g_V^2}\text{tr}[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})], \quad (2)$$

where

$$\mathcal{A}_\mu = \frac{1}{2}(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger), \quad \mathcal{V}_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \quad (3)$$

$$F_{\mu\nu}(\hat{\rho}) = \partial_\mu \hat{\rho}_\nu - \partial_\nu \hat{\rho}_\mu + [\hat{\rho}_\mu, \hat{\rho}_\nu], \quad (4)$$

$$u = \exp\,(i\Pi/f), \quad \hat{\rho}_\mu = i(g_V/\sqrt{2})\rho_\mu, \quad (5)$$

and Π and ρ are the usual 3×3 Hermitian pseudoscalar and vector matrices. $f = 130$ MeV is the pseudoscalar decay constant and $g_V = 5.9$ is determined by the values of the light vector meson masses.

Both the heavy pseudoscalar and the heavy vector mesons are described by the 4×4 matrix

$$H_a = \frac{1}{2}(1+\not{v})(P^*_{a\mu}\gamma^\mu - P_a \gamma_5), \quad (6)$$

where $a = 1,2,3$ is the $SU(3)_V$ index of the light flavors. $P^*_{a\mu}$ and P_a annihilate spin 1 and spin 0 heavy mesons $c\bar{q}_a$ having velocity v, respectively, and have mass dimension 3/2 so that the Lagrangian is explicitly mass independent in the heavy quark limit $m_c \to \infty$. Defining $\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0 = (P^{*\dagger}_{a\mu}\gamma^\mu + P_a^\dagger \gamma_5)(1+\not{v})/2$, we can write the leading order strong Lagrangian as

$$\mathcal{L}_{\text{even}} = \mathcal{L}_{\text{light}} + i\text{Tr}(H_a v_\mu(\partial^\mu + \mathcal{V}^\mu)\bar{H}_a) + ig\text{Tr}[H_b \gamma_\mu \gamma_5 (\mathcal{A}^\mu)_{ba}\bar{H}_a] + i\tilde{\beta}\text{Tr}[H_b v_\mu(\mathcal{V}^\mu - \hat{\rho}^\mu)_{ba}\bar{H}_a] + \frac{\tilde{\beta}^2}{4f^2}\text{Tr}(\bar{H}_b H_a \bar{H}_a H_b), \quad (7)$$

where $\mathcal{V}^\mu$ in the heavy meson kinetic term makes the derivative covariant and also ensures that the kinetic term is chiral invariant, since the heavy meson field transforms nonlinearly under chiral symmetry $SU(3)\times SU(3)$. The Lagrangian (7) is the most general even-parity Lagrangian to leading order in the heavy quark mass and the chiral symmetry limit. The first of the four terms in Eq. (7) is the kinetic term for the heavy field H_a and is thus properly normalized. The second term represents the strong interactions of the pseudoscalar meson field with the heavy meson field. The third term gives the interactions of the light vector mesons with the heavy field. These terms involve two unknown parameters, g and $\tilde{\beta}$, which are not determined by symmetry arguments, and must be determined empirically. As we will see below, only the parameter g will be relevant in the present investigation. Finally, the last term comes from the requirement [7] that the Lagrangian (7) reduce to Wise's Lagrangian [6] in the limit $g_V \to \infty$. The vector field $\hat{\rho}_\mu$ then has no derivatives and can be explicitly integrated out. With this requirement the coefficient of the last term is fixed. However, this convention is irrelevant for our calculations since the vertex with four heavy fields does not appear in any diagrams we need to consider.

We will also need the odd-parity Lagrangian for the heavy meson sector. The lowest order contribution to this Lagrangian is given by

$$\mathcal{L}_{\text{odd}} = i\lambda\,\text{Tr}[H_a \sigma_{\mu\nu} F^{\mu\nu}(\hat{\rho})_{ab}\bar{H}_b]. \quad (8)$$

The parameter λ is *a priori* free, but we do know that it is of the order $1/\Lambda_\chi$ with Λ_χ being the chiral perturbation theory scale.

III. WEAK LAGRANGIAN

The weak Lagrangian for the Cabibbo allowed D meson semileptonic decays is given at the quark level by

$$\mathcal{L}^{\text{eff}}_{\Delta C=\Delta S=1} = -\frac{G_F V^*_{cs}}{\sqrt{2}}[\bar{l}\gamma_\mu(1-\gamma^5)\nu_l][\bar{s}\gamma^\mu(1-\gamma^5)c], \quad (9)$$

where $G_F = 1.17\times 10^{-5}\ \text{GeV}^{-2}$ is the Fermi constant and $V_{cs} = 0.974$ is the relevant Kobayashi-Maskawa matrix element.

Of course, as usual, we have to interpret the quark current in terms of meson fields. We will present two different models, (A) and (B), for the weak part of the Lagrangian:

Model (A): In the first model, which is based on the HQET approach, we assume that the weak current transforms as $(\overline{3}_L,1_R)$ under chiral $SU(3)_L\times SU(3)_R$ and is linear in the heavy meson field. Using HQET one can then write the most general weak current contributing to D meson semileptonic decays to leading order in $1/M$ and to the next-to-leading order in the chiral expansion as

$$j_\lambda^{(A)}=+\frac{i\alpha}{2}J_\lambda u^\dagger-\alpha_1 J(\hat{\rho}-\mathcal{V})_\lambda u^\dagger-\alpha_2 J_\lambda v^\alpha(\hat{\rho}-\mathcal{V})_\alpha u^\dagger$$
$$+\alpha_3 J\mathcal{A}_\lambda u^\dagger+\alpha_4 J_\lambda v^\alpha\mathcal{A}_\alpha u^\dagger$$
$$+[J_\lambda v_\alpha-J_\alpha v_\lambda-i\epsilon_{\mu\lambda\alpha\beta}J^\mu v^\beta]$$
$$\times[\alpha_1(\hat{\rho}-\mathcal{V})^\alpha-\alpha_3\mathcal{A}^\alpha]u^\dagger, \tag{10}$$

where

$$J_\lambda=\mathrm{Tr}_D[\gamma_\lambda(1-\gamma_5)H] \tag{11}$$

and

$$J=\mathrm{Tr}_D[(1-\gamma_5)H]. \tag{12}$$

The first term in Eq. (10), i.e. the one proportional to α $(=f_D\sqrt{m_D})$, is $\mathcal{O}(E^0)$, while the rest is $\mathcal{O}(E)$ [10]. In the process $D\to Pl\nu$ [10] one takes into account only the first term in Eq. (10), which is formally the leading term. In $D\to Vl\nu$ [10] the terms proportional to α_1 and α_2 must be included as well, since the diagrams where they appear are of the same order as the diagrams with the terms proportional to α. Actually, the latter diagram has also a D^*DV vertex (8), which is $\mathcal{O}(E)$.

Model (B): In the decay $D(p_D)\to P_1(p_1)l\nu_l$ or $D(p_D)\to P_1(p_1)P_2(p_2)l\nu_l$ one would expect that the part of the amplitude proportional to p_D is the most important since $p_D^2=m_D^2$ while $p_i^2=m_i^2\ll m_D^2$. The procedure described in model (A) takes into account the leading α term in Eq. (10) and neglects the higher terms which contain derivatives of the light fields and are thus formally next-to-leading order terms. However, what is measured is not the amplitude but the matrix elements squared, and because the leptons are almost massless, the part of the amplitude proportional to $(p_D-\Sigma_i p_i)^\mu$ cannot contribute. Writing $p_D^\mu=(p_D-\Sigma_i p_i)^\mu+(\Sigma_i p_i)^\mu$ one sees that the formally large part of the amplitude proportional to p_D^μ contributes to the decay width only through the term $(\Sigma_i p_i)^\mu$. So unless the coefficients $\alpha_{1,2,3}$ in Eq. (10) are found to be numerically negligible compared to α they can contribute comparably, even if formally they are next-to-leading order terms. However, the term proportional to α_4 is of higher order and the terms in the last line of Eq. (10) do not contribute at all to $D^+\to K^-\pi^+l^+\nu_l$ decay. Consequently, the formal procedure described in model (A) does not take into account these possibly important contributions. However, if we assume that the lower dimensional coefficients of higher dimensional operators in Eq. (10) are naturally smaller (i.e. suppressed by powers of some large scale), then we can continue to use the usual approach (A), as was done in the past [6,10].

Another potential inadequacy of approach (A) is that the soft pion theorems are not satisfied by the weak current. This has its origin in the absence of all terms of order $1/m_D$ in Eq. (10). In our case the soft pion theorem requires that the D decay amplitude vanish in the limit $p_\pi\to 0$ [21]. The only term in the current (10) that does not satisfy this constraint is the first one, i.e. $-if_D\sqrt{m_D}v_\lambda Pu^\dagger$, since it does not have a derivative acting on the pion field. In HQET this term comes from the term $(f_D/\sqrt{m_D})D(\overleftarrow{\partial}-\mathcal{V})_\lambda u^\dagger$, where $D=e^{-im_D v.x}P$: one takes the derivative of only the exponent and neglects the rest, which is of higher order in $1/m_D$. However, since $-(\partial+\mathcal{V})_\lambda u^\dagger=\mathcal{A}_\lambda u^\dagger$ and the relevant component of $\mathcal{A}_\lambda$ in our case is proportional to the derivative of the pion field, it is clear how to modify the weak current in model (A) to satisfy the soft pion theorems: simply modify Eq. (10) by replacing (P,P^*) with (D,D^*) and $-im_D Dv_\lambda$ with $D(\overleftarrow{\partial}-\mathcal{V})_\lambda$. Keeping only the term relevant for our present purposes explicitly one finds this modification of Eq. (10) to be

$$j_\lambda^{(B)}=\frac{f_D}{\sqrt{m_D}}D(\overleftarrow{\partial}-\mathcal{V})_\lambda u^\dagger-2\alpha_3 D\mathcal{A}_\lambda u^\dagger$$
$$-2\alpha_1 D(\hat{\rho}-\mathcal{V})_\lambda u^\dagger-\frac{2\alpha_2}{m_D^2}D(\overleftarrow{\partial}-\mathcal{V})_\lambda(\overleftarrow{\partial}-\mathcal{V})^\alpha$$
$$\times(\hat{\rho}-\mathcal{V})_\alpha u^\dagger-f_D^*\sqrt{m_D}D_\lambda^* u^\dagger, \tag{13}$$

which we shall refer to as model (B). Note that this modification of the original model (A) based on HQET has come at the expense of the appearence of a new parameter, viz. α_3. As a result, the semileptonic $D\to Pl\nu$ data do not determine the parameter g in Eq. (7) as in model (A) [10], but only a combination of g and α_3.

IV. FORM FACTORS AND DECAY WIDTHS

Following [13] we write down the general form for the matrix element of the weak current:

$$\langle\pi(p_\pi)K(p_K)|\bar{s}\gamma_\mu(1-\gamma^5)c|D(p_D)\rangle$$
$$=ir(p_D-p_K-p_\pi)_\mu+iw_+(p_K+p_\pi)_\mu+iw_-(p_K-p_\pi)_\mu$$
$$-2h\epsilon_{\mu\alpha\beta\gamma}p_D^\alpha p_K^\beta p_\pi^\gamma. \tag{14}$$

The form factor r does not contribute to the decay width if the lepton mass is neglected and we will not consider it further. The following combinations of the remaining three form factors will be particularly convenient below:

$$F_1=Xw_++\left[\frac{\beta}{2}(m_D^2-s_{K\pi}-s_{l\nu})\cos\,\theta_K+\left(\frac{m_K^2-m_\pi^2}{s_{K\pi}}\right)X\right]w_-, \tag{15}$$

$$F_2=\beta(s_{K\pi}s_{l\nu})^{1/2}w_-, \tag{16}$$

$$F_3=\beta X(s_{K\pi}s_{l\nu})^{1/2}h. \tag{17}$$

Here θ_K is the angle between the kaon three-momentum in the $K\pi$ rest frame and the direction of the total momentum of the $K\pi$ center of mass in the D rest frame and

$$s_{K\pi}=(p_K+p_\pi)^2, \quad s_{l\nu}=(p_D-p_K-p_\pi)^2, \tag{18}$$

$$X=\frac{1}{2}[m_D^4+s_{K\pi}^2+s_{l\nu}^2-2m_D^2 s_{K\pi}-2m_D^2 s_{l\nu} -2s_{K\pi}s_{l\nu}]^{1/2}, \tag{19}$$

$$\beta=\frac{1}{s_{K\pi}}[s_{K\pi}^2+m_K^4+m_\pi^4 -2s_{K\pi}m_K^2-2s_{K\pi}m_\pi^2-2m_K^2 m_\pi^2]^{1/2}. \tag{20}$$

The D^+ meson differential semileptonic decay rate can then be written as

$$\frac{d^3\Gamma}{ds_{K\pi}ds_{l\nu}d\cos\theta_K}=\frac{G_F^2|V_{cs}|^2}{(4\pi)^5 m_D^3}\frac{X\beta}{3}[|F_1|^2 +\sin^2\theta_K(|F_2|^2+|F_3|^2)], \tag{21}$$

where the physical region of phase space is defined by $|\cos\theta_K|<1$, $0<s_{l\nu}<(m_D-\sqrt{s_{K\pi}})^2$ and $(m_K+m_\pi)^2<s_{K\pi}<m_D^2$. The form factors F_i in Eq. (21) have both resonant and nonresonant parts, which we separate by defining

$$F_i=F_i^r+F_i^{nr}, \quad i=1,2,3. \tag{22}$$

The resonant parts are given by

$$F_1^r=Cg_V\sqrt{\frac{m_D}{s_{K\pi}}}s_{l\nu}\left[(m_D^2-s_{K\pi}-s_{l\nu})\frac{\alpha_1}{2}-\frac{X^2}{m_D^2}\alpha_2\right]\cos\theta_K, \tag{23}$$

$$F_2^r=Cg_V\sqrt{m_D}\alpha_1, \tag{24}$$

$$F_3^r=Cg_V 2X\sqrt{\frac{m_{Ds*}}{m_D}}\frac{m_{Ds*}f_{Ds*}}{s_{l\nu}-m_{Ds*}^2}\lambda, \tag{25}$$

where

$$C=8\sqrt{\frac{\pi s_{l\nu}}{\beta}}\frac{\sqrt{m_{K*}\Gamma_{K*}(s_{K\pi})}}{s_{K\pi}-m_{K*}^2+im_{K*}\Gamma_{K*}(s_{K\pi})} \tag{26}$$

and

$$\Gamma_{K*}(s_{K\pi})=\frac{3}{2}\frac{g_V^2}{96\pi}\frac{\beta^3 s_{K\pi}}{m_{K*}}. \tag{27}$$

It is easy to see that for the resonant parts of the form factors, in the zero width approximation ($\Gamma_{K*}\to 0$), one obtains the previous expressions for the $D^+\to\bar{K}^{*0}l\nu_l$ decay [10].

The nonresonant contribution to the decay rate is

$$\frac{d^3\Gamma^{\rm nr}}{ds_{K\pi}ds_{l\nu}d\cos\theta_K} =\frac{G_F^2|V_{cs}|^2}{(4\pi)^5 m_D^3}\frac{X\beta}{3}\times\{|F_1^{nr}|^2+\sin^2\theta_K(|F_2^{nr}|^2+|F_3^{nr}|^2) +2{\rm Re}[F_1^{nr}F_1^{r*}+\sin^2\theta_K(F_2^{nr}F_2^{r*}+F_3^{nr}F_3^{r*})]\}. \tag{28}$$

The nonresonant form factors will be calculated from the leading order Feynman diagrams and given in the next section. Note that this nonresonant contribution contains not only the nonresonant amplitude itself, but also the interference terms with the resonant contribution.

V. RESULTS AND DISCUSSION

For the calculation of the Feynman diagrams we use the strong Lagrangian described in Sec. II and the weak Lagrangian from Sec. III for both currents in models (A) and (B). As briefly summarized in the Introduction, and discussed in detail in [10], we use the vertices as given by our Lagrangian, assuming that they do not vary appreciably away from the maximum recoil point, where HQET is applicable. *However, we use the complete heavy meson propagators, instead of the HQET approximation.* The nonresonant form factors can then be straightforwardly calculated from the Feynman diagrams with the result

$$w_+^{nr}=-\frac{g}{f_K f_\pi}\frac{f_{D*}m_{D*}\sqrt{m_D m_{D*}}}{[(p_D-p_\pi)^2-m_{D*}^2]}\left[1-\frac{p_\pi(p_D-p_\pi)}{m_{D*}^2}\right] +\frac{d_+}{2f_K f_\pi}-\sqrt{\frac{m_D}{f_K f_\pi}}\left[\beta\frac{X}{m_D^2}\alpha_2\cos\theta_K+\frac{m_K^2-m_\pi^2}{s_{K\pi}}\alpha_1\right], \tag{29}$$

$$w_-^{nr}=\frac{g}{f_K f_\pi}\frac{f_{D*}m_{D*}\sqrt{m_D m_{D*}}}{[(p_D-p_\pi)^2-m_{D*}^2]}\left[1+\frac{p_\pi(p_D-p_\pi)}{m_{D*}^2}\right] -\frac{d_-}{2f_K f_\pi}+\frac{\sqrt{m_D}\alpha_1}{f_K f_\pi}, \tag{30}$$

$$h^{nr}=\frac{-2g^2 f_{Ds*}m_{Ds*}\sqrt{m_D m_{Ds*}}}{f_K f_\pi[(p_D-p_\pi)^2-m_{D*}^2][(p_D-p_K-p_\pi)^2-m_{Ds*}^2]}. \tag{31}$$

In [10] the parameters λ, α_1 and α_2 were fitted to correctly reproduce the $D^+\to\bar{K}^{*0}$ decay. In the same reference it was found that the decay mode $D^0\to K^-$ fixes the parameters g. Because of the nonlinearity of the equations involved there are 8 possible sets of these 4 parameters. The same values for the decay constants and masses as in [10] will be used here, namely $f_D=f_{D*}=(0.24\pm0.05)$ GeV, $f_{Ds}=f_{Ds*}=(0.27\pm0.05)$ GeV [22,23] and $m_D=1.87$ GeV, $m_{D*}=2.01$ GeV , $m_{Ds*}=2.11$ GeV [24]. In deriving Eqs. (29)–(31), and in the following, the approximate relation $g_V=m_{K*}/\sqrt{f_K f_\pi}=5.9$ was used for simplicity and will not affect our conclusions.

TABLE I. The predictions in model (A) for the ratio $R = \Gamma^{nr}/(\Gamma^r + \Gamma^{nr})$ in the decay $D^+ \to K^- \pi^+ l^+ \nu_l$. Results are given for all the values of the input parameters determined from the D_{l3} decay data [10] . The experimental ratio is $R_{\rm expt} = (8.3 \pm 2.9)\%$.

g	λ [GeV^{-1}]	α_1 [GeV$^{1/2}$]	α_2 [GeV$^{1/2}$]	R [%]
0.08 ± 0.09	-0.34 ± 0.07	-0.14 ± 0.01	-0.83 ± 0.04	6.4 ± 0.6
0.08 ± 0.09	-0.34 ± 0.07	-0.14 ± 0.01	-0.10 ± 0.03	9.0 ± 1.1
0.08 ± 0.09	-0.74 ± 0.14	-0.064 ± 0.007	-0.60 ± 0.03	4.7 ± 0.5
0.08 ± 0.09	-0.74 ± 0.14	-0.064 ± 0.007	$+0.18 \pm 0.03$	6.4 ± 0.8
-0.90 ± 0.19	-0.34 ± 0.07	-0.14 ± 0.01	-0.83 ± 0.04	8.0 ± 5.2
-0.90 ± 0.19	-0.34 ± 0.07	-0.14 ± 0.01	-0.10 ± 0.03	3.0 ± 3.5
-0.90 ± 0.19	-0.74 ± 0.14	-0.064 ± 0.007	-0.60 ± 0.03	11.3 ± 6.2
-0.90 ± 0.19	-0.74 ± 0.14	-0.064 ± 0.007	$+0.18 \pm 0.03$	5.1 ± 4.7

The parameters d_+ and d_- depend on the choice of the weak current. In model (A), $d_+ = f_D$ and $d_- = 0$, while in model (B) $d_+ = d_- = f_D - 2\sqrt{m_D}\alpha_3$. In the soft pion limit, $m_\pi \to 0$ and $p_\pi \to 0$, the constraint

$$w_+^{nr} + w_-^{nr} = 0 \tag{32}$$

should be satisfied [21]. Combining Eqs. (29) and (30), indeed, one can easily verify that Eqs. (32) is satisfied in the soft pion limit only in model (B), where $d_+ = d_-$, but not in model (A), which was the main motivation for including model (B), also, in this analysis.

We note Eqs. (29)–(31) agree with the rsults of Lee *et al.* [13], when the differences off-shell between our phenomenological approach of model (A) in which $d_+ = f_D$ and $d_- = 0$ and the exact HQET [13] are taken into account, as discussed above. Next we present numerical results for both models (A) and (B).

In model (A) there are no unknown parameters. As we discussed above and showed in [10], there are 8 possible sets of parameters g, λ, α_1 and α_2 compatible with all the D_{l3} decay data. For the D_{l4} decay $D^+ \to K^- \pi^+ l^+ \nu_l$ we have calculated the ratio $R = \Gamma(\text{nres})/[\Gamma(\text{res}) + \Gamma(\text{nres})]$ for all eight possible sets and the results are given in Table I. We see from Table I that all the combinations of the allowed values of the input parameters predict a ratio R which is consistent with the experimental value (1) $R_{\rm expt} = (8.3 \pm 2.9)\%$. The errors quoted are because of the errors only in the model parameters. Unfortunately, due to the large experimental error in R, the parameters of the model are not restricted further. Indeed, the fact that all 8 sets of parameters that were determined from the D_{l3} data [10] give an acceptable prediction for R in this D_{l4} decay is remarkable, even given the large uncertainties. Perhaps, this merely indicates that R is not very model dependent, provided the model fits all the D_{l3} data.

Since phase space favors smaller $K\pi$ energies, the tree approximation, even though suspect for large energies, is adequate for our purposes. We explicitly checked that the nonresonant contribution does not become large at high values of $s_{K\pi}$ by calculating the distribution $d(\log \Gamma^{nr})/ds_{K\pi}$. To illustrate this point we show this distribution in Fig. 1, taking as input the mean values of the parameters displayed in the 5th row in Table I, which predict a value of R near to the central value of $R_{\rm expt}$. In Fig. 1 three curves are shown: the dashed line is the contribution of the nonresonant amplitude squared, which omitts the last line in Eq. (28), and the dotted line is the contribution of the resonance-nonresonance interferece terms, only the last line in Eq. (28), while the solid line is the sum of both these contributions (28). It is clearly seen that the nonresonant contribution to the decay width decreases considerably at large $s_{K\pi}$. After integrating the curves in Fig. 1 over the invariant mass squared of the $K\pi$ system, we found that almost 80% of the entire result (28) comes from the nonresonant amplitude, while only approximately 20% comes from the resonant-nonresonant interference term. Figure 1 is a prediction of our model (A), which can be tested in future experiments.

There is another distribution that is certainly interesting to compare with experiment to test this model. It is the distribution of the charged lepton energy E_l in the D^+ rest frame. From the kinematics,

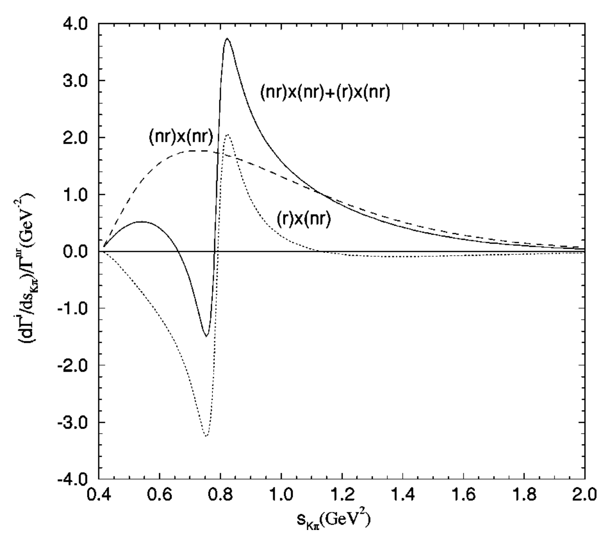

FIG. 1. The contribution to the $s_{K\pi}$ distribution for model (A) of the nonresonant and interference terms and their sums in Eq. (28). The dashed line denotes the term given by the square of the nonresonant amplitude $(nr)\times(nr)$, and the dotted line is the interference of the resonant and non-resonant amplitudes $(r)\times(nr)$, while the solid line gives their sum $(nr)\times(nr) + (r)\times(nr)$.

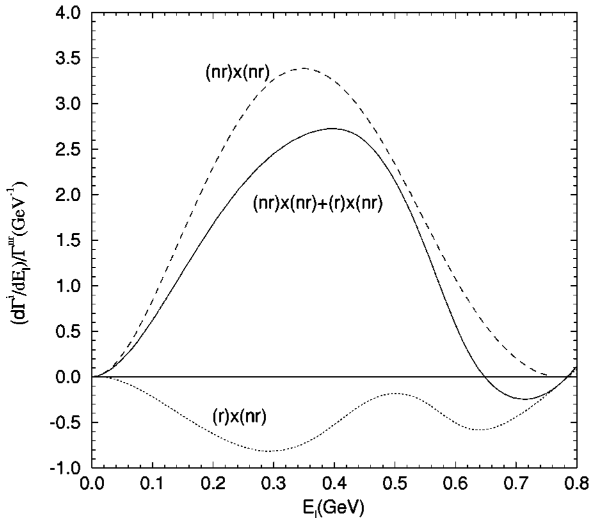

FIG. 2. The charged lepton energy distributions for model (A). The dashed line denotes the term given by the square of the non-resonant amplitude $(nr)\times(nr)$, and the dotted line is the interference of the resonant and non-resonant amplitudes $(r)\times(nr)$, while the solid line gives their sum $(nr)\times(nr)+(r)\times(nr)$.

$$E_l=\frac{1}{2m_D}\left(\frac{m_D^2+s_{l\nu}-s_{K\pi}}{2}+X\cos\theta_l\right), \tag{33}$$

where θ_l is the angle between the charged lepton momentum in the lepton pair center of mass system and the direction of the lepton pair center of mass momentum in the D^+ rest frame. X is defined above in Eq. (19).

In general, the distribution in E_l is defined by

$$\frac{d\Gamma}{dE_l}=\int ds_{K\pi}ds_{l\nu}d\cos\theta_K\frac{d^3\Gamma}{ds_{K\pi}ds_{l\nu}d\cos\theta_K}\frac{2m_D}{X} \tag{34}$$

with [13]

$$\frac{d^3\Gamma}{ds_{K\pi}ds_{l\nu}d\cos\theta_K}=\frac{G_F^2|V_{cs}|^2}{(4\pi)^5m_D^3}\frac{X\beta}{2}\Bigg\{\frac{1}{4}\left[|F_1|^2+\frac{3}{2}\sin^2\theta_K(|F_2|^2+|F_3|^2)\right]-\frac{1}{4}\left[|F_1|^2-\frac{1}{2}\sin^2\theta_K(|F_2|^2+|F_3|^2)\right]\cos 2\theta_l+\mathrm{Re}(F_2^*F_3)\sin^2\theta_K\cos\theta_l\Bigg\}. \tag{35}$$

In Eq. (34) the integration region is defined as in Eq. (28), but with the additional constraint $|\cos\theta_l|<1$ where $\cos\theta_l$ is given by Eq. (33). The range for E_l is $0<E_l<[m_D^2-(m_K+m_\pi)^2]/(2m_D)$.

In Fig. 2 this distribution $(d\Gamma^{nr}/dE_l)/\Gamma^{nr}$ is given by the solid line. In addition, the contribution of the nonresonant amplitude alone is given by the dashed line, while the contribution of the interference between the resonant and non-resonant amplitudes is given by the dotted line.

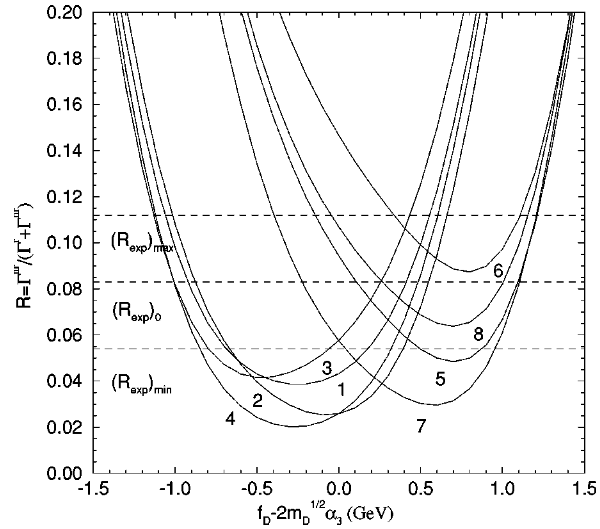

FIG. 3. The eight possible predictions in model (B) for the ratio $R=\Gamma^{nr}/(\Gamma^r+\Gamma^{nr})$ as a function of the parameter $d_+=d_-=f_D-2\sqrt{m_D}\alpha_3$ compared with the experimental central value $(R_{\rm expt})_0$ and the values allowed by one standard deviation $(R_{\rm expt})_{\rm min}$ and $(R_{\rm expt})_{\rm max}$.

Model (B) satisfies the soft pion theorems exactly and is defined by the current (13). It has the nonresonant amplitudes (29)–(31), but with the parameters $d_+=d_-=f_D-2\sqrt{m_D}\alpha_3$. Only one combination of d_+ and g is now determined by the D_{l3} semileptonic decay data. The relevant D_{l3} form factor [10] becomes

$$F_1(q^2)=\frac{1}{f_K}\left(-\frac{d_+}{2}+gf_{Ds*}\frac{m_{Ds*}\sqrt{m_Dm_{Ds*}}}{q^2-m_{Ds*}^2}\right), \tag{36}$$

and the D_{l3} decay width is given by [10]

$$\Gamma=\frac{G_F^2|V_{cs}|^2}{24\pi^3}\int_0^{(m_D-m_K)^2}dq^2|F_1(q^2)|^2\times\left[\frac{(m_D^2+m_K^2-q^2)^2}{4m_D^2}-m_K^2\right]^{3/2}. \tag{37}$$

After determining the relation between the parameters g and d_+ from the D_{l3} decay data we then fit the remaining parameter from the non-resonant D_{l4} decay data. There are, however, 8 possibilities: four possible sets of parameters λ, α_1 and α_2 for either of the two possible relations between g and d_+. From Eqs. (36) and (37) one can see there is a relation of the form

$$g_\pm f_{Ds*}=\frac{-bd_+\pm\sqrt{b^2d_+^2-4c(ad_+^2-\Gamma)}}{2c}. \tag{38}$$

Labelling the first four sets as given by the first, second, third and fourth columns in Table I with $g=g_-$ by $1-4$ and the same sets for λ, α_1 and α_2 but with $g=g_+$ by $5-8$, we present their predictions as functions of d_+ in Fig. 3. All the other parameters are the same as in model (A). The experimental band between $(R_{\rm expt})_{\rm min}$ and $(R_{\rm expt})_{\rm max}$ around the central value (1) is too broad to determine the model parameter $d_+ = f_D - 2\sqrt{m_D}\alpha_3$. We can only conclude that model (B) is also compatible with the available experimental data (1). Further precision is needed to make more conclusive statements and discriminate between models (A) and (B).

VI. CONCLUSIONS

We have used the effective model developed in [10] to calculate the nonresonant contribution to the $D^+ \to K^- \pi^+ l^+ \nu_l$ semileptonic decay. The result agrees with the experimental data, giving further support to the model considered. More precise experimental data are, however, needed in order to reduce the uncertainties in the parameters and further test the assumptions.

We have also calculated the distributions in the $K-\pi$ center-of-mass energy and in the charged lepton energy. Both will be useful in comparing with future data and will provide more sensitive tests of the model.

Since our original model [10] does not obey exactly the soft pion theorems, a slightly modified version, but involving another parameter, was introduced and explored. The predictions of this model are also in agreement with the present experimental data. Because of the large experimental errors, we are unable to distinguish between the two models at present.

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