# Coupling constant evolution in a universal seesaw mass matrix model

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Stimulated by a recent development of the universal seesaw mass matrix model, the evolutions of the gauge and Yukawa coupling constants are investigated under the gauge symmetries $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_Y$. Especially, an investigation is made as to whether this evolution can constrain the necessary intermediate scales in these types of models and its viability. [S0556-2821(98)05317-X]

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## I. INTRODUCTION

Recently, the so-called ''universal seesaw mass matrix model'' [1] has been revived [2,3] as a model which gives a unified description of masses and mixings of the quarks and leptons. The ''seesaw mechanism'' was first proposed [4] in order to answer the question of why neutrino masses are so invisibly small. Then, in order to understand that the observed quark and lepton masses are considerably smaller than the electroweak scale $\Lambda_L = \langle \phi_L^0 \rangle = 174$ GeV, the mechanism was applied to quarks [1]. However, the observation of the top quark in 1994 [5] raised doubt about the validity of the seesaw mechanism for quarks because the observed fact $m_t \sim \Lambda_L$ means that $M_F^{-1} m_R$ is of the order of 1 in the seesaw expression $M_f \simeq m_L M_F^{-1} m_R$. On the contrary, it has recently been found [2,3] that the model can give an interpretation for the question of why only the top quark acquires a mass of the order of $\Lambda_L$ if we take an additional condition $\det M_F = 0$ for the up-quark sector.

In the universal seesaw mass matrix model, the mass matrix for fermions $(f,F)$ is given by

$$M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z_L \\ \kappa Z_R & \lambda Y_F \end{pmatrix}, \tag{1.1}$$

where $f_i$ (fermion sector names $f = u,d,\nu,e$, family numbers $i = 1,2,3$) denote quarks and leptons, $F_i$ denote hypothetical heavy fermions $F = U$, $D$, $N$, and $E$ corresponding to $f = u$, $d$, $\nu$, and $e$, respectively, and they belong to $f_L = (2,1)$, $f_R = (1,2)$, $F_L = (1,1)$, and $F_R = (1,1)$ of $SU(2)_L \times SU(2)_R$. The matrices $Z_L$, $Z_R$, and $Y_F$ are those of the order of 1. The $3\times3$ matrices $m_L$ $(\sim m_0 = \Lambda_L)$ and $m_R$ $(\sim \kappa m_0 = \Lambda_R)$ are symmetry-breaking mass terms of $SU(2)_L$ and $SU(2)_R$, respectively, and those have common structures independent of the fermion sector names $f$. Only $M_F$ $(\sim \lambda m_0 = \Lambda_S)$ has a structure dependent on the sector name $f$. For the case $\lambda \gg \kappa \gg 1$, the mass matrix (1.1) leads to the well-known seesaw expression

$$M_f \simeq m_L M_F^{-1} m_R . \tag{1.2}$$

In contrast to the case (1.2), for the case with the additional condition

$$\det M_F = 0, \tag{1.3}$$

on the up-quark sector $(F = U)$, one of the heavy fermions $F_i$ (say, $F_3$) cannot acquire a mass of the order of $\Lambda_S \equiv \lambda m_0$, so that the seesaw mechanism does not work for the third fermion. Therefore, the mass generation at each energy scale is as follows: First, at the energy scale $\mu = \Lambda_S$, the heavy fermions $F$, except for $U_3$, acquire masses of the order of $\Lambda_S$. Second, at the energy scale $\mu = \Lambda_R$, the $SU(2)_R$ symmetry is broken, and the fermion $u_{R3}$ generates a mass term of the order of $\Lambda_R$ by pairing with $U_{L3}$. Finally, at $\mu = \Lambda_L$, the $SU(2)_L$ symmetry is broken, and the fermion $u_{L3}$ generates a mass term of the order of $\Lambda_L$ by pairing with $U_{R3}$. The other fermions $f$ acquire the well-known seesaw masses (1.2). The scenario is summarized in Table I. We regard the fermion pair $(u_{L3},\ U_{R3})$ as the top-quark state. Thus, we can understand why only the top quark $t$ acquires the mass $m_t \sim O(m_L)$ [2,3].

On the other hand, for neutrino mass generation, at present, we have the following two scenarios as summarized in Table II. One (scenario A) is a trivial extension of the present model: we introduce a further large energy scale $\Lambda_{\nu S}$ in addition to $\Lambda_S$, and we assume that $M_F \sim \Lambda_S$ $(F = U,D,E)$, while $M_N \sim \Lambda_{\nu S}$ $(\Lambda_{\nu S} \gg \Lambda_S)$. Another scenario (scenario B) (see Table III) [6] is one without introducing such an additional energy scale. The neutral heavy leptons are singlets of $SU(2)_L \times SU(2)_R$ and they do not have U(1) charge. Therefore, it is likely that they acquire Majorana masses $M_M$ together with the Dirac masses $M_D \equiv M_N$ at $\mu = \Lambda_S$. For example, we assume $M_M = M_D$ [7]. Then, the

TABLE I. Fermion mass generation scenario.

| Energy scale | $d$ and $e$ sectors | $u$ sector $(i \neq 3)$ |
|---|---|---|
| At $\mu = \Lambda_S \sim \lambda m_0$ | $m(F_L,F_R) \sim \Lambda_S$ | $m(U_{Li},U_{Ri}) \sim \Lambda_S$ |
| At $\mu = \Lambda_R \sim \kappa m_0$ | | $m(u_{R3},U_{L3}) \sim \Lambda_R$ |
| At $\mu = \Lambda_L \sim m_0$ | | $m(u_{L3},U_{R3}) \sim \Lambda_L$ |
| | $m(f_L,f_R) \sim \dfrac{\Lambda_L\Lambda_R}{\Lambda_S}$ | $m(u_{Li},u_{Ri}) \sim \dfrac{\Lambda_L\Lambda_R}{\Lambda_S}$ |

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TABLE II. Neutrino mass generation scenarios: $N_{\pm}=(N_L \pm N_R^c)/\sqrt{2}$.

| Energy scale | Scenario A | Scenario B |
|---|---|---|
| At $\mu=\Lambda_{\nu S}$ | $m(N_L,N_R)\sim\Lambda_{\nu S}$ | |
| At $\mu=\Lambda_S$ | | $m(N_+,N_+^c)\sim\Lambda_S$ |
| At $\mu=\Lambda_R$ | | $m(\nu_R,N_-)\sim\Lambda_R$ |
| At $\mu=\Lambda_L$ | $m(\nu_L,\nu_R)\sim\dfrac{\Lambda_L\Lambda_R}{\Lambda_{\nu S}}$ | $m(\nu_L,\nu_L^c)\sim\dfrac{\Lambda_L^2}{\Lambda_S}$ |

neutrino mass matrix for the conventional light neutrinos is given by $M_\nu=m_L M_N^{-1} m_L^T$, so that the masses $m_\nu$ are given with the order of

$$m_\nu\sim\frac{\Lambda_L^2}{\Lambda_S}=\frac{1}{\kappa}\,\frac{\Lambda_L\Lambda_R}{\Lambda_S}. \tag{1.4}$$

In order to explain the smallness of $m_\nu$, the model requires that the scale $\Lambda_R$ must be extremely larger than $\Lambda_L$ (for example, $\kappa\equiv\Lambda_R/\Lambda_L\sim 10^9$ [7]). This scenario seems to be very attractive from the theoretical point of view, because we can explain the mass hierarchy of the quarks and leptons by the three energy scales $\Lambda_L$, $\Lambda_R$, and $\Lambda_S$ only. On the other hand, in scenario A, there is no constraint on the value of $\kappa$ (however, the value must be larger than $\sim 10$ because of no observation of the right-handed weak bosons $W_R$ at present), so that the model allows a case with a lower value of $\Lambda_R$. Since we can expect abundant new physics effects for the case of $\kappa\sim 10$ [8], the case is also attractive from the phenomenological point of view.

One of the purposes of the present paper is to see whether a study of the evolutions of the gauge coupling constants of $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_Y$ and of the Yukawa coupling constants in the universal seesaw mass matrix model can give any hint of the value of the intermediate energy scale $\Lambda_R$ or not. For example, Shafi and Wetterich [9] and Rajpoot [10] have considered an O(10) model and an SO(10) model, respectively, with the symmetry breakings $SO(10) \to SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_Y$ at $\mu=\Lambda_{GUT}$ and $SU(2)_R\to U(1)_R$ at $\mu=\Lambda_R$, and they have demonstrated that the model with $\Lambda_{GUT}\sim 10^{19}$ GeV and $\Lambda_R\sim 10^9$ GeV is consistent with the low energy phenomenology. The value $\Lambda_R\sim 10^9$ GeV is favorable to the scenario B for neutrino masses. However, in the present model, since there are many new fermions $F$ above the intermediate energy scale $\Lambda_S$, their conclusion cannot be applied to the present seesaw mass matrix model straightforwardly.

TABLE III. Quantum numbers of the fermions $f$ and $F$ and Higgs scalars $\phi_L$, $\phi_R$, and $\Phi$.

| | $I_3^L$ | $I_3^R$ | $Y$ | | $I_3^L$ | $I_3^R$ | $Y$ |
|---|---|---|---|---|---|---|---|
| $u_L$ | $+\frac{1}{2}$ | 0 | $\frac{1}{3}$ | $u_R$ | 0 | $+\frac{1}{2}$ | $\frac{1}{3}$ |
| $d_L$ | $-\frac{1}{2}$ | 0 | $\frac{1}{3}$ | $d_R$ | 0 | $-\frac{1}{2}$ | $\frac{1}{3}$ |
| $\nu_L$ | $+\frac{1}{2}$ | 0 | $-1$ | $\nu_R$ | 0 | $+\frac{1}{2}$ | $-1$ |
| $e_L$ | $-\frac{1}{2}$ | 0 | $-1$ | $e_R$ | 0 | $-\frac{1}{2}$ | $-1$ |
| $U_L$ | 0 | 0 | $\frac{4}{3}$ | $U_R$ | 0 | 0 | $\frac{4}{3}$ |
| $D_L$ | 0 | 0 | $-\frac{2}{3}$ | $D_R$ | 0 | 0 | $-\frac{2}{3}$ |
| $N_L$ | 0 | 0 | 0 | $N_R$ | 0 | 0 | 0 |
| $E_L$ | 0 | 0 | $-2$ | $E_R$ | 0 | 0 | $-2$ |
| $\phi_L^+$ | $+\frac{1}{2}$ | 0 | 1 | $\phi_R^+$ | 0 | $+\frac{1}{2}$ | 1 |
| $\phi_L^0$ | $-\frac{1}{2}$ | 0 | 1 | $\phi_R^0$ | 0 | $-\frac{1}{2}$ | 1 |
| $\Phi$ | 0 | 0 | 0 | | | | |

On the other hand, a phenomenological study of the universal seesaw mass matrix model for the quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) [11] matrix parameters has successfully been given by the present author and Fusaoka [2]. In order to give explicit numerical predictions, they have used some working hypotheses that I will use here as well.

(i) The matrices $Z_L$ and $Z_R$, which are universal for quarks and leptons, have the same structure

$$Z_L=Z_R\equiv Z=\mathrm{diag}(z_1,z_2,z_3), \tag{1.5}$$

with $z_1^2+z_2^2+z_3^2=1$, where, for convenience, we have taken a basis in which the matrix $Z$ is diagonal.

(ii) The matrices $Y_F$, which have structures dependent on the fermion sector $f=u,d,\nu,e$, take a simple form [(unit matrix)+(a rank-1 matrix)]

$$Y_f=\mathbf{1}+3b_f X. \tag{1.6}$$

(iii) The rank-1 matrix $X$ is given by the democratic form

$$X=\frac{1}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}, \tag{1.7}$$

in the family basis where the matrix $Z$ is diagonal.

(iv) In order to fix the parameters $z_i$, we tentatively take $b_e=0$ for the charged lepton sector, so that the parameters $z_i$ are given by

$$\frac{z_1}{\sqrt{m_e}}=\frac{z_2}{\sqrt{m_\mu}}=\frac{z_3}{\sqrt{m_\tau}}=\frac{1}{\sqrt{m_e+m_\mu+m_\tau}}. \tag{1.8}$$

By taking $b_u=-1/3$ (then $\det M_U=0$), they have obtained the following top-quark mass enhancement without the suppression factor $\kappa/\lambda$:

$$m_t\simeq\frac{1}{\sqrt{3}}m_0, \tag{1.9}$$

together with the successful relation $m_u/m_c\simeq 3m_e/4m_\mu$. Furthermore, by taking $b_d=-e^{i\beta_d}$ ($\beta_d=18°$), they have succeeded in giving reasonable values of the CKM matrix parameters together with reasonable values of the quark mass ratios (not only $m_i^u/m_j^u$, $m_i^d/m_j^d$, but also $m_i^u/m_j^d$) with keeping the value of the parameter $(m_0\kappa/\lambda)_f$ in $(m_0\kappa/\lambda)_u=(m_0\kappa/\lambda)_d$. However, in order to fit the quark mass values (not the ratios) to the observed quark mass values at $\mu=m_Z$, they have taken the parameter $(m_0\kappa/\lambda)_f$ as

$$R(m_Z)\equiv\left(\frac{(m_0\kappa/\lambda)_u}{(m_0\kappa/\lambda)_e}\right)_{\mu=m_Z}\simeq 3. \tag{1.10}$$

It seems to be natural to consider that all Yukawa coupling constants become equal between quarks and leptons at a large energy scale $\Lambda_{YU}$. Therefore, another one of the purposes of the present paper is to see whether such a factor of 3 can be understood by the difference of the evolutions of the Yukawa coupling constants between quarks and leptons from the energy scale $\mu=\Lambda_{YU}$ to $\mu=m_Z$.

In Sec. II and Sec. III, we investigate evolution of the gauge and Yukawa coupling constants, respectively, under the gauge symmetries $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_Y$ at one loop. We will conclude that it is possible to find the energy scale $\Lambda_{YU}$ at which $R(\mu)$ takes $R=1$ only for a model with a value $\kappa<10^2$. Although in Sec. II and Sec. III we consider the case that the symmetries $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_Y$ are unbroken for the region $\mu>\Lambda_S$, in Sec. IV, we investigate a case that the symmetries $SU(3)_c\times U(1)_Y$ are embedded into the Pati-Salam symmetry [12] $SU(4)_{PS}$ at $\mu>\Lambda_S$, so that we consider the case of $SU(4)_{PS}\times SU(2)_L\times SU(2)_R$ in the region $\Lambda_S<\mu\leqslant\Lambda_{GUT}$. We will find that the model predicts $\Lambda_R\simeq 5\times 10^{12}$ GeV, $\Lambda_S\simeq 3\times 10^{14}$ GeV, and $\Lambda_{GUT}\simeq 6\times 10^{17}$ GeV. Finally, Sec. V is devoted to the conclusions and remarks. We will find that there is no model which satisfies $\Lambda_{YU}=\Lambda_{GUT}$.

## II. EVOLUTION OF THE GAUGE COUPLING CONSTANTS

The gauge symmetries $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_Y$ are broken into the gauge symmetries $SU(3)_c\times SU(2)_L\times U(1)_{Y'}$ at $\mu=\Lambda_R$. The electric charge operator $Q$,

$$Q=I_3^L+I_3^R+\frac{1}{2}Y, \tag{2.1}$$

at $\mu>\Lambda_R$ is changed into

$$Q=I_3^L+\frac{1}{2}Y', \tag{2.2}$$

in the region $\Lambda_L<\mu\leqslant\Lambda_R$. Hereafter, we call the regions $\Lambda_L<\mu\leqslant\Lambda_R$, $\Lambda_R<\mu\leqslant\Lambda_S$, and $\Lambda_S<\mu\leqslant\Lambda_X$ ($\Lambda_X\equiv\Lambda_{GUT}$ or $\Lambda_X\equiv\Lambda_{YU}$) regions I, II, and III, respectively.

The evolutions of the gauge coupling constants $g_i$ at one loop are given by the equations

$$\frac{d}{dt}\alpha_i(\mu)=-\frac{1}{2\pi}b_i\alpha_i^2(\mu), \tag{2.3}$$

where $\alpha_i\equiv g_i^2/4\pi$ and $t=\ln\mu$. Since the quantum numbers of the fermions $f$ and $F$ are assigned as those in Table III, the coefficients $b_i$ are given in Table IV. (Note that the heavy fermions $F_L$ and $F_R$ except for $U_{L3}$ and $U_{R3}$ are decoupled for $\mu\leqslant\Lambda_S$ and the fermions $u_{R3}$ and $U_{L3}$ are decoupled for $\mu\leqslant\Lambda_R$.) The boundary conditions at $\mu=\Lambda_L$ and $\mu=\Lambda_R$ are as follows:

TABLE IV. Coefficients in the evolution equations of gauge coupling constants.

| | $\Lambda_L<\mu\leqslant\Lambda_R$ | $\Lambda_R<\mu\leqslant\Lambda_S$ | $\Lambda_S<\mu\leqslant\Lambda_X$ |
|---|---|---|---|
| $SU(3)_c$ | $b_3^I=7$ | $b_3^{II}=19/3$ | $b_3^{III}=3$ |
| $SU(2)_L$ | $b_L^I=19/6$ | $b_L^{II}=19/6$ | $b_L^{III}=19/6$ |
| $SU(2)_R$ | | $b_R^{II}=19/6$ | $b_R^{III}=19/6$ |
| $U(1)_Y$ | $b_1^I=-41/10$ | $b_1^{II}=-43/6$ | $b_1^{III}=-41/2$ |

$$\alpha_{em}^{-1}(\Lambda_L)=\alpha_L^{-1}(\Lambda_L)+\frac{5}{3}\alpha_1'^{-1}(\Lambda_L) \tag{2.4}$$

and

$$\frac{5}{3}\alpha_1'^{-1}(\Lambda_R)=\alpha_R^{-1}(\Lambda_R)+\frac{2}{3}\alpha_1^{-1}(\Lambda_R), \tag{2.5}$$

respectively, correspondingly to Eqs. (2.2) and (2.1), where the normalizations of the $U(1)_{Y'}$ and $U(1)_Y$ gauge coupling constants have been taken as they satisfy $\alpha_1'=\alpha_L=\alpha_3$ in the SU(5) grand-unification limit and $\alpha_1=\alpha_L=\alpha_R=\alpha_3$ in the SO(10) grand-unification limit, respectively. For convenience, we use the initial values at $\mu=m_Z$ instead of those at $\mu=\Lambda_L$ in region I: $\alpha_1'(m_Z)=0.01683$, $\alpha_L(m_Z)=0.03349$, and $\alpha_3(m_Z)=0.118$. The values of $\alpha_1'$ and $\alpha_L$ have been derived from [13] $\alpha_{em}(m_Z)=(128.89\pm 0.09)^{-1}$ and $\sin^2\theta_W=0.23165\pm 0.00024$. The value of $\alpha_3$ has been quoted from Ref. [14]. We illustrate a typical case with $\Lambda_R=10^5$ GeV and $\alpha_R(\Lambda_R)=1/4\pi$ in Fig. 1.

In the numerical study, we have taken the value of the parameter $\Lambda_R/\Lambda_S\equiv\kappa/\lambda$ as

$$\kappa/\lambda=\Lambda_R/\Lambda_S=0.02, \tag{2.6}$$

which has been obtained from the observed value of the ratio $m_c/m_t$ in Ref. [2]. Although the value (2.6) has been obtained in the model with the specific matrix forms (1.5)–(1.7), the order of the value (2.6) will be valid for any other seesaw model with $\det M_U=0$ because in such a model the value of $\kappa/\lambda$ is given by the order of $m_b/m_t$.

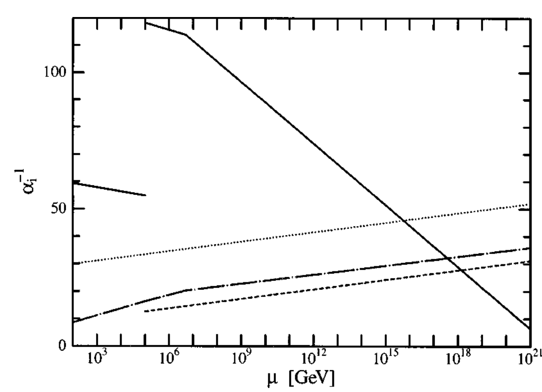


FIG. 1. Behaviors of $\alpha_3^{-1}(\mu)$ (dot-dashed line), $\alpha_L^{-1}(\mu)$ (dotted line), $\alpha_R^{-1}(\mu)$ (dashed line), and $\alpha_1^{-1}(\mu)$ $[\alpha_1'^{-1}(\mu)]$ (solid line) in the case with the input values $\Lambda_R=10^5$ GeV and $\alpha_R(\Lambda_R)=1/4\pi$.

As seen in Fig. 1, the U(1) coupling constant $\alpha_1(\mu)$ becomes rapidly strong in region III ($\mu > \Lambda_S$) because the heavy fermions $F$ become massless in region III. We consider that the unification energy scale $\Lambda_{YU}$ of the Yukawa coupling constants must be lower than an energy scale $\Lambda_1^\infty$ at which $\alpha_1(\mu)$ becomes infinity. This condition will impose a strong restriction on the possible $\Lambda_{YU}$ search as we discuss in the next section. Of course, in the grand-unification scenario, the U(1) symmetry will be embedded into a grand-unification symmetry $G$ before the U(1) coupling constant bursts. Such a case will be discussed in Sec. IV.

### III. EVOLUTION OF $y_L y_R / y_S$

The $3\times 3$ matrices $m_L$, $m_R$, and $M_F$ are given in terms of the vacuum expectation values $v_L = \sqrt{2}\langle \phi_L^0 \rangle$, $v_R = \sqrt{2}\langle \phi_R^0 \rangle$, and $v_S = \langle \Phi \rangle$, and the matrices $Z$ and $Y_F$ defined by Eqs. (1.5)–(1.7) as follows:

$$m_L^f = \frac{1}{\sqrt{2}} y_L^f v_L Z, \quad m_R^f = \frac{1}{\sqrt{2}} y_R^f v_R Z, \quad M_F = y_S^f v_S Y_F . \tag{3.1}$$

The evolution of the Yukawa coupling constants is given by

$$\frac{d}{dt}\ln(y_L^f Z) = \frac{1}{16\pi^2}(T_L^f - G_L^f + H_L^f), \tag{3.2}$$

$$\frac{d}{dt}\ln(y_R^f Z) = \frac{1}{16\pi^2}(T_R^f - G_R^f + H_R^f), \tag{3.3}$$

$$\frac{d}{dt}\ln(y_S^f Y_F) = \frac{1}{16\pi^2}(T_S^f - G_S^f + H_S^f), \tag{3.4}$$

where $T^f$, $G^f$, and $H^f$ denote contributions from fermion-loop corrections, vertex corrections due to the gauge bosons, and vertex corrections due to the Higgs boson, respectively.

What is of great interest to us is to see whether the evolutions can explain the value $R(m_Z) \simeq 3$ or not; i.e., our interest exists not in the hierarchy among $m_e$, $m_\mu$, and $m_\tau$, but in the hierarchy among up-quark, down-quark, charged lepton, and neutrino sectors. Therefore, we neglect the scale dependence of the matrix $Z$, because we can regard the value of $z_3$ as $z_3 \simeq 1$ from Eq. (1.8). We also neglect the scale dependence of the matrix $Y_F$ because the matrices $Y_F$ are expressed as

$$Y_F = \mathrm{diag}(1,1,1+3b_f), \tag{3.5}$$

on the basis of which the matrix $Y_F$ is diagonal, and we find that the forms $Y_E = \mathrm{diag}(1,1,1)$ and $Y_U = \mathrm{diag}(1,1,0)$ are scale invariant and $Y_D = \mathrm{diag}(1,1,1-3e^{i\beta_d})$ is almost scale invariant. For convenience, we still approximately use the evolution equation of $y_S Y_F$ at $\mu \leq \Lambda_S$ and that of $y_R Z$ at $\mu \leq \Lambda_R$. Then, the ratio $R(\mu)$ defined by Eq. (1.10) can be expressed in terms of the Yukawa coupling constants $y_L$, $y_R$, and $y_S$ as follows:

TABLE V. $G$ and $H$ terms in the evolution equation (3.8). $H_L^e$, $H_R^e$, and $H_S^e$ are given by the replacements $u \to e$ and $d \to \nu$ in $H_L^u$, $H_R^u$, and $H_S^u$, respectively.

| | I $(\Lambda_L < \mu \leq \Lambda_R)$ | II $(\Lambda_R < \mu \leq \Lambda_S)$ | III $(\Lambda_S < \mu \leq \Lambda_{YU})$ |
|---|---|---|---|
| $G=$ | $4\pi(8\alpha_3 - \frac{7}{5}\alpha_1')$ | $4\pi(8\alpha_3 - 2\alpha_1)$ | |
| $H_L^u =$ | $\frac{3}{2}(\lvert y_L^u\rvert^2 - \lvert y_L^d\rvert^2)$ | | |
| $H_R^u =$ | 0 | $\frac{3}{2}(\lvert y_R^u\rvert^2 - \lvert y_R^d\rvert^2)$ | |
| $H_S^u =$ | 0 | | $3\lvert y_S^u\rvert^2$ |

$$R(\mu) \equiv \frac{y_L^u(\mu) y_R^u(\mu)/y_S^u(\mu)}{y_L^e(\mu) y_R^e(\mu)/y_S^e(\mu)}. \tag{3.6}$$

The evolution of the ratio $R(\mu)$ is approximately given by

$$\frac{d}{dt}\ln R(\mu) \simeq -\frac{1}{16\pi^2}(G-H), \tag{3.7}$$

where

$$G = (G_L^u + G_R^u - G_S^u) - (G_L^e + G_R^e - G_S^e),$$
$$H = (H_L^u + H_R^u - H_S^u) - (H_L^e + H_R^e - H_S^e). \tag{3.8}$$

The $G$ and $H$ terms are given in Table V. Since in the present model, $|y_L^u|^2 \simeq |y_L^d|^2$, $|y_L^e|^2 \simeq |y_L^\nu|^2$, and so on, differently from other models where $|y_L^d/y_L^u| \simeq m_b/m_t$ and $|y_L^\nu/y_L^e| \simeq m_\nu/m_\tau$, we can neglect the $H_L$ and $H_R$ terms in Eq. (3.8). When we also neglect the $H_S$ terms, the ratio $R(\mu)$ is approximately evaluated as follows:

$$\frac{R(\mu)}{R(m_Z)} = \left(1 + \frac{b_3^I}{2\pi}\alpha_3(m_Z)\ln\frac{\mu}{m_Z}\right)^{-4/b_3^I} \times \left(1 + \frac{b_1^I}{2\pi}\alpha_1'(m_Z)\ln\frac{\mu}{m_Z}\right)^{7/10 b_1^I}, \tag{3.9}$$

$$\frac{R(\mu)}{R(\Lambda_R)} = \left(1 + \frac{b_3^{II}}{2\pi}\alpha_3(\Lambda_R)\ln\frac{\mu}{\Lambda_R}\right)^{-4/b_3^{II}} \times \left(1 + \frac{b_1^{II}}{2\pi}\alpha_1(\Lambda_R)\ln\frac{\mu}{\Lambda_R}\right)^{1/b_1^{II}}, \tag{3.10}$$

$$\frac{R(\mu)}{R(\Lambda_S)} = \left(1 + \frac{b_3^{III}}{2\pi}\alpha_3(\Lambda_S)\ln\frac{\mu}{\Lambda_S}\right)^{-4/b_3^{III}} \times \left(1 + \frac{b_1^{III}}{2\pi}\alpha_1(\Lambda_R)\ln\frac{\mu}{\Lambda_S}\right)^{1/b_1^{III}}, \tag{3.11}$$

for regions I, II, and III, respectively. By using Eqs. (3.9)–(3.11), we can obtain the energy scale $\mu = \Lambda_{YU}$ at which the ratio $R(\mu)$ takes the value $R(\Lambda_{YU}) = 1$.

In Fig. 2, we illustrate the behavior of $\Lambda_{YU}$ for a given value of $\Lambda_R$. For reference, we also illustrate the behavior of

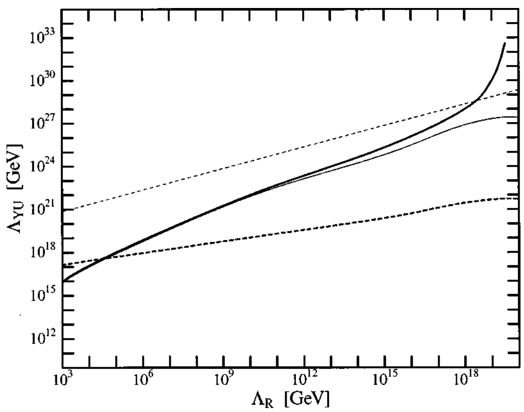


FIG. 2. Behavior of $\Lambda_{YU}$ versus $\Lambda_R$. The bold and thin solid lines denote the cases of the input values $\alpha_R(\Lambda_R)=\alpha_L(\Lambda_R)$ and $\alpha_R(\Lambda_R)=1/4\pi$, respectively. For reference, the behaviors of $\Lambda_1^\infty$ for the cases of the input values $\alpha_R(\Lambda_R)=\alpha_L(\Lambda_R)$ (bold dashed line) and $\alpha_R(\Lambda_R)=1/4\pi$ (thin dashed line) are illustrated. The physical value of $\Lambda_{YU}$ must be $\Lambda_{YU}<\Lambda_1^\infty$.

$\Lambda_1^\infty$, at which $\alpha_1^{-1}(\mu)$ takes the value $\alpha_1^{-1}(\Lambda_1^\infty)=0$. The value of $\Lambda_{YU}$ must be lower than the value of $\Lambda_1^\infty$. Therefore, as seen in Fig. 2, if we adhere to the constraint $\alpha_R(\Lambda_R)=\alpha_L(\Lambda_R)$, we must abandon a model with a higher $\kappa$ value ($\kappa>10^2$). Only a model with $\kappa\sim 10$ is acceptable. However, if we admit a strong coupling of the right-handed weak bosons at $\mu=\Lambda_R$, for example, $\alpha_R(\Lambda_R)\geqslant 1/4\pi$, a model with a higher $\kappa$ value also becomes acceptable.

Of course, from a similar study, we can find that the evolution of $R_{u/d}(\mu)\equiv(y_L^u y_R^u/y_S^u)/(y_L^d y_R^d/y_S^d)$ still keeps $R_{u/d}(m_Z)\simeq 1$. Therefore, the parametrization $(m_0\kappa/\lambda)_u=(m_0\kappa/\lambda)_d$ in Ref. [2] is justified.

## IV. EVOLUTION OF THE PATI-SALAM COLOR

In order to avoid the burst of the U(1) gauge coupling constant, we consider that the $U(1)_Y\times SU(3)_c$ symmetries are embedded into the Pati-Salam SU(4) symmetry [12] above $\mu=\Lambda_S$. In other words, the $SU(4)_{PS}$ gauge symmetry is broken into $SU(3)_c\times U(1)_Y$ at $\mu=\Lambda_S$. Indeed, the structures of the heavy fermion mass matrices $M_F$ are flavor dependent. The fermions $f$ and $F$ belong to $f_L=(2,1,4)$, $f_R=(1,2,4)$, $F_L=(1,1,4)$, and $F_R=(1,1,4)$ of $SU(2)_\times SU(2)_R\times SU(4)_{PS}$ at $\mu\geqslant\Lambda_S$.

In region III ($\Lambda_S<\mu\leqslant\Lambda_X$), $\alpha_L(\mu)$ and $\alpha_R(\mu)$ are evolved with the coefficients $b_L^{III}$ and $b_R^{III}$ given in Table IV, but $\alpha_1(\mu)$ and $\alpha_3(\mu)$ are replaced with $\alpha_4(\mu)$ which is evolved with the coefficient

$$b_4^{III}=20/3, \tag{4.1}$$

where the boundary condition at $\mu=\Lambda_S$ is

$$\alpha_1(\Lambda_S)=\alpha_3(\Lambda_S)=\alpha_4(\Lambda_S). \tag{4.2}$$

Since $\alpha_1(\Lambda_S)$ and $\alpha_3(\Lambda_S)$ are given by

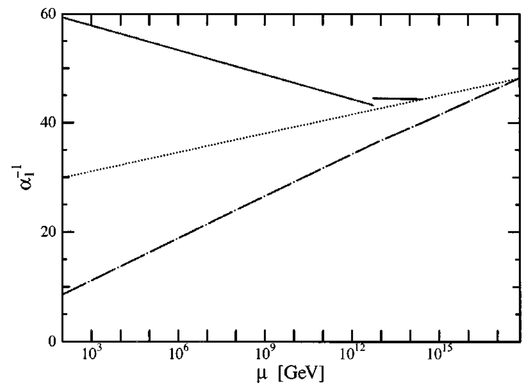


FIG. 3. Behaviors of $\alpha_3^{-1}(\mu)$ [$\alpha_4^{-1}(\mu)$ for $\Lambda_S<\mu\leqslant\Lambda_{GUT}$] (dot-dashed line), $\alpha_L^{-1}(\mu)$ [$=\alpha_R^{-1}(\mu)$ for $\mu>\Lambda_R$] (dotted line), and $\alpha_1^{-1}(\mu)$ [$\alpha_1'^{-1}(\mu)$ for $\Lambda<\mu\leqslant\Lambda_R$] (solid line) in the case of Pati-Salam type unification, where $\Lambda_R=5.46\times 10^{12}$ GeV, $\Lambda_S=2.37\times 10^{14}$ GeV, and $\Lambda_{GUT}=5.84\times 10^{17}$ GeV.

$$\alpha_1^{-1}(\Lambda_S)=\frac{5}{2}\left[\alpha_1^{-1}(\Lambda_L)+\frac{b_1^I}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}\right]-\frac{3}{2}\left[\alpha_L^{-1}(\Lambda_L)+\frac{b_L^I}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}\right]+\frac{b_1^{II}}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R}, \tag{4.3}$$

$$\alpha_3^{-1}(\Lambda_S)=\alpha_3^{-1}(\Lambda_L)+\frac{b_3^I}{2\pi}\ln\frac{\Lambda_R}{\Lambda_L}+\frac{b_3^{II}}{2\pi}\ln\frac{\Lambda_S}{\Lambda_R}, \tag{4.4}$$

respectively, the values of $\Lambda_R$ and $\Lambda_S$ are fixed at

$$\Lambda_R=5.46\times 10^{12}\ \text{GeV},\quad \Lambda_S=2.73\times 10^{14}\ \text{GeV}, \tag{4.5}$$

under the conditions (2.6) and (4.2). The unification scale $\Lambda_{GUT}$ is also fixed at

$$\Lambda_{GUT}=5.84\times 10^{17}\ \text{GeV} \tag{4.6}$$

by the condition

$$\alpha_L(\Lambda_{GUT})=\alpha_R(\Lambda_{GUT})=\alpha_4(\Lambda_{GUT}), \tag{4.7}$$

for example, for the embedding into SO(10) [8,9]. In Fig. 3, we illustrate the behaviors of $\alpha_i^{-1}(\mu)$. Roughly speaking, the value $\Lambda_R\sim 10^{12}$ is favorable to scenario B for the neutrino mass generation.

On the other hand, the evolution of $R(\mu)$ defined by Eq. (3.6) is almost constant at $\mu\geqslant\Lambda_S$, i.e., $R(\Lambda_S)\simeq R(\Lambda_{YU})$, because there is no difference between quarks and leptons in region III ($\Lambda_S<\mu\leqslant\Lambda_{YU}$). Therefore, we obtain

$$R(m_Z)/R(\Lambda_{YU})\simeq R(m_Z)/R(\Lambda_S)=2.3, \tag{4.8}$$

and we fail to obtain our desirable relation $R(m_Z)/R(\Lambda_{YU})\simeq 3$. If we adhere to the unification of the gauge symmetries $SU(3)_c\times SU(2)_L\times SU(2)_R\times U(1)_Y$ into a Pati-Salam type unification $G$, we must abandon the idea that the discrepancy $R(m_Z)\simeq 3$ between quarks and leptons in the model given in Ref. [2] comes from difference of evolutions between quarks and leptons, or we must consider that the magnitudes of the Yukawa coupling constants are different between quarks and leptons from the beginning at $\mu = \Lambda_{GUT}$.

## V. CONCLUDING REMARKS

In conclusion, we have investigated the evolution of the universal seesaw mass matrix model under the gauge symmetries $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_Y$. The symmetries can be embedded into a unification symmetry of the Pati-Salam type at $\Lambda_{GUT} = 5.84 \times 10^{17}$ GeV. Then, the value $\Lambda_R = 5.46 \times 10^{12}$ GeV is favorable to scenario B for neutrino mass generation. However, we cannot explain the discrepancy $R(m_Z) \simeq 3$ between quarks and leptons by the evolution of $R(\mu)$ starting from $R(\Lambda_{GUT}) = 1$.

On the other hand, if we abandon the grand-unification scenario, the model has the possibility that the value $R(m_Z) \simeq 3$ can be understood by the evolution of the Yukawa coupling constants. As seen in Fig. 2, we require $\alpha_L(\Lambda_R) = \alpha_R(\Lambda_R)$, and the value $\Lambda_R$ for the case which gives $R(m_Z) \simeq 3$ must be $\Lambda_R \leqslant 10^4$ GeV. If we accept a model with a strong $SU(2)_R$ force at $\mu = \Lambda_R$, for example, $\alpha_R(\Lambda_R) = 1/4\pi$, the region $\Lambda_R \leqslant 10^{18}$ GeV also becomes allowed. We consider that the model with $\kappa \sim 10$ is likely. Although this case rules out scenario B for neutrinos, phenomenologically we can expect an abundance of new physics effects [8], $t'$ production, FCNC effects, and so on, in the near future colliders.

Our numerical results have been obtained by the constraint $\Lambda_R/\Lambda_S = 0.02$ which has come from the observed ratio of $m_c/m_t$ under the special model with Eqs. (1.5)–(1.7). Since the ratio $\Lambda_R/\Lambda_S$ is fixed by the ratio $m_c/m_t$ (or $m_b/m_t$) as far as a universal seesaw model with $\det M_U = 0$ is concerned, the value of the ratio $\Lambda_R/\Lambda_S$ is, in general, of the order of $10^{-2}$. Therefore, our conclusions will be unchanged as far as the orders are concerned.

In the present paper, we have not discussed a supersymmetry (SUSY) version of the present model, although the case is attractive from the point of view of grand unification. In such a SUSY version, since the coefficient $8\alpha_3$ in $G$ terms in Eq. (3.8) (also in Table V) is changed for $(16/3)\alpha_3$, the case pushes the energy scale $\Lambda_{YU}$ to an unlikely ultrahigh energy scale ($> 10^{23}$ GeV). If we want to adopt a SUSY version of the present model, we must abandon the idea of the unification of the Yukawa coupling constants.

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