PHYSICAL REVIEW D, VOLUME 58, 051501

Perturbative pole mass in QCD

Andreas S. Kronfeld
Theoretical Physics Group, Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 1 May 1998; published 29 July 1998)

It is widely believed that the pole mass of a quark is infrared finite and gauge independent to all orders in perturbation theory. This seems not to have been proved in the literature. A proof is provided here. [S0556-2821(98)50515-2]

PACS number(s): 12.38.Bx, 11.15.Bt, 12.15.Ff

I. INTRODUCTION

Long ago Tarrach [1] showed that the pole mass of a quark is infrared finite and gauge independent through two-loop order in perturbation theory. It is widely believed that this result holds through any finite order in perturbation theory. There does not seem to be a reference in the literature containing a proof, however, and this paper aims to fill that gap. To be specific, I consider QCD with one massive quark and n_f-1 massless quarks, but extra massive quarks do not change the argument.

There are a few reasons why one might suspect infrared divergences to arise in the perturbative-QCD series for the pole mass. QCD contains massless self-interacting gluons, so the infrared behavior is often worse than for QED. In QCD and QED, infrared divergences *do* arise in the two-loop self-energy. Another worry is that the *non*perturbative pole mass, if at all defined, is clearly very sensitive to the infrared.

On the other hand, the kinematics of QCD, with the quark content under consideration, is like that of QED with a massive muon, some massless scalar bosons, and some massless electrons [2]. Here (or with n_f large enough to make QCD infrared-free) nonperturbative problems should not arise in the infrared. For the QED model, one can even pick boundary conditions so that gauge-invariant muon states are in the spectrum.[1] Also, the infrared divergence from the two-loop self-energy is cancelled by a term from evaluating the one-loop self-energy on the one-loop mass shell and iterating [1,4]; one could hope that this mechanism occurs at any order. Moreover, in asymptotically free QCD, a remnant of the anticipated infrared sensitivity appears through renormalons [5,6]; this implies that perturbation theory can ''know about'' the infrared behavior of nonperturbative QCD without having infrared divergences at fixed order.

The above discussion is nothing but a duel of fears and hopes, and it should be replaced by definitive results. Below it is shown that the infrared divergences cancel in the pole mass, as at two loops, to any order. It is then a simpler matter to confirm that the pole mass is independent of the gauge chosen for the calculation. The arguments are straightforward and can be found in textbooks, although, to the best of my knowledge, the specific application is not.

This paper is organized as follows: Some notation is in Sec. II. Section III proves that the pole mass is infrared finite at every order in perturbative QCD. Section IV proves that the pole mass does not depend on the gauge-fixing function through every finite order. Some concluding remarks are in Sec. V.

II. NOTATION

This paper uses the metric $g_{\mu\nu}=\mathrm{diag}(-1,1,1,1)$ in Minkowski space-time. In particular, the momentum p of a real particle with mass M satisfies $p^2=-M^2$.

The pole mass is derived by identifying the pole in the massive quark's full propagator $S(p)$. One has

$$S^{-1}(p)=i\not{p}+m_0-\Sigma(p), \tag{1}$$

where m_0 is the bare mass and the self-energy Σ is given by the sum of one-particle irreducible Feynman diagrams. One can write

$$\Sigma(p,m_0)=i\not{p}A(p^2,m_0)+m_0B(p^2,m_0), \tag{2}$$

exhibiting the parametric dependence of Σ on the bare mass m_0. In considering the dependence on p^2, it is convenient to use a slight abuse of notation, $\Sigma(p^2,m_0)$.

The propagator $S(p)$ has a pole at $p^2=-M^2$, where

$$M=m_0Z_m \tag{3}$$

and

$$Z_m=\lim_{p^2\to -M^2}\frac{1-B(p^2,m_0)}{1-A(p^2,m_0)}, \tag{4}$$

provided the limit is not infrared divergent. In perturbation theory, one applies Eq. (4) by expanding the right-hand side through Lth order and setting p^2 to $-M^2$ iteratively. A gauge-invariant ultraviolet regulator, such as dimensional regularization or a lattice, is assumed, but not made explicit. Because gauge theories are renormalizable, ultraviolet divergences of the coefficients $Z_m^{[l]}$ are compensated by the bare mass and gauge coupling.[2]

Perturbative series are written, for example, as

$$Z_m=1+\sum_{l=1}^{\infty} g_0^{2l}Z_m^{[l]}, \tag{5}$$

[1]The magnetic monopole considered by Ref. [3] is a prototype of such a state.

[2]For example, in dimensional regularization, one could introduce a renormalized mass $\overline{m}(\mu)=m_0Z_m^-$ by minimal subtraction. Then one could focus on the infrared behavior of Z_m/Z_m^-.

where g_0^2 is the bare gauge coupling. Below it is convenient to use a short-hand $[\bullet]^{[l]}$ for the lth term in the perturbative series of expressions abbreviated here with $\bullet$.

The momentum p is reserved for the external momentum of the quark. Loop momenta are denoted generically by k.

III. INFRARED FINITENESS

To prove infrared-finiteness, I follow the methods of Chapter 13 of Ref. [7]. First, I recall how, in perturbation theory, one finds singularities in Green functions. This analysis establishes that the propagator and the self-energy have a branch point at $p^2=-M^2$. Since the pole mass requires the self-energy functions to be evaluated here, one must check whether they diverge at the branch point or not. It turns out that the on-shell self-energy does suffer from infrared divergences, but I show that they drop out of the pole mass.

A. Location of singularities

Consider an arbitrary Feynman diagram (of any Green function), with quark propagators rationalized and all denominators combined with Feynman parameters α_i. If the diagram has n lines, the resulting denominator is

$$D=\left(\sum_{i=1}^{n}\alpha_i(q_i^2+m_i^2)\right)^n, \tag{6}$$

where $q_i=q_i(p,k)$ are the momenta of the internal lines. The Green function is an analytic function of Lorentz invariants of the external momenta, up to branch points. Branch points can arise only when D vanishes, but that is not enough. In addition, the contour of integration (over Feynman parameters α and loop momenta k) must be pinched [7]. This happens if and only if on each internal line

$$q_i^2+m_i^2=0 \quad \text{or} \quad \alpha_i=0 \tag{7}$$

and, furthermore, following any closed path $\mathscr{C}$ in the diagram

$$\sum_{i\in\mathscr{C}}\alpha_i q_i=0, \tag{8}$$

with the sign of q_i taken in the sense of the path $\mathscr{C}$. Equations (7) and (8) are the so-called Landau equations.

Solutions of the Landau equations have a physical interpretation [8]. Up to an overall factor the Feynman parameter α_i is the ratio of the time elapsed, from one end of the line to the other, to the energy propagating on the line. Thus, $\alpha_i q_i$ is the space-time separation between the two ends, and Eq. (8) says a loop in the diagram corresponds to a loop in space-time. Furthermore, Eq. (7) says that internal lines either are on shell ($q_i^2=-m_i^2$) or do not propagate ($\alpha_i=0$). For each diagram, one obtains the *reduced diagram* by shrinking off-shell lines to a point. Then branch points arise if and only if the reduced diagram represents a genuine physical process of on-shell states.

The physical picture given above is useful, because it is often easier to find solutions to Eqs. (7) and (8) with physical reasoning instead of with algebra. For example, a two-point function has branch points only at normal thresholds, that is, when p^2 is just right to produce a collection of on-shell particles. For the massive quark propagator, these branch points are at

$$p^2=-[(1+2r)M]^2, \tag{9}$$

corresponding to creation of the massive quark plus r massive pairs. These branch points are accumulations of infinitely many solutions to the Landau equations, because once a solution is found, others are given by adding zero-momentum massless lines. Physically, this is because it costs nothing to create an extra soft gluon or extra soft pair.[3] If the solutions accumulate too quickly, an infrared divergence will develop.

On the other hand, note that there are no collinear divergences. As soon as the massive quark radiates nonzero momentum, it is off shell, and the (un)physical picture disallows a singularity.

B. Infrared divergences

To examine the infrared properties, one performs a power-counting analysis. One scales some or all loop momenta by a factor λ; if the Feynman integral scales as λ^μ as $\lambda\to 0$, one says the degree of infrared divergence is μ. For example, in d dimensions the momentum-space volume element d^dk has $\mu(d^dk)=d$. If $\mu>0$, an integral is infrared convergent.

The conclusions derived above for arbitrary diagrams apply equally well to the one-particle irreducible ones contributing to the self-energy. It is convenient to route the external momentum p along the ''main line,'' the massive quark line that runs all the way through a self-energy diagram. Off the main line the momenta are independent of p, and the degrees of infrared divergence are straightforward. Soft gluon and ghost propagators contribute $\mu(\Delta)=-2$, and soft massless quark propagators $\mu(S)=-1$. Soft three-gluon and gluon-ghost vertices contribute $\mu(V_3)=+1$, and other soft vertices $\mu(V)=0$. In a closed loop, the massive quark propagator $S_0(k)=1/(i\not{k}+m_0)$ has degree of infrared divergence 0.

The internal parts of the main line have propagator $S_0(p+k)$. When k is soft

$$S_0(p+k)=\frac{1}{i(\not{p}+\not{k})+m_0}\to\frac{m_0-i\not{p}}{p^2+2p\cdot k+m_0^2}. \tag{10}$$

Off shell (away from the branch point) such lines have degree of infrared divergence 0. On shell, however,[4]

$$S_0(p+k)\to\frac{m_0-i\not{p}}{2p\cdot k}, \tag{11}$$

which gives degree -1.

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[3]If one of the other quarks had a mass M_l, the branch points would be at $p^2=-[(1+2r)M+2sM_l]^2$, so infinitely many collapse to the same point as $M_l\to 0$.

[4]With $p^2=-M^2$, one treats $m_0^2-M^2$ as higher order in g_0^2.

When all loop momenta are soft and $p^2=-M^2$, an arbitrary QCD (or QED) self-energy diagram G_L with L loops, but no massive quark loops, has degree of infrared divergence

$$\mu(G_L)=1+L(d-4) \tag{12}$$

in d dimensions. This holds at one loop. Higher-loop diagrams can be built by adding gluons, ghost loops, or (for now) massless quark loops. It is enough to insert the loops into gluon propagators; more gluons can be added later. Loop insertions give $\mu(G_{L+1})=\mu(G_L)+d-4$. New gluons give $\mu(G_{L+1})=\mu(G_L)+d-2+\mu_1+\mu_2$, where d comes from the new loop, -2 from the gluon propagator, and the μ_i are degrees associated with each end. The ends can be on a gluon or ghost line: $\mu_i=1-2=-1$ for vertex and propagator; on a three-gluon vertex: $\mu_i=-1$ from changing the three- to a four-gluon vertex; on the main line or a massless quark: $\mu_i=-1$ for the propagator.[5] Thus, these ways all give $\mu(G_{L+1})=\mu(G_L)+d-4$. Replacing an internal n-vertex polygon of a massless quark with a massive one increases $\mu(G_L)$ by n. Therefore, in four dimensions no infrared divergences can arise from the region with all loop momenta soft.

Infrared divergences may come, however, from regions with some loop momenta soft and others not. According to the physical picture of the self-energy, the non-soft lines can be shrunk to a point, augmenting the foregoing analysis with composite vertices. For $n>3$, n-point vertices V_n contribute $\mu(V_n)=0$ to the total. Additional soft gluons attached to such vertices come with a propagator and a loop integration, adding $d-2$ to the total degree. Composite three-point vertices have the same infrared power counting as their fundamental counterparts. For example, gauge invariance guarantees the beneficial $\mu(V_3)=+1$ for the (composite) vertex of three soft gluons, in the same way it safeguards renormalizability. Thus, multi-point composite vertices do not lower the degree of infrared divergence in four dimensions.

Internal, hard self-energy diagrams shrink to two-point vertices. In massless quark loops, the two-point vertex and extra propagator yield the harmless factor $\Sigma(k)/(i\not{k})=A(k)$, as required by chiral symmetry. In massive quark loops, the additional factor $\Sigma(k)/m_0\to B(k)$ is also harmless. (These self-energies are off shell and, therefore, well behaved.) Gauge symmetry provides two powers of soft momenta at two-point gluon and two-point ghost vertices, cancelling the extra propagator. Thus, these two-point vertices do not pose a problem.

What remains are two-point vertices of the massive quark on the main line. The extra factor from inserting such a two-point vertex is $\Sigma(p,m_0)[m_0-i\not{p}]/2p\cdot k$, which lowers the degree of infrared divergences to $\mu\leqslant 0$. For example, at two loops it is known [1] that the self-energy $\Sigma^{[2]}(-m_0^2,m_0)$ is infrared divergent. The origin of the divergence, as the above analysis implies, is sketched in Fig. 1. The problem worsens at higher and higher orders, as more and more two-point vertices can arise on the main line.

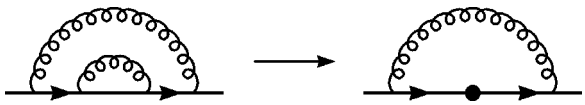

FIG. 1. Origin of the infrared divergence at two loops.

C. Infrared cancellation

For the pole mass at two loops, the infrared divergence in $\Sigma^{[2]}(-m_0^2,m_0)$ is cancelled by the $O(g_0^4)$ part of $\Sigma^{[1]}(-M^2,m_0)$. To examine this mechanism in general, it is convenient to solve for the bare mass that implies a desired pole mass, namely

$$m_0=MZ_m^{-1}, \tag{13}$$

with

$$Z_m^{-1}=\frac{1-A(-M^2,M/Z_m)}{1-B(-M^2,M/Z_m)} \tag{14}$$

showing the iterative nature of the solution. The Lth coefficient $[Z_m^{-1}]^{[L]}$ of the iterated expansion is infrared finite if and only if the coefficients $Z_m^{[l]}$, $l\leqslant L$, are infrared finite.

The coefficients of the iterated expansion involve certain combinations of the coefficients of the self-energy functions. Let

$$\Sigma(p^2,MZ_m^{-1})=\sum_l g_0^{2l}\overline{\Sigma}^{[l]}(p^2,M) \tag{15}$$

define the coefficients $\overline{\Sigma}^{[l]}(p^2,M)$ [and, by implication, $\overline{A}^{[l]}(p^2,M)$ and $\overline{B}^{[l]}(p^2,M)$]. Iterative expansion yields

$$\overline{\Sigma}^{[l]}(p^2,M)=\Sigma^{[l]}(p^2,M)+\sum_{i=1}^{l-1}\sum_{j=1}^{i}\frac{1}{j!}[(Z_m^{-1}-1)^j]^{[i]}\times M^j\left.\frac{\partial^j\Sigma^{[l-i]}}{\partial m_0^j}\right|_{m_0=M}. \tag{16}$$

Explicit calculation shows that $\overline{\Sigma}^{[1]}(p^2,M)=\Sigma^{[1]}(p^2,M)$ is infrared finite and, when $p^2=-M^2$, gauge independent. At two loops $\overline{\Sigma}^{[2]}(-M^2,M)$ is also infrared finite and gauge independent, even though $\Sigma^{[2]}(-M^2,M)$ is not [1,4].

Since

$$[Z_m^{-1}]^{[1]}=\overline{B}^{[1]}-\overline{A}^{[1]}, \tag{17}$$

$$[Z_m^{-1}]^{[2]}=\overline{B}^{[2]}-\overline{A}^{[2]}+[Z_m^{-1}]^{[1]}\overline{B}^{[1]}, \tag{18}$$

with all self-energy functions evaluated at $p^2=-M^2$ and $m_0=M$, one has a basis for a proof by induction. Let us assume that the $[Z_m^{-1}]^{[l]}$, $l<L$, are infrared finite. To show that $[Z_m^{-1}]^{[L]}$ is also infrared finite, it is enough to show that

[5]When the massive quark is off shell, the degree is increased by the number of main-line propagator segments—even more convergent.

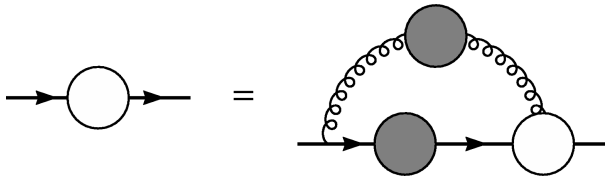

FIG. 2. Dyson-Schwinger equation for the self-energy. Grey blobs denote full propagators and white blobs denote one-particle irreducible functions.

$\overline{\Sigma}^{[L]}(-M^2,M)$ is infrared finite. The induction hypothesis implies that $\overline{\Sigma}^{[l]}(-M^2,M)$, $l<L$, is infrared finite (otherwise $[Z_m^{-1}]^{[l]}$ would not be).

It would be a nightmare to identify all infrared divergences and verify cancellation on the right-hand side of Eq. (16). Instead, it is more efficient to study $\overline{\Sigma}^{[L]}(p^2,M)$ directly. The Dyson-Schwinger equation for the self-energy, depicted in Fig. 2, is a useful tool. Power-counting and the induction hypothesis together say that the only new infrared divergence at L loops can come from diagrams with the $(L-1)$th order expansion of the quark propagator and an additional gluon:

$$\overline{\Sigma}^{[L]}_{\rm IR}=\int \frac{d^dk}{(2\pi)^4}\,\frac{\delta_{\mu\nu}-\xi k_\mu k_\nu/k^2}{k^2}\,\gamma_\mu[S(p+k)]^{[L-1]}\gamma_\nu. \quad (19)$$

Diagrams with a higher-order one particle irreducible vertex function or gluon propagator would have a quark propagator with $L-2$ (or fewer) loops; they can be infrared divergent only if $\overline{\Sigma}^{[l]}$, for some $l<L$, were too—contrary to the induction hypothesis.

The key to obtaining $\overline{\Sigma}^{[L]}_{\rm IR}$ from the right-hand side of Eq. (19) is to write $(q=p+k)$

$$S(q)=\frac{1}{i\not{q}+M-[\Sigma(q,MZ_m^{-1})-M(Z_m^{-1}-1)]}, \quad (20)$$

treating $i\not{q}+M$ as $O(g_0^0)$ and expanding the brackets (iteratively) in g_0^2. This expansion, for $p^2=-M^2$ and k soft, produces a sum of chains

$$[S(p+k)]^{[L-1]}=\frac{M-i\not{p}}{2p\cdot k}\sum\prod_j\cdots[\overline{\Sigma}(p+k,M) -M(Z_m^{-1}-1)]^{[l_j]}\frac{M-i\not{p}}{2p\cdot k}\cdots. \quad (21)$$

In any term of the sum the factors' superscripts l_j add up to $L-1$, and the sum is over all such partitions of $L-1$. Since k is soft the quantity in brackets reduces to

$$[\overline{\Sigma}(p+k,M)-M(Z_m^{-1}-1)]^{[l]}\to[i\not{p}A(p^2,M/Z_m) +MZ_m^{-1}B(p^2,M/Z_m)-M(Z_m^{-1}-1)]^{[l]} =(i\not{p}+M)\overline{A}^{[l]}(-M^2,M)+O(k). \quad (22)$$

The second step follows from Eq. (14) and setting $p^2\to-M^2$. In Eq. (21) the right-hand side of Eq. (22) multiplies $M-i\not{p}$; the $O(1)$ part of the product vanishes for $p^2=-M^2$, leaving a remainder of $O(k)$.[6] Consequently, $[S(p+k)]^{[L-1]}$ has degree of infrared divergence -1, just like $S_0(p+k)$. Thus, $\overline{\Sigma}^{[L]}(-M^2,M)$ is infrared-finite, as was to be proved.

Although the above formulas are a bit clumsy, the mechanism that cancels the infrared divergences is simple. Equation (20) says to split the bare mass into the pole mass plus a counterterm. The counterterm, like the shrunken self-energy subdiagram, produces a two-point vertex. Infrared divergences cancel in the (next order's) pole mass, because the combination of the two does not degrade the infrared power counting. In QED, this mechanism was identified in a footnote to Ref. [9].

IV. GAUGE INDEPENDENCE

The gauge invariance of the mass renormalization factor Z_m is ''nearly obvious.'' If the ultraviolet regulator respects gauge symmetry, the bare mass has a gauge-invariant meaning. From a physical point of view, it would be unsettling if the pole mass were to depend on the gauge. Thus, the ratio $Z_m=M/m_0$ ought to be gauge invariant too. Without the infrared-finiteness established above, however, a proof would require painstaking separation of infrared and ultraviolet regulators.

With infrared-finiteness, one can study the gauge dependence by treating the massive quark like a normal particle, as long as one discusses perturbation theory only. For p^2 near $-M^2$ the propagator takes the form

$$S(p)=Z_2/(i\not{p}+M), \quad (23)$$

where Z_2 is the field renormalization factor.[7] Because the pole mass is only defined perturbatively, M denotes here the pole mass through some finite order in perturbation theory.

The previous section assumed a gauge fixing term $(\lambda/2)\int(\partial\cdot A)^2d^4x$. [In Eq. (19), $\xi=1-\lambda^{-1}$.] Suppose M depends on λ. A shift $\Delta\lambda$ induces a first-order change

$$\Delta S(p)=\frac{\Delta\lambda}{i\not{p}+M}\left(\frac{\partial Z_2}{\partial\lambda}-\frac{\partial M}{\partial\lambda}\,\frac{Z_2}{i\not{p}+M}\right). \quad (24)$$

Note that $\partial M/\partial\lambda$ multiplies a double pole.

On the other hand, the propagator is given by

$$S(p)=\mathrm{FT}\langle 0|T\psi(x)\overline{\psi}(y)|0\rangle. \quad (25)$$

From Eq. (25), the change is

[6]One might worry whether the remainder is an infrared-divergent derivative of the self-energy. But because $p+k$ is off shell by k, the remainder is proportional to $k\ln$.

[7]Z_2 is not infrared divergent when p^2 is close to, but not equal to, $-M^2$.

$$\Delta S(p) = \frac{\Delta\lambda}{2}\,\mathrm{FT}\int d^d z \langle 0|T\psi(x)\bar{\psi}(y)(\partial\cdot A(z))^2|0\rangle. \quad (26)$$

A double pole would develop if the quark could scatter off (two) gluons with scalar polarization, which cannot happen because the scalar polarization decouples from the physical state space. The mass M cannot, therefore, depend on the gauge parameter, although the residue Z_2 certainly can.

(Conversely, one can see immediately that the insertions generated by a shift in the bare mass or gauge coupling *would* develop a double pole and, thus, a shift in the pole mass.)

For a more general gauge-fixing function $f^a(A(z))$ (and change Δf^a), the argument is similar. Let s denote the BRS operator and η ($\bar{\eta}$) the (anti)-ghost field. The change in the two-point function involves [10]

$$\langle s[\bar{\eta}_a(z)\Delta f^a(z)]\psi(x)\bar{\psi}(y)\rangle \sim \langle \bar{\eta}_a(z)\Delta f^a(z) s[\psi(x)\bar{\psi}(y)]\rangle \quad (27)$$

$$\sim \langle \bar{\eta}_a(z)\Delta f^a(z) t^b[\eta_b(x)-\eta_b(y)]\psi(x)\bar{\psi}(y)\rangle. \quad (28)$$

As before, but now because the ghosts decouple, this expression cannot develop a double pole.

V. CONCLUSIONS

The pole mass is widely used in the phenomenology of QCD and, when quark momenta are small compared to the mass, in nonrelativistic QCD and heavy-quark effective theory. In many of these contexts it is natural: it has considerable intuitive appeal, and it can be calculated with any ultraviolet regulator and in any effective theory. In some circles the pole mass—as an experimental quantity—has rightly fallen into disfavor, because infrared renormalons obstruct an unambiguous determination [5,6]. Had the pole mass turned out to be either infrared divergent or gauge dependent, one ought to have abandoned the pole mass for more basic reasons.

Fortunately, Sec. III shows that the perturbative pole mass is infrared finite, even though the on-shell self-energy is not. At higher orders the pole mass requires an iterative expansion and, thus, (infrared divergent) derivatives of the self-energy. In the pole mass the total infrared divergence vanishes. The cancellation mechanism and, indeed, the power counting in QCD are the same as in standard QED. Attaching a virtual photon to an on-shell massive electron line has the same effect as attaching a gluon anywhere in a massive-quark self-energy diagram. I cannot imagine that infrared-finiteness of the electron mass has never been proved, but, except for a footnote [9], I have not found a reference with a proof.

Because of its physical appeal, the pole mass remains valuable theoretically. In addition to matching to effective theories, mentioned above, it is similarly useful in relating lattice QCD to continuum renormalization schemes [11]. An example of considerable phenomenological interest is the application of (NR)QCD to threshold production of heavy quarks. There the pole mass is nearly irresistible, but it has been pointed out recently that infrared sensitivity in the mass's definition is conferred on the QCD potential as well [12–14].

Note added in proof. The gauge independence (but not the infrared finiteness) of the pole mass has been proven in QED [15,16] and QCD [16,17]; I thank V. Miransky, T. Steele, and G. Kilcup, respectively, for drawing my attention to these works. A proof of both infrared finiteness and gauge independence of the electron mass to all orders in QED is given in Ref. [18]; I thank A. Schreiber for drawing my attention to this text.

ACKNOWLEDGMENTS

I thank Tony Duncan and George Sterman for helpful conversations and Aida El-Khadra, Aneesh Manohar, and Scott Willenbrock for comments on the manuscript. I would also be grateful to anyone who would inform me of a prior (published) proof, even in QED. Fermilab is operated by Universities Research Association Inc., under contract with the U.S. Department of Energy.

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