

Electroweak origin of cosmological magnetic fields

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Magnetic fields may have been generated in the electroweak phase transition through spontaneous symmetry breaking or through the subsequent dynamical evolution of semiclassical field configurations. Here I demonstrate explicitly how magnetic fields emerge spontaneously in the phase transition also when no gradients of the Higgs field are present. Using a simple model, I show that no magnetic fields are generated, at least initially, from classical two-bubble collisions in a first-order phase transition. An improved gauge-invariant definition of the electromagnetic field is advocated which is more appropriate in the sense that it never allows electrically neutral fields to serve as sources for the electromagnetic field. In particular, semiclassical configurations of the Z field alone do not generate magnetic fields. The possible generation of magnetic fields in the decay of unstable Z strings is discussed. [S0556-2821(98)05414-9]

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I. INTRODUCTION

It is known that our galaxy and many other spiral galaxies possess a large-scale correlated magnetic field with a strength of the order of 10^{-6} G [1]. In each case the direction of the field seems to accord with the rotation axis of the galaxy, which suggests that it was generated by a dynamo mechanism in which an initial field was amplified by the turbulent motion of matter during the epoch of galaxy formation [2]. This mechanism usually requires a seed field of the order of 10^{-21} Gauss or larger which is primordial in nature (see, however, Ref. [3] for an alternative possibility). Various cosmological explanations for such a seed field have been suggested [4–12]. The present paper focuses on scenarios in which a strong magnetic field of magnitude 10^{20} – 10^{23} Gauss was generated during the electroweak phase transition and was thenceforth diluted by the expansion of the universe to values appropriate for a seed field at the time of onset of galaxy formation.

There have been several models proposed in which the strong magnetic field is produced by the turbulence of the conductive plasma during the phase transition [4,5]. In contrast, I shall restrict myself to mechanisms where the magnetic field would be generated directly from the dynamics of the order parameter (the Higgs field) and from the gauge fields in the process of breaking the electroweak symmetry $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$. Such mechanisms include the spontaneous generation of magnetic fields, collisions of bubbles of broken phase in a first-order phase transition, and the formation and dynamics of nontopological defects. In addition, there are scenarios in which magnetic fields are produced by bound pairs of monopoles in standard and extended electroweak models [7], but I shall not consider them here.

Vachaspati [8] has suggested that strong magnetic fields may emerge spontaneously in the phase transition because

the covariant derivatives of the Higgs field in causally disconnected regions must be uncorrelated. The electric current that produces these fields can receive contributions from gradients of the phases of the Higgs field and charged vector-boson currents or both, depending on which gauge is used. Recently the electric current from the Higgs field was calculated in Ref. [13] and was found always to be zero. For this reason, it was claimed that no coherent magnetic fields are generated by the rolling Higgs field in the electroweak phase transition. I will show below that these statements are incorrect.

A useful tool in the investigation of magnetic phenomena and magnetogenesis is the gauge-invariant definition of the electromagnetic field tensor introduced in Ref. [8]. It has recently been employed by Grasso and Riotto [9] in the study of semiclassical configurations of the Z and W fields. They discovered a set of puzzling paradoxes in which the electrically neutral Z field appears to act as a source for magnetic fields. In particular, it seemed that a magnetic field would always be present along the internal axis of an electroweak Z string.

These surprising and counterintuitive results have prompted me to reexamine the gauge-invariant definition of the electromagnetic field tensor. I find that it is indeed not suited to situations where the magnitude of the Higgs field deviates from its vacuum value. I propose a different definition of this tensor which, in addition to resolving the paradoxes, proves to be a potent calculational tool. For example, it follows immediately that no magnetic field is generated initially from the classical dynamics of the Higgs field in a collision between two bubbles in a first-order electroweak phase transition.

The paper is organized as follows. In Sec. II I describe the problems with the conventional gauge-invariant definition of the electromagnetic field tensor and argue why it should be modified. I then present an improved definition and describe its general properties. In Sec. III I point out that the contribution to the electric current from the Higgs field actually does not vanish. I go on to demonstrate that in an arbitrary gauge one can always construct electrically charged field directions in the Lie algebra and corresponding charged vector-

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boson fields. The current resulting from these fields is in general nonzero and will give rise to electromagnetic fields.

In Sec. IV I present an alternative description of the spontaneous generation of magnetic fields where the unitary gauge is imposed. In this gauge there are no angular degrees of freedom of the Higgs field. Instead, the magnetic fields arise from SU(2) and U(1) vector potentials that were present already in the ground state of the symmetric phase. As the SU(2)_L × U(1)_Y symmetry breaks, the vector potentials find themselves having random nonvanishing components along new physical directions of the Lie algebra which are the eigenstates of mass and electric charge. This reinterpretation confirms Vachaspati's original proposal that magnetic fields can be generated spontaneously in the electroweak phase transition [8].

In Sec. V, I show that no magnetic field is generated initially from the classical dynamics of the Higgs field in a collision between two bubbles in a first-order electroweak phase transition. This is shown for arbitrary difference and relative orientation of the phases of the Higgs field in the two bubbles. The result is in stark contrast to that of the Abelian U(1) model, in which a field strength is present from the instant of collision [14,15].

In Sec. VI the field configurations of the electroweak Z string [16] and W string [17] are investigated, using the redefined electromagnetic field tensor. I verify that they carry neither magnetic fields nor electric currents. In Ref. [9] it was suggested that magnetic fields may be generated in the decay of electroweak strings. In the case of the Z string, the source of the magnetic field would be charged W fields which are initially present in the decay. By constructing the unstable W mode responsible for the decay, I verify explicitly that a magnetic field is indeed generated. Estimates of the strength and correlation length of the generated magnetic field are provided, for each mechanism separately, at the end of Secs. IV, V, and VI, respectively.

II. GAUGE-INVARIANT DEFINITION OF THE ELECTROMAGNETIC FIELD

The conventional gauge-invariant definition of the electromagnetic field tensor in the SU(2) × U(1) Yang-Mills-Higgs system is given by [8]

$$F_{\mu\nu}^{\text{em}} \equiv -\sin\theta_w \hat{\phi}^a F_{\mu\nu}^a + \cos\theta_w F_{\mu\nu}^Y - i \frac{\sin\theta_w}{g} \frac{2}{\Phi^\dagger \Phi} [(\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}_\nu \Phi - (\mathcal{D}_\nu \Phi)^\dagger \mathcal{D}_\mu \Phi], \quad (1)$$

where

$$\hat{\phi}^a \equiv \frac{\Phi^\dagger \tau^a \Phi}{\Phi^\dagger \Phi}, \quad \mathcal{D}_\mu \equiv \partial_\mu - i \frac{g}{2} \tau^a W_\mu^a - i \frac{g'}{2} Y_\mu \equiv \partial_\mu - i \Lambda_\mu.$$

This definition of $F_{\mu\nu}^{\text{em}}$ has the attractive property that, in a ‘unitary’ gauge where $\Phi = (0, \rho)^\top$, $\hat{\phi}^a = -\delta^{a3}$, with ρ real and positive, it reduces to the usual expression $A_{\mu\nu} \equiv$

$\partial_\mu A_\nu - \partial_\nu A_\mu$, where $A_\mu = \sin\theta_w W_\mu^3 + \cos\theta_w Y_\mu$. This holds true, however, only when the magnitude ρ is a constant. For a general (positive) $\rho = \rho(x)$, it is easy to show that

$$F_{\mu\nu}^{\text{em}} = A_{\mu\nu} - 2 \tan\theta_w (Z_\mu \partial_\nu \ln \rho - Z_\nu \partial_\mu \ln \rho) \quad (\text{unitary gauge}) \quad (2)$$

with $Z_\mu = \cos\theta_w W_\mu^3 - \sin\theta_w Y_\mu$.

While such a definition certainly is possible, its physical consequences become highly disturbing when one considers the dynamical equation for $F_{\mu\nu}^{\text{em}}$ in this gauge, which takes the form [9]

$$\begin{aligned} \partial^\mu F_{\mu\nu}^{\text{em}} = & -ie [W^{\mu\dagger} (\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu) - (\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu)^\dagger W^\mu] - ie \partial^\mu (W_\mu^\dagger W_\nu - W_\nu^\dagger W_\mu) \\ & - 2 \tan\theta_w \partial^\mu [Z_\mu \partial_\nu \ln \rho(x) - Z_\nu \partial_\mu \ln \rho(x)] \quad (\text{unitary gauge}). \end{aligned} \quad (3)$$

Here W_μ^\dagger and $W_\mu \equiv (W_\mu^1 - iW_\mu^2)/\sqrt{2}$ are the charged vector bosons, and $\mathcal{D}_\mu W_\nu \equiv (\partial_\mu - igW_\mu^3)W_\nu$.

From Eq. (3) one would infer that an electromagnetic field could be generated from currents involving the fields Z_ν and ρ . From most points of view such a result seems absurd since, in the unitary gauge, Z_ν and ρ are electrically neutral. In fact, the charge operator $(1 + \tau^3)/2$ annihilates $(0, \rho)^\top$ and commutes with the Z direction in the Lie algebra, $T_Z \propto \cos^2\theta_w \tau^3 - \sin^2\theta_w 1$. The fields Z_ν and ρ remain neutral also when ρ is coordinate dependent because the form of the charge operator can depend only on the choice of gauge. The change from $\rho = \text{const}$ to $\rho = \rho(x)$ does not constitute a change of gauge, since no angular degrees of freedom of the Higgs field are involved.

The definition (1) thus implies that electromagnetic fields can be produced by neutral currents. A more reasonable and practical definition should exclude this possibility.

Through a slight modification of a definition given by 't Hooft [18] for the SO(3) Georgi-Glashow model one obtains an improved gauge-invariant definition of the electromagnetic field tensor

$$\mathcal{F}_{\mu\nu}^{\text{em}} \equiv -\sin\theta_w \hat{\phi}^a F_{\mu\nu}^a + \cos\theta_w F_{\mu\nu}^Y + \frac{\sin\theta_w}{g} \epsilon^{abc} \hat{\phi}^a (D_\mu \hat{\phi})^b (D_\nu \hat{\phi})^c, \quad (4)$$

where $(D_\mu \hat{\phi})^a = \partial_\mu \hat{\phi}^a + g \epsilon^{abc} W_\mu^b \hat{\phi}^c$. This definition depends on the Higgs field only through the unit vector $\hat{\phi}^a$ which is independent of the magnitude $\rho = (\Phi^\dagger \Phi)^{1/2}$. Therefore, the problematic terms in Eqs. (2) and (3) involving gradients of ρ will not appear in the unitary gauge, where the field tensor now always reduces to the familiar expression $\mathcal{F}_{\mu\nu}^{\text{em}} = A_{\mu\nu}$. An intricate interplay between the first and last term in Eq. (4) ensures that the electrically charged SU(2) vector fields cancel (in any gauge), leaving only the neutral component $-\sin\theta_w \hat{\phi}^a (\partial_\mu W_\nu^a - \partial_\nu W_\mu^a)$. The definition (4) has been pro-

posed earlier by Hindmarsh [19], but has not been applied before in the study of magnetic fields in the electroweak phase transition.

It can be shown that the Bianchi identity $\epsilon^{\mu\nu\alpha\beta}\partial_\nu\mathcal{F}_{\alpha\beta}^{\text{em}}=0$ is satisfied everywhere except along world lines around which $\hat{\phi}^a$ takes ‘‘hedgehog’’ configurations [18]. This ensures that there is no magnetic charge or magnetic current in the absence of magnetic monopoles. The conventional definition, Eq. (1), does not have this property.

Repeating the calculation done in Ref. [9] for the field tensor of Eq. (1), one may derive the field equation for the redefined field tensor $\mathcal{F}_{\mu\nu}^{\text{em}}$ using the equations of motion for $F_{\mu\nu}^a$ and $F_{\mu\nu}^Y$ and a few Fierz identities. One thus obtains

$$\begin{aligned}\partial^\mu\mathcal{F}_{\mu\nu}^{\text{em}} &= j_\nu^e \\ &\equiv -\sin\theta_w(D^\mu\hat{\phi})^a F_{\mu\nu}^a \\ &\quad + \frac{\sin\theta_w}{g}\partial^\mu[\epsilon^{abc}\hat{\phi}^a(D_\mu\hat{\phi})^b(D_\nu\hat{\phi})^c],\end{aligned}\quad (5)$$

where j_ν^e is the gauge-invariant electric current.

It should be remarked that no physics is affected by using one definition of the electromagnetic field rather than the other. In fact, in any chosen gauge the field configuration is completely specified by the components W_μ^a and Y_μ of the gauge potentials, as defined by their occurrence in the covariant derivative \mathcal{D}_μ , together with the four real components of the Higgs field. Two observers, using different definitions (1) and (4) of the electromagnetic field, may then disagree on whether this same field configuration constitutes an electromagnetic field or not. Clearly, this does not affect the subsequent evolution of the field configuration. In the absence of topological defects, it will evolve into a state with uniform magnitude of the Higgs field, where the two definitions coincide.

The choice of definition is, however, important for the interpretation, description, and understanding of physical processes whenever $\Phi^\dagger\Phi$ is not constant. In particular, one should be aware that it may be meaningless to make strong claims about the presence or absence of magnetic fields in situations that involve a nonuniform magnitude of the Higgs field, unless one is careful to specify which definition of the electromagnetic field tensor is used.

In this paper, I adopt the modified definition (4) which ensures that there is no magnetic charge or magnetic current and that no electromagnetic field is generated from electrically neutral sources. Even so, one should remember that there is no exact standard by which definition (1) would be incorrect.

In Ref. [9] it was stated that, because of the last term of Eq. (3), the formation of a magnetic field is always associated to the appearance of a semiclassical Z configuration. As is seen from the above arguments, such a statement depends on the definition of the electromagnetic field. In the view of the modified definition, Eq. (4), no magnetic field would accompany the neutral-charge configuration.

III. A NONVANISHING ELECTRIC CURRENT

It was originally suggested by Vachaspati [8] that electromagnetic fields may emerge in the electroweak phase transition through the process of spontaneous symmetry breaking. The principal idea is that as the Higgs field magnitude $\rho=(\Phi^\dagger\Phi)^{1/2}$ becomes nonzero in the phase transition, the covariant derivative $\mathcal{D}_\mu\Phi\equiv(\partial_\mu-iA_\mu)\Phi$ cannot remain everywhere zero, because that would imply an inexplicable correlation of phases and gauge fields over distances greater than the causal horizon distance at the time of the phase transition.

In the much simpler case of a global U(1) symmetry (i.e., with the gauge potential A_μ set to zero), an instructive analogy can be made with the phase transition in superfluid He⁴ [20]. When such a system is rapidly quenched, the complex field Φ emerges from the false $\Phi\equiv 0$ ground state attempting to find a new true minimum on the circle $|\Phi|=v$ but is forced to assign values for its phase more quickly than the time it takes information to propagate across the container (given by the speed of ‘‘second sound’’). Gradients of the phase thus appear and, because the fluid velocity is proportional to the gradient of the phase, a flow is generated.

The analogy with the superfluid has sometimes led to the misinterpretation that magnetic fields in the electroweak phase transition are generated only by gradients of the phases of the Higgs field. Recently, it was claimed [13] that the electric current resulting from Higgs gradients is always zero, and that for this reason no magnetic field would be produced during the phase transition due to spontaneous symmetry breaking. As I will explain below, these conclusions were contingent upon using an incomplete expression for the electric current from the Higgs field as well as neglecting the electric current from charged vector bosons.¹ In general the electric current receives contributions both from charged vector fields and from gradients of the phases of the Higgs field. For example, in Sec. IV it is shown that magnetic fields emerge spontaneously in the electroweak phase transition also when no gradients of the Higgs field are present.

Let us begin by considering the gauge-covariant charge operator proposed in Ref. [13],

$$Q = -\frac{1}{2}\hat{\phi}^a\tau^a + \frac{Y}{2}, \quad \hat{\phi}^a = \frac{\Phi^\dagger\tau^a\Phi}{\Phi^\dagger\Phi}, \quad (6)$$

where I define the hypercharge Y of the Higgs doublet to be +1. This operator has the property that $Q\Phi=0$, which can be understood as follows. Due to gauge freedom, one may represent the Higgs field of the vacuum state in any ‘‘coordinate system’’ of choice through applying a gauge transformation to $(0,v)^\top$. This would not constitute an active, physical change of the state, but merely a change of basis of the Lie algebra and its representations. In the unitary gauge the vacuum state is represented by

¹The latter point was also made in Ref. [9].

$$\Phi_0 = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad A_\mu = \partial_\mu \lambda(x) Q, \quad \lambda \in \mathbf{R},$$

$$Q = Q_0 \equiv \frac{1}{2}(1 + \tau^3), \quad (7)$$

where the $u(1)$ ‘‘pure-gauge’’ form of A_μ is the most general expression for which $\mathcal{D}_\mu \Phi_0$ and the field tensors $F_{\mu\nu}^a$ and $F_{\mu\nu}^Y$ vanish. This vacuum state can be equivalently reexpressed as

$$\Phi_0 = \frac{\Phi}{(\Phi^\dagger \Phi)^{1/2}} v, \quad \Phi(x) \text{ arbitrary},$$

$$A_\mu = \partial_\mu \lambda(x) Q - i(\partial_\mu V) V^\dagger \quad (8)$$

through a gauge transformation $\Phi_0 \rightarrow V \Phi_0$ with $V \in \text{SU}(2)$ defined by

$$V = \frac{1}{(\Phi^\dagger \Phi)^{1/2}} [(i\tau^2 \Phi)^* \Phi]. \quad (9)$$

Under this transformation, $Q_0 \rightarrow Q \equiv V Q_0 V^\dagger$. It can be checked that this definition of Q agrees with Eq. (6).

We see that when the charge operator Q is defined covariantly as in Eq. (6), Φ is always proportional to the vacuum Higgs field Φ_0 with a real factor. Thus, Φ in this formulation is always electrically neutral. The end result is a reformulation of the unitary gauge in an arbitrary basis.

Let us now focus on the issue of the electric current. In Ref. [13] a current involving the Higgs field was derived using the relation $j^\nu \equiv -\partial \mathcal{L}_H / \partial A_\nu$, where $\mathcal{L}_H = (\mathcal{D}_\mu \Phi)^\dagger \mathcal{D}^\mu \Phi$ is the Higgs kinetic term in the Lagrangian and A_ν is the Lie-algebra component along Q . The resulting expression,

$$j^\nu = ie[\Phi^\dagger Q \mathcal{D}^\nu \Phi - (\mathcal{D}^\nu \Phi)^\dagger Q \Phi], \quad (10)$$

is zero by virtue of $Q\Phi = 0$. This current, however, is not the electric current, because the A_ν used here is not the vector potential whose curl gives $\mathcal{F}_{\mu\nu}^{\text{em}}$. A vector potential with such a property can in fact be constructed [21]. The electric current corresponding to $\mathcal{F}_{\mu\nu}^{\text{em}}$ is given by Eq. (5) and includes a contribution from gradients of the phases of the Higgs field. Therefore, magnetic fields can be produced by the classically evolving Higgs field in the electroweak phase transition.

Magnetic fields also arise from charged vector-boson currents in the absence of gradients of the Higgs field. In the remainder of this section I shall construct the charged vector fields for an arbitrary choice of Φ in Eq. (8) and proceed to show that they give rise to an electric current which in general is nonzero.

The charged vector-boson fields can be found by determining the $\text{SU}(2) \times \text{U}(1)$ Lie-algebra eigenstates under the adjoint action of the gauge-covariant charge operator Q . After some algebra and using a series of Fierz identities, one can readily verify that

$$[Q, T_\pm] = \pm T_\pm, \quad [Q, T_3] = [Q, 1] = 0, \quad (11)$$

$$[T_3, T_\pm] = \pm T_\pm, \quad [T_+, T_-] = 2T_3, \quad (12)$$

where

$$T_+ \equiv \frac{(-i\Phi^\dagger \tau^2)^\top \Phi^\dagger}{\Phi^\dagger \Phi}, \quad T_- \equiv \frac{\Phi(i\tau^2 \Phi)^\top}{\Phi^\dagger \Phi} = T_+^\dagger,$$

$$T_3 \equiv -\frac{1}{2} \frac{\Phi^\dagger \tau^a \Phi}{\Phi^\dagger \Phi} \tau^a, \quad (13)$$

and $Q = T_3 + Y/2$. Thus T_+ and T_- are the generators of the Lie algebra corresponding to charged field directions. Using $T_\pm = T_1 \pm iT_2$ we can write

$$A_\mu = g \tilde{W}_\mu^a T_a + \frac{g'}{2} \tilde{Y}_\mu 1 + \partial_\mu \lambda(x) Q - i(\partial_\mu V) V^\dagger, \quad (14)$$

where $\tilde{W}_\mu^a = \tilde{Y}_\mu = 0$ corresponds to the vacuum, Eq. (8). Under an $\text{SU}(2)$ gauge transformation $\Phi \rightarrow U\Phi$ the generators T_a , $a = 1, 2, 3, (+, -)$, transform according to the adjoint representation $T_a \rightarrow UT_a U^\dagger$, and it can be shown that the fields \tilde{W}_μ^a are gauge invariant. Furthermore, the field tensor components $\tilde{F}_{\mu\nu}^a = 2\text{Tr}(T_a \mathbb{F}_{\mu\nu})/g$ are invariant under general $\text{SU}(2) \times \text{U}(1)$ gauge transformations.

The important point is that, in general, there will be charged vector-boson fields $\tilde{W}_\mu \equiv (\tilde{W}_\mu^1 - i\tilde{W}_\mu^2)/\sqrt{2}$ and \tilde{W}_μ^\dagger present regardless of what gauge we choose for the vacuum, corresponding to the components of the Lie algebra along T_+ and T_- . I shall now show that these charged fields give rise to an electric current and therefore magnetic fields. First, let us evaluate the electromagnetic field tensor. Inserting the components of A_μ and $\mathbb{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]$ into Eq. (4), one finds after rather lengthy calculations that the derivatives $\partial_\mu T_a$ in the first term cancel against the last term, and we retrieve

$$\mathcal{F}_{\mu\nu}^{\text{em}} = \sin\theta_w (\partial_\mu \tilde{W}_\nu^3 - \partial_\nu \tilde{W}_\mu^3) + \cos\theta_w F_{\mu\nu}^Y. \quad (15)$$

Turning next to the field equation for $\mathcal{F}_{\mu\nu}^{\text{em}}$, Eq. (5), insertion and yet more algebra produces

$$\partial^\mu \mathcal{F}_{\mu\nu}^{\text{em}} = -ie[W^{\mu\dagger} (\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu) - (\mathcal{D}_\mu W_\nu - \mathcal{D}_\nu W_\mu)^\dagger W^\mu] - ie\partial^\mu (W_\mu^\dagger W_\nu - W_\nu^\dagger W_\mu), \quad (16)$$

where, as a final step, the tildes were omitted. This is exactly the expression (3) obtained in the unitary gauge, but without the objectionable last term, as was discussed in the previous section.

I have thus established that the treatment of Ref. [13] is equivalent to a treatment in the unitary gauge, where the Higgs field possesses no angular degrees of freedom. These degrees of freedom are absorbed into the vector bosons.

However, the current from charged vector bosons was omitted in Ref. [13]. In general this current, given by Eq. (16), is nonzero and will give rise to electromagnetic fields. In the next section we shall see an example of how this can happen.

IV. SPONTANEOUS GENERATION OF MAGNETIC FIELDS

Previous descriptions [8,13] of the spontaneous generation of magnetic fields in the electroweak phase transition have borrowed from the analogy with superfluids in that they attribute the magnetic fields to the presence of gradients of phases of the Higgs field. I present here an alternative description of magnetogenesis where the unitary gauge is imposed. In this gauge, there are no angular degrees of freedom of the Higgs field. Instead, the magnetic fields arise from SU(2) and U(1) vector potentials that were already present in the ground state of the symmetric phase. As the SU(2)_L × U(1)_Y symmetry breaks, these vector potentials find themselves having random nonvanishing components along new physical directions of the Lie algebra which are the eigenstates of mass and electric charge.

When the symmetry breaks, unstable nontopological defects such as W strings and Z strings typically form carrying large fluxes of gauge fields. In the core of these defects the Higgs field Φ goes to zero, at which points the unitary gauge is ill defined. For now, I shall consider a region of space where such defects are absent. Nontopological defects will be considered in more detail in Sec. VI.

In the symmetric phase, the vacuum state of the electroweak model is characterized by $\Phi \equiv 0$, $F_{\mu\nu}^a = F_{\mu\nu}^Y = 0$. Surely, in the high-temperature electroweak plasma there will be fluctuations around the vacuum values, but these fluctuations are expected to have a small correlation length of the order $(2\pi T)^{-1}$, and we are primarily interested in a mechanism that may generate magnetic fields correlated on a larger scale. The macroscopic spatial average of $F_{\mu\nu}^a$ and $F_{\mu\nu}^Y$ on such a scale will also vanish, and therefore the Lie-algebra valued vector potential is a Maurer-Cartan form

$$A_\mu = -i(\partial_\mu \Omega) \Omega^\dagger \equiv -i[\partial_\mu U(x)]U(x)^\dagger + \partial_\mu \chi(x), \quad (17)$$

where $\Omega \in \text{SU}(2) \times \text{U}(1)$, $U \in \text{SU}(2)$, and $\chi \in \mathbf{R}$. The group-valued function Ω maps to the group manifold $S^3 \times S^1$, the direct product of a three-sphere and a circle, and is completely arbitrary. Because the energy is independent of the space dependence of $\Omega(x)$, there is no reason that Ω should be uniform over space.

Let us now consider the process of symmetry breaking, and for simplicity use the unitary gauge in the broken state. We shall assume that the temporal component A_0 is a continuous function of the space coordinates and require in addition that the ‘‘electric fields’’ F_{0i}^a and F_{0i}^Y be everywhere finite. Then the spatial components A_i , $i = 1, 2, 3$, are continuous functions of time, and the initial A_i immediately after the phase transition are given by Eq. (17). In general, A_i will not be aligned with the vector potential of the broken-symmetry vacuum, Eq. (7). This would happen only in the special case

when Ω is restricted to the embedded circle $\Omega = e^{i\lambda(x)Q}$ as x covers space. For other choices of Ω , for example, $\Omega = e^{i\xi(x)\tau^1}$, it is easy to check that there will be physical, electrically charged W -boson fields present immediately after the phase transition. The ensuing state is a coherent semiclassical field configuration which cannot be constructed from the new vacuum by perturbative means.

Let us now look at a concrete example of how the magnetic field is generated. The condition

$$0 = F_{kl}^3 \equiv (\partial_k W_l^3 - \partial_l W_k^3) + g(W_k^1 W_l^2 - W_l^1 W_k^2) \quad (18)$$

can be satisfied if both the first term and the second term are nonzero but cancel exactly. The first term enters in the unitary-gauge definition of the magnetic field in the broken phase:

$$A_{kl} = \sin\theta_w (\partial_k W_l^3 - \partial_l W_k^3) + \cos\theta_w F_{kl}^Y, \quad (19)$$

where in our case $F_{kl}^Y = 0$.

The emerging magnetic field can therefore be traced to a ‘‘random’’ partitioning of fields into the two cancelling terms of Eq. (18). In the symmetric phase, these terms had no independent physical meaning, and fields could be moved from one to the other through arbitrary gauge transformations while keeping F_{kl}^3 zero. When the symmetry is broken, the terms take on a new physical meaning. The first term in Eq. (18) has components along A_{kl} as well as along $Z_{kl} = \partial_k Z_l - \partial_l Z_k$. The second term in Eq. (18) can be written

$$ig(W_k^\dagger W_l - W_l^\dagger W_k) \quad (20)$$

in terms of the charged W fields. It is now apparent that there can be no spontaneous generation of magnetic fields in the electroweak phase transition without the simultaneous generation of charged W -boson currents which act as the only source (in the unitary gauge) for that magnetic field. In fact, the field equation for the electromagnetic field in the unitary gauge, when $F_{\mu\nu}^a = 0$, is²

$$\partial^\mu A_{\mu\nu} = -ie\partial^\mu (W_\mu^\dagger W_\nu - W_\nu^\dagger W_\mu). \quad (21)$$

The term on the right-hand side of this equation is the magnetization current corresponding to the anomalous magnetic dipole moment of the W boson [22–24]. The initial magnetic field can therefore be viewed as being entirely comprised of magnetization of the vacuum due to W bosons. This state has previously been investigated in the context of the QCD vacuum [25].

Let us now see explicitly how the two terms of Eq. (18) obtain nonzero values from a random vector potential in the symmetric ground state. Because $F_{\mu\nu}^a = F_{\mu\nu}^Y = 0$ the initial gauge potential must be given by Eq. (17). The most general SU(2) × U(1)-valued function Ω can be written

²It should be noted that when the two terms in Eq. (18) are nonzero, a state with $F_{\mu\nu}^a = 0$ does not remain an exact solution in the broken phase because of the mass terms that appear there.

$$\begin{aligned}\Omega(x) &= e^{i\lambda/2} \begin{pmatrix} e^{i(\lambda/2-\beta)} \cos \omega & -e^{i(\alpha-\lambda/2)} \sin \omega \\ e^{i(\lambda/2-\alpha)} \sin \omega & e^{i(\beta-\lambda/2)} \cos \omega \end{pmatrix} \\ &= e^{i\lambda/2} U, \quad U \in \text{SU}(2).\end{aligned}\quad (22)$$

The $\text{su}(2)$ algebra part of the gauge potential is given by $W_\nu^a \tau^a = -(2i/g)(\partial_\nu U)U^\dagger$. The curl of its components can be calculated from

$$\partial_{[\mu} W_{\nu]}^a \tau^a = \frac{2i}{g} (\partial_{[\mu} U)(\partial_{\nu]} U^\dagger), \quad (23)$$

where $[\mu \cdots \nu]$ indicates antisymmetrization, using the trace identity $\text{Tr} \tau^a \tau^b = 2\delta^{ab}$. One then finds

$$\begin{aligned}\partial_k W_l^3 - \partial_l W_k^3 &= \frac{2}{g} \sin 2\omega (\omega_{[k} \alpha_{,l]} + \omega_{[k} \beta_{,l]} - \omega_{[k} \lambda_{,l]}) \\ &= -g(W_k^1 W_l^2 - W_l^1 W_k^2),\end{aligned}\quad (24)$$

where a comma denotes partial differentiation. Here we see that the two terms of Eq. (18) have the opposite sign and in general assume non-zero values that vary as one changes the group-valued function Ω . One of these terms gives rise to the magnetic field, according to Eq. (19):

$$A_{kl} = \frac{2 \sin \theta_w}{g} \sin 2\omega (\omega_{[k} \alpha_{,l]} + \omega_{[k} \beta_{,l]} - \omega_{[k} \lambda_{,l]}). \quad (25)$$

Thus, in this semiclassical description it is a random vector potential in the symmetric phase that gives rise to a magnetic field in the broken phase. In this sense, the magnetic field was already present in the ground state of the symmetric phase, but took on a different physical meaning after the symmetry was broken and eigenstates of mass and electric charge became well defined.

We must now address the issue of gauge invariance. So far we have used the unitary gauge to calculate the magnetic field resulting from a vector potential A_μ expressed as the Maurer-Cartan form (17). This potential is, however, gauge dependent, so it is necessary to show that the magnetic field generated is independent of our choice of gauge.

Let us therefore pick an arbitrary gauge in which the vector potential in the symmetric phase is some particular function $A_k = -i(\partial_k \Lambda)\Lambda^\dagger$ with $\Lambda \in \text{SU}(2) \times \text{U}(1)$. Then Λ is uniquely determined up to right multiplication ($\Lambda \rightarrow \Lambda M$) by an arbitrary constant group element M . By continuity, the vector potential is the same in the broken phase immediately after the phase transition. In addition, we obtain in the broken phase some isospin orientation of the Higgs field, which in the same gauge can be characterized by the matrix $V \in \text{SU}(2)$ defined by Eq. (9), such that $\Phi = V(0, \rho)^\top$. Now we can evaluate the magnetic field, either directly from the gauge-invariant definition (4), or equivalently by making a gauge transformation to the unitary gauge using the $\text{SU}(2)$ element $V^\dagger = V^{-1}$. In this gauge we obtain the vector poten-

tial $A'_k = -i[\partial_k(V^\dagger \Lambda)](V^\dagger \Lambda)^\dagger \equiv -i(\partial_k \Omega)\Omega^\dagger$, where $\Omega = V^\dagger \Lambda$. Let us now show that the magnetic field resulting from A'_k is gauge invariant.

Under a general gauge transformation g given by $g = e^{i\xi/2} U$ with $U \in \text{SU}(2)$, we have that $\Lambda \rightarrow g\Lambda$, while $V \in \text{SU}(2)$ transforms as $V \rightarrow gVh^\dagger$, where $h \equiv e^{i\xi Q}$ is an electromagnetic $\text{U}(1)$ gauge transformation. Here $Q = (1 + \tau^3)/2$. Therefore, under the full gauge transformation, $\Omega \rightarrow h\Omega$. The electromagnetic part of the vector potential then changes only by a pure gradient, corresponding to the remaining gauge symmetry of the broken phase. Furthermore, A'_k is invariant under the transformation $\Lambda \rightarrow \Lambda M$ for constant M , so this ambiguity in the definition of Λ has no significance. The resulting magnetic field, obtained from Eqs. (22) and (25) with $\Omega = V^\dagger \Lambda$, is therefore independent of which gauge was used originally to express A_k and the Higgs field.

Given a gauge potential A_μ of the form (17) in a unitary gauge, with Ω given by Eq. (22), we may conversely set the gauge potential to zero by means of a gauge transformation with group element Ω^{-1} . The phases will then reappear in the Higgs field, which becomes

$$\Phi = \rho \begin{pmatrix} e^{i\alpha} \sin \omega \\ e^{-i\beta} \cos \omega \end{pmatrix}. \quad (26)$$

The phase λ does not appear here because the broken vacuum still has the electromagnetic $\text{U}(1)$ symmetry.

Therefore, as long as $F_{\mu\nu}^a$ can be considered to vanish, one can give two *equivalent* descriptions of magnetogenesis in two different gauges. (a) In a gauge where all vector potentials are identically zero, the magnetic field arises spontaneously from the angular degrees of freedom of the Higgs field and is given by the last term of Eq. (4). (b) In the unitary gauge, with $\Phi = (0, \rho)^\top$, the initial magnetic field is the result of $\text{SU}(2) \times \text{U}(1)$ vector-potential remnants of the symmetric phase whose associated field tensor finds itself with a nonzero projection along the electromagnetic field after symmetry breaking.

There are several reasons to prefer the second gauge. One is that in this gauge the constant operator $Q = (1 + \tau^3)/2$ defines simple charge eigenstates for all fields, while in the first gauge there is no simple global definition of electric charge. More importantly, the equivalence of gauges holds only as long as $F_{\mu\nu}^a = 0$. As soon as the symmetry breaks, mass terms appear for the charged W bosons and for the Z field, and the fields will start evolving into states with nonzero $F_{\mu\nu}^a$. Vector-field degrees of freedom of this type can no longer be transferred into the Higgs field by a gauge transformation. Even in the simplest case of a $\text{U}(1)$ symmetry, only the longitudinal degree of freedom of the vector field can be exchanged with a phase of the Higgs field, while the transverse degrees of freedom are unaffected by gauge transformations. The vector fields thus contain more dynamical degrees of freedom than does the Higgs field. Therefore, treating the issue of generation of magnetic fields from the point of view of the vector-boson fields is more appropriate.

Having shown that a magnetic field can be generated spontaneously in the phase transition, it remains to determine

the initial strength and correlation length of the field. Estimates of these two quantities are required in order to predict the properties of the magnetic seed field at the onset of galaxy formation.

To begin with, let us assume that the phase transition is second order with critical temperature T_c , so that the Higgs expectation value for $T < T_c$ has the generic temperature dependence $v(T) = v(1 - T^2/T_c^2)^{1/2}$ where $v = 174.1$ GeV, and $100 \text{ GeV} \lesssim T_c \lesssim 300 \text{ GeV}$. The magnetic field will “freeze out,” i.e., become insensitive to thermal fluctuations, at some temperature $T_B < T_c$, where T_B is to be determined in what follows.

Although the correlation length and strength of the magnetic field can be calculated from the gauge-invariant expression (5), the computations simplify considerably in the unitary gauge, where one may use Eq. (16) with $\mathcal{F}_{\mu\nu}^{\text{em}} \equiv A_{\mu\nu}$. Since the electromagnetic field A_μ is massless, no natural scale emerges from the homogeneous part of the equation, $\partial^\mu A_{\mu\nu} = 0$. The correlation length ξ of the magnetic field is instead determined by the source terms of Eq. (16). Because they are at least quadratic in the charged fields W_μ and W_μ^\dagger , and since the correlation length of each of these fields is $M_W^{-1}(T_B)$ at the temperature T_B , we find $\xi = [2M_W(T_B)]^{-1}$.

Next, let us estimate the magnetic field strength. As the temperature is lowered from T_c to T_B , the Higgs potential energy density decreases by the amount $\lambda[v(T_B)]^4 = [v(T_B)]^2[M_H(T_B)]^2/4$, where λ is the quartic Higgs coupling. The lost potential energy is redistributed to the other positive definite terms in the energy density. Let us assume that each such term receives approximately the same fraction $\gamma \sim 10^{-1}$ of the energy density. In particular, we then have

$$|\mathcal{D}_{(\mu)}W_{(\nu)} - \mathcal{D}_{(\nu)}W_{(\mu)}|^2 \sim \frac{1}{2}g^2v^2(T_B)|W_{(\mu)}|^2 \sim \gamma\lambda[v(T_B)]^4, \quad (27)$$

where indices enclosed in parentheses indicate that there is no summation. Here W_μ and $\mathcal{D}_\mu W_\nu$ are defined below Eq. (3). Inserting these estimates and the expression for ξ into Eq. (16) we obtain

$$B \sim \frac{\sin \theta_w}{g} \gamma M_H^2(T_B). \quad (28)$$

In Ref. [26] it was argued that magnetic fields become stable to thermal fluctuations when the temperature drops below the so-called Ginzburg temperature T_G , defined as the temperature below which thermal fluctuations become too weak to restore the symmetry locally. Here we shall provide a more conservative estimate, and assume that the magnetic field freezes out at a temperature $T_B < T_G$ when the typical energy of thermal fluctuations drops below the magnetic field energy contained within a correlated domain of approximate volume ξ^3 . Hence, T_B is determined by the condition $\xi^3 B^2/2 \sim T_B$. Inserting the characteristic temperature dependence of the masses, we obtain for a second-order phase transition

$$\frac{T_B^2}{T_c^2} \sim \left[1 + \frac{T_c^2}{M_W^2} \left(\frac{4g}{\gamma \sin \theta_w} \frac{M_W^2}{M_H^2} \right)^4 \right]^{-1}. \quad (29)$$

Thus, except in the case of a very high value of the Higgs-boson mass, T_B/T_c is not near unity, and masses at temperature T_B are well approximated by their values at zero temperature.

If the phase transition is first order, magnetic fields can still be generated spontaneously within the bubbles of broken phase. The masses for $T < T_c$ are close to their values at zero temperature. From the condition $\xi^3 B^2/2 \sim T_B$ we obtain for a first-order phase transition

$$T_B \sim \left(\frac{\gamma \sin \theta_w}{4g} \frac{M_H^2}{M_W^2} \right)^2 M_W. \quad (30)$$

This temperature approximates that of Eq. (29) in the limit of a low Higgs-boson mass. Note that the magnetic freeze-out temperature T_B is generally lower than the Ginzburg temperature, which may be very close to the critical temperature [9]. This is to be expected, since the magnetic field contains only part of the energy released by the Higgs potential in the phase transition and thus may be destroyed by smaller thermal fluctuations.

In summary, for spontaneously generated magnetic fields in either a first- or second-order electroweak phase transition, the above estimates of the magnetic field strength B and correlation length ξ give

$$B_{\text{sp}} \sim \left[\frac{M_H}{100 \text{ GeV}} \right]^2 \times 10^{22} \text{ G}, \quad \xi_{\text{sp}} \sim 10^{-2} \text{ GeV}^{-1}. \quad (31)$$

These estimates are in rough agreement with those derived in Ref. [8].

V. MAGNETIC FIELDS FROM BUBBLE COLLISIONS

Let us now consider the possibility of forming magnetic fields in the collision of two bubbles of broken vacuum in a first-order electroweak phase transition. Such collisions were investigated in Refs. [27,9] for some special cases. Using the same model as those references for the initial evolution, I shall show here that no magnetic field is generated for arbitrary difference and relative orientation of the Higgs phases of the two bubbles.

For initial times, the Higgs field configurations of two disjoint bubbles of arbitrary shape and size are, respectively,

$$\Phi_i^1(\mathbf{x}) = \begin{pmatrix} 0 \\ \rho_1(\mathbf{x}) \end{pmatrix} \quad \text{and} \quad \Phi_i^2(\mathbf{x}) = \exp \left[i \frac{\theta_0}{2} n^a \tau^a \right] \begin{pmatrix} 0 \\ \rho_2(\mathbf{x}) \end{pmatrix}, \quad (32)$$

where $\hat{\mathbf{n}} = (n^1, n^2, n^3)$ is a constant unit vector. The phases and orientations of the Higgs field within each bubble have equilibrated to constant values. A constant $U(1)_Y$ factor $e^{i\varphi_0}$ was excluded from Φ_i^2 , since φ_0 can be absorbed into

$\theta_0 n^3/2$. Because $n^a \tau^a$ is the only Lie-algebra direction involved, one may write the initial complete Higgs field as [27]

$$\Phi_i(\mathbf{x}) = \exp\left[i \frac{\theta(\mathbf{x})}{2} n^a \tau^a\right] \begin{pmatrix} 0 \\ \rho(\mathbf{x}) \end{pmatrix}. \quad (33)$$

Furthermore, the authors of Refs. [27,9] have assumed that all gauge potentials and their derivatives vanish initially. As we learned in the preceding section, one is free to choose such a gauge as long as the field tensors $F_{\mu\nu}^a$ and $F_{\mu\nu}^Y$ also vanish.

Proceeding as the references, we assume that the above expressions are valid until the two bubbles collide. One may easily evaluate $\hat{\phi}^a$ which may be written as $\hat{\phi} = \cos\theta \hat{\phi}_0 + \sin\theta \hat{n} \times \hat{\phi}_0 + 2 \sin^2(\theta/2) (\hat{n} \cdot \hat{\phi}_0) \hat{n}$ where $\hat{\phi}_0 = (0, 0, -1)^\top$. Then $\partial_\mu \hat{\phi}^a$ takes the particularly simple form $\partial_\mu \hat{\phi} = \partial_\mu \theta \hat{n} \times \hat{\phi}$. The last term of the electromagnetic field $\mathcal{F}_{\mu\nu}^{\text{em}}$ [Eq. (4)] thus vanishes, and since $F_{\mu\nu}^a$ and $F_{\mu\nu}^Y$ are zero, the electromagnetic field vanishes. Similarly, the electric current (5) vanishes.

It is instructive to check this result by transforming the Higgs field into the unitary gauge, using the group-valued function $\Omega = U = \exp[-in^a \tau^a \theta/2]$. This leads to a vector potential of the form (17). It follows easily from Eqs. (23), or alternatively from Eqs. (22) and (25), that their contribution to the magnetic field is zero. From the latter of these equations it is apparent that the phases of our U are rather special, and that there in general would be a magnetic field. The absence of a magnetic field can be traced directly to the fact that the unit vector \hat{n} is a constant or, more precisely, that the Higgs phases depend on only one parameter θ . In Ref. [27] it was proven that the Higgs field in any two-bubble collision can be written in the form (33) for constant n^a . We thus conclude that no magnetic field is generated from the initial classical evolution of the Higgs field in an electroweak two-bubble collision. One should remember, though, that the expression (33) is probably too simplistic to describe what takes place once the bubbles overlap significantly. Magnetic fields could then emerge gradually if \hat{n} develops a spatial dependence.

The present result is in stark contrast to that of the Abelian U(1) model [14,15], in which a field strength is present from the instant of collision. The principal difference is that the U(1) vector field in that model is massive and the corresponding field strength is generated as a result of the coupling of the U(1) field to the Higgs field. In contrast, the electromagnetic U(1) field in the broken electroweak theory is distinguished as that direction of the Lie algebra that does *not* couple to the Higgs field.

In the electroweak theory, in order to generate a magnetic field already at the instant of bubble collision, one would need an initial configuration in which \hat{n} has a spatial dependence, i.e., where the Higgs phases are generated by at least two elements of the Lie algebra. The simplest example of this would be a three-bubble collision [28].

If one relaxes the assumption that the gauge potentials are zero initially, magnetic fields may also emerge spontane-

ously within each bubble by the mechanism described in the previous section. When the presence of the plasma is taken into account, other processes may lead to the creation of magnetic fields. In particular, magnetic fields may stem from the motion of dipole charge layers that develop on bubble walls because of the baryon asymmetry [5]. It is also possible that bubble collisions give rise to field configurations which indirectly produce magnetic fields. This will be investigated in Sec. VI.

Let us now provide some coarse estimates of the strength and correlation length of the magnetic field, assuming only that it arises from some mechanism directly associated with bubble collisions. The growth of nucleated bubbles in the electroweak phase transition has recently been studied numerically by Kurki-Suonio and Laine [29], using model parameters obtained from lattice computations. For a weak first-order transition with $M_H > 68$ GeV, they find that the average radius of the bubbles at the time of collision is $R \lesssim 10^{-7} t_{\text{EW}} \sim 10^6 \text{ GeV}^{-1}$, where t_{EW} is the Horizon scale at the time of the phase transition. Under the assumption that the magnetic field is coherent on the scale of a bubble radius, we may take $\xi \sim R$ as the correlation length. One can then derive a naive estimate of the magnetic field strength, using $\oint A_i dx_i$ as the expression for the magnetic flux enclosed by a loop the size of a bubble. Noting that $A_i \sim \partial_i \vartheta/g$ for some angle ϑ on the Higgs vacuum manifold S^3 , and that the average difference in the value of ϑ between two adjacent bubbles is of the order of π , we obtain $BR^2 \sim \pi/g$, and thus a lower bound on the magnetic field, $B \gtrsim 10^8$ Gauss. The correlation length of this field is much larger, and the strength much smaller, than for fields arising from other mechanisms described in this article. Nevertheless, the results of Ref. [29] indicate that an even weaker first-order phase transition leads to smaller bubbles, and hence a larger magnetic field.

On the other hand, taking into account the effects of the finite conductivity of the plasma after two of the bubbles have initially touched, Ahonen and Enqvist [15] argue, subject to some approximations, that diffusion causes magnetic flux to become concentrated around the expanding circle of most recent intersection of the two bubbles. In this way, they predict a correlation length of $\xi \sim 10^4 \text{ GeV}^{-1}$ and a magnetic field strength of $B \sim 10^{20} \text{ G}$.

We see that the presence of the plasma may have a dramatic effect on the order of magnitude of the magnetic field. A numerical field-theory simulation of multibubble collisions in the full electroweak theory is currently in progress, initially neglecting the plasma. Once the results of this simulation are known and understood, the various plasma effects can be incorporated. It should then become possible to provide more precise predictions of the strength and correlation length of magnetic fields that arise in electroweak bubble collisions.

VI. MAGNETIC FIELDS FROM NONTOPOLOGICAL DEFECTS

It was recently suggested by Grasso and Riotto [9] that magnetic fields may arise from nontopological defects

formed in the electroweak phase transition, such as Z strings [16] and W strings [17]. These are stringlike embedded vortex solutions of the electroweak theory characterized by the winding of a phase of the Higgs field around a core where the Higgs field goes to zero. The core encloses a flux quantum of one of the gauge-field components which attains considerable field strength, since the characteristic width is given by the inverse vector-boson mass. In a $U(1)$ model, these defects are topologically stable, but in the electroweak theory the phase can unwind by slipping over the simply connected vacuum manifold, and the defect decays to the vacuum.

Saffin and Copeland [27] have shown that W -string and Z -string configurations may be generated during bubble collisions in the $SU(2)_L \times U(1)_Y$ theory. In terms of the notation of the previous section, this occurs in the two special cases when the unit vector \hat{n} is perpendicular or parallel to $\hat{\phi}_0$, respectively. In these cases, the effective symmetry group of the problem reduces to $U(1)$, for which vortex production in bubble collisions has been studied earlier [14,15]. In simulations the strings form as circular loops along the circle of intersection of the two bubbles, with the axis of the loop coinciding with the line through the two bubble centers.

There are three important questions that need be answered in connection with the possible generation of magnetic fields from nontopological defects: (1) Do the defects themselves carry magnetic fields? (2) Do the defects contain electrically charged fields which could produce electric currents? (3) Are electromagnetic fields generated when these unstable defects decay? I shall defer the last question to the end of this section and begin instead to address the first two questions. For a reasonable set of definitions, and in the absence of magnetic monopoles, they should be equivalent.

In defiance of such expectations, some surprising results were recently obtained in Ref. [9]. The results seemed to indicate that a magnetic field would always be present along the internal axis of a Z string, which is known to contain only neutral fields. This interpretation was based on the conventional gauge-invariant definition of the electromagnetic field tensor, Eq. (1), which led to the inclusion of the last term of Eqs. (2), (3) in the unitary gauge.

As we have learned in Sec. II, there exist alternative definitions of the electromagnetic field tensor which coincide only when the magnitude of the Higgs field is constant. I have argued that the definitions of the field tensor and electric current given in Eqs. (4) and (5) are more appropriate, in that $\mathcal{F}_{\mu\nu}^{\text{em}}$ always reduces to $A_{\mu\nu}$ in the unitary gauge and electrically neutral fields never serve as sources for the electromagnetic field. Indeed, with the new definitions everything becomes perfectly consistent with naive expectations. In order to illustrate this, let us investigate the field configurations for the Z and W strings in some detail. They can be written in the form

$$\Lambda_\varphi^Z = \frac{m\nu(r)}{r} \begin{pmatrix} \cos 2\theta_w & 0 \\ 0 & -1 \end{pmatrix}, \quad \Phi^Z = \rho(r) \begin{pmatrix} 0 \\ e^{im\varphi} \end{pmatrix} \quad (34)$$

and

$$\Lambda_\varphi^W = \frac{m\tilde{\nu}(r)}{r} \begin{pmatrix} 0 & e^{i\delta} \\ e^{-i\delta} & 0 \end{pmatrix}, \quad \Phi^W = \tilde{\rho}(r) \begin{pmatrix} ie^{i\delta} \sin m\varphi \\ \cos m\varphi \end{pmatrix}, \quad (35)$$

where r, φ are cylindrical coordinates, δ is an arbitrary real number labeling a family of gauge-equivalent W vortex solutions, and m is the integer winding number. Because of its particular phase singularity at $r=0$, there is no nonsingular expression for the W vortex in a gauge, where the upper component of the Higgs field is zero [30].

For the Z -string configuration, we obtain $\hat{\phi}^a = -\delta^{a3}$, and thus the last term of Eq. (4) vanishes. The first two terms combine to give $\sin\theta_w \partial_{[\mu} W_{\nu]}^3 + \cos\theta_w F_{\mu\nu}^Y = 0$ and so $\mathcal{F}_{\mu\nu}^{\text{em}}$ vanishes. With the electric current, Eq. (5), we find that $(D^\mu \hat{\phi})^3 = F_{\mu\nu}^1 = F_{\mu\nu}^2 = 0$, and the last term is just a derivative of the term we previously found to be zero, so there is no electric current.

Next, let us investigate the W -string solution. It is convenient to recognize that it is of the form $\Lambda_\varphi = mn^a \tau^a \tilde{\nu}(r)/r$ and $\Phi = \exp[im\varphi n^a \tau^a][0, \tilde{\rho}(r)]^\top$ for $\hat{n} = (\cos \delta, -\sin \delta, 0)$. Using the method of the previous section, we find $\hat{\phi} = \cos(2m\varphi)\hat{\phi}_0 + \sin(2m\varphi)\hat{n} \times \hat{\phi}_0 + 2\sin^2(m\varphi)(\hat{n} \cdot \hat{\phi}_0)\hat{n}$ where $\hat{\phi}_0 = (0, 0, -1)^\top$. The only nonzero field-tensor components are $F_{r\varphi}^a = [m\tilde{\nu}'(r)/r]n^a$. Because $n^a \hat{\phi}^a \equiv n^a \hat{\phi}_0^a = 0$, we have that the term $\hat{\phi}^a F_{r\varphi}^a = 0$ in Eq. (4) vanishes. In the last term of this equation, one of the factors is $\partial \hat{\phi}^b / \partial r = 0$. Thus $\mathcal{F}_{r\varphi}^{\text{em}}$ vanishes.

The issue of whether there is an electric current is more interesting in the case of the W string, since its gauge fields involve charged fields W_φ^1 and W_φ^2 . On the other hand, also the phases of the Higgs field are charged, as compared with the unitary-gauge vacuum. We find the last term of the current (5) to be zero as before. Since $\partial \hat{\phi}^a / \partial r = 0$ and there is no radial component Λ_r , only the r component of the current may be nonvanishing. We now make use of the relation $\partial_\varphi \hat{\phi} = 2m\hat{n} \times \hat{\phi}$ and can write $(D_\varphi \hat{\phi}) = 2m\{[1 + \nu(r)]/r\}\hat{n} \times \hat{\phi}$. This is perpendicular to \hat{n} , and so the term $(D_\varphi \hat{\phi})^a F_{r\varphi}^a$ vanishes, and there is no electric current.

Although this section has so far only confirmed what was expected, it has served as a nice illustration of the properties and applicability of the new definition of the electromagnetic field tensor $\mathcal{F}_{\mu\nu}^{\text{em}}$. We have established that it works and that it gives results that are reasonable in cases where the conventional definition appears to lead to absurdities.

Finally, I shall discuss the suggestion made in Ref. [9] that magnetic fields may be generated in the decay of Z strings. It is well known that the unstable Z string decays initially through charged W -boson fields [31,30]. The idea is that these W fields form a ‘‘condensate’’ which then in turn would act as a source of magnetic fields. One extremely important caveat is that the presence of W fields is highly transient, as the Z string is known to decay to a vacuum configuration [32]. It is conceivable, however, that the large conductivity of the plasma in the early universe [2,11,33] may cause the magnetic field lines to freeze into the fluid so

that it remains preserved at later times.

The instability of the Z string is a result of the occurrence in the energy density of a term

$$ig \cos \theta_w Z_{12} (W_1^\dagger W_2 - W_2^\dagger W_1) \quad (36)$$

which couples the field strength $Z_{12} = \partial_1 Z_2 - \partial_2 Z_1$ with the magnetic dipole moment of the W boson. The energy is lowered through a suitable alignment of this magnetic moment, corresponding to $W_1 = -iW_2 \equiv W$ for $Z_{12} > 0$. The instability is greatest at the center of the vortex, where Z_{12} is largest and where the W mass term is reduced by the vanishing of the Higgs field. Let us make the simplified assumption that Z_{12} is approximately uniform in the core of the vortex. This is actually justified if the Higgs-boson mass is considerably larger than the Z -boson mass. In such a case, the unstable modes of the W field are well known [22,23]. The mode that peaks in the center of the vortex is given by

$$W(r) = W(0) \exp\left(-\frac{1}{4} g C r^2\right), \quad (37)$$

where $C = \cos \theta_w Z_{12}$. For this mode, it is easy to check that $F_{ij}^1 = F_{ij}^2 = 0$. This is in fact true for any unstable mode [22,23]. Neglecting back reactions on the Higgs field, we still have $\hat{\phi}^a = -\delta^{a3}$. The last term of Eq. (4) evaluates to $2e|W|^2$ which cancels against parts of the first term, leaving $\mathcal{F}_{ij}^{\text{em}} = A_{ij}$ as usual. In the current Eq. (5) something more interesting happens. Since $(D_i \hat{\phi})^3 = 0$, we are left only with the last term, and the equation for the magnetic field can be written

$$\partial_i (\mathcal{F}_{12}^{\text{em}} - 2e|W|^2) = 0. \quad (38)$$

The (nonuniform) magnetic field $B = \mathcal{F}_{12}^{\text{em}}$ is thus entirely comprised of the magnetization from the W bosons. It is apparent that the W bosons initially present in the decay of the Z string do indeed generate a magnetic field.

Let us now compute the strength and correlation length of the magnetic field produced by this mechanism. An upper bound on the magnitude of $|W|^2$ can be obtained by studying the growth of the W field in a fixed Z -string background, which is limited by the quartic term in the energy density, $g^2 |W_1^\dagger W_2 - W_2^\dagger W_1|^2 / 2$. The instability ceases at a maximal value $|W|^2 = \cos \theta_w Z_{12} / g$. From Eq. (38) one then obtains the bound

$$B \lesssim \sin 2\theta_w Z_{12}. \quad (39)$$

To find an estimate for Z_{12} one can use the flux quantization condition $\int d^2x Z_{12} = 4\pi \cos \theta_w / g$, which arises from requiring the covariant derivative of the Higgs field to vanish asymptotically. The integral here is evaluated over a surface perpendicular to the Z string. Assuming that the flux is confined to an approximate cross-sectional area πM_Z^{-2} , we find $Z_{12} \sim 4 \cos \theta_w M_Z^2 / g$, and therefore

$$B \lesssim \frac{8 \cos^2 \theta_w \sin \theta_w}{g} M_Z^2. \quad (40)$$

The growth of W fields, and therefore of a correlated magnetic field, is limited to the region where Z_{12} is large and $\Phi^\dagger \Phi$ is small. These regions have characteristic widths M_Z^{-1} and L_H , respectively, where $L_H^{-1} = \min(M_H, 2M_Z)$ [34,30]. The correlation length of the magnetic field is thus $\xi \sim \min(M_Z^{-1}, L_H) \sim M_Z^{-1}$.

In summary, for magnetic fields generated by decaying nontopological defects, we obtain the following numerical estimates:

$$B_{\text{ntop}} \lesssim 10^{24} \text{ Gauss}, \quad \xi_{\text{ntop}} \sim 10^{-2} \text{ GeV}^{-1}. \quad (41)$$

VII. CONCLUSIONS

The main results of this paper are as follows: I have established that magnetic fields are indeed generated classically from Higgs and gauge fields in the electroweak phase transition through the mere process of spontaneous symmetry breaking, as was originally suggested by Vachaspati [8]. Reformulating the problem in the unitary gauge, I have explicitly constructed the magnetic field thus generated. Previous claims that no such magnetic field is produced were based on an investigation in which an incomplete expression for the electric current from the Higgs field was used, and currents from charged vector bosons were neglected.

Moreover, I have shown that no magnetic field results initially from the classical evolution of the Higgs field in a collision of two bubbles in a first-order electroweak phase transition. This was shown for arbitrary difference and relative orientation of the phases of the Higgs field. The reason is that only one constant direction in the Lie algebra is involved. Nevertheless, one should not exclude the possibility that later evolution of the fields could give rise to magnetic fields. These issues are currently being investigated.

Furthermore, I have pointed out that the notion of an electromagnetic field tensor is ambiguous whenever the magnitude of the Higgs field is not constant. With the conventional gauge-invariant definition, Eq. (1), electrically neutral currents may give rise to electromagnetic fields. In particular, magnetic fields may be present inside electrically neutral configurations such as the Z string. In order to remedy this, I have proposed a different gauge-invariant definition of the electromagnetic field, Eq. (4), which ensures that no electromagnetic fields are generated from neutral sources and which coincides with the other definition for constant Higgs-field magnitude.

The issue of the definition of the electromagnetic field tensor is important for the interpretation and description of physical phenomena, but should have no bearing on the physics, as the various fields evolve independently of how we interpret them. One particular example concerns the simultaneous collision of multiple similar-sized bubbles at the time of percolation, after which the Higgs magnitude is expected to fluctuate violently [35]. In the presence of Z fields one would then conclude from Eq. (3), which follows from definition (1), that electromagnetic fields are created from the gradients of this magnitude. In such a context it is important to realize that any statement about the presence or absence of electromagnetic fields will depend on which definition of the

electromagnetic field tensor is used, and agreement will only be reached if the evolution of all fields is traced to a later time when the Higgs magnitude has assumed a uniform value. Nevertheless, if one makes the assumption that the Higgs field relaxes to a uniform value without exciting any new dynamics in the angular degrees of freedom, the new definition (4) has the property that it predicts the same magnetic field during the fluctuating stage as it does after the fluctuations of the Higgs magnitude have subsided.

Finally, I have verified that a magnetic field is produced in the initial decay of the Z string, as was suggested in Ref. [9]. Although such a field is transient in the pure Yang-Mills-Higgs model, it is conceivable that it may survive until later times due to the high conductivity of the plasma in the early universe.

Estimates of the strength and correlation length of the initial magnetic field have been provided for each of the three mechanisms of production: Spontaneous generation, bubble collisions, and decay of nontopological defects. The subsequent evolution of correlated domains may be calculated according to the recipe presented in Ref. [36]. The correlation length may increase faster than the scale factor $a(t)$ due to the presence of magnetohydrodynamic Alfvén waves [36]. Such waves serve to bring two initially uncorrelated domains into causal contact, so that the magnetic field may untangle and smoothen.

With the possible exception of bubble collisions, one finds in all cases that the correlation length at the time of equal matter and radiation energy densities, $t_{\text{eq}} \sim 10^{11}$ sec, is

smaller than the magnetic diffusion length $l_d \sim 10^{23} \text{ GeV}^{-1}$. This remains true also when enhancement due to Alfvén waves is taken into account. In order to evolve into a seed field of sufficient correlation length and strength at the onset of galaxy formation, fields of such weak correlation may require, depending on how the root-of-mean-square average of the magnetic field is calculated [37], some additional mechanism which stretches the correlation length, such as nonlinear inverse cascade [10].

In the case of bubble collisions it is still an open question whether one may produce a correlated, strong magnetic field without the need to invoke complicated models of magneto-hydrodynamic turbulence such as nonlinear inverse cascade. This issue is likely to be resolved with the results from current and future computer simulations of bubble collisions in the electroweak theory.

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