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$D_s^+ \to \phi\rho^+$ decay

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Motivated by the experimental measurement of the decay rate Γ and longitudinal polarization P_L in the Cabibbo favored decay $D_s^+ \to \phi\rho^+$, we study theoretical predictions within the context of the factorization approximation invoking several form factor models. We obtain agreement with experiment for both Γ and P_L by using experimentally measured values of the form factors $A_1^{D_s\phi}(0)$, $A_2^{D_s\phi}(0)$, and $V^{D_s\phi}(0)$ in the semileptonic decay $D_s^+ \to \phi l^+ \nu_l$. We also include in our calculation the effect of the final state interaction by working with the partial wave amplitudes S, P, and D. A numerical calculation shows that the decay amplitude is dominated by the S wave, and that the polarization is sensitive to the interference between S and D waves. The range of the phase difference $\delta_{SD} = \delta_S - \delta_D$ accommodated by experimental error in P_L is large. [S0556-2821(98)04413-0]

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I. INTRODUCTION

The branching ratio and the longitudinal polarization in $D_s^+ \to \phi\rho^+$ have now been measured:

$$B(D_s^+ \to \phi\rho^+) = (6.7 \pm 2.3)\% \ \ [1],$$

$$P_L(D_s^+ \to \phi\rho^+) \equiv \Gamma_L/\Gamma = (0.370 \pm 0.035 \pm 0.038) \ \ [2]. \tag{1}$$

Theoretically, Gourdin *et al.* [3] studied the ratio

$$R_h \equiv B(D_s^+ \to \phi\rho^+)/B(D_s^+ \to \phi\pi^+) = 1.86 \pm 0.26 \pm^{0.29}_{0.40} \ \ [4]. \tag{2}$$

Within the context of the factorization scheme, which the authors of [3] adopt, this ratio is independent of the normalization of the form factor $A_1(0)$. It depends on the ratios

$$x \equiv A_2(0)/A_1(0), \quad y \equiv V(0)/A_1(0), \tag{3}$$

and the q^2 dependence of the form factors. For the definitions of the form factors, see Wirbel, Stech, and Bauer [5]. No particular model for the form factors was assumed in [3]. Instead, R_h was studied as a function of x and y in three different scenarios for the q^2 dependence of the form factors.

The result of [3] was that the (x,y) domain allowed by R_h was inconsistent with the measurement of x and y from the semileptonic data in Ref. [6], and just barely consistent with that of Ref. [7]. The allowed domain of x and y was also inconsistent with the theoretical prediction of [5]. Reference [3] also concluded that within the factorization scheme, the allowed range of x and y implied the following limits on the longitudinal polarization:

monopole form factors with pole mass 2.53 GeV:

$$0.43 \leq P_L \leq 0.55;$$

monopole form factors with pole mass 3.50 GeV:

$$0.33 \leq P_L \leq 0.55;$$

flat form factors:

$$0.36 \leq P_L \leq 0.55. \tag{4}$$

Subsequently, the authors of [8] incorporated nonfactorized contributions in the decay matrix elements, and using the average of x and y from data [6,7,9], showed that R_h of Eq. (2) and

$$R_{sl} \equiv B(D_s^+ \to \phi l^+ \nu_l)/B(D_s^+ \to \phi\pi^+) = 0.54 \pm 0.10 \ \ [10] \tag{5}$$

could be understood within a scenario where the form factors have a monopole dependence as in [5]. However, there had to be a significant nonfactorization contribution to $D_s^+ \to \phi\pi^+$, though factorization need not be violated in $D_s^+ \to \phi\rho^+$. Reference [8] did not study longitudinal polarization.

An important point to be made is that there are three partial waves in $P \to VV$ decays, S, P, and D, and though the decay rate does not depend on their phases, the longitudinal polarization does depend on the phase difference $\delta_S - \delta_D$. Reference [3] did not consider the effect of partial wave amplitude phases.

In this paper we have studied the data shown in Eq. (1) within the context of factorization invoking several form factor models (to be revealed in the next section), and allowing for nonzero S-, P-, and D-wave phases. This paper is organized as follows: Section II deals with the details and the calculation. A discussion of the results follows in Sec. III.

II. DETAILS OF THE CALCULATION

The decay $D_s^+ \to \rho^+\phi$ is Cabibbo favored and is induced by the effective weak Hamiltonian given by

$$H = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* [C_1(\overline{u}d)(\overline{s}c) + C_2(\overline{u}c)(\overline{s}d)], \tag{6}$$

where $V_{qq'}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements; C_1 and C_2 are the Wilson coefficients. The brackets $(\bar{u}d)$ represent $(V-A)$ color-singlet Dirac bilinears. Fierz transforming in color space with $N_c=3$,

$$(\bar{u}c)(\bar{s}d)=\frac{1}{3}(\bar{u}d)(\bar{s}c)+\frac{1}{2}\sum_{a=1}^{8}(\bar{u}\lambda^a d)(\bar{s}\lambda^a c), \tag{7}$$

the relevant part of the Hamiltonian can be written in the following form:

$$H=\frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^{*}[a_1(\bar{u}d)(\bar{s}c)+C_2O_8], \tag{8}$$

where $a_1=C_1+C_2/3=1.09\pm0.04$ [8] and $O_8=\frac{1}{2}\Sigma_{a=1}^{8}(\bar{u}\lambda^a d)(\bar{s}\lambda^a c)$. λ^a are the Gell-Mann matrices. In a factorization approximation one neglects the contribution from the octet current part O_8, and the matrix element of the first term is written as a product of two current matrix elements. It should be pointed out that there are no W-annihilation or W-exchange terms in $D_s^+\to\rho^+\phi$ decay. However, hairpin graphs are allowed. We neglect them in what follows. The decay amplitude then takes the following form:

$$A(D_s^+\to\rho^+\phi)=\frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^{*}a_1\langle\phi|\bar{s}c|D_s^+\rangle\langle\rho^+|\bar{u}d|0\rangle. \tag{9}$$

Each of the current matrix elements can be expressed in terms of meson decay constants and invariant form factors. We use the following definitions:

$$\langle\rho^+|\bar{u}d|0\rangle=m_\rho f_\rho \varepsilon_\mu^{*}, \tag{10}$$

$$\begin{aligned}\langle\phi|\bar{s}c|D_s\rangle=&\frac{2}{m_D+m_\phi}\epsilon_{\mu\nu\rho\sigma}\varepsilon_\phi^{*\nu}P_D^\rho P_\phi^\sigma V(q^2)\\&+i\Bigg\{\varepsilon_{\phi\mu}^{*}(m_D+m_\phi)A_1(q^2)\\&-\frac{\varepsilon_\phi^{*}.q}{m_D+m_\phi}(P_\phi+P_D)_\mu A_2(q^2)\\&-\frac{\varepsilon_\phi^{*}.q}{q^2}2m_\phi q_\mu A_3(q^2)\\&+\frac{\varepsilon_\phi^{*}.q}{q_2}2m_\phi q_\mu A_0(q^2)\Bigg\},\end{aligned} \tag{11}$$

where $q=P_D-P_\phi$ is the momentum transfer, f_ρ (for which we use 212.0 MeV) is the decay constant of the ρ meson, $\varepsilon_{\phi(\rho)}$ is the polarization vector of the vector mesons $\phi(\rho)$, and $A_i(q^2)$ $(i=1,2,3)$ and $V(q^2)$ are invariant form factors defined in [5]. The decay rate is given by

$$\Gamma(D_s^+\to\rho^+\phi)=\frac{p}{8\pi m_D^2}\{|A_{00}|^2+|A_{++}|^2+|A_{--}|^2\}, \tag{12}$$

where p is the center of mass momentum in the final state. A_{00}, A_{++}, and A_{--} are the longitudinal and transverse helicity amplitudes given by

$$\begin{aligned}A_{00}(D_s^+\to\rho^+\phi)=&-i\frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^{*}m_\rho f_\rho(m_D+m_\phi)\\&\times a_1\{aA_1(m_\rho^2)-bA_2(m_\rho^2)\},\end{aligned} \tag{13}$$

where the parameter a and b are defined as follows:

$$a=\frac{1-r^2-t^2}{2rt},\quad b=\frac{k^2}{2rt(1+r)^2},$$

with

$$r=\frac{m_\phi}{m_D},\quad t=\frac{m_\rho}{m_D},$$

$$k^2=(1+r^4+t^4-2r^2-2t^2-2r^2t^2). \tag{14}$$

The other two helicity amplitudes are

$$\begin{aligned}A_{\pm\pm}(D_s^+\to\rho^+\phi)=&i\frac{G_F}{\sqrt{2}}V_{cs}V_{ud}^{*}m_\rho f_\rho(m_D+m_\phi)a_1\\&\times\left\{A_1(m_\rho^2)\pm\frac{k}{(1+r)^2}V(m_\rho^2)\right\}.\end{aligned} \tag{15}$$

The longitudinal polarization is defined by the ratio of the longitudinal decay rate to the total decay rate:

$$P_L=\frac{\Gamma_{00}}{\Gamma}=\frac{|A_{00}|^2}{|A_{++}|^2+|A_{--}|^2+|A_{00}|^2}. \tag{16}$$

One can work with the helicity amplitudes or the partial wave amplitudes. We prefer to work with the latter as the dependence of the polarization on the partial wave phases is more obvious in that basis. The helicity and partial wave amplitudes are related by [11]

$$A_{00}=-\frac{1}{\sqrt{3}}S+\sqrt{\frac{2}{3}}D,\quad A_{\pm\pm}=\frac{1}{\sqrt{3}}S\pm\frac{1}{\sqrt{2}}P+\frac{1}{\sqrt{6}}D. \tag{17}$$

The partial waves are in general complex and can be expressed in terms of their phases as follows:

$$S=|S|\exp(i\delta_S),\quad P=|P|\exp(i\delta_P),\quad D=|D|\exp(i\delta_D). \tag{18}$$

The decay rate is given by an incoherent sum, $\Gamma\propto|A_{++}|^2+|A_{--}|^2+|A_{00}|^2=|S|^2+|P|^2+|D|^2$, and is independent of the phases. But the polarization does depend on the phase difference $\delta_{SD}=\delta_S-\delta_D$ arising from the interference between S and D waves:

$$P_L=\frac{1}{3}\frac{|S|^2+2|D|^2-2\sqrt{2}|S||D|\cos\,\delta_{SD}}{|S|^2+|P|^2+|D|^2}. \tag{19}$$

TABLE I. Decay rate and longitudinal polarization for $D_s^+ \to \rho^+ \phi$. The values of Γ must be multiplied by 10^{12} s^{-1}. $\delta_{SD} = \delta_S - \delta_D$ is the value needed to get agreement with P_L data to one standard deviation. The last column uses experimentally measured form factors. ''Expt.FF'' stands for ''Experimental form factors.''

	WSBI	WSBII	AW	CDDFGN	ISGW	Expt.FF [1]
Γ	0.32	0.32	0.35	0.15	0.37	0.18 ± 0.04
δ_{SD}	135 ± 45	138 ± 43	122 ± 32	140 ± 40	120 ± 35	134 ± 46
$\frac{\lvert S\rvert}{\lvert P\rvert}$	4.3	3.7	3.8	2.8	5.5	4.7
$\frac{\lvert S\rvert}{\lvert D\rvert}$	11.9	13.5	7.4	8.2	8.6	16.5
Experimental values of Γ and P_L				$\Gamma = 0.14 \pm 0.05$ [1]	$P_L = 0.370 \pm 0.052$ [2]	

S, P, and D waves were calculated first by using the amplitudes (13) and (15) in Eqs. (17) and feeding in the phases by hand as shown in Eq. (18). To continue with the numerical analysis of the decay rate Γ and the longitudinal polarization P_L, we have used form factors from six different sources: (i) Wirbel-Stech-Bauer (WSBI) model [5], where an infinite momentum frame is used to calculate the form factors at $q^2 = 0$, and a monopole form (pole masses are as in [5]) for the q^2 dependence is assumed to extrapolate all the form factors to the desired value of q^2; (ii) the WSBII model is a modification of the WSBI model, where, while $F_0(q^2)$ and $A_1(q^2)$ are the same as in the WSBI model, a dipole q^2 dependence is assumed for $A_2(q^2)$ and $V(q^2)$; (iii) Altomari-Wolfenstein (AW) model [12], where the form factors are evaluated in the limit of zero recoil, and a monopole form is used to extrapolate to the desired value of q^2; (iv) Casalbuoni–Deandrea–Di Bartolomeo–Feruglio–Gatto–Nardulli (CDDFGN) model [13], where the form factors are evaluated at $q^2 = 0$ in an effective Lagrangian satisfying heavy-quark spin-flavor symmetry in which light vector particles are introduced as gauge particles in a broken chiral symmetry; a monopole form is used for the q^2 dependence (we mention here that we have updated this model by using more recent experimental results of the form factors $A_1^{DK^*}(0)$, $A_2^{DK^*}(0)$, and $V^{DK^*}(0)$ [1], and $f_{D_s} = 241 \pm 37$ MeV [14] in calculating the weak couplings constant, of the model, at $q^2 = 0$ [13], which are subsequently used in evaluating the required form factors); (v) Isgur-Scora-Grinstein-Wise (ISGW) model [15], where a nonrelativistic quark model is used to calculate the form factors at zero recoil and an exponential q^2 dependence is used to extrapolate them to the desired q^2; (vi) using experimental values [1] of the form factors at $q^2 = 0$ and extrapolating them using monopole forms.

III. RESULTS AND DISCUSSION

The results are summarized in Table I. For the entries in the last column of Table I we have used the experimental values of the form factors at $q^2 = 0$: $A_1^{D_s\phi}(0) = 0.62 \pm 0.06$, $A_2^{D_s\phi}(0) = 1.0 \pm 0.3$, and $V^{D_s\phi}(0) = 0.9 \pm 0.3$ [1] and extrapolated them with monopole forms. First, we note from Table I that all models, except the CDDFGN model and the one where experimentally measured form factors are used, overestimate the decay rate. This fact arises from an overestimate of the form factor A_1. Reference [16] has noted this fact and attributes it to the imposing of chiral symmetry. Further, as Ref. [17] has argued, more theoretical as well as experimental studies are needed for a better understanding of the q^2 dependence of form factors. Second, we observe that all six sources of form factors allow a range for the polarization which overlaps with experiment with $\delta_{SD} \neq 0$. Note that the polarization is independent of the normalization of A_1. It is also found that most of the final state in the decay $D_s^+ \to \rho^+ \phi$ is in the S wave. It is also seen from Table I that the hierarchy of the partial wave amplitudes is $|S| > |P| > |D|$. If we consider the final state to get a contribution only from the S wave, the decay rates would only be reduced by (5–12)%, while the polarization would be $P_L = 0.33$. The hierarchy of the sizes of the partial wave amplitudes is in accordance with intuitive expectations based on threshold arguments. It is the S-wave dominance which makes an accurate determination of δ_{SD} difficult (the errors in δ_{SD} are large despite the fact that the errors in P_L are small) since the D wave is an order of magnitude smaller than the S wave. The interference term is, consequently, small.

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