Weak phase γ from color-allowed $B\to DK$ rates

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The ratios of partial rates for charged B decays to the recently observed D^0K mode and to the two $D_{CP}K$ final states ($CP=\pm$) are shown to constrain the weak phase $\gamma\equiv\mathrm{Arg}(V^*_{ub})$. The smaller color-suppressed rate, providing further information about the phase, can be determined from these rates alone. Present estimates suggest that, while the first constraints can already be obtained in a high luminosity e^+e^- B factory, measuring the color-suppressed rate would require dedicated hadronic B production experiments.
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The CLEO Collaboration has recently observed the decay $B^-\to D^0K^-$ and its charge conjugate [1]. This is the first observation of a decay mode described by the quark subprocess $b\to c\bar{u}s$ involving the Cabibbo-Kobayashi-Maskawa (CKM) factor $V_{cb}V^*_{us}$. The reported branching ratio, $0.055\pm0.014\pm0.005$, measured relative to $B^-\to D^0\pi^-$, is in agreement with the standard model expectation. The observed decay plays a crucial role in a method proposed some time ago [2,3] to determine the CP violating weak phase γ, the relative phase between $V_{cb}V^*_{us}$ and $V_{ub}V^*_{cs}$. The purpose of this Brief Report is to reexamine this method in view of its importance, and to suggest some variants to overcome its difficulties. A complementary variant was proposed in Ref. [4].

The other processes involved in the method are $B^-\to D_{CP}K^-$, $B^-\to\bar{D}^0K^-$ and their charge conjugates. Partial D decay rates into CP eigenstates (such as K^+K^-) are about an-order-of-magnitude smaller than into states of specific flavor (such as $K^-\pi^+$). Thus, by combining a few CP modes, the decays $B^-\to D_{CP}K^-$ should be observed in near future high statistics experiments. The third process, $B^-\to\bar{D}^0K^-$, mediated by $b\to u\bar{c}s$ and involving the CKM factor $V_{ub}V^*_{cs}$, is harder to measure. It is usually assumed to have a ''color-suppressed'' branching ratio, about two orders of magnitude smaller than that of $B^-\to D^0K^-$. Let us recall the arguments on which this estimate is based.

The effective Hamiltonians for $b\to c\bar{u}s$ and $b\to u\bar{c}s$ transitions are

$$H_{\rm eff}(b\to c\bar{u}s)=\frac{G_F}{\sqrt{2}}V_{cb}V^*_{us}[c_1(\mu)(\bar{s}u)(\bar{c}b)+c_2(\mu)(\bar{c}u)(\bar{s}b)], \quad (1)$$

and

$$H_{\rm eff}(b\to u\bar{c}s)=\frac{G_F}{\sqrt{2}}V_{ub}V^*_{cs}[c_1(\mu)(\bar{s}c)(\bar{u}b)+c_2(\mu)(\bar{u}c)(\bar{s}b)], \quad (2)$$

respectively, where $c_1(m_b)=1.13$, $c_2(m_b)=-0.29$ [5]. $(\bar{c}b)=\bar{c}\gamma^\mu(1-\gamma_5)b$ etc. are left-handed color-singlet quark currents. The ratio of the corresponding CKM factors is $|V_{ub}V^*_{cs}/V_{cb}V^*_{us}|=0.4\pm0.1$ [6]. The hadronic matrix elements of the four-fermion operators, depending on the scale μ, are very difficult to calculate. The conventional description of strangeness-conserving decays such as $B^0\to D^-\pi^+$ assumes that ''color-allowed'' operator matrix elements factorize [7]. Nonperturbative effects, arising from soft gluon exchange [8], require the use of a free parameter to describe decay amplitudes. This parameter, fitted by data, determines the ratio of color-suppressed and color-allowed amplitudes, $a_2/a_1\approx0.26$ [9]. This value depends on unmeasured form factors of B mesons into light mesons for which a model must be assumed. These form factors dominate color-suppressed amplitudes of processes such as $B^0\to\bar{D}^0\pi^0$. Using flavor SU(3) [10], this value of a_2/a_1 can also be used to study $B\to\bar{D}K$ decays. An application to relations between $B\to\bar{D}K$ (given by a $B\to D$ form factor) and $B\to DK$ (given by a $B\to K$ form factor), in which final states carry opposite charm, is more questionable. Nevertheless, one often assumes that

$$r\equiv\frac{|A(B^-\to\bar{D}^0K^-)|}{|A(B^-\to D^0K^-)|}\approx\frac{|V_{ub}V^*_{cs}|}{|V_{cb}V^*_{us}|}\frac{a_2}{a_1}\approx0.1. \quad (3)$$

It is difficult to associate a theoretical uncertainty with this estimate, which is based largely on empirical observations in the $\Delta S=0$ sector, rather than on firm theoretical grounds. We will usually assume that the ratio of amplitudes r cannot be greater or smaller than 0.1 by a factor larger than two. We note, however, that larger values cannot be excluded. As we will see, the precision of determining the weak phase γ improves as r increases. One of the questions addressed in the present report is how to determine this quantity experimentally.

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An essential difficulty in measuring the rate of $B^- \to \bar{D}^0 K^-$ was pointed out by Atwood, Dunietz and Soni [4]. When a $\bar{D}^0$ from $B^- \to \bar{D}^0 K^-$ is identified through its hadronic decay mode (such as $K^+\pi^-$), the decay amplitude interferes with a comparable doubly Cabibbo suppressed decay amplitude of a D^0 from $B^- \to D^0 K^-$. [Here Eq. (3) is assumed.] This forbids a direct measurement of $\Gamma(B^- \to \bar{D}^0 K^-)$. Using two different final states to identify a neutral D meson (e.g. $K^+\pi^-$ and $K^+\pi^-\pi^0$), may allow a determination of r and γ from the branching ratios of these processes and their charge conjugates [4]. The products of corresponding B and D decay branching ratios are expected to be about two orders of magnitude smaller than $B(B^- \to D^0 K^-)B(D^0 \to K^-\pi^+)$, at a level of 10^{-7}. The number of events, expected in future e^+e^- colliders, is likely to be too small to allow a precise determination of r and γ [11]. Such precision can potentially be achieved in dedicated hadronic B production experiments [12], which are expected to yield an order of a few thousand events of this kind [13].

Let us study the information about γ obtained from measuring only the more abundant processes $B^- \to D^0 K^-$, $B^- \to D_{CP} K^-$ and their charge conjugates. We will derive a simple sum rule from which the suppressed rate of $B^- \to \bar{D}^0 K^-$ can, in principle, be determined from the above less suppressed rates, without involving an interference with $B^- \to D^0 K^-$. New constraints on the weak phase γ will be shown to be obtained by measuring only the two ratios of partial decay rates into CP-even and -odd and into flavor states, combining particles and antiparticles. We will look into the prospects of carrying out these studies in a future very high luminosity $e^+e^- B$ factory.

Defining decay amplitudes by their magnitudes, strong and weak phases,

$$A(B^+ \to \bar{D}^0 K^+) = \bar{A} e^{i\bar{\Delta}}, \quad A(B^+ \to D^0 K^+) = A e^{i\Delta} e^{i\gamma}, \tag{4}$$

we find (disregarding common phase space factors)

$$\Gamma(B^+ \to \bar{D}^0 K^+) = \Gamma(B^- \to D^0 K^-) = \bar{A}^2, \tag{5}$$

$$\Gamma(B^+ \to D^0 K^+) = \Gamma(B^- \to \bar{D}^0 K^-) = A^2, \tag{6}$$

$$\Gamma(B^\pm \to D_1 K^\pm) = \frac{1}{2}[\bar{A}^2 + A^2 + 2\bar{A}A \cos(\delta \pm \gamma)], \tag{7}$$

$$\Gamma(B^\pm \to D_2 K^\pm) = \frac{1}{2}[\bar{A}^2 + A^2 - 2\bar{A}A \cos(\delta \pm \gamma)], \tag{8}$$

where $\delta \equiv \Delta - \bar{\Delta}$. $D_{1,2}$ are the two neutral D meson CP eigenstates, $D_{1,2} = (D^0 \pm \bar{D}^0)/\sqrt{2}$.

One obtains the following sum rule:

$$\Gamma(B^- \to D_1 K^-) + \Gamma(B^- \to D_2 K^-) = \Gamma(B^- \to D^0 K^-) + \Gamma(B^- \to \bar{D}^0 K^-). \tag{9}$$

A similar sum rule is obeyed by the charge-conjugated processes. In principle these sum rules allow a determination of $\Gamma(B^- \to \bar{D}^0 K^-) = \Gamma(B^+ \to D^0 K^+)$ from measurements of the other larger rates. Using Eq. (3) we note, however, that the second rate on the right-hand side is expected to be about two orders of magnitude smaller than the first rate. Therefore, a useful determination of $\Gamma(B^- \to \bar{D}^0 K^-)$ requires very precise measurements of $\Gamma(B^- \to D^0 K^-)$ and of $\Gamma(B^- \to D_{CP} K^-)$.

The ratio of amplitudes r can be obtained from the charge-averaged ratio for decays into D meson CP and flavor states:

$$S \equiv \frac{\Gamma(B^+ \to D_1 K^+) + \Gamma(B^- \to D_1 K^-) + \Gamma(B^+ \to D_2 K^+) + \Gamma(B^- \to D_2 K^-)}{\Gamma(B^+ \to \bar{D}^0 K^+) + \Gamma(B^- \to D^0 K^-)},$$

$$S = 1 + r^2. \tag{10}$$

The CP asymmetries of decays into $D_1 K$ and $D_2 K$, normalized by the rate into the D meson flavor state,

$$\mathcal{A}_i \equiv \frac{\Gamma(B^+ \to D_i K^+) - \Gamma(B^- \to D_i K^-)}{\Gamma(B^+ \to \bar{D}^0 K^+) + \Gamma(B^- \to D^0 K^-)}, \quad i = 1,2, \tag{11}$$

are equal in magnitude and have opposite signs. They yield a combined asymmetry

$$\mathcal{A} \equiv \mathcal{A}_2 - \mathcal{A}_1 = 2r \sin\delta \sin\gamma. \tag{12}$$

It is convenient to define two charge-averaged ratios for the two CP eigenstates

$$R_i \equiv \frac{2[\Gamma(B^+ \to D_i K^+) + \Gamma(B^- \to D_i K^-)]}{\Gamma(B^+ \to \bar{D}^0 K^+) + \Gamma(B^- \to D^0 K^-)}, \quad i = 1,2, \tag{13}$$

for which we find

$$R_{1,2} = 1 + r^2 \pm 2r \cos\delta \cos\gamma. \tag{14}$$

The factor of 2 in the numerator of $R_{1,2}$ is used to normalize these ratios to values of approximately one. Rewriting

$$R_{1,2} = \sin^2\gamma + (r \pm \cos\delta \cos\gamma)^2 + \sin^2\delta \cos^2\gamma, \tag{15}$$

one obtains the two inequalities [14]

$$\sin^2\gamma \leqslant R_{1,2}, \quad i = 1,2. \tag{16}$$

The quantities S, $\mathcal{A}$ and R_i hold information from which r, δ and γ can be determined up to discrete ambiguities. r is given by S, and γ is obtained from R_i and $\mathcal{A}$:

$$R_i = 1 + r^2 \pm \sqrt{4r^2\cos^2\gamma - \mathcal{A}^2\cot^2\gamma}. \tag{17}$$

Plots of R_i as function of γ for a few values of r and $\mathcal{A}$, and the precision in r, R_i and $\mathcal{A}$ required to measure γ to a given level, are given in Ref. [15]. The accuracy of this method of determining γ depends on the actual value of r. The larger this ratio, the more precisely can γ be determined.

If r is as small as estimated in Eq. (3), then a useful determination of this quantity from S requires that the rates in the numerator and denominator of Eq. (10) are measured to better than 1% which is unattainable in near future experiments. This demonstrates the difficulty of looking for the color-suppressed process.

Consider, for instance, a sample of 300 million B^+B^- pairs thought to be produced in an upgraded version of the Cornell Electron Storage Ring (CESR) [16]. Using $\mathcal{B}(B^- \to D^0K^-) = 3\times10^{-4}$ [1] and $\mathcal{B}(D^0\to K^-\pi^+) = 0.04$ [17], one expects a total of about 7000 identified D^0K^- and $\bar{D}^0K^+$ events. (Use of other D decay modes compensates for suppression due to detection efficiencies). This would yield a 1.2% measurement of the sum of rates in the denominator of Eq. (10). To estimate the precision of the numerator, in which the D meson decays to CP eigenstates, we use [17] $\mathcal{B}(D^0\to\pi^+\pi^-) + \mathcal{B}(D^0\to K^+K^-) = 6\times10^{-3}$ for two positive CP states. This implies a combined sample of about 1000 identified D_1K^+ and D_1K^- events. (Detection efficiencies may decrease this number somewhat.) Let us assume a similar number of D_2K^+ and D_2K^- events, identified by D decay final states such as $K_S\pi^0$, $K_S\rho^0$ and $K_S\phi$. (The combined decay branching ratio into these states and others are actually larger than into positive CP eigenstates [17], however detection efficiencies are smaller due to the larger number of final particles.) This determines the numerator to within 2.2%, so that the combined statistical error on S is 2.5%. Systematic uncertainties are likely to increase this error, although some of them cancel in the ratio of rates. Assuming that the total error in S is 5%, a 90% C.L. upper limit $r<0.25$ could be obtained from this measurement.

Another way for learning r is by measuring the rate of the rare process $B^-\to(K^+\pi^-)_DK^-$ combined with its charge conjugate. 300 million B^+B^- pairs lead to a few tens of events, for which a large error in the combined branching ratio is expected. The amplitude of this process consists of two interfering contributions carrying an unknown relative phase. The two terms describe the color suppressed process $B^-\to\bar{D}^0K^-$ followed by Cabibbo-allowed D decay, and $B^-\to D^0K^-$ followed by a doubly Cabibbo suppressed D decay. The magnitude of the second amplitude is expected to be known to a few percent at the time of the experiment. Comparison of this amplitude with the measured one could be used to constrain r. In the likely case that the two amplitudes are equal within experimental errors, destructive interference would be assumed to obtain an upper limit on r, $r < 2\sqrt{\mathcal{B}(D^0\to K^+\pi^-)/\mathcal{B}(\bar{D}^0\to K^+\pi^-)} = 0.18$. Here current central values were used and all experimental errors were neglected. This upper limit, increased somewhat by experimental errors in branching ratios, is about the limit obtained from S.

Assuming that r is too small to be measured from S (i.e. $r\leq0.2$), one may still obtain useful constraints on the weak phase γ from the asymmetries $\mathcal{A}_i$ and the two ratios R_i. The information obtained from these pairs of quantities is complementary to each other. While the asymmetries become larger for large values of $\sin\delta\sin\gamma$, the deviation of R_i from $1+r^2\approx1$ increases with $\cos\delta\cos\gamma$. One thousand identified $D_iK^\pm$ events allow a 3σ asymmetry measurement at a level of 10% or larger. For $r\sim0.1$, such asymmetries are expected if δ is sizable, namely $\delta>30°$. It is needless to emphasize the importance of nonzero CP asymmetry measurements, however one should foresee a possibility of small final state phases. Upper limits on the corresponding final state phase difference in $B\to\bar{D}\pi$ decays are already at a level of 20° [18]. Assuming that the final state phase difference between $B\to DK$ and $B\to\bar{D}K$ is not larger, the only information about γ would be derived from R_i.

A particularly interesting case is $R_i<1$, holding for either $i=1$ or $i=2$. Using Eq. (16), this implies new bounds on γ. The condition $R_i<1$ ($i=1$ or 2), equivalent to $|\cos\delta\cos\gamma| > r/2$, holds for values of δ and γ which are not too close to 90 degrees. Taking $\cos\delta\approx1$ and $r\sim0.1$, this condition is fulfilled by all the currently allowed vallues of γ, $30°\leq\gamma\leq150°$ [19], excluding a narrow band around $\gamma=90°$. That is, *for all values outside this narrow band one of the two ratios of rates R_1 or R_2 must be smaller than one*, $R_1<1$ for $\gamma>90°$ and $R_2<1$ for $\gamma<90°$.

Using $r=0.1$, $\delta=0$ in Eq. (14), we find for $\gamma = 150(30)$, 140(40), 130(50), 120(60), 110(70), 100(80) degrees the following values of $R_1(R_2)$: 0.84, 0.86, 0.88, 0.91, 0.94, 0.98, respectively. Measuring these values for R_1 or R_2 would exclude by Eq. (16) the following ranges of γ: 66°–114°, 68°–112°, 70°–110°, 73°–107°, 76°–104°, 81°–99°, respectively. For another choice of parameters, $r=0.2$, $\delta=20°$, the measurements of R_i corresponding to the above values of γ would be 0.71, 0.75, 0.80, 0.85, 0.91, 0.97, respectively. These values exclude somewhat larger ranges of γ than in the case $r=0.1$. Assuming the above number of events, the statistical error of measuring R_1 and R_2 is 3.4%. Particularly interesting is the ratio of rates R_1. The systematic errors in this ratio, in which the numerator and denominator involve similar three charged pion and kaon final states, are expected to cancel. A few percent accuracy in R_i is sufficient for excluding a sizable range of values of γ for the above two choices of parameters. The excluded range grows with increasing values of r, for which smaller values of R_i are obtained.

In conclusion, we have shown that the ratios of rates R_i, for charged B decays to the two $D_{CP}K$ final states and to the already observed D^0K mode, lead to new constraints on the weak phase γ. The smaller color-suppressed rate, which would lend further information about this phase, can be determined from a sum rule involving these rates. The estimate of Eq. (3) suggests that this may be beyond the capability of future e^+e^-B factories, and would have to await dedicated hadronic B production experiments. This method is complementary to the one suggested in Ref. [4]. The two methods seem to be comparable in their statistical power for determining γ from the color-suppressed rate of $B^- \to \overline{D}^0 K^-$, which requires in both cases very high statistics hadronicly produced B experiments. With less statistics, already available in high luminosity e^+e^- experiments, the present method can be used to set new bounds on γ through measurements of the more abundant processes $B^{\pm} \to D_{CP} K^{\pm}$.

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