Possibility of large direct CP violation in $B\to K\pi$-like modes because of long distance rescattering effects and implications for the angle γ

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(Received 17 December 1997; published 8 July 1998)

We consider the strong rescattering effects that can occur in decays such as $B\to K\pi$, $K^*\pi$, $K\rho$, . . . , and their impact on direct CP violation in these modes. First we discuss, in general, how the CPT theorem constrains the resulting pattern of partial rate asymmetries (PRA's) leading to different brands of direct CP violation. Traditional discussions have centered around the absorptive part of the penguin graph which has $\Delta I=0$ in $b\to s$ transitions and as a result causes ''simple'' CP violation; long-distance final state rescattering effects, in general, will lead to a different pattern of CP violation: ''compound'' CP violation. Predictions of simple CP violation are quite distinct from that of compound CP violation. Final state rescattering phases in B decays are unlikely to be small possibly causing large compound CP-violating partial rate asymmetries in these modes. The CPT theorem requires a cancellation of PRA's due to compound CP violation among the $K\pi$ states themselves; thus there can be no net cancellation with other states such as $K^*\pi$, $K\rho$, etc. Therefore, each class of such modes, namely, $K\pi$, $K\rho$, $K^*\pi$, Ka_1, etc., can have large direct CP violation emanating from rescattering effects. Various repercussions for the angle γ are also discussed. [S0556-2821(98)06215-8]

PACS number(s): 11.30.Er, 13.25.Hw

I. INTRODUCTION

Recent evidence from CLEO [1] indicates that the long sought after penguin dominated decays $B^0\to K^+\pi^-$ and $B^+\to K^0\pi^+$ occur with branching ratios (B_r) on the order of 10^{-5}:

$$B_r(\overline{B}^0\to K^-\pi^+)=1.5{}^{+0.5+0.1}_{-0.4-0.1}\pm 0.1\times 10^{-5},$$

$$B_r(B^+\to \overline{K}^0\pi^-)=2.3{}^{+1.1+0.2}_{-1.0-0.2}\pm 0.2\times 10^{-5}, \tag{1}$$

where both modes have been averaged with their conjugates.

Using the short-distance (SD) Hamiltonian [2], there have been several recent theoretical calculations [3] of such exclusive modes. While these calculations are rather unreliable, the relative contributions to these processes from penguin and tree graphs suggest that penguin operators, i.e., $b\to sg^*$, will be the dominant contributors. Nonetheless, tree processes, i.e., $b\to W^*u\to u\overline{u}s$ could be an important feature of these decays through, for example, interference effects with the penguin amplitudes.

In this work we will primarily explore the possibility of relatively large direct CP violation driven by long-distance (LD) rescattering effects in any of the following modes:

$$B^-\to K^-\pi^0,\quad B^-\to \overline{K}^0\pi^-,$$

$$\overline{B}^0\to K^-\pi^+,\quad \overline{B}^0\to \overline{K}^0\pi^0. \tag{2}$$

In order to understand how CP violation will manifest itself in these modes, we first prove general theorems that show how such large effects may come about consistent with the constraints of the CPT theorem. Applying this in the specific case of $K\pi$ final states, we shall see that to have large partial rate asymmetries (PRA's) there must be a significant amount of inelastic rescattering with other light (e.g., $K+n\pi$) states [4]. In particular, it is required that in the case of B^- decay the process $b\to u\overline{u}s$ contribute to the final state $\overline{K^0}\pi^-$ (which has a different quark content, i.e., $d\overline{d}s$). These strong rescattering effects may not in general be reliably calculated, but there are good reasons to believe that at the scale of the B mass their contributions are unlikely to be small.

CP violation emerging from these LD rescattering effects is rather distinct from those governing the effects of penguin transitions, which have been the focus of discussion for many years. In the latter case the final states have $I=1/2$. The rescattering effects, on the other hand, lead to states that are mixtures of $I=1/2$ and 3/2. CPT consideration along with unitarity of the S matrix lead us to categorize these as two brands of CP: simple CP and compound CP violation. It is also useful to further subdivide simple CP violation into two categories type I and type II, to be defined below. The partial rate asymmetry cancellations in the various cases are quite different with rather distinctive experimental predictions.

In particular, CP violation (i.e., simple CP type I) driven by penguin transitions [5] (i.e., $\Delta I=0$) may be regarded as a partial rate asymmetry of the quark level decay $b\to u\overline{u}s$. In this case the partial rate asymmetry cancels with $b\to c\overline{c}s$. At the meson level this leads to a cancellation between $u\overline{u}s$ states such as $K\pi$ and $c\overline{c}s$ states such as $D\overline{D}_s+n\pi$. On the other hand, simple CP violation (type II) arising from rescattering effects, such as in Ref. [4], lead to cancellations

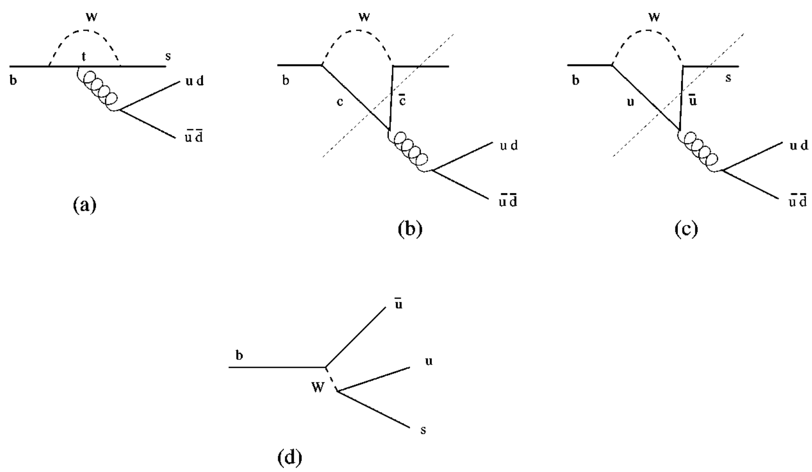

FIG. 1. Lowest order Feynman diagrams for various quark level processes which lead to $B \to K\pi$. (a) A penguin diagram with an intermediate t quark. (b) A penguin diagram with an intermediate c quark. (c) A penguin diagram with an intermediate u quark. (d) A tree graph $b \to u\overline{u}s$. In graphs (c) and (d) cuts are shown where there can be intermediate on-shell states.

between mesonic states of light quark content; for instance, $K\pi$ cancels against $K+n\pi$. Finally compound CP violation is the result of interference between different isospins and can only result in a cancellation of partial rate asymmetry between different charge exchange modes, for example, $B^- \to K^-\pi^0$ cancels against $B^- \to \overline{K}^0\pi^-$.

Using these interference effects we will then try to obtain information about the angle γ of the Cabbibo-Kobayashi-Maskawa (CKM) matrix. Bounds on γ may be deducible either assuming no LD rescattering effects or by assuming a specific value for such effects.

It is important to note that while our discussion is largely in terms of $K\pi$, it applies more generally to similar states which involve only one amplitude. This would include all decays to a kaonic resonance and an isospin 1 meson where one of the two is a scalar or a pseudoscalar, for instance,

$$B \to K^*\pi, \quad K\rho, \quad K(1^+)\pi, \quad Ka_1, \quad K(0^+)\rho, \quad K(0^+)\pi, \quad Ka_2, \quad K^*a_0, \quad \text{etc.} \tag{3}$$

In the case of compound CP violation, the CPT theorem dictates that partial rate asymmetries arising from LD charge exchange rescattering effects in the $K\pi$ system cannot cancel against those of the $K^*\pi$ system (for example). Thus each such class of final state can independently have large direct CP violation emanating from long-distance rescattering effects.

Note also that the PRA's from each of the three sources mentioned in the above discussion are additive. This means that the net PRA in some of the modes given above would be numerically bigger than that due only to compound CP violation, for example; whereas in other cases it would be smaller, as a result of partial cancellations.

Some of these modes have the useful property that the kaonic state is self-tagging in the case where all the mesons are neutral. For instance, in the decay $\overline{B}^0 \to \overline{K}^0\pi$ it is not possible to tell directly whether a B^0 or a $\overline{B}^0$ initially decayed. However in the case $\overline{B}^0 \to \overline{K}^{0*}\pi$ followed by the subsequent decay $\overline{K}^{0*} \to K^-\pi^+$ the final state could only come from a $\overline{B}^0$.

II. QUARK LEVEL PROCESSES

In all of our subsequent discussion, we will be probing the same four quark level processes depicted in Fig. 1. Figures 1(a)–1(c) represent penguin processes all of which mediate the decay $b \to u\overline{u}s(d\overline{d}s)$. Figure 1(d) is a tree process which also gives $b \to u\overline{u}s$.

Each of these quark level processes will take place in a meson where the b quark is combined with a light u or d quark to form a B^- or $\overline{B}^0$ meson. For definiteness, we take the final state to be $K\pi$ so that we have one of the decays in Eq. (2) though the generalization of our discussion to the modes in Eq. (3) is straightforward.

Of primary concern to us is the possibility of CP violation due to the interference of these diagrams. Indeed in the phase convention of Ref. [6] Figs. 1(c), 1(d) have a weak phase of γ with respect to Figs. 1(a) or 1(b). In order to have CP violation manifested in the interference between these graphs, however, it is also necessary that there be a strong rescattering phase.

As pointed out in Ref. [5] one form of rescattering phase which exists at the quark level arises in Fig. 1(b). In this case if the invariant mass of the $u\overline{u}$ is larger than $2m_c$, the indicated cut through the $c\overline{c}$ intermediate state will lead to an imaginary part for this diagram. The diagram in Fig. 1(c), which is the higher order contribution to the rescattering in Fig. 1(d) will likewise generate an absorptive phase. However, as discussed in the context of perturbation theory in Ref. [7] and more generally in Ref. [8], the CPT theorem prevents the diagonal rescattering $u\overline{u} \to u\overline{u}$ from contributing to CP-violating asymmetries at the quark level. In addition this particular higher order correction is likely to be numerically a small contribution to the amplitude.

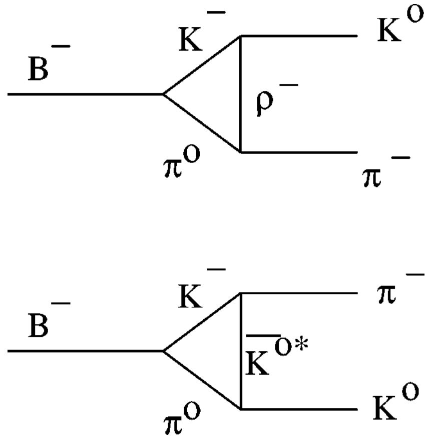

FIG. 2. Examples of meson level diagrams for the rescattering of a $K^-\pi^0$ final state to the final state $\pi^-\overline{K}^0$.

In the paper [9] a model for the contribution of the rescattering effects of Fig. 1(b) to CP violation in $B\to K\pi$ is considered and it is found that the resulting asymmetries are only a few percent, although it must be kept in mind that these calculations have significant uncertainties. In Ref. [10] $K\pi$, KK, and $\pi\pi$ final states were considered in the context of an SU(3) analysis which, as pointed out in Refs. [11,12] implicitly assumed that long distance rescattering of the final state was small. Specifically, in Ref. [10] it was assumed that the tree graph, Fig. 1(d), could not contribute to the decay $B^-\to\overline{K}^0\pi^-$.

If one accepts this assumption, there is an important implication concerning the extraction of the CKM parameter γ from experimental data. As is suggested in Ref. [12], the isospin amplitudes extracted from the relations of Ref. [10] imply that given the total rates for the decays in Eq. (2), even if one ignores CP-violating information (by adding each decay rate to its charge conjugate), one can place a bound on the CP odd angle γ which under some conditions may be quite restrictive when compared with other experimental bounds on γ. In Ref. [13] an interesting suggestion is made (as will be discussed below) that with experimental data only slightly more precise than the current data (1), an upper bound on the value of $\sin^2\gamma$ could be established which would likely be fairly restrictive if the actual rates are similar to the current central values. Unfortunately this bound is based on the assumption, similar to Ref. [10] that either long-distance rescattering effects are not important or that all amplitudes are affected by such rescattering according to a constant factor [14]. We believe there is good reason to think that this assumption may not hold.

Consider, for instance, the meson level Feynman diagrams in Fig. 2 [15]. If one naively calculates this diagram, because of the essentially massless meson exchange in the t channel, one obtains an answer which does not make sense in the context of perturbation theory since contributions become so large that perturbation theory is not trustworthy. In particular one obtains the result that the loop contribution is larger than the initial $B\to K\pi$ amplitude. This would suggest that the LD rescattering phases are unlikely to be small unless there are large cancellations.

In particular, as was also pointed out in Ref. [12], in order for the tree not to contribute to the $\overline{K}^0\pi^-$ final state, there would have to be a remarkable coincidence of such long-distance rescattering effects adjusting both the isospins $I=1/2$ and $I=3/2$ by multiplying them by the same magnitude and phase. Unless the long-distance rescattering phases are vanishingly small it is highly implausible that such effects are independent of isospin.

Indeed, there is an analogous situation in D decays [16], where a similar set of isospin amplitudes govern the Cabibbo allowed decay $D\to K\pi$. Here there are two isospin amplitudes $T_{3/2}$ and $T_{1/2}$ in terms of which the decay amplitudes are

$$A(D^+\to\pi^+\overline{K}^0)=\sqrt{3}T_{3/2},$$

$$A(D^0\to\pi^+K^-)=T_{3/2}+\sqrt{2}T_{1/2},$$

$$A(D^0\to\pi^0\overline{K}^0)=\sqrt{2}T_{3/2}-T_{1/2}. \tag{4}$$

Using the branching ratios from Ref. [17] $\sqrt{\Gamma(D^+\to\pi^+\overline{K}^0)/\Gamma(D^0)}=0.104$, $\sqrt{\Gamma(D^0\to\pi^+K^-)/\Gamma(D^0)}=0.196$, and $\sqrt{\Gamma(D^0\to\pi^0\overline{K}^0)/\Gamma(D^0)}=0.149$ from which one can solve for $|\arg(T_{1/2}T^*_{3/2})|\approx 86°$ [18].

Another example of a D^0 decay which shows how long-distance effects can effectively annihilate one $q\overline{q}$ pair into another is $D^0\to K^0\overline{K}^0$ [19]. In the usual singly Cabibbo suppressed charm decay $c\to s\overline{s}u$, the final quark content of such a D^0 decay is $u\overline{u}s\overline{s}$ which can only form K^+K^- and so some rescattering must be involved in the formation of the $K^0\overline{K}^0$ state. The branching ratio

$$\frac{B_r(D^0\to K^0\overline{K}^0)}{B_r(D^0\to K^+K^-)}\approx 0.3 \tag{5}$$

indicates that such effects are prominent. It is important to emphasize that the annihilation ($u\overline{u}\to d\overline{d}$) in the above process is through long-distance effects and cannot, in general, be calculated reliably through perturbation theory. Therefore, in the analogous decay $B^-\to\overline{K}^0\pi^-$ we must seriously consider the possibility that the tree graph has a substantial contribution to the final state which cannot be estimated by short-distance perturbative methods [15].

One might hope that since the energies in the B decay are much larger than in D, the rescattering effects through any given channel may be greatly reduced. It has, however, been suggested in Ref. [4] that there are in fact many multibody intermediate states which may contribute to the rescattering process and they argue that the cumulative effect of all such states does not decrease with m_B.

The argument of Ref. [4] is essentially as follows. First, from the optical theorem one can relate the forward scattering amplitude of $K\pi$ to the total cross section for $K\pi$:

$$\mathrm{Im}[\mathcal{M}_{K\pi\to K\pi}(s,t=0)]\approx s\sigma(K\pi). \tag{6}$$

It is reasonable to assume that $\sigma(K\pi)$ follows the phenomenological scaling law of other total hadronic cross sections [4,20,21]:

$$\sigma(s)=X(s/s_0)^{0.08}+Y(s/s_0)^{-0.56}, \tag{7}$$

where $s_0\approx 1\ \text{GeV}^2$. From this it follows that the above imaginary part of the amplitude scales similar to

$$\text{Im}(\mathcal{M})\propto s^{1.08}. \tag{8}$$

If we now assume that the behavior of the amplitude $\mathcal{M}$ as a function of t for $t\leqslant 0$ is an exponential decrease $\mathcal{M}\propto\exp(-b|t|)$ (where $b\approx 0.25\ \text{GeV}^{-2}$) then it can be shown that the imaginary part of $\mathcal{M}(B\to\pi K)\propto(M_b^2)^{0.08}$ just from integrating over the $K\pi$ intermediate states. When taking into account a more detailed argument involving Regge theory in the rescattering this is modified only slightly to $\mathcal{M}(B\to\pi K)\propto(M_B^2)^{0.08}/\ln(M_B^2/s_0)$. The point is that there is little scaling with M_B^2.

Since the amplitude for the elastic scattering is dominated by the imaginary part of the amplitude, unitarity of the strong S matrix can be shown to imply that rescattering through other states, such as $K+n\pi$ will give an even bigger contribution to the strong phase of $B\to K\pi$ than the elastic rescattering channel. Thus the elastic rescattering will be large and the inelastic rescattering [22] will be even larger giving rise to a totally incalculable rescattering phase which could well be appreciable even at the scale (m_B) of the B mass. Again, it is important to note that the large contributions to the rescattering amplitude assessed here cannot be estimated via perturbation theory.

In the following sections, we consider the impact of such large phase shifts on the forms of CP violation which can occur in $B\to K\pi$. In particular the proportion of CP violation in pairs of charge exchange modes (e.g., $\bar{K}^0\pi^-$ versus $K^-\pi^0$) tells us about the nature of the rescattering processes involved. First though, let us consider the implications of the CPT theorem in a very general situation where some symmetry of the strong interaction is present.

III. IMPLICATIONS OF THE *CPT* THEOREM: SIMPLE AND COMPOUND *CP* VIOLATION

The CPT theorem is an important prediction of relativistic quantum field theory [23] and indeed all experimental information to date affirm that CPT is an exact symmetry of nature [23]. An important consequence of this theorem is that the total decay rate of a particle A and its antiparticle $\bar{A}$ are identical:

$$\Gamma(A)=\Gamma(\bar{A}). \tag{9}$$

It does not, however, follow that the partial decay rate to a specific final state $\Gamma(A\to X)$ is the same as its CP conjugate $\Gamma(\bar{A}\to\bar{X})$. In fact, defining

$$\Delta\Gamma(A\to X)=\Gamma(A\to X)-\Gamma(\bar{A}\to\bar{X}) \tag{10}$$

(where we will use Δ generally to mean the difference between a quantity and its CP conjugate) if $\Delta\Gamma\neq 0$ CP is clearly violated but CPT need not be. CP violation of the form in Eq. (10) is referred to as a partial rate asymmetry. Clearly, if CPT is not to be violated, all of the different partial rate asymmetries [24] present in a given decay must cancel. Thus there must exist $n\geqslant 2$ states $\{X_1,\ldots,X_n\}$ such that

$$\sum_{i=1}^{n}\Delta\Gamma(A\to X_i)=\Delta\Gamma(A)=0. \tag{11}$$

Let us start by considering the specific case of B^- decay, i.e., $B^-\to K^-\pi^0$ and $B^-\to\bar{K}^0\pi^-$. As explained above, if there is a partial rate asymmetry (PRA) in these modes, they must exchange PRA with some other state and indeed, the state which it exchanges PRA with will depend fundamentally on the mechanism which gives rise to the PRA in the first place.

The state $B^-\to K^-\pi^0$ may exchange partial rate asymmetry in (at least) two specific ways. First, there may be some net exchange of the two states $K^-\pi^0$ and $\bar{K}^0\pi^-$ with some other states and/or PRA in $K^-\pi^0$ may balance against the PRA in $\bar{K}^0\pi^-$. We can characterize these two possibilities with the quantities

$$\begin{aligned}\Delta^+(B^-)&=\Delta\Gamma(B^-\to K^-\pi^0)+\Delta\Gamma(B^-\to\bar{K}^0\pi^-),\\ \Delta^-(B^-)&=\Delta\Gamma(B^-\to K^-\pi^0)-\Delta\Gamma(B^-\to\bar{K}^0\pi^-).\end{aligned} \tag{12}$$

Similarly, for the case of the $\bar{B}^0$ we have

$$\begin{aligned}\Delta^+(\bar{B}^0)&=\Delta\Gamma(\bar{B}^0\to K^-\pi^+)+\Delta\Gamma(\bar{B}^0\to\bar{K}^0\pi^0),\\ \Delta^-(\bar{B}^0)&=\Delta\Gamma(\bar{B}^0\to K^-\pi^+)-\Delta\Gamma(\bar{B}^0\to\bar{K}^0\pi^0).\end{aligned} \tag{13}$$

We will refer to CP-violating effects which cause $\Delta^+\neq 0$ as ''simple CP violation'' since, as we shall see, in this case the exchange is between states of the same isospin while CP violation which causes $\Delta^-\neq 0$ we will refer to as ''compound CP violation'' since it can only result from the interference of two different isospin states. *CP violation which maintains* $\Delta^+=0$ *is pure compound CP violation*. In general, however, it would be expected that both simple and compound CP violations would be present.

There is a further distinction among the states which compensate for Δ^+, namely, that portion which is exchanged with other final states containing only light quarks and that portion which is exchanged with states containing $c\bar{c}$ such as $D\bar{D}_s+n\pi$. Let us define $\Delta^+_{u\bar{u}}$ to be that portion of Δ^+ exchanged with other light quark states such as $K+n\pi$ and $\Delta^+_{c\bar{c}}$ to be that portion exchanged with states containing a $c\bar{c}$. Thus, we can write

$$\Delta^+=\Delta^+_{c\bar{c}}+\Delta^+_{u\bar{u}}. \tag{14}$$

For convenience, we are subdividing simple CP violation further. The case $\Delta^{+}_{c\bar{c}}\neq 0$ is being dubbed type I, whereas $\Delta^{+}_{u\bar{u}}\neq 0$ is type II. The quantities $\Delta^{+}_{u\bar{u}}$ and $\Delta^{+}_{c\bar{c}}$ are not, however, separately experimentally observable. On the other hand, the net exchange between all light quark states and all states containing $c\bar{c}$ which we denote $\Delta^{+}(u\bar{u})$ can be obtained from experiment. We can define this quantity by

$$\Delta^{+}(u\bar{u})=\sum_{i}\Delta\Gamma(X_i^{u\bar{u}}), \tag{15}$$

where the sum is over all states $X_i^{u\bar{u}}$ which contain only light quarks. In effect, $\Delta^{+}(u\bar{u})$ is the partial rate asymmetry for $b\to u\bar{u}s$ and so we expect it to correspond to the perturbative calculation [5] of the partial rate asymmetry exchange between $b\to u\bar{u}s$ and $b\to c\bar{c}s$.

In any case, among the family of $K\pi$ final states (2), the quantities $\Delta^{+}_{u\bar{u}}$ and Δ^{-} do not correspond in a simple way to a quark level perturbative calculation. This is because in terms of purely quark topologies there is no simple compensating process for them provided by states consisting entirely of light quarks. The long-distance rescattering effects which, from the discussion in the last section, may be large, do provide such a mechanism. We will argue later that such LD effects lead to large CP violation of the form $\Delta^{+}_{u\bar{u}}$ and particularly Δ^{-} which may be as big as $O(20\%)$ assuming $\gamma\simeq 90°$.

Let us now consider in very general terms some theorems which we can apply to this case to show how the symmetries of the strong interactions select which kinds of interference effects can contribute to either Δ^{+} or Δ^{-}. One way of understanding the CPT cancellation is to suppose that the Hamiltonian contains a strong piece which is CP invariant and a weak piece with terms with different complex phases (we will consider two different such phases here for the purposes of illustration):

$$\mathcal{H}=\mathcal{H}_s+\mathcal{H}_{w1}e^{i\lambda_1}+\mathcal{H}_{w2}e^{i\lambda_2}+\text{H.c.} \tag{16}$$

In this formulation we refer to the strong force as the force which predominates in the rescattering which in the cases we will be interested in will be generated by QCD. The weak forces are those that cause the initial decay and violate CP which in this case are electro-weak interactions. All of the results we will consider will be to lowest order in the weak interactions.

Clearly we can therefore assume that the strong Hamiltonian does not contain any terms that allow the decay of $A\to X$. Now if T is the weak transition matrix and we expand it to first order in the weak Hamiltonian,

$$\mathcal{A}(A\to X_i)=\langle X_i|T|A\rangle=U_ie^{i\lambda_1}+V_ie^{i\lambda_2}$$

$$\mathcal{A}(\bar{A}\to\bar{X}_i)=\langle\bar{X}_i|T|\bar{A}\rangle=U_ie^{-i\lambda_1}+V_ie^{-i\lambda_2}, \tag{17}$$

where in general U_i and V_i are complex numbers and their phases $\phi_U^i=\arg(U_i)$ and $\phi_V^i=\arg(V_i)$ are usually referred to as strong phases since they may be regarded as being the result of rescattering effects of the strong interaction. Below, we will see in more detail how the unitarity of the S matrix relates these phases to strong effects. The partial rate asymmetry is thus given (up to phase space factors) by

$$\Delta\Gamma(B\to X_i)=-4|U_i||V_i|\sin(\lambda_1-\lambda_2)\sin(\phi_U^i-\phi_V^i). \tag{18}$$

In Ref. [8] it is shown that if there are only two states X_1 and X_2 with partial rate asymmetries, the cancellation embodied in Eq. (11) can be understood through the application of the Cutkoski theorem. In each case the total strong phase results from the rescattering through all possible intermediate states. It can, however, be shown that the part of the rescattering phase difference $\phi_U^1-\phi_V^1$ which is due to the contribution to ϕ_U^1 resulting from rescattering through X_2 is equal and opposite to the contribution to ϕ_V^2 which results from rescattering through X_1. A similar statement applies to the relation between contributions to ϕ_V^2 and ϕ_U^1 and thus Eq. (11) is realized. If there are more than two states, the contribution to the PRA of state X_i resulting from rescattering through X_j cancels the contribution to the partial rate asymmetry of X_j resulting from the rescattering through X_i and thus the requirement of the CPT theorem is affirmed. In this case we will say that X_i exchanges PRA with X_j.

Note that if there are more than two states, it cannot be experimentally determined in detail how much PRA is exchanged between any given pair. For instance, if there are four states, there are only four partial rate asymmetries that may be observed but there are six possible pairs of states which may exchange PRA.

As the above description implies, it is a necessary condition that X_i and X_j can rescatter into each other for them to exchange PRA. In this paper we wish to consider, in rather general terms, the role that the symmetries of the strong interactions play in the pattern of PRA exchanges between different final states.

Let us suppose that R is a Hermitian operator which commutes with the strong Hamiltonian $\mathcal{H}_s$ and that R is invariant under CPT [i.e., $(CPT)R^T(CPT)^\dagger=R$]. Then the eigenspaces corresponding to the various eigenvalues of R will be invariant subspaces under $\mathcal{H}_s$. The decomposition of the various possible final states into the eigenspaces of R will allow us to understand which possible exchanges of partial rate are allowed through the three theorems which we will prove below

Theorem 1. If R is an operator invariant under CPT and $[R,\mathcal{H}_s]=0$ then, to first order in the weak interaction, for each eigenvalue r_i of R, $\Sigma_j\Delta\Gamma(A\to X_j)=0$, where the sum is taken over eigenstates of R with eigenvalue r_i.

Proof. This theorem is a simple generalization of Eq. (11). To prove it let us decompose following the formalism of Ref. [23] and write the S matrix as follows:

$$S=S_s+iT_W. \tag{19}$$

Here S_s is the strong rescattering matrix and is unitary and S_s does not connect the initial state A to the final states so $\langle A|S_s|A\rangle=1$; T_W is the first order transition matrix for the weak interaction.

If we apply unitarity to the above expression, we obtain the standard result (where **1** is the identity matrix):

$$\mathbf{1}=S^\dagger S=S_s^\dagger S_s-iT_W^\dagger S_s+iS_s^\dagger T_W+T_W^\dagger T_W. \tag{20}$$

By unitarity $S^\dagger S=S_s^\dagger S_s=\mathbf{1}$ and so if we drop the last term which is higher order in the weak interactions, we obtain

$$T_W^\dagger=S_s^\dagger T_W S_s^\dagger. \tag{21}$$

If we multiply this expression on the left by a final state $\langle X_i|$ and on the right by $|A\rangle$ we obtain

$$\langle X_i|S_s^\dagger T_W|A\rangle=\langle X_i|T_W^\dagger|A\rangle\equiv\langle A|T_W|X_i\rangle^*. \tag{22}$$

If we apply the CPT invariance to the right-hand side of Eq. (22) and since by assumption $\langle A|S_s|A\rangle=1$, we obtain

$$\langle\overline{X_i}|T_W|\overline{A}\rangle^*=\langle X_i|S_s^\dagger T_W|A\rangle, \tag{23}$$

where the overbar indicates the CPT transform of a given state, i.e., particles are transformed into their antiparticles with their spin degrees of freedom reversed but momentum degrees of freedom the same. This equation is identical to Eq. (1) of Ref. [8].

Since $[R,H_s]=0$, $[R,S_s]=0$ and so if r_i is an eigenvalue of R, the space of eigenvectors $\mathcal{R}_i$ is an invariant subspace of S_s. In particular, if Π_i is the orthogonal projector onto $\mathcal{R}_i$, $[\Pi_i,S_s]=0$. If $\Gamma(A\to\mathcal{R}_i)$ is the total decay rate of A to states in $\mathcal{R}_i$ then

$$\begin{aligned}\Gamma(A\to\mathcal{R}_i)&=\sum_j|\langle X_j|\Pi_i T_W|A\rangle|^2\\&=\langle A|T_W^\dagger\Pi_i T_W|A\rangle=\langle A|T_W^\dagger S_s S_s^\dagger\Pi_i T_W|A\rangle\\&=\langle A|T_W^\dagger S_s\Pi_i S_s^\dagger T_W|A\rangle,\end{aligned} \tag{24}$$

where the sum over j in the above indicates the sum over a complete set of states.

The corresponding decay rate for the antiparticle is

$$\begin{aligned}\Gamma(\overline{A}\to\mathcal{R}_i)&=\sum_j|\langle\overline{X_j}|\Pi_i T_W|\overline{A}\rangle|^2\\&=\sum_j|\langle X_j\Pi_i|S_s^\dagger T_W|A\rangle|^2\\&=\langle A|T_W^\dagger S_s\Pi_i S_s^\dagger T_W|A\rangle\\&=\Gamma(A\to\mathcal{R}_i),\end{aligned} \tag{25}$$

which are therefore identical hence $\Delta\Gamma(A\to\mathcal{R}_i)=0$.

QED. This theorem is more specific than Eq. (11) in that the PRA cancellations to first order in the weak interactions are shown to be between states that can rescatter into each other under the strong interaction. In particular, states that are connected by the strong interactions must share all possible quantum numbers preserved by H_s, for instance, r_i as above.

Even if there is no PRA for any eigenstate of R, partial rate asymmetries may still be present in states that are quantum-mechanical mixtures of such eigenstates. For these mixed states, we can regard the PRA which will be present as being the result of a separate mechanism due to the interference of the two eigenstate channels. As the theorem below shows, this will result in a distinctive pattern of net PRA exchange which we will refer to as compound CP violation since two or more eigenstates of R must be involved. In general both simple and compound CP violations will be present, however, to understand what the features of compound CP violation will be, let us consider the ideal case where no simple CP violation is present, i.e., that for each eigenstate X_i of R, $\Delta\Gamma(A\to X_i)=0$.

In this case, then, let Y be a general state which is a mixture of various eigenstates of R. Let us define $\mathcal{T}(Y)$ to be the smallest invariant subspace R which includes Y. In particular, $\mathcal{T}(Y)$ will be spanned by $\{|Y\rangle,R|Y\rangle,R^2|Y\rangle,\ldots\}$, where the space is exhausted after n terms if Y can be expressed as a linear combination of n eigenstates of R(n will be finite in all examples we will consider). The partial rate asymmetries which may be present in such a case is, however, restricted by the following theorem.

Theorem 2. Let R be an operator invariant under CPT and $[R,\mathcal{H}_s]=0$ and for all eigenstates of R, X_i, $\Delta\Gamma(A\to X_i)=0$. If Y is a state which is not an eigenstate of R and $\Delta\Gamma(A\to Y)\neq 0$ then Y has a net exchange of partial rate asymmetry only with states in $\mathcal{T}(Y)$, where $\mathcal{T}(Y)$ is the smallest invariant subspace of R which contains Y. Equivalently, $\Delta\Gamma[A\to\mathcal{T}(Y)]=0$.

Proof. Let us denote by $\Pi_\mathcal{T}$ the orthogonal projector onto the subspace $\mathcal{T}(Y)$. Since it is an invariant subspace of R, $[R,\Pi_\mathcal{T}]=0$ and thus for all r_i, $[\Pi_i,\Pi_\mathcal{T}]=0$ and in fact $\Pi_i\Pi_\mathcal{T}$ is an orthogonal projector onto the subspace $\mathcal{T}(Y)\cap\mathcal{R}_i$ which we will denote $\mathcal{T}_i(Y)$. Let the states $\{X_j^i\}$ be an orthonormal basis of $\mathcal{T}_i(Y)$. These states are eigenstates of R so by assumption none of these final states has a partial rate asymmetry. Thus

$$\Delta\Gamma[A\to\mathcal{T}(Y)]=\sum_i\sum_j\Delta\Gamma(A\to X_j^i)=0. \tag{26}$$

QED. We can also restate this theorem in terms of the expectation values for observables of the final state. In particular if $\mathcal{O}$ is some observable we define the expectation value $\langle\mathcal{O}\rangle=\langle A|S^\dagger\mathcal{O}S|A\rangle$, $\overline{\langle\mathcal{O}\rangle}=\overline{\langle A|S^\dagger\mathcal{O}S|A\rangle}$, and $\Delta\langle\mathcal{O}\rangle=\langle\mathcal{O}\rangle-\overline{\langle\mathcal{O}\rangle}$.

Theorem 3. If $\mathcal{O}$ is a CPT invariant operator on the final state where $[\mathcal{O},R]=0$ and for all eigenstates X_i of R, $\Delta\Gamma(A\to X_i)=0$ then the expectation value $\Delta\langle\mathcal{O}\rangle=0$.

Proof. This follows if we make an eigenstate decomposition in terms of states that are both eigenstates of R and of $\mathcal{O}$ (since $[R,\mathcal{O}]=0$). Using the CPT invariance of $\mathcal{O}$, we can write this as

$$\mathcal{O}=\sum_k \lambda_k(|X_k\rangle\langle X_k|+|\bar{X}_k\rangle\langle \bar{X}_k|). \tag{27}$$

Since each of the above X_i is an eigenstate of R, by assumption the contribution to $\Delta\langle\mathcal{O}\rangle$ of each of the terms in the above expansion vanishes:

$$\Delta\langle\mathcal{O}\rangle=\sum_k \lambda_k\Delta\langle(|X_k\rangle\langle X_k|)\rangle=0. \tag{28}$$

QED. For our purposes, it is useful to state the above theorem in the following logically equivalent form:

Corollary. Let $\mathcal{O}$ be a CPT-invariant operator. If for all eigenstates X_i of R, $\Delta\Gamma(X_i)=0$, and $[R,H_S]=0$, then $[\mathcal{O},R]\neq 0$ is a necessary condition for $\Delta\langle\mathcal{O}\rangle\neq 0$.

IV. EXAMPLES OF COMPOUND CP VIOLATION

In summary we may restate the implication of the above theorems in terms of what kinds of exchanges of PRA are possible. Thus we have the following possibilities.

(1) If X_1 is a state with a definite quantum number r_i under some symmetry R which is conserved under the strong interactions, it can only exchange PRA with a state X_2 which has the same quantum number r_i. We will refer to this kind of PRA as being simple with respect to R.

(2) If no eigenstate X_i of R has a PRA, then a general state Y can only exchange PRA with other states related to it by the application of R. We will refer to this kind of PRA as being compound with respect to R.

(3) In general there may be both mechanisms of CP violation present. In this case simple CP violation will account for the exchange of PRA between $\mathcal{T}(Y)$ and other states not in $\mathcal{T}(Y)$ while compound CP violation can only lead to exchanges within the states of $\mathcal{T}(Y)$.

Of course these are only necessary conditions for the presence of each kind of partial rate asymmetry; the specifics of the physical situation will determine if either of these kinds of PRA's will actually be present. In this paper we wish to emphasize that even in the case of compound CP violation, partial rate asymmetries can potentially be large.

Before proceeding to our main example, decays of the form $B\to K\pi$, let us consider, for instance, the CP-violating effects discussed in Ref. [25]. Here decays such as $B^-\to\gamma K^*\pi$ and $B^-\to\gamma K\pi$ were considered. The analysis of the CP violation which is discussed in Ref. [25] may be facilitated by labeling which instances are simple or compound with respect to the operator $R=J_h^2$. Here J_h denotes the total angular momentum of the hadronic part of the final state, i.e., the $K\pi$ or $K^*\pi$ system.

The model adopted in that paper is that the production of the final hadronic final state is dominated by kaonic resonances. In this model it is possible that there is a PRA of specific angular momentum states which would correspond to a PRA of the form $\Delta\Gamma(B^-\to\gamma k_i)\neq 0$ for some kaonic resonance k_i. For this to happen, though, there must be at least two states with the same J^{PC} which exchange PRA, which is possible for the two 1^{++} states considered, i.e., $K_1(1270)$ and $K_1(1400)$. This is an explicit example of simple CP violation with respect to the symmetry operator J_h^2

It was shown [25], however, that this effect is likely to be quite small. In fact in the $K\pi$ case it cannot occur in this model since the decay $1^+\to K\pi$ is forbidden. The larger effects which may occur in this system are of the second type where we assume that for each specific eigenstate of J_h^2, i.e., for each specific kaonic resonance channel, there is no PRA but partial rate asymmetries result from the interference of two such channels.

To understand the type of CP violation which is present in the case of $B^+\to\gamma K\pi$, let us define the angle θ between the π and γ in the $K\pi$ rest frame. Clearly a final state with a specific value of θ is, in general a mixture of all possible values of J_h. Thus in accord with theorem 2 even if no single angular momentum state has a PRA, partial rate asymmetries may be exchanged from one value of θ to another. The CP violation will thus be manifested as a difference in the distribution in θ between B^+ and B^- decays even though the rate of decay integrated over θ will be the same for both. This would therefore be an example of compound CP violation with respect to J_h^2.

Let us now consider the case of primary interest, CP violation in the various instances of $B\to K\pi$ listed in Eq. (2). Here, the operator which provides the most useful characterization of various forms of CP violation is the total isospin, thus we take $R=\vec{I}^2$.

The final observed states (e.g., $\bar{K}^0\pi^-$ and $\bar{K}^-\pi^0$) are not pure eigenstates of $\vec{I}^2$ but are linear combinations of $I=1/2$ and $I=3/2$ states. We will denote these eigenstates as $(K\pi)_{1/2}$ and $(K\pi)_{3/2}$.

Following the discussion above, CP violations may cause partial rate asymmetries among the states in Eq. (2) either as simple CP violation in the $(K\pi)_{1/2}$ or $(K\pi)_{3/2}$ channels or compound CP violation due to the interference of these channels. Simple CP violation thus contributes to Δ^+ while compound CP violation contributes to Δ^-.

Suppose that there is only compound CP violation so that $\Delta^+=0$ and $\Delta^-\neq 0$. This would clearly mean that the CP violation in, for instance, $K^-\pi^0$ is exactly compensated by the CP violation in $\bar{K}^0\pi^-$. It would be a mistake, however, to jump to the conclusion that the strong phase involved in this CP violation $K^-\pi^0$ is due to an intermediate $\bar{K}^0\pi^-$ state and vice versa. In general, if n states are present there are $n(n-1)/2$ instances where PRA exchange is possible. If $n\geqslant 3$, observing all n partial rate asymmetries does not fix the $n(n-1)/2$ instances of exchanges. Thus compound CP violation in $K^-\pi^0$ could imply either that the only exchange is with $\bar{K}^0\pi^-$ or that it has some net exchange with various other, perhaps multibody states while $\bar{K}^0\pi^-$ has an equal and opposite exchange with these or similar states. The argument in Ref. [4] together with theorem 2 suggests that the latter case would be the more likely scenario.

As we shall discuss below, it is unlikely that simple CP violation in $(K\pi)_{3/2}$ will be large, i.e., unless electroweak penguins are very significant or physics beyond the standard model makes a large contribution to this decay. If simple CP violation is present in the $(K\pi)_{1/2}$ channel, there are two possible kinds of states which PRA may be exchanged with, either states which contain a $c\bar{c}$ pair such as $D\bar{D}_s+n\pi$ which we will refer to as charm pair states or multibody states that do not contain a $c\bar{c}$ pair but only light quarks such as $K+n\pi$. The first of these is being called simple CP type I and the second is type II.

In the case where the PRA is exchanged with $c\bar{c}$ states, it has been argued [5] that the inclusive sum of the PRA exchange between all $c\bar{c}$ states with all light quark final states may be estimated perturbatively as the quark level PRA exchange between $b\to c\bar{c}s$ and $b\to u\bar{u}s$. Models where this contribution to the PRA of $K\pi$ states has been estimated through simple models of hadronization [9] suggest that it tends to be quite small, i.e., O (a few %) but with large uncertainties. PRA exchange with multibody light quark states cannot be calculated perturbatively. Furthermore, as we have stressed, due to LD effects the rescattering phases involved in $B\to K\pi$ can remain large even at the high mass of the B resulting in large PRA's.

If the LD rescattering phases are large, another important consequence which we wish to emphasize in this paper is that compound CP violation in the modes (2) is also likely to be large. Following the reasoning in Ref. [4] it seems that there is no simple argument which places an *a priori* limit on the size of such effects. We will argue below that it is not unreasonable to have partial rate asymmetries on the order of O (20%) assuming $\sin\gamma\sim 1$, though, again there is no reliable way of calculating the phases that they depend on.

V. CP VIOLATION IN $B\to K\pi$

Let us now consider the physical mechanisms which may produce these simple and compound CP violations in $B\to K\pi$. To do this, we decompose the amplitudes in terms of the weak phases of the theory as in Eq. (17). Each amplitude is therefore expanded in terms of elements of the Cabibbo-Kobayashi-Maskawa matrix as follows:

$$\mathcal{A}=v_t\mathcal{A}_t+v_c\mathcal{A}_c+v_u\mathcal{A}_u,$$
$$\bar{\mathcal{A}}=v_t^*\mathcal{A}_t+v_c^*\mathcal{A}_c+v_u^*\mathcal{A}_u, \tag{29}$$

where

$$v_t=V_{tb}V_{ts}^*,\quad v_c=V_{cb}V_{cs}^*,\quad v_u=V_{ub}V_{us}^*. \tag{30}$$

In this expression, $\mathcal{A}$ is the amplitude for a given $\bar{B}$ meson decay and $\bar{\mathcal{A}}$ is the amplitude for the charge conjugate B decay. As is well known, the equality of the factors $\mathcal{A}_i$ in both the B and $\bar{B}$ amplitudes is a consequence of time reversal invariance of the strong interaction. Complex phases which are present in $\mathcal{A}_i$ are the strong phases due to rescattering. Because of the essential nonperturbative origin of these rescattering phases it is unlikely that they may be accurately calculated.

At the quark level, $\mathcal{A}_t$ is generated by penguin graphs with an internal t quark as show in Fig. 1(a) and related higher order corrections. Likewise $\mathcal{A}_c$ is generated by graphs with an internal c quark as show in Fig. 1(b). The amplitude $\mathcal{A}_u$ may be generated either by a penguin graph with a internal u quark as show in Fig. 1(c) or by a tree graph as shown in Fig. 1(d). Indeed any distinction between these two kinds of contributions is artificial since the penguin graph simply represents the strong rescattering of the tree graph to light quark ($u\bar{u}$ or $d\bar{d}$) states. In addition, unitarity of the CKM matrix implies that $v_t+v_c+v_u=0$ therefore if we add an arbitrary constant (i.e., independent of the quark mass) to each of the amplitudes $\mathcal{A}_i$, i.e., $\mathcal{A}_i\to\mathcal{A}_i+C$, the physics remains unaffected. Which arbitrary constant one adds is purely a matter of convention; for our purpose, we will choose to set $\mathcal{A}_t\to 0$ so that we can write the amplitudes as

$$\mathcal{A}=v_c\hat{\mathcal{A}}_c+v_u\hat{\mathcal{A}}_u,$$
$$\bar{\mathcal{A}}=v_c^*\hat{\mathcal{A}}_c+v_u^*\hat{\mathcal{A}}_u, \tag{31}$$

where $\hat{\mathcal{A}}_c=\mathcal{A}_c-\mathcal{A}_t$ and $\hat{\mathcal{A}}_u=\mathcal{A}_u-\mathcal{A}_t$.

In the approximation of the CKM matrix used in Ref. [6] the phase difference between the CKM angles in Eq. (31) is

$$\arg(v_u^*v_c)\approx\arg(-V_{ud}V_{ub}^*V_{cd}^*V_{cb})=\gamma, \tag{32}$$

where γ is the phase of v_u^* in this convention. This CP odd angle may combine with a strong phase difference between $\hat{\mathcal{A}}_c$ and $\hat{\mathcal{A}}_u$, resulting in CP-violating effects proportional to $\sin\gamma$.

In particular, for each of the $B\to K\pi$ states let us define

$$\sigma=\frac{1}{2}(B_r+\bar{B}_r),\quad \delta=\frac{1}{2}(B_r-\bar{B}_r),\quad x_{cp}=\delta/\sigma, \tag{33}$$

where B_r is the branching ratio for the case involving the b quark while $\bar{B}_r$ is the conjugate involving the anti-b quark and x_{cp} is the partial rate asymmetry (PRA) as it is traditionally defined [24]. In terms of the amplitudes above, in units normalized to the total branching ratio of the B meson,

$$\sigma=|v_u\hat{\mathcal{A}}a_u|^2+|v_c\hat{\mathcal{A}}a_c|^2+2|v_u||v_c|\mathrm{Re}(\hat{\mathcal{A}}_u\hat{\mathcal{A}}_c^*)\cos(\gamma),$$
$$\delta=+2|v_u||v_c|\mathrm{Im}(\hat{\mathcal{A}}_u\hat{\mathcal{A}}_c^*)\sin(\gamma). \tag{34}$$

Let us now write these amplitudes in terms of their isospin components. Here, one must realize that the penguin diagrams Fig. 1(a)–1(c) are $\Delta I=0$ transitions while the tree diagram Fig. 1(d) has both $\Delta I=0$ and $\Delta I=1$ components. The most general form of the amplitudes $\hat{\mathcal{A}}_c$ and $\hat{\mathcal{A}}_u$ allowed by these isospin constraints are thus

$$\hat{\mathcal{A}}_c(K^-\pi^0)=-A,\quad \hat{\mathcal{A}}_u(K^-\pi^0)=-B+\sqrt{2}D,$$
$$\hat{\mathcal{A}}_c(\bar{K}^0\pi^-)=\sqrt{2}A,\quad \hat{\mathcal{A}}_u(\bar{K}^0\pi^-)=\sqrt{2}B+D,$$
$$\hat{\mathcal{A}}_c(K^-\pi^+)=-\sqrt{2}A,\quad \hat{\mathcal{A}}_u(K^-\pi^+)=-\sqrt{2}C+D,$$

$$\hat{\mathcal{A}}_c(\overline{K}^0\pi^0)=A, \quad \hat{\mathcal{A}}_u(\overline{K}^0\pi^0)=C+\sqrt{2}D. \tag{35}$$

Here each of $\{A, B, C, D\}$ is an amplitude which will in general contain a strong phase $\{A, B, C\}$ connect to $I=1/2$ final states of $K\pi$ and D connects to the $I=3/2$ final state of $K\pi$. The assumption that $|\hat{\mathcal{A}}_u(\overline{K}^0\pi^-)|=0$ used in Ref. [13] corresponds to

$$B=-D/\sqrt{2}. \tag{36}$$

As emphasized also in Refs. [12,11] this identity can only hold if B and D have the same phase and, indeed, their magnitudes are required to have the same ratio as would be the case in the absence of any large rescattering effects.

Of course the description in Ref. [4] implies that this will not be the case. On the other hand, in Ref. [14] it is argued that the penguin topologies are an accurate enough description of all QCD rescattering effects in that the phase contributions from long-distance effects average out and such final state interaction effects only modify the magnitude by a constant factor. In particular they suggest that $B^-\to\overline{K}^0\pi^-$ will have its magnitude altered via long-distance contributions but will still receive no tree contribution as such.

In view of the description in terms of the isospin amplitudes, however, this seems unlikely. First of all, the rescattering implicit in Fig. 1(c) only affects the $I=1/2$ amplitudes so if the effects were sizable in terms of even only the magnitude then there would be a significant tree contribution to $B^-\to\overline{K}^0\pi^-$. It is also hard to see how the phase shift in the $I=1/2$ and $I=3/2$ could be locked together. If indeed the phase shifts in the $I=1/2$ and $I=3/2$ fail to be the same as would be implied by Eq. (36), that would in turn imply at the very least, of compound CP violation. It seems therefore more likely that either rescattering effects in the $K\pi$ channel of B decay are generally large, in which case both simple and compound CP violation would be present or else only CP-violating effects proportional to $\Delta_{c\bar{c}}$ are present and the description in Refs. [10,13,14] of the decay $B^-\to\overline{K}^0\pi^-$ is substantially correct.

If we denote the amplitudes for the decays as $m_1=\mathcal{A}(K^-\pi^0)$, $m_2=\mathcal{A}(\overline{K}^0\pi^-)$, $m_3=\mathcal{A}(K^-\pi^+)$ and $m_4=\mathcal{A}(\overline{K}^0\pi^0)$ then we can write the amplitudes as

$$m_1=-Av_c-Bv_u+\sqrt{2}Dv_u,$$

$$\overline{m}_1=-Av_c^*-Bv_u^*+\sqrt{2}Dv_u^*,$$

$$m_2=\sqrt{2}Av_c+\sqrt{2}Bv_u+Dv_u,$$

$$\overline{m}_2=\sqrt{2}Av_c^*+\sqrt{2}Bv_u^*+Dv_u^*,$$

$$m_3=-\sqrt{2}Av_c-\sqrt{2}Cv_u+Dv_u,$$

$$\overline{m}_3=-\sqrt{2}Av_c^*-\sqrt{2}Cv_u^*+Dv_u^*,$$

$$m_4=Av_c+Cv_u+\sqrt{2}Dv_u,$$

$$\overline{m}_4=Av_c^*+Cv_u^*+\sqrt{2}Dv_u^*. \tag{37}$$

Consider first the amplitudes $\{A, B, C\}$ [26] which generate the $K\pi$ states of $I=1/2$ which we will denote (for the $S=-1$ cases) $(K\pi)^0_{1/2}$ and $(K\pi)^-_{1/2}$ (where the superscript indicates the total charge). Substituting into Eq. (34), we obtain the rates for decays to these states and their conjugates described by

$$\begin{aligned}
\frac{1}{3}\sigma[(K\pi)^-_{1/2}] &= |v_c|^2|A|^2+|v_u|^2|B^2| \\
&\quad +2|v_u||v_c||A||B|\cos\gamma\cos\phi_-, \\
\frac{1}{3}\delta[(K\pi)^-_{1/2}] &= +2|v_u||v_c||A||B|\sin\gamma\sin\phi_-, \\
\frac{1}{3}\sigma[(K\pi)^0_{1/2}] &= |v_c|^2|A|^2+|v_u|^2|C^2| \\
&\quad +2|v_u||v_c||A||C|\cos\gamma\cos\phi_0, \\
\frac{1}{3}\delta[(K\pi)^0_{1/2}] &= +2|v_u||v_c||A||C|\sin\gamma\sin\phi_0,
\end{aligned} \tag{38}$$

where $\phi_-=\arg(BA^*)$ and $\phi_0=\arg(CA^*)$.

Clearly this effect is an example of simple CP violation. According to theorem 1, therefore, there must be an $I=1/2$ state which these states exchange PRA with, whether they be $c\bar{c}$ states or light quark states.

In the case of $c\bar{c}$ states we can understand what is happening at the quark level [5] from the schematic Feynman diagrams in Figs. 3(a) and 3(b). In Fig. 3(a) we have a $c\bar{c}$ penguin contributing to $\delta[(K\pi)_{1/2}]$ where the phase difference is generated by the rescattering of $c\bar{c}$ through all possible on-shell states indicated by the cut. This is simple CP type I. In Fig. 3(b) we have the related diagram contributing to $\delta(c\bar{c}s)$ through a $u\bar{u}$ penguin operator; here the cut may include, among other states, the $(K\pi)_{1/2}$ state. The contribution that can be attributed to the $(K\pi)_{1/2}$ state is precisely the one required to balance off the PRA in $(K\pi)_{1/2}$ final states.

Let us turn our attention now to the case of an intermediate state which is composed entirely of light quarks. In this case we must consider all states which have isospin $I=1/2$ (e.g., $K+n\pi$ or $K\eta'+n\pi$, etc.). An exchange of PRA in this case (simple CP type II) will result from a difference, ϕ_- and ϕ_0 above, between the interaction phase of charm penguin processes contributing to A and the predominantly tree processes contributing to B and C. This can occur because the effective Hamiltonian for these two processes at the quark level has a different Dirac structure and so each process will couple differently to different intermediate light quark states giving contributions to ϕ_- and ϕ_0. Diagrammatically, this is shown in Fig. 3(c), where the hexagon represents the contribution of the penguin operator to a multibody intermediate state including a kaon (e.g., $\overline{K}+n\pi$) and the circle represents the tree contribution to the two body

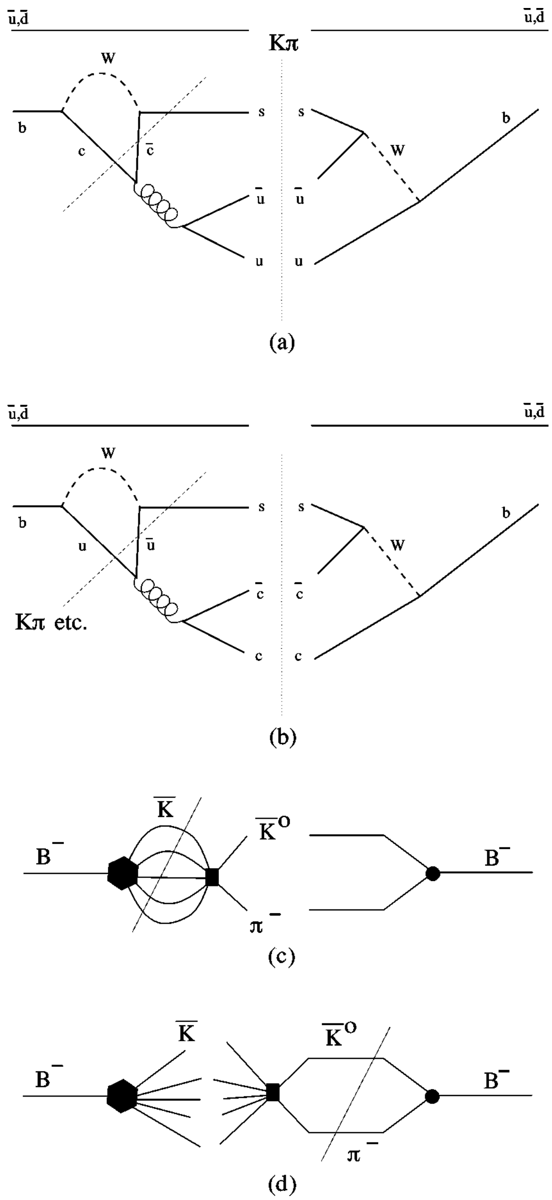

FIG. 3. Quark level Feynman diagrams that contribute to the partial rate asymmetry for for simple CP violation of type I at the quark level, decays involving $b \to u\bar{u}s$ and $b \to c\bar{c}s$, and CP violation of type II at the meson level. (a) shows a penguin contribution which generates a partial rate asymmetry in $(K\pi)_{1/2}$ which is simple CP violation of type I through the interference with the tree diagram where the strong phase of the penguin contribution is generated by the $c\bar{c}$ cut indicated. (b) shows a contribution to the partial rate asymmetry for $b \to c\bar{c}s$ through the interference of a penguin contribution with an internal u quark and the tree. The cut here includes $K\pi$ states and the contribution of those $K\pi$ states will be exactly opposite to the partial rate asymmetry of $K\pi$ in (a). (c) shows a contribution to simple CP violation of type II where the hexagon indicates a penguin process, the circle indicates a tree process, and the box indicates strong rescattering. In this case the intermediate state is a multibody, e.g., $K+n\pi$. (d) shows the process which compensates for the partial rate asymmetry in (c).

state $\overline{K^0}\pi^-$. Since this is simple CP violation, all of the states will be of the same isospin, in this case $I=1/2$. In Fig. 3(d) we show the compensating process which gives an asymmetry to $B^- \to \bar{K}+n\pi$. In the preceding pages we have argued that these simple type-II contributions may be large or, at least, are not bounded in any way.

For the $I=3/2$ final state there can be no simple PRA since the penguin diagrams produce only $I=1/2$ final states. Thus the total PRA summed over all $K\pi$ final states is given by the $(K\pi)_{1/2}$ result above. In particular,

$$\delta(K^-\pi^0)+\delta(\overline{K^0}\pi^-)=\delta[(\overline{K}\pi)^-_{1/2}],$$

$$\delta(K^-\pi^+)+\delta(\overline{K^0}\pi^0)=\delta[(\overline{K}\pi)^0_{1/2}]. \tag{39}$$

Since the physical states that are actually detected are those in Eq. (2) which are mixtures of the isospin eigenstates, compound CP violation becomes possible giving $\Delta^- \neq 0$.

To see this, consider as in theorem 2, what happens in the limit of ϕ_-, $\phi_0 \to 0$, i.e., in the limit that there is no simple CP violation and all of it is compound CP violation. In this case,

$$\begin{aligned}\delta(\overline{K^0}\pi^-) &= -\delta(K^-\pi^0)\\ &= \delta(\overline{K^0}\pi^0) = -\delta(K^-\pi^+)\\ &= 2\sqrt{2}|v_u||v_c||A||D|\sin\gamma\sin\Phi,\end{aligned} \tag{40}$$

where $\Phi=\arg(DA^*)$. The equality in the first line and the second line follow from theorem 2 while the equality of all three lines follows from the general isospin considerations, in particular from the fact that the penguin process here is $\Delta I=0$, again ignoring the effects of electroweak penguin processes.

The isospin structure of the strong penguin process also determines the pattern of the simple CP violation given in Eq. (38). In particular, if *only simple CP violation* (i.e., $\Phi=0$) is present, these equations become

$$\delta(K^-\pi^0)=\frac{1}{2}\,\delta(\overline{K^0}\pi^-)=2|v_u v_c|\sin\gamma\sin\phi_-,$$

$$\delta(\overline{K^0}\pi^0)=\frac{1}{2}\,\delta(K^-\pi^+)=2|v_u v_c|\sin\gamma\sin\phi_0. \tag{41}$$

Another way of expressing the pattern in Eq. (41) is to write it in terms of x_{cp} where $x_{cp}(X_i)=\delta(X_i)/\sigma(X_i)$, i.e., the PRA. If we assume that the penguin processes (i.e., $v_c A$) dominates σ then

$$x_{cp}(K^-\pi^0)=x_{cp}(\overline{K^0}\pi^-),\quad x_{cp}(\overline{K^0}\pi^0)=x_{cp}(K^-\pi^+), \tag{42}$$

where if $B=C$ then all four values of x_{cp} are equal.

If both simple and compound CP violations are present, from the combination of Eqs. (39) and (40) we find that

$$2\,\delta(K^-\pi^0)-\delta(\bar{K}^0\pi^-)-\delta(K^-\pi^+)+2\,\delta(\bar{K}^0\pi^0)=0. \tag{43}$$

Since Eq. (43) came from assuming that the penguin contribution is $\Delta I=0$, a violation of this relation would imply that the light quark pair is not made in a $I=0$ state. If a penguin-type process were generating such a contribution this could mean that instead of being produced via a virtual g^* the quark–anti-quark pair is produced via a γ^* or a Z^* through either unexpectedly large electroweak penguin process or new physics penguin processes with large contributions. Alternatively, tree processes involving, perhaps, extra W bosons, charged or neutral Higgs scalars could also lead to amplitudes with $\Delta I\neq 0$ that could violate Eq. (43).

In any case, in the context of the standard model, the important point to note is that Φ is totally unconstrained by the CPT theorem. Furthermore it is driven by LD rescattering effects in the $K\pi$ system so we cannot say that it is small.

VI. NUMERICAL ESTIMATES

Let us now estimate numerically the largest magnitude of CP-violating effects which might be present. For the purposes of this illustration, we will consider the case where there is no simple CP violation but only compound CP violation $\phi_-=\phi_0=0$. In order to obtain such a rough estimate recall that the relations in Eq. (36) would be true if final state rescattering were turned off. If we assume as suggested by Ref. [12] that the main effect of such rescattering is to adjust the respective isospin amplitudes by a phase, then the magnitudes, but not the phases, obey the relation Eq. (36), $|B|\approx|D|/\sqrt{2}$. The largest CP-violating effects would occur when $\arg(AD^*)\approx 90°$. Let us suppose that $B\approx C$ and define

$$r=|v_u\hat{\mathcal{A}}_u(K^-\pi^+)|/|v_c\hat{\mathcal{A}}_c(K^-\pi^+)|. \tag{44}$$

Thus, we find $|Dv_u/(Av_c)|\approx(\sqrt{2/5})r$ so that

$$\begin{aligned}|x_{cp}(K^-\pi^0)|&=|x_{cp}(\bar{K}^0\pi^0)|\\&=2|x_{cp}(\bar{K}^0\pi^-)|=2|x_{cp}(K^-\pi^+)|\\&\approx\sqrt{2}r\ \sin\gamma\ \sin\Phi.\end{aligned} \tag{45}$$

Thus, if we suppose that $\Phi=\gamma=90°$ then if $r=0.3$ [27], the above yields

$$\begin{aligned}|x_{cp}(K^-\pi^0)|&=|x_{cp}(\bar{K}^0\pi^0)|\approx 0.42,\\|x_{cp}(\bar{K}^0\pi^-)|&=|x_{cp}(K^-\pi^+)|\approx 0.21.\end{aligned} \tag{46}$$

As one can see, the isospin structure determines the pattern of CP violation as discussed in the last section. If the CP violation were simple and if $B=C$, x_{cp} would be the same for each of the four modes assuming the denominator is dominated by the penguin process. On the other hand, for the example of compound CP discussed above, PRA's in $K^-\pi^0$ and $\bar{K}^0\pi^0$ mode are twice that in the $\bar{K}^0\pi^-$ and $K^-\pi^+$ modes [see Eq. (45)].

VII. BOUNDING γ FROM EXPERIMENTAL DATA

Let us now consider how information may be obtained about γ through the measurement of the rates of $B\to K\pi$. First let us consider what may be learned about γ from the two modes that have actually been recently observed (1), namely, $\bar{B}^0\to K^-\pi^+$ and $B^-\to\bar{K}^0\pi^-$. For each of these modes let us define the following parameters which characterize the relative magnitudes of various amplitudes:

$$\begin{aligned}r&=|v_u\hat{\mathcal{A}}_u(K^-\pi^+)|/|v_c\hat{\mathcal{A}}_c(K^-\pi^+)|,\\\rho&=|\hat{\mathcal{A}}_u(\bar{K}^0\pi^-)|/|\hat{\mathcal{A}}_u(K^-\pi^+)|,\\R&=\sigma(K^+\pi^-)/\sigma(\bar{K}^0\pi^-).\end{aligned} \tag{47}$$

In the paper [13] assuming $\rho=0$ it is shown that an accurate measurement of R may lead to a lower bound on $\cos\gamma$ especially if information about r from some other source is known.

This bound comes about since, by isospin symmetry (the $u\bar{u}$ and $d\bar{d}$ pair from the gluon must be in an $I=0$ state)

$$\hat{\mathcal{A}}_c(K^-\pi^+)=\hat{\mathcal{A}}_c(\bar{K}^0\pi^-) \tag{48}$$

and since it is assumed that $\rho=0$,

$$\sigma(\bar{K}^0\pi^-)=|v_c\hat{\mathcal{A}}_c(K^-\pi^+)|^2. \tag{49}$$

Thus the observed ratio R is given by

$$R=1+r^2+2r\ \cos\gamma\ \cos\phi_-. \tag{50}$$

From the fact that $|\cos\phi_-|\leqslant 1$ we can in this case infer that

$$|\cos\gamma|\geqslant\left|\frac{1+r^2-R}{2r}\right|. \tag{51}$$

Since this bound provides a lower bound on $\cos\gamma$, if γ is in the first or second quadrants (which is required by consistency with CP violation in the K_L^0), there is some angle $\gamma_{\max}$ such that only $\gamma\leqslant\gamma_{\max}$ and $\gamma\geqslant\pi-\gamma_{\max}$ are allowed. In Fig. 4 we show the allowed region for γ in the first quadrant as a function of r given the values of $R=0.25$, 0.65, and 1.05 (as shown by the solid curves). The current experimental value is $R=0.65\pm 0.40$. From the graph it is clear that for the smaller values of R there is a lower bound on $\cos\gamma$ independent of any information about r corresponding to the peak in the curve. In fact if $R<1$, then

$$\cos\gamma\geqslant\sqrt{1-R}. \tag{52}$$

As pointed out in Refs. [13,27] one can argue that even the current data from CLEO [1] would indicate an upper

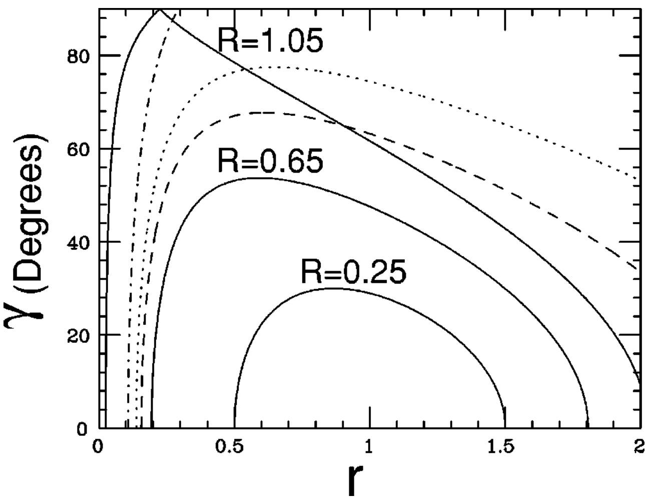

FIG. 4. An example of the bounds that may be obtained on γ from the observation of $\sigma(K^+\pi^-)$ and $\sigma(\pi^-\overline{K}^0)$ under various assumptions as a function of r. In all cases the allowed region is below the curve. The solid curves correspond to the case $\rho=0$ for the values of $R=0.25$, 0.65 and 1.05 as indicated. The dashed curve is the bound given $R=0.65$ and $\rho_{\max}=0.3$, the dotted curve is for $R=0.65$ and $\rho_{\max}=0.5$, while the dot-dashed curve is for $R=0.65$ and $\rho_{\max}=1$.

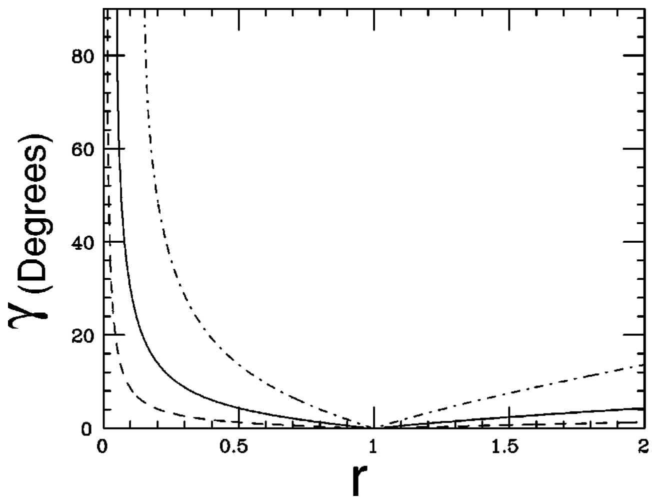

FIG. 5. The bounds that may be obtained on γ from the observation of x_{cp} for some mode $B\to K\pi$. Here the allowed region is above the curves. The bound if $x_{cp}=0.03$ is shown in the dashed curve, the bound if $x_{cp}=0.1$ is shown in the solid curve, and the bound if $x_{cp}=0.3$ is shown in the dot-dashed curve.

bound on r if we assume that $B^\pm\to\pi^\pm\pi^0$ is dominated by tree processes. If this is true, then SU(3) arguments would suggest that

$$r\approx\lambda\,\frac{f_K}{f_\pi}\sqrt{\frac{2\,\sigma(\pi^-\pi^0)}{\sigma(\overline{K}^0\pi^-)}} \tag{53}$$

(where $\lambda=\theta_c$ is one of the CKM parameters from Ref. [6]). Thus given the current bound of $\sigma(\pi^\pm\pi^0)<2\times10^{-5}$ it follows that $r\lesssim0.5$. Factorization arguments in Ref. [13] suggest that $r\approx0.2$ though this estimate has considerable uncertainty.

VIII. GENERAL BOUND IN THE PRESENCE OF LONG-DISTANCE RESCATTERING EFFECTS

It is probably unreasonable to assume that $\rho\to0$. If, however, some argument or indirect evidence allows a bound on ρ to be known, $\rho\leqslant\rho_{\max}$, then a bound on cos γ may still be obtained in some cases if $\rho_{\max}\leqslant1$. This is because

$$(1-r\rho_{\max})^2\leqslant\frac{\sigma(\overline{K}^0\pi^-)}{|v_c\hat{\mathcal{A}}_c(K^+\pi^-)|^2}\leqslant(1+r\rho_{\max})^2 \tag{54}$$

so that

$$\left|\frac{1+r^2-R(1+r\rho_{\max})^2}{2r}\right|\leqslant|\cos\,\gamma|\quad\text{if}\quad 1+r^2\geqslant(1+r\rho_{\max})^2R,$$

$$\left|\frac{1+r^2-R(1-\rho_{\max}r)^2}{2r}\right|\leqslant|\cos\,\gamma|\quad\text{if}\quad 1+r^2\leqslant(1-r\rho_{\max})^2R. \tag{55}$$

If $(1+r\rho_{\max})^2R>1+r^2>(1-r\rho_{\max})^2R$ then there is no bound on cos γ. If $\rho_{\max}=0.3$ the bounds for various values of R are shown in Fig. 5 with dashed lines.

There is some prospect of obtaining information about the value of ρ through the study of the analogous process $B^0\to K^+K^-$. In this case neither a tree decay nor a penguin decay may lead to the final state quark content $u\bar{u}s\bar{s}$. The tree decay $b\to u\bar{u}d$ can, however, produce, for instance, a $\pi\pi$ state that can rescatter to K^+K^- and likewise a penguin decay $b\to s\bar{s}d$, $u\bar{u}d$, and $d\bar{d}d$ can lead to a $\pi\pi$ or $K^0\overline{K}^0$ state which may rescatter to K^+K^-. Thus, by comparing the rate of $B^0\to K^+K^-$ to $K^0\overline{K}^0$, $\pi^+\pi^-$, or $\pi^0\pi^0$, it may be possible to put a bound on ρ. In particular if $B^0\to K^+K^-$ is much smaller than the other processes then the assumption of Refs. [13,14] would be vindicated.

IX. CONSTRAINTS ON γ VIA DIRECT CP VIOLATION IN $B\to K\pi$

In view of the fact that it may not be possible to derive a bound on ρ, it would be useful to have another way to find a bound on γ. If CP violation is discovered in any of the four modes $B\to K\pi$ (i.e., $\delta\neq0$) then a lower bound can be placed on sin γ.

To understand how this works, suppose that γ and r were known. Then, the system of equations (34) can be solved for a positive real value of $|v_u\hat{\mathcal{A}}_u|$ if and only if

$$|\sin\,\gamma|\geqslant\frac{x_{cp}}{2\sqrt{1-x_{cp}^2}}\left|\frac{1-r^2}{r}\right|, \tag{56}$$

where $x_{cp}=\delta/\sigma$. Thus, if γ is in the first or second quadrant this bound will mean that there is a value of $\gamma_{\min}$ such that only $\gamma_{\min}\leqslant\gamma\leqslant\pi-\gamma_{\min}$ is allowed as a function of r. In Fig. 5 we show this bound as a function of r in the first quadrant.

From Eq. (56) (see Fig. 5) if $r=1$ there is no lower bound on $|\sin\gamma|$. This corresponds to a situation where the penguin and tree diagrams happen to almost exactly cancel so that a small value of γ is amplified to a large value of δ due to almost total destructive interference. Since r is likely to be smaller than 1, this singular configuration is probably not a problem, future experimental measurement of $\pi\pi$ modes together with SU(3) arguments should help to clarify what a reasonable value of r is. If an overall upper bound on the value of r, $r\leqslant r_{\rm max}\leqslant 1$ is known, then the lower bound on $|\sin\gamma|$ for all values of $r\leqslant r_{\rm min}$ will be obtained by substituting $r_{\rm min}$ into Eq. (56). A similar statement is true if a lower bound on $r\geqslant r_{\rm max}\geqslant 1$ is known.

For instance, as a numerical example, if it were true that the restriction on r of $r_{\rm max}=0.2$ can be obtained, a value of $x_{cp}=0.3$ would lead to the bound $50°\leqslant\gamma\leqslant 130°$ while if $x_{cp}=0.1$ gives $14°\leqslant\gamma\leqslant 176°$. One can see that to put bounds on γ that are interesting from the perspective of the standard model, one must have an instance of $x_{cp}\geqslant O(0.1)$ for at least one of the modes.

X. EXTRACTING INFORMATION ABOUT γ FROM DIRECT CP IN $B\to K\pi$ -LIKE MODES

Let us now consider the case where full experimental information about this system (2) is available. If all four branching ratios and their conjugates may be observed, it is still not, in general, possible to solve for γ without making some additional assumption. One can, however, obtain the combination

$$Q=|v_c A|\sin\gamma. \tag{57}$$

The experimental determination of the branching ratios for each of the four modes and their conjugates allows us to determine $|m_i|$ and $|\overline{m_i}|$ of Eq. (37), i.e., eight quantities in all subject to one constraint, i.e., Eq. (43).

We can most easily obtain information about the amplitudes from the observable quantities by noting that a common strong phase (ϕ_D) is not soluble and by rewriting Eq. (37) in terms of the expressions

$$\begin{aligned}
f&=3e^{-i(-\gamma+\phi_D)}v_u D,\\
g_1&=3e^{-i(-\gamma+\phi_D)}(v_u B+v_c A),\\
\overline{g}_1&=3e^{-i(+\gamma+\phi_D)}(v_u^* B+v_c^* A),\\
g_2&=-3e^{-i(-\gamma+\phi_D)}(v_u C+v_c A),\\
\overline{g}_2&=-3e^{-i(+\gamma+\phi_D)}(v_u^* C+v_c^* A),
\end{aligned} \tag{58}$$

where $\phi_D=\arg(D)$ so f is real and g_i and $\overline{g_i}$ are general complex numbers which satisfy

$$g_1+g_2-\overline{g}_1-\overline{g}_2=0. \tag{59}$$

We then obtain

$$\begin{aligned}
3|m_1|&=|-g_1+\sqrt{2}f|, & 3|\overline{m}_1|&=|-\overline{g}_1+\sqrt{2}f|,\\
3|m_2|&=|\sqrt{2}g_1+f|, & 3|\overline{m}_2|&=|\sqrt{2}\overline{g}_1+f|,\\
3|m_3|&=|\sqrt{2}g_2+f|, & 3|\overline{m}_3|&=|\sqrt{2}\overline{g}_2+f|,\\
3|m_4|&=|-g_2+\sqrt{2}f|, & 3|\overline{m}_4|&=|-\overline{g}_2+\sqrt{2}f|.
\end{aligned} \tag{60}$$

These equations may be solved to obtain the complex values of g_i, $\overline{g}_i$, as well as the real number f, though the solutions will have some discrete ambiguities since they require the solution to polynomial equations.

The quantity Q may thus be expressed as

$$Q=|v_c A|\sin\gamma=|g_1-\overline{g}_1|/6. \tag{61}$$

Furthermore, from g_1 and $\overline{g}_1$ we may also discover if there is indeed a strong phase difference $\Phi=\arg(DA^*)$ because

$$\Phi=\arg[i(g_1-\overline{g}_1)]. \tag{62}$$

In addition we can learn the phase of $|B-C|$ since

$$\frac{1}{3}|g_1+g_2|=(B-C)|v_u|. \tag{63}$$

The simple point is that there are seven independent quantities that are measured since the eight values of $|m_i|$ and $|\overline{m_i}|$ are subject to the constraint Eq. (43). On the other hand, the right-hand side of Eq. (58) depends on eight unknowns: γ, $|v_c|{\rm Re}(A)$, $|v_c|{\rm Im}(A)$, $|v_u|{\rm Re}(B)$, $|v_u|{\rm Im}(B)$, $|v_u|{\rm Re}(C)$, and $|v_u|{\rm Im}(C)$ (note that the observables do not depend on an overall strong phase, here taken as ϕ_D). Thus γ cannot be determined from these equations.

However, if we know the value of ρ we may obtain the ratio

$$r_B=|B/D|=\left|\frac{\sqrt{2}\rho^2\pm 3\rho+\sqrt{2}}{1-2\rho^2}\right|, \tag{64}$$

where the $\pm$ in the above represents a twofold ambiguity. From this

$$\gamma=\arg[(1-i\lambda)g_1-(1-i\lambda)\overline{g}_1], \tag{65}$$

where λ is one of the two solutions to

$$|(1+i\lambda)g_1+(1-i\lambda)\overline{g}_1|=2fr_B. \tag{66}$$

In the above we assume that the decays of B^0 are self-tagging and so oscillation effects need not be taken into account. This would not be true for $\overline{B}^0\to\overline{K}^0\pi^0$, however, in the analogous case where the K^0 is replaced with the K^{0*} which decays to a charged $K^\pm$ the decay chain will be self-tagging. Thus $\delta(\overline{K}^{*0}\pi^0)$ may be determined through the comparison of the decay chain $\overline{B}^0\to\overline{K}^{*0}\pi^0\to K^-\pi^+\pi^0$ to $B^0\to K^{*0}\pi^0\to K^+\pi^-\pi^0$.

In the case where the decay is not self tagging such as $\overline{B^0}\to\overline{K^0}\pi^0$ or $\overline{B^0}\to\overline{K^0}\rho^0$, we can still carry out the analysis through the use of Eq. (43). Consider, for instance, the case $\overline{B^0}\to\overline{K^0}\pi^0$. In this case oscillation effects will not alter the observed value of $\sigma(\overline{K^0}\pi^0)$ while $\delta(\overline{K^0}\pi^0)$ may be obtained through Eq. (43).

Of course, using this equation assumes the isospin structure due to the presence on the quark level of only the tree and strong penguin diagrams. In order to confirm this one can independently check the value of $\delta(\overline{K^0}\pi^0)$ by factoring in the oscillation effects. Let us consider the experimental situation as it exists at an e^+e^- collider where a $B^0\overline{B^0}$ pair is produced, one of the pair undergoes a tagging decay and the other one decays (for instance) to $K_s\pi^0$. Here we will consider the situation where the times of the decay cannot be determined (as would likely be true for $K_s\pi^0$) and so we consider only time integrated quantities.

Let us denote a tagging decay that indicates a B^0 meson (such as $e^+\nu D^-$) by $B^0\to$tag and a tagging decay that indicates a $\overline{B^0}$ meson (such as $e^-\overline{\nu}D^+$) by $\overline{B^0}\to\overline{\text{tag}}$. If we designate the neutral B meson that undergoes the tagging decay as B_1 and the neutral B meson which undergoes the decay to $K\pi$ as B_2 then we can define the following observable time integrated quantities:

$$\frac{1}{2}\hat{\sigma}(K_s\pi^0)=\frac{1}{2}[B_r(\overline{B^0}\to K_s\pi^0)+B_r(B^0\to K_s\pi^0)],$$

$$\frac{1}{2}\hat{\delta}(K_s\pi^0)=\frac{B_r(B_1\to\text{tag};B_2\to K_s\pi^0)-B_r(B_1\to\overline{\text{tag}};B_2\to K_s\pi^0)}{B_r(B_1\to\text{tag})+B_r(B_1\to\overline{\text{tag}})}. \tag{67}$$

These may be related to σ and δ via

$$\hat{\sigma}(K_s\pi^0)=\sigma(\overline{K^0}\pi^0),$$

$$\hat{\delta}(K_s\pi^0)=\frac{1}{1+x_d^2}\delta(\overline{K^0}\pi^0), \tag{68}$$

where $x_d=\Delta m_B/\Gamma_B$.

Thus if $\hat{\sigma}(K_s\pi^0)$ and $\hat{\delta}(K_s\pi^0)$ are observed experimentally, the quantities $\sigma(\overline{K^0}\pi^0)$ and $\delta(\overline{K^0}\pi^0)$ may be found from Eq. (68) which gives us $|m_4|$ and $|\overline{m}_4|$. The analysis for extracting Q then proceeds as given above. For the B^0, the experimental value for x_d is about 0.73 hence the factor $1/(1+x_d^2)$ in Eq. (68) is about 0.65.

XI. SEVERAL SHOTS AT LARGE DIRECT (COMPOUND) *CP* VIOLATION

It is important to understand that because of theorem 2, the partial rate asymmetries in $B\to K\pi$ that are driven by LD rescattering effects leading to compound CP violation cannot cancel with similar PRA's in the $B\to K^*\pi$ system, for instance. Since, as a rule, we should anticipate LD effects to cause possibly large, unpredictable, phases in all such modes [see Eq. (3)] therefore experimentally we get several independent shots at the consequences of large direct CP violation by searching for all of these modes. We note, in passing, in this context that large final state rescattering phases have been seen in $D\to K\pi$, $K^*\pi$, and in $K\rho$ [16].

XII. CONCLUSIONS

Traditional discussions of direct CP violation in B decays [9,28] have been centered around that emerging from the absorptive part of the penguin graph [5]. We are labeling this ''simple CP violation'' as, for $b\to s$ transitions, it involves $\Delta I=0$ effective interaction only. Simple CP violation of type I entails partial width cancellation against $c\overline{c}$ states whereas for type II the cancellation is with light quark states which contribute through final state interactions [4]. Long-distance rescattering effects can cause another brand of CP violation, ''compound CP violation,'' involving mixtures of eigenstates of isospin. We have discussed CPT constraints governing the PRA's in the various cases. In particular, the pattern of asymmetries in $B\to K\pi$ modes in these cases is quite different.

We have also examined the repercussions of the long-distance rescattering effects for constraints on the CKM angle γ. Since at m_B LD rescattering effects in $B\to K\pi$-like modes are unlikely to be small they need to be taken into account. Full experimental information in the $K\pi$ helps in deducing useful constraints on γ. Since PRA due to compound CP violation in $B\to K\pi$ cannot cancel with those (say) in $B\to K\rho$, each class of these final states would exhibit PRA dictated by the corresponding rescattering effects in the respective channel.

PRA's from different sources of *CP* violation, discussed herein, are additive. Thus in some of these modes the net PRA will be bigger than that only due to compound *CP* violation, for example, in other cases, due to partial cancellations, it could be smaller.

Note added: In the final stages of preparation of this paper we became aware of a few recent works that discuss some of the same issues as this paper [29].

ACKNOWLEDGMENTS

This research was supported in part by U.S. DOE Contract Nos. DE-AC02-76CH00016 (BNL) and DE-FG02-94ER40817 (ISU).

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