Relativistic quantum model of confinement and the current quark masses

L. D. Soloviev
Department of Physics, University of Michigan, Ann Arbor, Michigan 48109-1120
and Institute for High Energy Physics, 142284, Protvino, Russia
(Received 4 April 1997; revised manuscript received 2 March 1998; published 30 June 1998)

We consider a relativistic quantum model of confined massive spinning quarks and antiquarks which describes the leading Regge trajectories of mesons. The quarks are described by the Dirac equations and the gluon contribution is approximated by the Nambu-Goto straight-line string. The string tension and the current quark masses are the main parameters of the model. Additional parameters are phenomenological constants which approximate nonstring short-range contributions. A comparison of the measured meson masses with the model predictions allows one to determine the current quark masses (in MeV) to be $m_s=227\pm5$, $m_c=1440\pm10$, and $m_b=4715\pm20$. The chiral SU_3 model makes it possible to estimate from here the u- and d-quark masses to be $m_u=6.2\pm0.2$ Mev and $m_d=11.1\pm0.4$ Mev. [S0556-2821(98)03313-X]

PACS number(s): 12.60.Rc, 12.15.Ff

It has been believed for a long time that the properties of quarks confined in a meson are closely related to those of the relativistic string with a Nambu-Goto self-interaction [1]. The anomaly in quantum string theory in four-dimensional space-time has led to other important applications of string theory [1]. Nevertheless, hadron theory can use particular simple configurations of the string for an approximate description of the hadrons if these configurations admit relativistic quantization. If the approximate hadron model obtained in this way appears to be in acceptable agreement with the experiment one can try next more complicated string configurations, having in mind that at some step the whole notion of a string may fail, especially when more experimental information about hadron daughter trajectories will be available.

The simplest string configuration, a straight-line string, was quantized in [2,3] in accordance with Poincaré invariance and gave good agreement with the spectrum of the light-quark mesons lying on the leading Regge trajectory. The next approximation was to take into account the masses and the spins of the quarks attached to the ends of the string. This has been done in [4–16] with various assumptions.

The distinctive features of the present approach as compared with those of Refs. [4–16] are the consistent treatment of the quark spins and the canonical quantization. The gauge invariant formalism is used throughout the paper. We also show that there is no radial motion of the quarks along the rotating straight-line string. This means that the daughter meson states correspond to higher modes of the string (vibrations).

The advantage of the present approach as compared with the potential models ([17] for example) is relativistic invariance (in [17] it is only approximate) and use of current quark masses (in [17] constituent quark masses are used). The disadvantage of the present paper is restriction to the leading Regge trajectories, i.e., in the potential model language, to the lowest radial excitations.

We consider the Nambu-Goto straight-line string with pointlike massive spinning quarks attached to its ends. This is an extended relativistic object [18,19] called a rotator for which the explicitly relativistic description introduces auxiliary variables resulting in a symmetry of the rotator Lagrangian. The Hamiltonian of the rotator is given by an implicit function which can be calculated numerically. For important particular cases (light or heavy quarks) series expansions for the Hamiltonian are obtained.

The quark spins are described by anticommuting spin variables obeying constraints [20]. Special care has been taken to ensure conservation of these constraints [21,22].

Canonical quantization of this system preserving the Poincaré invariance yields meson states with different spins and parities lying on Regge trajectories which depend on the quark masses. The 16-component wave function of a composite meson satisfies two Dirac equations and a spectral condition which can be compared with the experimental mass spectrum.

The spectral condition contains a contribution of the universal string confining mechanism together with nonstring short-range contribution which is treated phenomenologically. The dominant part of the short-range contribution do not depend on the meson spin J and its decreasing with J part is seen only in low-J quarkonia. The string contribution dominates when at least one quark is light and grows with the meson spin. On the other hand, it is near threshold for low-spin heavy-quark mesons. The string contribution to the $\Upsilon(1S)$ mass is about 20 MeV and to the $\chi_{b2}(1P)$ mass – 320 MeV.

So, the present approach in its simple form is applicable to mesons containing at least one light quark where the nonrelativistic potential models are not applicable. For heavy quarkonia the string mechanism should be supplemented with other small (compared to heavy-quark masses) contributions to account for the fine structure of the levels.

We compare the model with experiment for the trajectories with $P=C=(-1)^J$ and lowest states having $J^{PC}=1^{--}$. For these trajectories mesons with highest spins were observed and mixing with other trajectories is negligible.

This comparison with meson masses allows to estimate the current s-, c- and b-quark masses assuming that the current u- and d-quark masses are zero within error bars. We then use the chiral SU_3 model [23] to estimate the u- and d-quark masses through the s-quark mass to check the consistency of the calculations.

To check the model we have used the obtained quark masses to calculate the masses of mesons not used in the input. We compare the predicted masses with experiment and with the results of the potential model [17] and discuss a possible interpretation of the gluon string in terms of the potential model.

So, let us consider a simplest extended relativistic object, a straight-line:

$$x(\tau,\sigma)=r(\tau)+f(\tau,\sigma)q(\tau), \tag{1}$$

where r is a 4-vector corresponding to a point on the straight-line, q is an affine 4-vector of its direction, f is a scalar monotonic function of σ labelling points on the line, and τ is a scalar evolution parameter. We shall not fix the coordinates $f_i(\tau)=f(\tau,\sigma_i(\tau))$ of the end points of the string considering them as dynamical variables to be determined from extremum of an action. Then the explicit Poincaré covariance of Eq. (1) introduces superfluous variables not necessary for description of the straight-line as a physical object, so that theory in terms of Eq. (1) must be invariant under a group of three sets of τ-dependent transformations (gauge transformations).

(1) The shift of r along q:

$$r\to r+f(\tau)q. \tag{2}$$

(2) The multiplication of q by an arbitrary scalar function:

$$q\to g(\tau)q. \tag{3}$$

(3) The reparametrization of τ, which means that the Lagrangian must satisfy the condition

$$\mathcal{L}(h(\tau)\dot{z},h(\tau)(h(\tau)\dot{z})^{\cdot})=h(\tau)\mathcal{L}(\dot{z},\ddot{z}), \tag{4}$$

where $\dot{z}$ and $\ddot{z}$ mean every τ-derivative in the Lagrangian.

This symmetry implies that the phase-space variables of our system obey three constraints which are in involution with respect to their Poisson brackets; the canonical Hamiltonian is zero and the total Hamiltonian is a linear combination of the constraint functions.

Invariants of a symmetry play an important role in the description of a symmetric system. In our case they are orthonormal vectors along line direction, velocity of the line rotation, and velocity of its movement as a whole:

$$n=(-q^2)^{-1/2}q,\quad v^1=b^{-1}\dot{n},\quad v^0=(\dot{r}_\perp^2)^{-1/2}\dot{r}_\perp, \tag{5}$$

where

$$b=(-\dot{n}^2)^{1/2} \tag{6}$$

and

$$\dot{r}_\perp^k=(g^{kl}+n^kn^l+v^{1k}v^{1l})\dot{r}_l. \tag{7}$$

The angular velocity b is invariant under Eqs. (2) and (3) and transforms as the Lagrangian under Eq. (4). The scalar invariant of the symmetry is

$$l=b^{-1}(\dot{r}_\perp^2)^{1/2}. \tag{8}$$

We shall label points on the string with respect to the instant center of its rotation z:

$$(-q^2)^{1/2}f=z+y, \tag{9}$$

$$z=b^{-1}\dot{r}v^1 \tag{10}$$

(velocity of the point $r+zn$, orthogonal to q, is orthogonal to v^1). The length of the rotator at fixed τ is $|y_2-y_1|$. From $\dot{x}_i^2\geqslant 0$ it follows that $|y_i|\leqslant l$.

We shall take quark spins into account later on. Without quark spins the Lagrangian of our model is a sum of the Nambu-Goto Lagrangian for an open string with a string-tension parameter a and two Lagrangians for free pointlike particles with masses m_1 and m_2 and velocities of the ends of the string

$$\mathcal{L}=-a\int_{\sigma_1}^{\sigma_2}g^{1/2}d\sigma-\sum_i m_i(\dot{x}_i^2)^{1/2}, \tag{11}$$

where $g=(\dot{x}x')^2-\dot{x}^2x'^2$ is minus determinant of the induced metric of the string worldsheet and $\dot{x}_i=dx(\tau,\sigma_i(\tau))/d\tau$, $i=1,2$ are velocities of the string ends. Using the notations introduced above we can rewrite Eq. (11) for the straight-line string (1,9) in the form

$$\mathcal{L}=-bF, \tag{12}$$

where F is a gauge and Poincaré invariant function

$$F=a\int_{y_1}^{y_2}(l^2-x^2)^{1/2}dx+\sum m_i(l^2-y_i^2-w_i^2)^{1/2}, \tag{13}$$

$$w_i=b^{-1}(\dot{y}_i+\dot{z}-\dot{r}n). \tag{14}$$

We shall consider the case when

$$b\neq 0 \tag{15}$$

(this is a gauge invariant condition). Then we must consider w_i as independent variables and the stationary condition with respect to them yields

$$w_i=0. \tag{16}$$

The other way to obtain this result [16] is to consider the Euler-Lagrange equations following directly from Eq. (11) which give for the straight-line string

$$\dot{y}=0,\quad \dot{z}-\dot{r}n=0. \tag{17}$$

Equation (16) follows from here by continuity.

We conclude that for our model

$$F=a\int_{y_1}^{y_2}(l^2-x^2)^{1/2}dx+\sum m_i(l^2-y_i^2)^{1/2} \tag{18}$$

with y_i satisfying the stationary condition

$$\partial F/\partial y_i = 0, \tag{19}$$

or

$$(-1)^i y_i = (l^2 + (m_i/2a)^2)^{1/2} - (m_i/2a). \tag{20}$$

Calculating the momenta p and π canonically conjugate to r and q,

$$p = -\partial\mathcal{L}/\partial\dot{r}, \quad \pi = -\partial\mathcal{L}/\partial\dot{q}, \tag{21}$$

we get three constraints $\phi_i = 0, \quad i = 1,2,3$ where the constraint functions are

$$\phi_1 = pq, \quad \phi_2 = \pi q, \tag{22}$$

$$\phi_3 = L - K. \tag{23}$$

Here

$$L = ((q^2 - (qp)^2/p^2)\pi^2)^{1/2} \tag{24}$$

is the magnitude of the conserved orbital spin

$$L_\mu = \epsilon_{\mu\nu\rho\sigma} p^\nu M^{\rho\sigma}/2m, \tag{25}$$

where

$$M^{\mu\nu} = r^{[\mu} p^{\nu]} + q^{[\mu} \pi^{\nu]} \tag{26}$$

is the angular momentum tensor. K is a function of $m = (p^2)^{1/2}$, implicitly given by the equations

$$K = lm - F, \tag{27}$$

$$\partial F/\partial l = m. \tag{28}$$

The rotator Hamiltonian is a linear combination of the constraint functions

$$H = \sum_{i=1,2,3} c_i \phi_i . \tag{29}$$

It determines the dynamical equations for any variable X

$$\dot{X} = \{X, H\}, \tag{30}$$

$\phi_i = 0$ after calculating the brackets and the nonzero Poisson brackets are

$$\{p^\mu, r^\nu\} = \{\pi^\mu, q^\nu\} = g^{\mu\nu}. \tag{31}$$

We can choose gauge conditions to fix $c_{1,2} = 0$ in Eq. (29):

$$p\pi = 0, \quad q^2 + 1 = 0. \tag{32}$$

To obtain the Poisson brackets in this gauge we introduce new variables having vanishing brackets with the constraints (22) and (32):

$$p, \quad r_0 = r + ((p\pi)q - (pq)\pi)/p^2, \quad v = (-q_p^2)^{-1/2} q_p, \quad L \tag{33}$$

$[q_p^\mu = (g^{\mu\nu} - p^\mu p^\nu/p^2) q_\nu]$. To have zero brackets of the external coordinate of the rotation center r_0 with the internal coordinates v and L we use four orthonormal vectors $e_\alpha, \alpha = 0,1,2,3,$

$$e_0 = p/m, \quad e_\alpha e_\beta = g_{\alpha\beta}, \tag{34}$$

and introduce new variables

$$n^a = -e_a v, \quad L^a = -e_a L, \quad a = 1,2,3, \tag{35}$$

$$z = r_0 + \frac{1}{2}\epsilon_{abc} e_{a\nu} \frac{\partial e_b^\nu}{\partial p} L^c. \tag{36}$$

The nonzero Poisson brackets of the new variables are

$$\{p^k, z^l\} = g^{kl}, \quad \{L^a, L^b\} = \epsilon_{abc} L^c, \quad \{L^a, n^b\} = \epsilon_{abc} n^c. \tag{37}$$

The constraint function ϕ_3 now takes the form

$$\phi_3 = ((L^a)^2)^{1/2} - K(m) \tag{38}$$

and the solution of the dynamical equations (30) can be easily obtained to be

$$z = z_0 + lVp/m, \tag{39}$$

$$n = n_0 \cos V - n_1 \sin V, \tag{40}$$

$$V = \int c_3 d\tau. \tag{41}$$

From Eq. (39) the laboratory time of the rotation center

$$t = z^0 - z_0^0 = lVp^0/m \tag{42}$$

and the space coordinates of this point

$$z^a = z_0^a + p^a t/p^0 \tag{43}$$

correspond to its movement in the laboratory with constant velocity p^a/p^0. The direction of the rotator rotates with constant angular velocity

$$\omega = \frac{m}{p^0 l}, \tag{44}$$

where $l = l(m)$ from Eq. (28).

The canonical quantization can now be performed quite easily. We replace our variables by operators and their Poisson brackets (37) by commutators. The constraint equation now holds for the wave function

$$[((L^a)^2)^{1/2} - K(m) - a_0]\psi = 0, \tag{45}$$

where in the operator form of Eq. (38) we have added a term a_0 to account for nonstring short-range contributions.

Our quantum system is relativistic because the quantization procedure transforms the classical Poisson brackets of p^μ and $M^{\nu\sigma}$ into commutators without any change in their form, so that the Poincaré algebra is fully preserved.

Quark spins are important especially for small L. They were taken into account in [21,22] where the spinless-particle Lagrangians in Eq. (11) were replaced by those of Berezin and Marinov [20] and a special term was added to preserve conservation of the spin constraints, with the result that for the leading Regge trajectories one can simply replace the orbital spin L in Eq. (45) by the total meson spin J. This yields

$$(J(J+1))^{1/2}=K(m)+a_0 \tag{46}$$

for the physical eigenstates with fixed dependence of space and charge-conjugation parities P and C on J.

The function $K(m)$ is given by Eqs. (18), (20), (27), and (28). We must solve Eq. (28) to find l as a function of m and put this function into Eq. (27). This can be done numerically for any quark masses. For important particular cases K can be expanded into a series. For light quarks,

$$y_i=\pi m_i/m\ll 1, \tag{47}$$

$$K(m)=\frac{m^2}{2\pi a}\left[1-\frac{4}{3\pi}\sum y_i^{3/2}\left(1-\frac{3}{20}y_i\right)+\frac{1}{(3\pi)^2}\left(\sum y_i^{3/2}\right)^2+O(y_i^{7/2})\right]. \tag{48}$$

For heavy quarks

$$D=m-m_1-m_2\ll m_i, \tag{49}$$

$$K(m)=\frac{1}{a}\left(\frac{2}{3}D\right)^{3/2}\nu_1^{-1/2}\left[1+\frac{7}{36}\frac{\nu_3}{\nu_1^2}D+O\left(\left(\frac{D}{m_i}\right)^2\right)\right], \tag{50}$$

$$\nu_n=\sum m_i^{-n}. \tag{51}$$

For light and heavy quarks

$$d=m-m_2,\quad y_1=\frac{\pi m_1}{2d}\ll 1,\quad x_2=\frac{2d}{\pi m_2}\ll 1, \tag{52}$$

$$K(m)=\frac{d^2}{\pi a}\left[1-\frac{8}{3\pi}y_1^{3/2}-\frac{2}{\pi}x_2+\frac{9}{\pi^2}x_2^2-\left(\frac{54}{\pi^3}-\frac{7}{6\pi}\right)x_2^3+\left(\frac{378}{\pi^4}-\frac{35}{2\pi^2}\right)x_2^4+O(y_1^{5/2})+O(y_1^{3/2}x_2)+O(x_2^5)\right]. \tag{53}$$

We see that the slope of the trajectory for mesons formed by a heavy and a light quark (antiquark) is twice as big as for light-quark mesons.

The term a_0 in Eq. (46) can in general depend on J, but it cannot grow with J. An analysis of Coulomb-like short-range interaction suggests the following dependence of a_0 on J [or on m, what is practically the same when Eq. (46) is satisfied]:

$$a_0=A+\left(\frac{16m_1m_2}{(m_1+m_2)m(2J+1)^2}\right)^2 B, \tag{54}$$

where A and B do not depend on J. In all cases considered below the first term in Eq. (54) dominates, so the precise form of the second term is not important for our conclusions. As a first approximation one could neglect the second term to get the quark masses within error bars following from comparison with experiment. On the other hand the second term allows one to get good agreement with the experimental heavy-quarkonia spectrum. The errors in the quark masses in this case formally reduce and to estimate their values one has to go outside of the model and to analyze the interaction between mesons and their decay channels. An approximate analysis of this problem was performed in Ref. [17] with the result that the error in the heavy-quark meson masses is about 10 MeV. We tentatively take this value as an error in the heavy-quark masses deduced from a precise fit to experimental meson masses with the help of the second term in Eq. (54).

Assuming B in Eq. (54) to be of order 1 we see that the second term in Eq. (54) is negligible when one or both quarks are light. It is negligible also for the $s\bar{s}$-mesons below.

We shall apply Eq. (46) to the leading trajectories with $P=C=(-1)^J$ and the lowest states having $J^{PC}=1^{--}$. Estimates show they do not mix with other trajectories with the same J^{PC} having much heavier states.

Applying Eq. (46) to the leading ρ and $K^\star$ trajectories we have

$$K(m_{\rho J})=K(m_{K^\star J}), \tag{55}$$

or, neglecting the u- and d-quark masses,

$$\frac{m_s}{m_{K^\star J}}=\frac{1}{\pi}z_J^{2/3}\left(1+\frac{1}{10}z_J^{2/3}+\frac{1}{18\pi}z_J+O(z_J^{4/3})\right), \tag{56}$$

$$z_J=\frac{3\pi}{4}\left(1-\frac{m_{\rho J}^2}{m_{K^\star J}^2}\right). \tag{57}$$

The error from neglecting the u- and d-quark masses can be estimated from the ω- and ρ-mass difference to be 1.8%. Using experimental data for the meson masses from [24] we obtain the corresponding values for the strange quark mass shown in Table I. The error in the average m_s corresponds to the accuracy of calculations and, partly, to the accuracy of the model.

We get the following values for the other model parameters:

$$a=0.176\ \text{GeV}^2,\quad 2\pi a\equiv\alpha'^{-1}=1.11\ \text{GeV}^2, \tag{58}$$

$$a_0=A=0.88. \tag{59}$$

TABLE I. The input meson masses and the predicted current quark masses in the present model.

Meson spin J	Input meson masses [24]	Quark masses in MeV Calculated in the present model	Other estimates [24]
1	$\rho,K^\star$	$m_s=220\pm4$	
2	$a_2,K_2^\star$	$m_s=234\pm4$	
3	$\rho_3,K_3^\star$	$m_s=204\pm18$	
	average	$m_s=227\pm5$	m_s =100 to 300
1	$D^\star,D_2^\star$	$m_c=1440\pm10$	$m_c=1.0$ to 1.6 GeV
1	$\Upsilon,B^\star,\chi_{b2}$	$m_b=4715\pm20$	$m_b=4.1$ to 4.5GeV

The parameter (59) is the same for the light and the strange quarks and corresponds to the intercept parameter (of J with the $K=0$ axis) $J_0=0.51$.

Knowing the strange-quark mass we can estimate the light-quark masses from the linear approximation of the chiral SU_3 model [23]:

$$m_u/m_d=0.554\pm0.002,\quad m_s/m_d=20.13\pm0.03. \tag{60}$$

Using m_s from Table I here we get (in MeV)

$$m_u=6.2\pm0.2,\quad m_d=11.1\pm0.4. \tag{61}$$

We see that neglecting these masses in the above calculations does not introduce any noticeable error.

To check these results we can use them to calculate masses of mesons consisting of $s\bar{s}$, Table II. They are in good agreement with the experimental values.

To obtain the c-quark mass we consider Eqs. (46),(54) for the $D^\star$ and $D_2^\star$ mesons. The second term in Eq. (54) is negligible and

$$\sqrt{2}=K(D^\star)+A(c),\quad \sqrt{6}=K(D_2^\star)+A(c), \tag{62}$$

what allows us to calculate the c-quark mass through those of $D^\star$ and $D_2^\star$ (Table I) and to estimate $A(c)$:

$$A(c)=0.90. \tag{63}$$

We see that it is close to the constant A for the light quarks (59). To describe this closeness let us remark that the shift 0.02 in a_0 yields the shift from -10 to -17 MeV in the vector-meson masses. This shift decreases for higher meson spins.

Application of Eqs. (46),(54) to the $c\bar{c}$-mesons J/ψ and $\chi_{c2}(1P)$ gives the constants

$$A(c\bar{c})=0.90, \tag{64}$$

which coincides with Eq. (63), and

$$B(c\bar{c})=1.43. \tag{65}$$

For the b-quark we cannot carry out a similar analysis because the mass of $B_2^\star$ is not known. To get an estimate of the b-quark mass we have to rely on an assumption. The safest assumption seems to be

$$A(b)=A(b\bar{b}) \tag{66}$$

similar to the case of the c-quark (63),(64). Using the masses of $B^\star,\Upsilon(1S)$ and $\chi_{b2}(1P)$ mesons we get the b-quark mass in Table I and

$$A(b)=A(b\bar{b})=0.77, \tag{67}$$

$$B(b\bar{b})=3.14. \tag{68}$$

Experimental measurement of the $B_2^\star$ mass is important for checking the assumption (66).

Now we can calculate masses of other mesons belonging to our trajectory. Some of them are presented in Table II, together with experimental data available and predictions of the potential model of Ref. [17]. This model is based upon linear rising potential, Coulomb-like short-range potential from perturbative QCD, approximate relativistic corrections, and constituent quark masses among other parameters.

It is tempting to conclude from Table II that the present model agrees slightly better with the data and that future precise measurements might distinguish both models. But far more impressive is the similarity of the results of apparently quite different calculations. This similarity confirms the main physical motivation for considering the gluon string, namely, the string describes two separate mechanisms of the potential

TABLE II. The model predictions for meson masses (in MeV) and comparison with the potential model predictions of Ref. [17] (q stands for u or d).

Quark content	Meson spin J^{PC}	Present model	Experimental values	Potential model [17]
$q\bar{q}$	2^{++}	1317	1318.1 ± 0.7	1310
	3^{--}	1690	1691 ± 5	1680
	4^{++}	1993		2010
	5^{--}	2255		2300
$q\bar{s}$	4^{+}	2080	2045 ± 9	2110
$s\bar{s}$	1^{--}	1019	1019.413 ± 0.008	1020
	2^{++}	1520	1525 ± 5	1530
	3^{--}	1873	1854 ± 7	1900
	4^{++}	2160		2200
$c\bar{q}$	3^{-}	2780		2830
$c\bar{s}$	1^{-}	2134	2112.4 ± 0.7	2130
	2^{+}	2561	2573.5 ± 1.7	2590
	3^{-}	2870		2920
$c\bar{c}$	3^{--}	3830		3850
$b\bar{q}$	2^{+}	5720		5800
$b\bar{s}$	1^{-}	5430		5450
$b\bar{c}$	1^{-}	6410		6340
$b\bar{b}$	3^{--}	10110		10160

approach, confining potential, and the constituent quark masses.

In conclusion, let us discuss the relation between quark masses in this model and in QCD. The present model is a quantum mechanical model of free quarks bound in mesons. Since it agrees with experimental data it is reasonable to assume that the quark masses of this model are the current quark masses entering as parameters into the QCD Lagrangian when one uses the on-mass-shell perturbative renormalization procedure summed to all orders.

It would be interesting to check the obtained values of the current quark masses in other applications.

The author is grateful to V.A. Petrov, Yu.F. Pirogov, and A.V. Razumov for discussions and to Professor A. D. Krisch for the kind hospitality at the University of Michigan where this work was finished.

[1] M. Green, J. Schwartz, and E. Witten, *Superstring Theory* (Cambrige University Press, Cambridge, England, 1987).
[2] G. P. Pron'ko and A. V. Razumov, Theor. Math. Phys. **56**, 192 (1983).
[3] V. I. Borodulin *et al.*, Theor. Math. Phys. **65**, 119 (1985).
[4] A. Chodos and C. B. Thorn, Nucl. Phys. **B72**, 509 (1974).
[5] P. Frampton, Phys. Rev. D **12**, 538 (1975).
[6] I. Bars, Phys. Rev. Lett. **36**, 1521 (1976).
[7] I. Bars and A. Hanson, Phys. Rev. D **13**, 1744 (1976).
[8] R. C. Giles and S.-H. H. Tye, Phys. Rev. D **16**, 1079 (1977).
[9] K. Kikkawa and M. Sato, Phys. Rev. Lett. **38**, 1309 (1977); K. Kikkawa, T. Kotani, and M. Sato, Phys. Rev. D **18**, 2606 (1978).
[10] M. Ida, Prog. Theor. Phys. **59**, 1661 (1978).
[11] K. Johnson and C. Nohl, Phys. Rev. D **19**, 291 (1979).
[12] F. Lizzi and C. Rosenzweig, Phys. Rev. D **31**, 1685 (1985).
[13] I. Yu. Kobzarev, B. V. Martemianov, and M. G. Shchepkin, Yad. Fiz. **44**, 475 (1986); I. Yu. Kobzarev, N. A. Kondratiuk, B. V. Martemianov, and M. G. Shchepkin, *ibid.* **46**, 1552 (1987).
[14] A. V. Batunin and O. L. Zorin, IHEP Report No. 89-87, Serpukhov, 1989.
[15] B. M. Barbashov, Report No. JINR E2-94-444, Dubna, 1994; *Strong Interactions at Long Distances*, Proceedings of the Conference, edited by L. Jenkovsky (Hadronic, Palm Harbor, FL, 1995), p. 257.
[16] V. V. Nesterenko, Z. Phys. C **47**, 111 (1990). In this paper the conclusion about absence of the radial quark motion on a straight-line string in the three-dimensional Minkowski space was also made.
[17] S. Godfrey and N. Isgur, Phys. Rev. D **32**, 189 (1985).
[18] N. Mukunda, H. van Dam, and L. C. Biedenharn, Phys. Rev. D **22**, 193 (1980).
[19] R. R. Aldinger *et al.*, Phys. Rev. D **29**, 2828 (1984).
[20] F. A. Berezin and M. S. Marinov, Ann. Phys. (N.Y.) **104**, 336 (1977).
[21] L. D. Soloviev, ''Problems of Quantum Field Theory,'' 10th International Conference (JINR E-2-96-369, Dubna, 1996), p. 75.
[22] L. D. Soloviev, IHEP Report No. 96-67, Protvino, 1996.
[23] S. Weinberg, Trans. NY Acad. Sci. **38**, 185 (1977).
[24] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).