Predictions from an anomalous U(1) model of Yukawa hierarchies

Nikolaos Irges, Stéphane Lavignac,* and Pierre Ramond

Institute for Fundamental Theory, Department of Physics, University of Florida, Gainesville, Florida 32611

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We present a supersymmetric standard model with three gauged Abelian symmetries of a type commonly found in superstrings. One is anomalous; the other two are E_6 family symmetries. It has a vacuum in which only these symmetries are broken by stringy effects. It reproduces all observed quark and charged lepton Yukawa hierarchies and the value of the Weinberg angle. It predicts three massive neutrinos, with mixing that can explain both the small angle MSW effect and the atmospheric neutrino anomaly. The Cabibbo angle is expressed in terms of the gauge couplings at unification. It conserves R parity and proton decay is close to experimental bounds. [S0556-2821(98)06513-8]

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I. INTRODUCTION

Over the last few years there has been growing interest in relating generic features of superstring models to low-energy phenomenology. Prominent among these are models which contain an anomalous U(1) with anomalies cancelled by the Green-Schwarz mechanism [1] and in which the dilaton gets a vacuum value, generating a Fayet-Iliopoulos term that triggers the breaking [2] of at least the anomalous gauged symmetry at a large computable scale.

Through the anomalous U(1), the Weinberg angle at the cutoff is related to anomaly coefficients [3]. This allows for possible relations between fundamental string quantities (in the ultraviolet) and experimental parameters (in the infrared). A simple model [4] with one family-dependent anomalous U(1) beyond the standard model was the first to exploit these features to produce Yukawa hierarchies and fix the Weinberg angle. It was soon realized that some features could be abstracted from the presence of the anomalous U(1): expressing the ratio of downlike quarks to charged lepton masses in terms of the Weinberg angle [5–7], the suppression of the bottom to the top quark masses [8], relating the uniqueness of the vacuum to Yukawa hierarchies and the presence of minimal supersymmetric standard model (MSSM) invariants in the superpotential, and finally relating the seesaw mechanism [9] to R-parity conservation [10].

Recently, many of these ideas were incorporated in a model [11] with one anomalous and two nonanomalous U(1) symmetries spontaneously broken by stringy effects. It contained only the three standard model chiral families, three right-handed neutrinos, and the fields necessary to break the extra phase symmetries. It reproduced all quark and charged lepton hierarchies, and the Weinberg angle, but failed in some other aspects: the proton decayed faster than observed, and the three light neutrinos had an inverse mass hierarchy, which could not account for the solar neutrino deficit.

In this paper, we propose an alteration of this model, in which there are two nonanomalous U(1) family symmetries

contained within E_6 and one anomalous family-independent U(1) symmetry. All three are spontaneously broken by the dilaton-generated FI term. To cancel anomalies, it contains vectorlike matter with standard model charges, and hidden sector fields and interactions, some of the many features encountered in superstring models. It is expressed as an effective low-energy supersymmetric theory with a cutoff scale M. It has some distinctive features, such as all quark and charged lepton Yukawa hierarchies and mixing, including the bottom to top Yukawa suppression, the value of the Weinberg angle at unification, three massive neutrinos with mixings that give the small-angle Mikheyev-Smirnov-Wolfenstein (MSW) effect for the solar neutrino deficit and the large angle mixing necessary for the atmospheric neutrino effect, natural R-parity conservation, proton decay into $K^0 + \mu^+$ near the experimental limit, and a hidden sector that contains strong gauge interactions. It is heavily constrained by the requirement that the vacuum, in which the three U(1)'s are broken by stringy effects, be free of flat directions associated with the MSSM invariants. Our model's vacuum is demonstrably free of the flat direction associated with each invariant.

The theoretical consistency of the model is tested by the many ways in which its cutoff is ''measured.'' First, the renormalization group evolution of the standard model gauge couplings yields their unification scale. Its value depends on the number of standard-model vectorlike matter at intermediate masses; in our model we find $M_U \sim 3 \times 10^{16}$ GeV. Second, assuming that all couplings in the superpotential are of order 1, it is measured by fitting the neutrino mass scale. A fit to both the small-angle MSW and the atmospheric neutrino deficit yields $10^{16} < M < 4 \times 10^{17}$ GeV. A fit only to the MSW effect yields a larger value, $M \sim 10^{18}$ GeV. Third, the lack of experimental evidence for proton decay sets a lower bound for M consistent with these estimates.

In all the above estimates, we have used the Cabibbo angle as the expansion parameter. However, the Green-Schwarz relation yields a natural expansion parameter in terms of the gauge coupling at unification. In our model, we find it to be $\lambda \sim 0.28$, clearly of the same order of magnitude but larger than the Cabibbo angle, but this value depends on the standard-model vectorlike matter, about which we have no direct experimental information. Thus we have used the

*Permanent address: Laboratoire de Physique Théorique et Hautes Energies, Université Paris-Sud, Bât. 210, F-91405 Orsay Cedex, France.

experimental value of the Cabibbo angle in all estimates of the suppression factors.

Furthermore, our determination [3] of the Weinberg angle assumes that the cutoff is close to the unification scale. Thus theoretical and numerical considerations imply that if our theory is to be derived from a theory in higher dimensions, its ''string'' cutoff must be near the unification scale.

To complete our model, we need to include mechanisms that break both supersymmetry and electroweak symmetry. The hidden sector contains a gauge theory with strong coupling, capable of breaking supersymmetry through the Binétruy-Dudas mechanism [12]. Unfortunately, it cannot be the main agent of supersymmetry breaking. The reason is that squarks get soft masses through the D terms of the gauge symmetries, and while the D term of the family-independent anomalous U(1) yields equal squark masses, the D terms of the other two U(1)'s give generically flavor-dependent contributions.[1] Since our model does not align [13] the quark and squark mass matrices sufficiently to account for the flavor-changing constraints, we are left with the usual flavor problem associated with supersymmetry breaking. Also, this mechanism does not generate large gaugino masses. We note that in some free-fermion superstring models [14], the flavor-dependent D terms can vanish.

In the following, we present the details of the model. Section II details the gauge sector, followed in Sec. III by a discussion of the gauge anomalies and their cancellations. This is followed in Sec. IV by a discussion of the general features of its vacuum. The phenomenology of quark and lepton masses is presented in Sec. V, followed in Sec. VI by a thorough discussion of the neutrino phenomenology of our model. In Sec. VII, we analyze the consequences of the matter with vectorlike standard-model charges. The discussion of the matter content concludes in Sec. VIII with the hidden sector needed to cancel anomalies. We then describe in Sec. IX how R-parity conservation arises in our model, followed in Sec. X by the analysis of proton decay interactions. Finally we close with a detailed analysis of the vacuum flat directions associated with the invariants of the model.

II. THE GAUGE SECTOR

In the visible sector, the gauge structure of our model is that of the standard model, augmented by three Abelian symmetries:

$$\text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times \text{U}(1)_X \times \text{U}(1)_{Y^{(1)}} \times \text{U}(1)_{Y^{(2)}}. \tag{2.1}$$

One of the extra symmetries, which we call X, is anomalous; its charges are assumed to be family independent. The other two symmetries $Y^{(1)}$ and $Y^{(2)}$ are not anomalous, but have specific dependence on the three chiral families, designed to reproduce the Yukawa hierarchies. This theory is inspired by models generated from the superstring $E_8 \times E_8$ heterotic theory, and its chiral matter lies in broken-up representations

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[1]E. Dudas (private communication).

of E_6, resulting in the cancellation of many anomalies. This also implies the presence of both matter that is vectorlike with respect to standard model charges and right-handed neutrinos, which trigger neutrino masses through the seesaw mechanism [9].

The three symmetries X, $Y^{(1,2)}$ are spontaneously broken at a high scale by the Fayet-Iliopoulos term generated by the dilaton vacuum. This Dine-Seiberg-Witten (DSW) vacuum [2] is required to preserve both supersymmetry and the standard-model symmetries. Below its scale, our model displays only the standard-model gauge symmetries.

To set our notation and explain our charge assignments, let us recall some basic E_6 [16]. It contains two Abelian symmetries outside of the standard model: The first U(1), which we call V', appears in the embedding

$$E_6 \subset \text{SO}(10) \times \text{U}(1)_{V'}, \tag{2.2}$$

with

$$\mathbf{27} = \mathbf{16}_1 + \mathbf{10}_{-2} + \mathbf{1}_4, \tag{2.3}$$

where the U(1) value appears as a subscript. The second U(1), called V, appears in

$$\text{SO}(10) \subset \text{SU}(5) \times \text{U}(1)_V, \tag{2.4}$$

corresponding to

$$\mathbf{16} = \bar{\mathbf{5}}_{-3} + \mathbf{10}_1 + \mathbf{1}_5, \quad \mathbf{10} = \bar{\mathbf{5}}_2 + \mathbf{5}_{-2}. \tag{2.5}$$

The familiar hypercharge Y appears in

$$\text{SU}(5) \subset \text{SU}(2) \times \text{SU}(3) \times \text{U}(1)_Y, \tag{2.6}$$

with the representation content

$$\bar{\mathbf{5}} = (\mathbf{2}, \mathbf{1}^c)_{-1} + (\mathbf{1}, \bar{\mathbf{3}}^c)_{2/3}, \tag{2.7}$$

$$\mathbf{10} = (\mathbf{1}, \mathbf{1}^c)_2 + (\mathbf{2}, \mathbf{3}^c)_{1/3} + (\mathbf{1}, \bar{\mathbf{3}}^c)_{-4/3}. \tag{2.8}$$

The two U(1)'s in SO(10), can also be identified with baryon number minus lepton number and right-handed isospin as

$$B - L = \frac{1}{5}(2Y + V), \quad I_{3R} = \frac{1}{10}(3Y - V). \tag{2.9}$$

The first combination is $B - L$ only on the standard-model chiral families in the **16**; on the vectorlike matter in the **10** of SO(10) it cannot be interpreted as their baryon number minus their lepton number.

We postulate the two nonanomalous symmetries to be

$$Y^{(1)} = \frac{1}{5}(2Y + V) \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{2.10}$$

$$Y^{(2)} = \frac{1}{4}(V + 3V') \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \qquad (2.11)$$

The family matrices run over the three chiral families, so that $Y^{(1,2)}$ are family traceless.

We further assume that the X charges on the three chiral families in the **27** are of the form

$$X = (\alpha + \beta V + \gamma V') \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad (2.12)$$

where α, β, γ are as-of-yet undetermined parameters. Since $\mathrm{Tr}(YY^{(i)}) = \mathrm{Tr}(YX) = 0$, there is no appreciable kinetic mixing between the hypercharge and the three gauged symmetries. The matter content of this model is the smallest that reproduces the observed quark and charged lepton hierarchy, cancels the anomalies associated with the extra gauge symmetries, and produces a unique vacuum structure.

Three chiral families each with the quantum numbers of a **27** of E_6. This means three chiral families of the standard model, $\mathbf{Q}_i$, $\bar{\mathbf{u}}_i$, $\bar{\mathbf{d}}_i$, L_i, and $\bar{e}_i$, together with three right-handed neutrinos $\bar{N}_i$, three vectorlike pairs denoted by $E_i + \bar{\mathbf{D}}_i$ and $\bar{E}_i + \mathbf{D}_i$, with the quantum numbers of the $\bar{\mathbf{5}} + \mathbf{5}$ of SU(5). Our model does not contain the singlets S that make up the rest of the **27**. With our charges, they are not required by anomaly cancellation, and their presence would create unwanted flat directions in the vacuum.

One standard-model vectorlike pair of Higgs weak doublets.

Chiral fields that are needed to break the three extra U(1) symmetries in the DSW vacuum. We denote these fields by θ_a. In our minimal model with three symmetries that break through the FI term, we just take $a = 1,2,3$. The θ sector is necessarily anomalous.

Hidden sector gauge interactions and their matter, together with singlet fields, needed to cancel the remaining anomalies.

III. ANOMALIES

In a four-dimensional theory, the Green-Schwarz anomaly compensation mechanism occurs through a dimension-5 term that couples an axion to all the gauge fields. As a result, any anomaly linear in the X symmetry must satisfy the Green-Schwarz relations

$$(XG_iG_j) = \delta_{ij}C_i, \qquad (3.1)$$

where G_i is any gauge current. The anomalous symmetry must have a mixed gravitational anomaly, so that

$$(XTT) = C_{\mathrm{grav}} \neq 0, \qquad (3.2)$$

where T is the energy-momentum tensor. In addition, the anomalies compensated by the Green-Schwarz mechanism satisfy the universality conditions

$$\frac{C_i}{k_i} = \frac{C_{\mathrm{grav}}}{12} \quad \text{for all} \quad i. \qquad (3.3)$$

A similar relation holds for $C_X \equiv (XXX)$, the self-anomaly coefficient of the X symmetry. These result in important numerical constraint, which can be used to restrict the matter content of the model.

All other anomalies must vanish:

$$(G_iG_jG_k) = (XXG_i) = 0. \qquad (3.4)$$

In terms of the standard model, the vanishing anomalies are therefore of the following types.

The first involve only standard-model gauge groups G_{SM}, with coefficients $(G_{\mathrm{SM}}G_{\mathrm{SM}}G_{\mathrm{SM}})$, which cancel for each chiral family and for vectorlike matter. Also the hypercharge mixed gravitational anomaly (YTT) vanishes.

The second type is where the new symmetries appear linearly of the type $(Y^{(i)}G_{\mathrm{SM}}G_{\mathrm{SM}})$. The choice of family-traceless $Y^{(i)}$ ensures their vanishing over the three families of fermions with the standard model. Hence they must vanish on the Higgs fields: with $G_{\mathrm{SM}} = \mathrm{SU}(2)$, it implies that the Higgs pair is vectorlike with respect to the $Y^{(i)}$. It follows that the mixed gravitational anomalies $(Y^{(i)}TT)$ are zero over the fields with standard model quantum numbers. They must therefore vanish as well over all other fermions in the theory.

The third type involve the new symmetries quadratically of the form $(G_{\mathrm{SM}}Y^{(i)}Y^{(j)})$. These vanish automatically except for those of the form $(YY^{(i)}Y^{(j)})$. Two types of fermions contribute: the three chiral families and standard-model vectorlike pairs

$$0 = (YY^{(i)}Y^{(j)}) = (YY^{(i)}Y^{(j)})_{\mathrm{chiral}} + (YY^{(i)}Y^{(j)})_{\mathrm{real}}. \qquad (3.5)$$

By choosing $Y^{(1,2)}$ in E_6, overall cancellation is assured, but the vectorlike matter is necessary to cancel one of the anomaly coefficients, since we have

$$(YY^{(1)}Y^{(2)})_{\mathrm{chiral}} = -(YY^{(1)}Y^{(2)})_{\mathrm{real}} = 12. \qquad (3.6)$$

The fourth type are the anomalies of the new symmetries of the form $(Y^{(i)}Y^{(j)}Y^{(k)})$. Since standard-model singlet fermions can contribute, it is not clear, without a full theory, how to determine how the cancellations come about. We know that over the fermions in an E_6 representation, they vanish, but, as we shall see, the θ sector is necessarily anomalous. In the following we will present a scenario for these cancellations, but it is the least motivated sector of the theory since it involves the addition of fields whose sole purpose is to cancel anomalies.

The remaining vanishing anomalies involve the anomalous charge X.

Since both X and Y are family independent and $Y^{(i)}$ are family traceless, the vanishing of the $(XYY^{(1,2)})$ coefficients over the three families is assured; so they must vanish over the Higgs pair. This means that X is vectorlike on the Higgs pair. It follows that the standard-model invariant H_uH_d (the μ term) has zero X and $Y^{(i)}$ charges; it can appear by itself in

the superpotential, but we are dealing with a string theory, where mass terms do not appear in the superpotential: it can appear only in the Kähler potential. This results, after supersymmetry breaking, in an induced μ term of weak strength, as suggested by Giudice and Masiero [17].

Since the Higgs pair does not contribute to anomaly coefficients, we can compute the standard-model anomaly coefficients. We find

$$C_{\text{color}} = 18\alpha, \quad C_{\text{weak}} = 18\alpha, \quad C_Y = 30\alpha. \quad (3.7)$$

Applying these to the Green-Schwarz relations we find the Kac-Moody levels for the color and weak groups to be the same,

$$k_{\text{color}} = k_{\text{weak}}, \quad (3.8)$$

and through the Ibáñez relation [3], the value of the Weinberg angle at the cutoff,

$$\tan^2 \theta_w = \frac{C_Y}{C_{\text{weak}}} = \frac{5}{3}, \quad (3.9)$$

not surprisingly the same value as in SU(5) theories.

The coefficients $(XY^{(1)}Y^{(2)})$. Since standard-model singlets can contribute, we expect its cancellation to come about through a combination of hidden sector and singlet fields. Its contribution over the chiral fermions (including the right-handed neutrinos) is found to be

$$(XY^{(1)}Y^{(2)})_{\text{chiral + real}} = 18\alpha. \quad (3.10)$$

The coefficient (XXY). With our choice for X, it is zero.

The coefficients $(XXY^{(i)})$ vanish over the three families of fermions with standard-model charges, but contributions are expected from other sectors of the theory.

The vanishing of these anomaly coefficients is highly nontrivial, and it was the main motivator for our (seemingly arbitrary) choices of X, and $Y^{(i)}$.

IV. DSW VACUUM

The X, $Y^{(1)}$, and $Y^{(2)}$ Abelian symmetries are spontaneously broken below the cutoff. Phenomenological considerations require that neither supersymmetry nor any of the standard-model symmetries be broken at that scale. This puts severe restrictions on the form of the superpotential and the matter fields [10].

Since three symmetries are to be broken, we assume that three fields θ_a acquire a vacuum value as a result of the FI term. They are singlets under the standard-model symmetries, but not under X and $Y^{(1,2)}$. If more fields than broken symmetries assume nonzero values in the DSW vacuum, we would have undetermined flat directions and hierarchies, and Nambu-Goldstone bosons associated with the extra symmetries.

We express their charges in terms of a 3×3 matrix $\mathbf{A}$, whose rows are the X, $Y^{(1)}$, and $Y^{(2)}$ charges of the three θ fields, respectively.

Assuming the existence of a supersymmetric vacuum

where only the θ fields have vacuum values implies, from the vanishing of the three D terms,

$$\mathbf{A} \begin{pmatrix} |\theta_1|^2 \\ |\theta_2|^2 \\ |\theta_3|^2 \end{pmatrix} = \begin{pmatrix} \xi^2 \\ 0 \\ 0 \end{pmatrix}. \quad (4.1)$$

It follows that the matrix $\mathbf{A}$ must not only have an inverse but that the entries in the first row of its inverse be positive.

Since $\mathbf{A}$ is invertible, its rows consist of three linearly independent (but not orthogonal) basis vectors, $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$, whose components are the X, $Y^{(1)}$, and $Y^{(2)}$ charges of the θ fields. The charges of any standard-model invariant S (or any standard-model singlet χ) form a vector which can be expressed in that basis:

$$\mathbf{w} = -(n_1 \mathbf{v}_1 + n_2 \mathbf{v}_2 + n_3 \mathbf{v}_3). \quad (4.2)$$

If all n_α, $\alpha = 1,2,3$ are positive integers, then $S \, \theta_1^{n_1} \theta_2^{n_2} \theta_3^{n_3}$ is a holomorphic invariant and can be present in the superpotential. It is quite remarkable that the invertibility of $\mathbf{A}$, which ensures the existence of the DSW vacuum, is the same condition required for invariants of the form $S \, \theta_1^{n_1} \theta_2^{n_2} \theta_3^{n_3}$ to exist. Those invariants are precisely the ones needed to generate mass hierarchies in the DSW vacuum, with S being Yukawa invariants. If all n_α are positive, but some of them are fractional, the invariant appears at higher order: $(S \, \theta_1^{n_1} \theta_2^{n_2} \theta_3^{n_3})^m$. Finally, if some n_α is negative, one cannot form any holomorphic invariant out of S and the θ fields.

We have found no fundamental principle that fixes the charges of the θ fields. However, by requiring that they all get the same vacuum value and reproduce the quark hierarchies, we arrive at the simple assignment

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 1 \\ 1 & -1 & 0 \end{pmatrix}. \quad (4.3)$$

Forming its inverse

$$\mathbf{A}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}, \quad (4.4)$$

we see that all three θ fields have the same vacuum expectation value:

$$|\langle \theta_1 \rangle| = |\langle \theta_2 \rangle| = |\langle \theta_3 \rangle| = \xi. \quad (4.5)$$

The presence of other fields that do not get values in the DSW vacuum severely restricts the form of the superpotential. In particular, when the extra fields are right-handed neutrinos, the uniqueness of the DSW vacuum is attained only after adding to the superpotential terms of the form $\bar{N}^p \mathcal{P}(\theta)$, where p is an integer ≥ 2, and $\mathcal{P}$ is a holomorphic polynomial in the θ fields. If $p=1$, its F term breaks supersymmetry at the DSW scale.

The case $p=2$ is more desirable since it translates into a Majorana mass for the right-handed neutrino, while the cases $p \geq 3$ leave the $\bar{N}$ massless in the DSW vacuum. To single out $p=2$ we simply choose the X charge of the $\bar{N}_i$ to be a negative half-odd integer. Since right-handed neutrinos couple to the standard model invariants $L_i H_u$, it implies that $X_{L_i H_u}$ is also a half-odd integer.

The same analysis can be applied to the invariants of the MSSM. Since they must be present in the superpotential to give quarks and leptons their masses, their X charges must be negative integers. Remarkably, these are the very same conditions necessary to avoid flat directions along which these invariants do not vanish: with negative charge, these invariants cannot be the only contributors to D_X in the DSW vacuum. The presence of a holomorphic invariant, linear in the MSSM invariant multiplied by a polynomial in the θ fields, is necessary to avoid a flat direction where both the invariant and the θ fields would get DSW vacuum values. The full analysis of the DSW vacuum in our model is rather involved, but it is greatly simplified by using the general methods introduced by two of us [18]. We postpone the discussion of the uniqueness of the vacuum until the end of this paper.

Finally, we note a curious connection between the DSW vacuum and the anomalies carried by the θ fields. Assume that the θ sector does not contribute to the mixed gravitational anomalies

$$(Y^{(i)} TT)_\theta = 0. \tag{4.6}$$

This means that the charges $Y^{(i)}$ are traceless over the θ sector. They are therefore generators of the global SU(3) under which the three θ fields form the **3** representation. However, SU(3) is anomalous, and it contains only one nonanomalous U(1) that resides in its SU(2) subgroup. Thus to avoid anomalies, the two charges $Y^{(1,2)}$ need to be aligned over the θ fields, but this would imply $\det \mathbf{A} = 0$, in contradiction with the necessary condition for the DSW vacuum. It follows that the vacuum structure *requires* the θ sector to be anomalous. Indeed we find that, over the θ fields,

$$(Y^{(1)} Y^{(1)} Y^{(2)})_\theta = (Y^{(1)} Y^{(2)} Y^{(2)})_\theta = -1. \tag{4.7}$$

In a later section we discuss how these anomalies might be compensated for.

V. QUARK AND CHARGED LEPTON MASSES

To account for the top quark mass, we assume that the superpotential contains the invariant

$$\mathbf{Q}_3 \bar{\mathbf{u}}_3 H_u. \tag{5.1}$$

Since X is family independent, it follows that the standard-model invariant operators $\mathbf{Q}_i \bar{\mathbf{u}}_j H_u$, where i,j are family indices, have zero X charge. Together with the anomaly conditions, this fixes the Higgs charges

$$X_{H_u} = -X_{H_d} = -X_{\mathbf{Q}} - X_{\bar{\mathbf{u}}} \tag{5.2}$$

and

$$Y_{H_u}^{(1)} = -Y_{H_d}^{(1)} = 0, \quad Y_{H_u}^{(2)} = -Y_{H_d}^{(2)} = 2, \tag{5.3}$$

$$X(\mathbf{Q}_i \bar{\mathbf{u}}_j H_u) \equiv X^{[u]} = 0. \tag{5.4}$$

The superpotential contains terms of higher dimensions. In the charge 2/3 sector, they are

$$\mathbf{Q}_i \bar{\mathbf{u}}_j H_u \left(\frac{\theta_1}{M} \right)^{n_{ij}^{(1)}} \left(\frac{\theta_2}{M} \right)^{n_{ij}^{(2)}} \left(\frac{\theta_3}{M} \right)^{n_{ij}^{(3)}}, \tag{5.5}$$

in which the exponents must be positive integers or zero. Invariance under the three charges yields

$$n_{ij}^{(1)} = 0, \quad n_{ij}^{(2)} = Y_{ij}^{(2)[u]}, \quad n_{ij}^{(3)} = -Y_{ij}^{(1)[u]} + Y_{ij}^{(2)[u]}, \tag{5.6}$$

where $Y_{ij}^{(1)[u]}$ and $Y_{ij}^{(2)[u]}$ are the charges of $\mathbf{Q}_i \bar{\mathbf{u}}_j H_u$, respectively. They are determined by our choice for the charges $Y^{(1,2)}$. A straightforward computation yields the orders of magnitude in the charge 2/3 Yukawa matrix

$$Y^{[u]} \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, \tag{5.7}$$

where $\lambda = |\theta_a|/M$ is the expansion parameter.

A similar computation is now applied to the charge $-1/3$ Yukawa standard-model invariants $\mathbf{Q}_i \bar{\mathbf{d}}_j H_d$. The difference is the absence of dimension-3 terms, so that its X charge, which we denote by $X^{[d]}$, need not vanish. We find that if $X^{[d]} > -3$, one exponent in the (33) position is negative, resulting in a supersymmetric zero [13] and spoiling the quark hierarchy. Hence, as long as $X^{[d]} \leq -3$, we deduce the charge $-1/3$ Yukawa matrix

$$Y^{[d]} \sim \lambda^{-3X^{[d]} - 6} \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda & 1 & 1 \end{pmatrix}, \tag{5.8}$$

and diagonalization of the two Yukawa matrices yields the Cabibbo-Kobayashi-Maskawa (CKM) matrix

$$\mathcal{U}_{\text{CKM}} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}. \tag{5.9}$$

This shows the expansion parameter to be of the same order of magnitude as the Cabibbo angle λ_c. For definiteness in what follows we take them to be equal, although as we show later the Green-Schwarz evaluation of λ gives a slightly higher value.

The eigenvalues of these matrices reproduce the geometric interfamily hierarchy for quarks of both charges,

$$\frac{m_u}{m_t} \sim \lambda_c^8, \quad \frac{m_c}{m_t} \sim \lambda_c^4, \tag{5.10}$$

$$\frac{m_d}{m_b} \sim \lambda_c^4, \quad \frac{m_s}{m_b} \sim \lambda_c^2, \tag{5.11}$$

while the quark intrafamily hierarchy is given by

$$\frac{m_b}{m_t} = \cot \beta \lambda_c^{-3X^{[d]}-6}, \tag{5.12}$$

implying the relative suppression of the bottom to top quark masses, without large $\tan \beta$. These quark-sector results are the same as in a previously published model [11], but our present model is different in the lepton sector.

The analysis is much the same as for the down quark sector. No dimension-3 term appears and the standard-model invariants $L_i \bar{e}_j H_d$ have charges $X^{[e]}, Y_{ij}^{(1,2)[e]}$. The pattern of eigenvalues depends on the $X^{[e]}$: if $X^{[e]} > -3$, we find a supersymmetric zero in the (33) position, and the wrong hierarchy for lepton masses; if $X^{[e]} = -3$, there are supersymmetric zeros in the (21) and (31) position, yielding

$$Y^{[e]} \sim \lambda_c^3 \begin{pmatrix} \lambda_c^4 & \lambda_c^5 & \lambda_c^3 \\ 0 & \lambda_c^2 & 1 \\ 0 & \lambda_c^2 & 1 \end{pmatrix}. \tag{5.13}$$

Its diagonalization yields the lepton interfamily hierarchy

$$\frac{m_e}{m_\tau} \sim \lambda_c^4, \quad \frac{m_\mu}{m_\tau} \sim \lambda_c^2. \tag{5.14}$$

Our choice of X ensures that $X^{[d]} = X^{[e]}$, which guarantees through the anomaly conditions the correct value of the Weinberg angle at the cutoff, since

$$\sin^2 \theta_w = \frac{3}{8} \leftrightarrow X^{[d]} = X^{[e]}; \tag{5.15}$$

it sets $X^{[d]} = -3$, so that

$$\frac{m_b}{m_\tau} \sim 1, \quad \frac{m_b}{m_t} \sim \cot \beta \lambda_c^3. \tag{5.16}$$

It is a remarkable feature of this type of model that both inter- and intrafamily hierarchies are linked not only with one another but with the value of the Weinberg angle as well. In addition, the model predicts a natural suppression of m_b/m_τ, which suggests that $\tan \beta$ is of order 1.

VI. NEUTRINO MASSES

Our model, based on E_6, has all the features of SO(10); in particular, neutrino masses are naturally generated by the seesaw mechanism [9] if the three right-handed neutrinos $\bar{N}_i$ acquire a Majorana mass in the DSW vacuum. The flat direction analysis then indicates that their X charges must be negative half-odd integers, that is, $X_{\bar{N}} = -1/2, -3/2, \ldots$.

Their standard-model invariant masses are generated by terms of the form

$$M \bar{N}_i \bar{N}_j \left(\frac{\theta_1}{M} \right)^{p_{ij}^{(1)}} \left(\frac{\theta_2}{M} \right)^{p_{ij}^{(2)}} \left(\frac{\theta_3}{M} \right)^{p_{ij}^{(3)}}, \tag{6.1}$$

where M is the cutoff of the theory. In the (ij) matrix element, the exponents are computed to be equal to $-2X_{\bar{N}}$ plus

$$\begin{pmatrix} (0,4,0) & (0,2,1) & (0,0,-1) \\ (0,2,1) & (0,0,2) & (0,-2,0) \\ (0,0,-1) & (0,-2,0) & (0,-4,-2) \end{pmatrix}. \tag{6.2}$$

If $X_{\bar{N}} = -1/2$, this matrix has supersymmetric zeros in the (23), (32), and (33) elements. While this does not result in a zero eigenvalue, the absence of these invariants from the superpotential creates flat directions along which $\langle \bar{N}_3 \rangle \neq 0$; such flat directions are dangerous because they can lead to vacua other than the DSW vacuum. If $X_{\bar{N}} \leq -5/2$, none of the entries of the Majorana mass matrix vanishes, but then the vacuum analysis indicates that flat directions are allowed which involve MSSM fields. For those reasons, we choose $X_{\bar{N}} = -3/2$, which still yields one harmless supersymmetric zero in the Majorana mass matrix, now of the form

$$M \lambda_c^7 \begin{pmatrix} \lambda_c^6 & \lambda_c^5 & \lambda_c \\ \lambda_c^5 & \lambda_c^4 & 1 \\ \lambda_c & 1 & 0 \end{pmatrix}. \tag{6.3}$$

Its diagonalization yields three massive right-handed neutrinos with masses

$$m_{\bar{N}_e} \sim M \lambda_c^{13}, \quad m_{\bar{N}_\mu} \sim m_{\bar{N}_\tau} \sim M \lambda_c^7. \tag{6.4}$$

By definition, right-handed neutrinos are those that couple to the standard-model invariants $L_i H_u$, and serve as Dirac partners to the chiral neutrinos. In our model,

$$X(L_i H_u \bar{N}_j) \equiv X^{[\nu]} = 0. \tag{6.5}$$

The superpotential contains the terms

$$L_i H_u \bar{N}_j \left(\frac{\theta_1}{M} \right)^{q_{ij}^{(1)}} \left(\frac{\theta_2}{M} \right)^{q_{ij}^{(2)}} \left(\frac{\theta_3}{M} \right)^{q_{ij}^{(3)}}, \tag{6.6}$$

resulting, after electroweak symmetry breaking, in the orders of magnitude (we note $v_u = \langle H_u^0 \rangle$)

$$v_u \begin{pmatrix} \lambda_c^8 & \lambda_c^7 & \lambda_c^3 \\ \lambda_c^5 & \lambda_c^4 & 1 \\ \lambda_c^5 & \lambda_c^4 & 1 \end{pmatrix} \tag{6.7}$$

for the neutrino Dirac mass matrix. The actual neutrino mass matrix is generated by the seesaw mechanism. A careful calculation yields the orders of magnitude

$$\frac{v_u^2}{M\lambda_c^3}\begin{pmatrix} \lambda_c^6 & \lambda_c^3 & \lambda_c^3 \\ \lambda_c^3 & 1 & 1 \\ \lambda_c^3 & 1 & 1 \end{pmatrix}. \tag{6.8}$$

A characteristic of the seesaw mechanism is that the charges of the $\bar{N}_i$ do not enter in the determination of these orders of magnitude as long as there are no massless right-handed neutrinos. Hence the structure of the neutrino mass matrix depends only on the charges of the invariants L_iH_u, already fixed by phenomenology and anomaly cancellation. In the few models with two nonanomalous horizontal symmetries based on E_6 that reproduce the observed quark and charged lepton masses and mixings, the neutrino mass spectrum exhibits the same hierarchical structure: the matrix (6.8) is a very stable prediction of our model. Its diagonalization yields the neutrino mixing matrix [19]

$$\mathcal{U}_{\text{MNS}}=\begin{pmatrix} 1 & \lambda_c^3 & \lambda_c^3 \\ \lambda_c^3 & 1 & 1 \\ \lambda_c^3 & 1 & 1 \end{pmatrix}, \tag{6.9}$$

so that the mixing of the electron neutrino is small, of the order of λ_c^3, while the mixing between the μ and τ neutrinos is of order 1. Remarkably enough, this mixing pattern is precisely the one suggested by the nonadiabatic MSW [20] explanation of the solar neutrino deficit and by the oscillation interpretation of the reported anomaly in atmospheric neutrino fluxes (which has been recently confirmed by the Super-Kamiokande [21] and Soudan [22] Collaborations). It should be stressed here that the model of Ref. [11], which differs from the present one by the fact that $Y^{(1)}$ is along $B+L$ instead of $B-L$, predicts the same lepton mixing matrix. However, it cannot accommodate the MSW effect, because it yields an inverted mass hierarchy in the neutrino sector. The change of $B+L$ into $B-L$ restores the natural hierarchy, but requires the addition of vectorlike matter to cancel anomalies.

Whether the present model actually fits the experimental data on solar and atmospheric neutrinos or not depends on the eigenvalues of the mass matrix (6.8). A naive order of magnitude diagonalization gives a μ and τ neutrinos of comparable masses and a much lighter electron neutrino:

$$m_{\nu_e}\sim m_0\,\lambda_c^6,\quad m_{\nu_\mu},\,m_{\nu_\tau}\sim m_0,\quad m_0=\frac{v_u^2}{M\lambda_c^3}. \tag{6.10}$$

The overall neutrino mass scale m_0 depends on the cutoff M. Thus the neutrino sector allows us, in principle, to measure it.

At first sight, this spectrum is not compatible with a simultaneous explanation of the solar and atmospheric neutrino problems, which requires a hierarchy between m_{ν_μ} and m_{ν_τ}. However, the estimates (6.10) are too crude: since the (2,2), (2,3), and (3,3) entries of the mass matrix all have the same order of magnitude, the prefactors that multiply the powers of λ_c in Eq. (6.8) can spoil the naive determination

of the mass eigenvalues. In order to take this effect into account, we rewrite the neutrino mass matrix, expressed in the basis of charged lepton mass eigenstates, as

$$m_0\begin{pmatrix} a\lambda_c^6 & b\lambda_c^3 & c\lambda_c^3 \\ b\lambda_c^3 & d & e \\ c\lambda_c^3 & e & f \end{pmatrix}, \tag{6.11}$$

where the prefactors a, b, c, d, e, and f, unconstrained by any symmetry, are assumed to be of order 1, say, $0.5<a,\ldots,f<2$. Depending on their values, the two heaviest neutrinos may be either approximately degenerate (scenario 1) or well separated in mass (scenario 2). It will prove convenient in the following discussion to express their mass ratio and mixing angle in terms of the two parameters $x=(df-e^2)/(d+f)^2$ and $y=(d-f)/(d+f)$:

$$\frac{m_{\nu_2}}{m_{\nu_3}}=\frac{1-\sqrt{1-4x}}{1+\sqrt{1-4x}},\quad \sin^2 2\theta_{\mu\tau}=1-\frac{y^2}{1-4x}. \tag{6.12}$$

Scenario 1 corresponds to both regimes $4x\sim1$ and $(-4x)\gg1$, while scenario 2 requires $|x|\ll1$. Let us stress that small values of $|x|$ are very generic when d and f have same sign, provided that $df\sim e^2$. Since this condition is very often satisfied by arbitrary numbers of order 1, a mass hierarchy is not less natural, given the structure (6.8), than an approximate degeneracy.

Scenario 1. $m_{\nu_2}\sim m_{\nu_3}$. In this scenario, the oscillation frequencies $\Delta m_{ij}^2=m_{\nu_j}^2-m_{\nu_i}^2$ are roughly of the same order of magnitude, $\Delta m_{12}^2\sim\Delta m_{23}^2\sim\Delta m_{13}^2$. There is no simultaneous explanation of the solar and atmospheric neutrino data. A strong degeneracy between ν_2 and ν_3, which would result in two distinct oscillation frequencies $\Delta m_{23}^2\ll\Delta m_{12}^2\simeq\Delta m_{13}^2$, would be difficult to achieve in this model,[2] as it would require one either to fine-tune $d\simeq f$ and to allow for $e\ll1$ (case $4x\sim1$) or to fine-tune $d\simeq-f$ [case $(-4x)\gg1$].

Thus, this scenario yields only the MSW effect, with $\Delta m_{12}^2\sim\Delta m_{13}^2\sim10^{-6}$ eV^2, and a total electron neutrino oscillation probability

$$P(\nu_e\to\nu_{\mu,\tau})=4\,u^2\lambda_c^6\sin^2\!\left(\frac{\Delta m_{12}^2 L}{4E}\right)+4v^2\lambda_c^6\sin^2\!\left(\frac{\Delta m_{13}^2 L}{4E}\right), \tag{6.13}$$

where the parameters u and v are defined to be $u=(bf-ce)/(df-e^2)$ and $v=(be-cd)/(df-e^2)$. If Δm_{12}^2 is close enough to Δm_{13}^2, Eq. (6.13) can be viewed as a two-flavor oscillation with a mixing angle $\sin^2 2\theta=4(u^2+v^2)\lambda_c^6$. The solar neutrino data then require $(u^2+v^2)\sim10-20$ [24], which is still reasonable in our approach.

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[2]This is to be contrasted with the models of Ref. [23], in which the close degeneracy is linked to the structure of the neutrino mass matrix.

Although the mixing between μ and τ neutrinos is of order 1, they are too light to account for the atmospheric neutrino anomaly.

Scenario 2. $m_{\nu_2} \ll m_{\nu_3}$. The two distinct oscillation frequencies Δm_{12}^2 and $\Delta m_{13}^2 \simeq \Delta m_{23}^2$ can explain both the solar and atmospheric neutrino data: nonadiabatic MSW $\nu_e \to \nu_{\mu,\tau}$ transitions require [24]

$$4 \times 10^{-6} \ \text{eV}^2 \lesssim \Delta m^2$$
$$\lesssim 10^{-5} \ \text{eV}^2 \quad (\text{best fit:} \quad 5 \times 10^{-6} \ \text{eV}^2), \tag{6.14}$$

while an oscillation solution to the atmospheric neutrino anomaly requires [25]

$$5 \times 10^{-4} \ \text{eV}^2 \lesssim \Delta m^2$$
$$\lesssim 5 \times 10^{-3} \ \text{eV}^2 \quad (\text{best fit:} \quad 10^{-3} \ \text{eV}^2). \tag{6.15}$$

To accommodate both, we need $0.03 \lesssim m_{\nu_2}/m_{\nu_3} \simeq x \lesssim 0.15$ (with $x = 0.06$ for the best fits), which can be achieved without any fine-tuning in our model. Interestingly enough, such small values of x generically push $\sin^2 2\theta_{\mu\tau}$ towards its maximum, as can be seen from Eqs. (6.12). Indeed, since d and f have the same sign and are both of order 1, y^2 is naturally small compared with $(1-4x)$. This is certainly a welcome feature, since the best fit to the atmospheric neutrino data is obtained precisely for $\sin^2 2\theta = 1$.

To be more quantitative, let us fix x and try to adjust y to make $\sin^2 2\theta_{\mu\tau}$ as close to 1 as possible. With $x = 0.06$, one obtains $\sin^2 2\theta_{\mu\tau} = 0.9$ for $y \simeq 0.3$, $\sin^2 2\theta_{\mu\tau} = 0.95$ for $y \simeq 0.2$, and $\sin^2 2\theta_{\mu\tau} = 0.98$ for $y \simeq 0.1$. This shows that very large values of $\sin^2 2\theta_{\mu\tau}$ can be obtained without any fine-tuning (note that $y = 1/3$ already for $d/f = 2$). Thus, in the regime $x \ll 1$, $\nu_\mu \leftrightarrow \nu_\tau$ oscillations provide a natural explanation for the observed atmospheric neutrino anomaly. As for the solar neutrino deficit, it can be accounted for by MSW transitions from the electron neutrinos to both μ and τ neutrinos, with parameters $\Delta m^2 = \Delta m_{12}^2$ and $\sin^2 2\theta = 4 u^2 \lambda^6$. To match the mixing angle with experimental data, one needs $u \sim 3-5$; we note that such moderate values of u are favored by the fact that $df \sim e^2$.

In both scenarios, the scale of the neutrino masses measures the cutoff M. In scenario 1, the MSW effect requires $m_0 \sim 10^{-3}$ eV, which gives $M \sim 10^{18}$ GeV. In scenario 2, the best fit to the atmospheric neutrino data gives m_0 ($d + f) = m_{\nu_2} + m_{\nu_3} \simeq 0.03$ eV, which corresponds to a slightly lower cutoff 10^{16} GeV $\lesssim M \lesssim 4 \times 10^{17}$ GeV (assuming $0.2 \lesssim d + f \lesssim 5$). It is remarkable that those values are so close to the unification scale obtained by running the standard-model gauge couplings. This result depends of course on our choice for $X_{\bar{N}}$, since

$$m_0 = \frac{v_u^2}{M} \lambda_c^{6(1+X_{\bar{N}})}, \tag{6.16}$$

but the value $X_{\bar{N}} = -3/2$ is precisely that favored by the flat direction analysis. As a comparison, $X_{\bar{N}} = -1/2$ would give $M \sim 10^{22}$ GeV, and $X_{\bar{N}} \lesssim -5/2$ corresponds to $M < 10^{14}$ GeV.

Turning the argument the other way, had we set $M = M_U$ *ab initio*, the value of $X_{\bar{N}}$ favored by the flat direction analysis would yield precisely the neutrino mass scale needed to explain the solar neutrino deficit, $m_0 \sim 10^{-3}$ eV. Other values of $X_{\bar{N}}$ would give mass scales irrelevant to the data: $X_{\bar{N}} = -1/2$ corresponds to $m_0 \sim 10^{-7}$ eV, which is not interesting for neutrino phenomenology, and $X_{\bar{N}} \lesssim -5/2$ to $m_0 > 10$ eV, which, given the large mixing between μ and τ neutrinos (and assuming no fine-tuned degeneracy between them), is excluded by oscillation experiments.

To conclude, our model can explain both the solar neutrino deficit and the atmospheric neutrino anomaly, depending on the values of the order-1 factors that appear in the neutrino mass matrices. The cutoff M, which is related to the neutrino mass scale, is determined to be close to the unification scale. Finally, our model predicts neither a neutrino mass in the few eV range, which could account for the hot component of the dark matter needed to understand structure formation, nor the LSND result [26]. The upcoming flood of experimental data on neutrinos will severely test our model.

VII. VECTORLIKE MATTER

To cancel anomalies involving hypercharge, vectorlike matter with standard-model charges must be present. Its nature is not fixed by phenomenology, but by a variety of theoretical requirements: vectorlike matter must not affect the unification of gauge couplings, must cancel anomalies, must yield the value of the Cabibbo angle, must not create unwanted flat directions in the DSW vacuum, and of course must be sufficiently massive to have avoided detection. As we shall see below, our E_6-inspired model, with vectorlike matter in $5 - \bar{5}$ combinations, comes close to satisfying these requirements, except that it produces a high value for the expansion parameter.

The masses of the three families of standard-model vectorlike matter are determined through the same procedure, namely, operators of the form

$$M \bar{\mathbf{D}}_i \mathbf{D}_j \left(\frac{\theta_1}{M}\right)^{s_{ij}^{(1)}} \left(\frac{\theta_2}{M}\right)^{s_{ij}^{(2)}} \left(\frac{\theta_3}{M}\right)^{s_{ij}^{(3)}}$$
$$+ M \bar{E}_i E_j \left(\frac{\theta_1}{M}\right)^{t_{ij}^{(1)}} \left(\frac{\theta_2}{M}\right)^{t_{ij}^{(2)}} \left(\frac{\theta_3}{M}\right)^{t_{ij}^{(3)}}. \tag{7.1}$$

The X charges of the standard-model invariant mass terms are the same:

$$X(\bar{\mathbf{D}}_i \mathbf{D}_j) = X(\bar{E}_i E_j) = 2\alpha - 4\gamma \equiv -\mathbf{n}. \tag{7.2}$$

Its value determines the X charge, since $X^{[d]} = -3$ and $X_{\bar{N}_i} = -3/2$ already fix $\beta = -3/20$ and $\alpha + \gamma = -3/4$. It also fixes the orders of magnitude of the vectorlike masses.

First we note that **n** must be a non-negative integer. The reason is that the power of θ_1 is **n**, the X charge of the invariant, and by holomorphy, it must be zero or a positive integer. Thus if **n** is negative, all vectorlike matter is massless, which is not acceptable. The exponents for the heavy quark matrix are given by the integer **n** plus

$$\begin{pmatrix} (0,-3,-3) & (0,-1,-3) & (0,1,-1) \\ (0,-2,0) & (0,0,0) & (0,2,2) \\ (0,-1,1) & (0,1,1) & (0,3,3) \end{pmatrix} : \quad \bar{\mathbf{D}}_i \mathbf{D}_j$$

(7.3)

and those of the heavy leptons by **n** plus

$$\bar{E}_i E_j : \begin{pmatrix} (0,-3,-3) & (0,-2,-2) & (0,-1,-1) \\ (0,-1,-1) & (0,0,0) & (0,1,1) \\ (0,1,1) & (0,2,2) & (0,3,3) \end{pmatrix} .$$

(7.4)

Since these particles carry standard-model quantum numbers, they can affect gauge coupling unification. As these states fall into complete SU(5) representations, the gauge couplings unify at one loop like in the MSSM, provided that the mass splitting between the doublet and the triplet is not too large.

n$=0$. We obtain the mass matrices

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \lambda_c^4 \\ 0 & \lambda_c^2 & \lambda_c^6 \end{pmatrix} M = M_{\bar{\mathbf{D}}\mathbf{D}}, \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & \lambda_c^2 \\ \lambda_c^2 & \lambda_c^4 & \lambda_c^6 \end{pmatrix} M = M_{\bar{E}E} .$$

(7.5)

Diagonalization of these matrices yields one zero eigenvalue for both matrices and nonzero (order of magnitude) eigenvalues M and $\lambda_c^7 M$ for $M_{\bar{\mathbf{D}}\mathbf{D}}$ and M and $\lambda_c^2 M$ for $M_{\bar{E}E}$. The pair of zero eigenvalues is clearly undesirable and furthermore the mass splitting between the second family E and $\mathbf{D}$ destroys gauge coupling unification. This excludes **n**$=0$.

n$=1$. The mass matrices are

$$M_{\bar{\mathbf{D}}\mathbf{D}} = M \begin{pmatrix} 0 & 0 & \lambda_c^3 \\ 0 & \lambda_c^3 & \lambda_c^7 \\ \lambda_c^3 & \lambda_c^5 & \lambda_c^9 \end{pmatrix}, \quad M_{\bar{E}E} = M \begin{pmatrix} 0 & 0 & \lambda_c \\ \lambda_c & \lambda_c^3 & \lambda_c^5 \\ \lambda_c^5 & \lambda_c^7 & \lambda_c^9 \end{pmatrix} .$$

(7.6)

The eigenvalues for $M_{\bar{\mathbf{D}}\mathbf{D}}$ are $\lambda_c^3 M$, $\lambda_c^3 M$, and $\lambda_c^3 M$ and, for $M_{\bar{E}E}$, $\lambda_c M$, $\lambda_c M$, and $\lambda_c^9 M$. The splitting between the members of the third family vectorlike fields is too large and, as a consequence, gauge coupling unification is spoiled.

n$=2$. The mass matrices are

$$M_{\bar{\mathbf{D}}\mathbf{D}} = M \begin{pmatrix} 0 & 0 & \lambda_c^6 \\ \lambda_c^4 & \lambda_c^6 & \lambda_c^{10} \\ \lambda_c^6 & \lambda_c^8 & \lambda_c^{12} \end{pmatrix},$$

$$M_{\bar{E}E} = M \begin{pmatrix} 0 & \lambda_c^2 & \lambda_c^4 \\ \lambda_c^4 & \lambda_c^6 & \lambda_c^8 \\ \lambda_c^8 & \lambda_c^{10} & \lambda_c^{12} \end{pmatrix} .$$

(7.7)

The eigenvalues are now $\lambda_c^4 M$, $\lambda_c^6 M$, $\lambda_c^8 M$ and $\lambda_c^2 M$, $\lambda_c^4 M$, $\lambda_c^{12} M$, respectively. There is again splitting between the families of the doublet and the triplet and therefore the gauge couplings do not unify at one loop. The splitting in this case is not too big and a two-loop analysis may actually prove this case viable from the gauge coupling unification point of view.

n$=3$. We obtain the mass matrices

$$M_{\bar{\mathbf{D}}\mathbf{D}} = M \begin{pmatrix} \lambda_c^3 & \lambda_c^5 & \lambda_c^9 \\ \lambda_c^7 & \lambda_c^9 & \lambda_c^{13} \\ \lambda_c^9 & \lambda_c^{11} & \lambda_c^{15} \end{pmatrix},$$

$$M_{\bar{E}E} = M \begin{pmatrix} \lambda_c^3 & \lambda_c^5 & \lambda_c^7 \\ \lambda_c^7 & \lambda_c^9 & \lambda_c^{11} \\ \lambda_c^{11} & \lambda_c^{13} & \lambda_c^{15} \end{pmatrix},$$

(7.8)

with eigenvalues

$$M_{\mathbf{D}} = \{\lambda_c^3 M, \quad \lambda_c^9 M, \quad \lambda_c^{15} M\}$$

(7.9)

and

$$M_E = \{\lambda_c^3 M, \quad \lambda_c^9 M, \quad \lambda_c^{15} M\},$$

(7.10)

respectively. The unification of couplings in this case is preserved. For **n**≥ 3, there are no supersymmetric zeros in the mass matrices and the mass eigenvalues are just the diagonal entries; so there is no splitting between masses of the same family of **D** and E. A simple one-loop analysis using self-consistently $M = M_U$ in the mass of the vectorlike particles and for the unification scale yields unified gauge couplings at the unification scale M_U:

$$n=3: \quad \alpha(M_U) \sim \frac{1}{19}, \quad M_U \sim 3 \times 10^{16} \text{ GeV.} \quad (7.11)$$

For **n** large, other problems arise as the vectorlike matter becomes too light. This can easily spoil gauge coupling unification by two-loop effects [28] and cause significant deviations from precision measurements of standard-model parameters [27,28]. Thus the unification of the gauge couplings favors **n**$=3$.

The value of **n** also determines the mixing between the chiral and vectorlike matter. Indeed, the quantum numbers of the vectorlike matter allow for mixing with the chiral fami-

TABLE I. Operators that mix MSSM fields with vectorlike matter with $\beta = -3/20$.

Class 1		Class 2		Class 3	
$E H_u$	$\left[\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$	$L\bar{E}$	$\left[-\dfrac{\mathbf{n}}{2}\right]$	$\mathbf{\bar{u}D\bar{D}}$	$\left[-\mathbf{n} - \dfrac{3}{2}\right]$
$\bar{E} H_d$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$	$\mathbf{D\bar{d}}$	$\left[-\dfrac{\mathbf{n}}{2}\right]$	$E Q \bar{D}$	$\left[-\mathbf{n} - \dfrac{3}{2}\right]$
$\mathbf{QQD}$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$	$\mathbf{Q\bar{D}} H_d$	$\left[-3 - \dfrac{\mathbf{n}}{2}\right]$	$E E \bar{e}$	$\left[-\mathbf{n} - \dfrac{3}{2}\right]$
$\mathbf{\bar{u}d\bar{D}}$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$	$E \bar{e} H_d$	$\left[-3 - \dfrac{\mathbf{n}}{2}\right]$		
$\mathbf{Q\bar{u}}\bar{E}$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$				
$E Q \bar{d}$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$				
$L Q \mathbf{\bar{D}}$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$				
$L E \bar{e}$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$				
$\mathbf{D}\bar{u}\bar{e}$	$\left[-\dfrac{3}{2} - \dfrac{\mathbf{n}}{2}\right]$				

lies, since (E_i, L_j, H_d), $(\bar{E}_i$ with $H_u)$, and $(\mathbf{\bar{D}}_i$ with $\bar{\mathbf{d}}_j)$ have the same standard-model quantum numbers. This generates new standard-model invariants. In Table I, we give a set of mixed operators up to superfield dimension 3. Next to the operator we show its X charge in brackets. One notices that the operators fall into three classes.

For $\mathbf{n}$ odd only the operators of the first class can appear in the superpotential and for $\mathbf{n}$ even only operators of the second class appear. The third class is excluded for any integer value of $\mathbf{n}$. Let us examine these two possibilities in more detail.

$\mathbf{n} = 2,4,6,\ldots$. Only operators of the second class are allowed in W and $\mathbf{\bar{D}}$ mixes with $\bar{\mathbf{d}}$. The mixing is computed by diagonalizing the down-type quark mass matrices. To see this, we give a one-family example where the operators $\mathbf{\bar{D}D}$, $\mathbf{Q\bar{d}}H_d$, $\mathbf{Q\bar{D}}H_d$, and $\mathbf{D\bar{d}}$ are all present in the superpotential. After electroweak breaking the masses of the down-type quark fields come from diagonalizing the matrix

$$\begin{pmatrix} v_d Y^{[d]} & v_d Y^{[D]} \\ M_{\mathbf{D\bar{d}}} & M_{\mathbf{\bar{D}D}} \end{pmatrix}. \tag{7.12}$$

The extra quark fields affect the down quark mass matrices of Sec. V and modify our previous order of magnitude estimates. The same type of mixing happens in the lepton sector due to the operators $\bar{E}E$, $L\bar{E}$, and $E\bar{e}H_d$. If allowed, this type of mixing produces phenomenologically unacceptable mass patterns for quarks and charged leptons.

$\mathbf{n} = 3,5,\ldots$. Operators of the first class are allowed since their X charges are all negative integers. Because of the mixing of the heavy leptons with the Higgs doublets, we have to diagonalize the following mass matrix (we give again a simple one-family example):

$$\begin{pmatrix} \mu & M_{\bar{E}H_d} \\ M_{EH_u} & M_{\bar{E}E} \end{pmatrix}. \tag{7.13}$$

The 11 entry is the μ term generated by the Giudice-Masiero mechanism and is naturally of the order of 1 TeV. The Higgs eigenstates will be modified to

$$H_u' = H_u + \sum_i c_i^u \cdot \bar{E}_i \tag{7.14}$$

and

$$H_d' = H_d + \sum_i c_i^d \cdot E_i, \tag{7.15}$$

where $c_i^{u,d}$ are mixing angles to be obtained upon diagonalization. With both off-diagonal entries present, this matrix has two large eigenvalues and consequently the Higgs mass is driven to the Planck scale. If one of the off-diagonal entries is missing, then the matrix has one small and one large eigenvalue and the mixing is harmless as long as the angles $c_i^{u,d}$ are small (see later).

There are several ways to evade these problems. One is to relax the simple but very restrictive assumption that X is the same for both the MSSM and the vectorlike fields and another is to assume the existence of a discrete symmetry that prohibits the dangerous operators.

A. Shift X

The vectorlike matter could come from a different **27** than the MSSM fields so that the X charges of the vectorlike fields are shifted relative to the fields in the **16**:

$$X_{VL} = \bar{\alpha} + \bar{\beta} V + \bar{\gamma} V'. \tag{7.16}$$

In Table II we show the different operators with their X charges. It is interesting to notice that the X charges of these operators depend only on $\bar{\beta}$ and $\mathbf{n} = -2\bar{\alpha} + 4\bar{\gamma}$. We have two possibilities.

1. No MSSM vectorlike mixing

We can choose $\bar{\beta}$ in such a way that none of the X charges of the operators appearing in Table II is an integer for any integer $\mathbf{n}$. None of them will appear in W and therefore we avoid the mixing problem. Then, the lightest of the vectorlike fields will be stable. To avoid cosmological problems, this requires a reheating temperature lower than the lowest vectorlike mass in order to dilute their abundance during inflation. Recall that the mass of the lightest pair of **D** and E for $\mathbf{n} = 3$ is $\lambda_c^{15} M \sim 10^{6-7}$ GeV, and therefore a re-

TABLE II. Operators that mix MSSM fields with vectorlike matter with $X_{VL} = \bar{\alpha} + \bar{\beta}V + \bar{\gamma}V'$.

Class 1	Class 2	Class 3
EH_u $\left[-\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}-\dfrac{18}{10}\right)\right]$	$L\bar{E}$ $\left[\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{3}{10}\right)\right]$	$\mathbf{u}\bar{\mathbf{D}}\bar{D}$ $\left[-\left(-4\bar{\beta}+\mathbf{n}+\dfrac{9}{10}\right)\right]$
$\bar{E}H_d$ $\left[-\left(2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{18}{10}\right)\right]$	$\mathbf{D}\bar{\mathbf{d}}$ $\left[-\left(2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{3}{10}\right)\right]$	$EQ\bar{\mathbf{D}}$ $\left[-\left(-4\bar{\beta}+\mathbf{n}+\dfrac{9}{10}\right)\right]$
$\mathbf{QQD}$ $\left[-\left(2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{18}{10}\right)\right]$	$\mathbf{Q}\bar{\mathbf{D}}H_d$ $\left[-\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{27}{10}\right)\right]$	$EE\bar{e}$ $\left[-\left(-4\bar{\beta}+\mathbf{n}+\dfrac{9}{10}\right)\right]$
$\bar{\mathbf{u}}\mathbf{d}\bar{\mathbf{D}}$ $\left[-\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{12}{10}\right)\right]$	$E\bar{e}H_d$ $\left[-\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{27}{10}\right)\right]$	
$\mathbf{Q}\bar{\mathbf{u}}\bar{E}$ $\left[-\left(2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{18}{10}\right)\right]$		
$EQ\bar{\mathbf{d}}$ $\left[-\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{12}{10}\right)\right]$		
$LQ\bar{\mathbf{D}}$ $\left[-\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{12}{10}\right)\right]$		
$LE\bar{e}$ $\left[-\left(-2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{12}{10}\right)\right]$		
$\mathbf{D}\bar{\mathbf{u}}\bar{e}$ $\left[-\left(2\bar{\beta}+\dfrac{\mathbf{n}}{2}+\dfrac{18}{10}\right)\right]$		

heating temperature of at most this order of magnitude is required. $\mathbf{n}=4$ or higher result in lower eigenvalues and thus lower reheating temperatures. We therefore favor in this case $\mathbf{n}=3$. Similar arguments apply to any other scenario with stable heavy vectorlike states.

2. Partial MSSM vectorlike mixing

Let us take $\mathbf{n}=3$ which avoids the dangerous $\bar{\mathbf{d}}$-$\bar{\mathbf{D}}$ and L-$\bar{E}$ mixing. The X charges of the operators that could give rise to mixing are $X(EH_u) = 2\bar{\beta} + 3/10$ and $X(\bar{E}H_d) = -2\bar{\beta} - 33/10$. We can choose $\bar{\beta}$ in a way that $X(EH_u)$ is positive and $X(\bar{E}H_d)$ is negative and so prohibit EH_u from appearing but allow $\bar{E}H_d$. This yields the mass matrix (7.10) with its 21 element being zero. As we mentioned before, the mixing is harmless if the angles $c_i^{u,d}$ are small which is indeed the case.

We still have to check if the proton decays due to mixed operators slowly enough to avoid conflict with experimental data. Proton decay due to operators consisting only of MSSM fields will be discussed in a separate section, since it is independent of the choice of the charges of the vectorlike matter.

We find that the dominant proton decay channels come from the operators

$$LQ\bar{\mathbf{D}} \quad \text{and} \quad \bar{\mathbf{u}}\mathbf{d}\bar{\mathbf{D}} \tag{7.17}$$

and

$$\mathbf{QQD} \quad \text{and} \quad \mathbf{D}\bar{\mathbf{u}}\bar{e} \tag{7.18}$$

via an intermediate heavy quark. They appear after DSW breaking as

$$\lambda_{ijk}L_i\mathbf{Q}_j\bar{\mathbf{D}}_k + \bar{\lambda}_{ijk}\bar{\mathbf{u}}_i\bar{\mathbf{d}}_j\bar{\mathbf{D}}_k \tag{7.19}$$

and

$$\rho_{ijk}\mathbf{Q}_i\mathbf{Q}_j\mathbf{D}_k + \bar{\rho}_{ijk}\mathbf{D}_i\bar{\mathbf{u}}_j\bar{e}_k, \tag{7.20}$$

where

$$\lambda_{ijk} \sim \left(\frac{\langle\theta_1\rangle}{M}\right)^{n^{(1)}}\left(\frac{\langle\theta_2\rangle}{M}\right)^{n^{(2)}}\left(\frac{\langle\theta_3\rangle}{M}\right)^{n^{(3)}} \tag{7.21}$$

is the suppression factor in the DSW vacuum in front of the corresponding operator with flavor indices i,j,k. Similar expressions hold for $\bar{\lambda}_{ijk}$, ρ_{ijk}, and $\bar{\rho}_{ijk}$. The experimental constraint on these is [29]

$$\lambda_{ijk}\bar{\lambda}_{ijk} \lesssim M_{\mathbf{D}}^2 \ 10^{-32} \ \text{GeV}^{-2} \qquad (7.22)$$

and similarly

$$\rho_{ijk}\bar{\rho}_{ijk} \lesssim M_{\mathbf{D}}^2 \ 10^{-32} \ \text{GeV}^{-2}, \qquad (7.23)$$

We computed the suppression factors of these operators in the DSW vacuum that the model gives for $\bar{\beta}=7/20$ and we found that the above constraints are very difficult to satisfy. Notice that this choice amounts to shifting the X charge of the vectorlike matter by half a unit of V. Interestingly enough, a similar mechanism occurs in some superstring models, as a result of Wilson line breaking [30].

B. Discrete symmetry

It is known that superstring models usually contain discrete symmetries. If present, they could forbid the dangerous mixed operators, leaving the mass terms for the vectorlike matter intact.

As an example, consider the discrete symmetry where

$$E \rightarrow -E, \quad \bar{E} \rightarrow -\bar{E}, \quad \mathbf{D} \rightarrow -\mathbf{D}, \quad \bar{\mathbf{D}} \rightarrow -\bar{\mathbf{D}}. \quad (7.24)$$

This additional symmetry, indeed, completely decouples the MSSM fields from the vectorlike matter. No operator with an odd number of vectorlike fields is allowed for any value of $\mathbf{n}$. Specifically, all operators that mix MSSM fields and vectorlike matter and that can cause proton decay are also prohibited. Such are the dimension-3 operators

$$L\mathbf{Q}\bar{\mathbf{D}} \quad \text{and} \quad \bar{\mathbf{u}}\mathbf{d}\bar{\mathbf{D}}, \qquad (7.25)$$

which belong to class 1 and the dimension-4 operators

$$\mathbf{Q}\mathbf{Q}\mathbf{Q}E, \quad \bar{\mathbf{u}}\bar{\mathbf{u}}\bar{\mathbf{D}}\bar{e}. \qquad (7.26)$$

As a consequence of this discrete symmetry, the vectorlike matter has no available decay channels. This can have undesired cosmological implications except if inflation takes place at a temperature lower than the lightest of the vectorlike particles. For this reason we strongly favor the value $\mathbf{n}=3$. Also in this case we can keep the simple universal X charge assignment $X=\alpha+\beta V+\gamma V'$ for both the MSSM and the vectorlike fields which makes the flat direction analysis particularly simple because the superpotential has a very small number of supersymmetric zeros corresponding to standard-model invariants with vectorlike fields.

C. Summary

To summarize, we have given three alternative ways to fix the X charges of the vectorlike fields.

The solution of Sec. VII A 1 is viable for a reheating temperature $\sim 10^{6-7}$ GeV for $\mathbf{n}=3$. Lower reheating tempera-

tures are required as $\mathbf{n}$ increases, and so in this case $\mathbf{n}=3$ is clearly favored.

The solution of Sec. VII A 2 ($\bar{\beta}=7/20$) is not viable even if the mixing angles $c_i^{u,d}$ are small because proton decay is too fast. The vectorlike particles can decay.

The solution of Sec. VII B involves a discrete symmetry. Stable heavy quarks and leptons require a reheating temperature $\sim 10^{6-7}$ GeV for $\mathbf{n}=3$ and lower temperatures for higher values of $\mathbf{n}$; so $\mathbf{n}=3$ is again favored. In this case the flat direction analysis is particularly simple.

We do not have any physical motivation that can tell us which of the above proposed mechanisms is the correct one. The simplest is the scenario with the discrete symmetry and from now on we will continue our discussion on flat directions and proton decay in this context.

VIII. THE HIDDEN SECTOR

So far we have described the matter necessary to satisfy the anomaly conditions that involve standard-model quantum numbers, the breaking of the extra gauge symmetries, and phenomenology. These are the three chiral families, the three right-handed neutrinos, the three vectorlike families just described, and three θ fields necessary to produce the DSW vacuum. We refer to this as *visible* matter. By fixing the value of $X(\bar{E}E)=X(\bar{\mathbf{D}}\mathbf{D})=-\mathbf{n}$, the X charge is totally determined. Since gauge unification favors $\mathbf{n}=3$, the weak and color anomalies are fixed, $C_{\text{color}}=C_{\text{weak}}=-18$.

This enables us to ''predict'' the value of the Cabibbo angle through the relation

$$\lambda_c \sim \lambda = \frac{\langle \theta \rangle}{M} = \sqrt{\frac{-g_{\text{string}}^2}{192\pi^2}C_{\text{grav}}}. \qquad (8.1)$$

Using the Green-Schwarz relation

$$\frac{C_{\text{grav}}}{12} = \frac{C_{\text{weak}}}{k_{\text{weak}}} \qquad (8.2)$$

and the identification

$$g_{\text{string}}^2 = k_{\text{weak}}g_{\text{weak}}^2, \qquad (8.3)$$

we relate the Cabibbo angle to the gauge couplings at the cutoff $\alpha(M)$, using only *visible* matter contributions:

$$\lambda_c \sim \sqrt{-\frac{C_{\text{weak}}}{4\pi}\alpha(M)}. \qquad (8.4)$$

For $\mathbf{n}=3$, the couplings unify with $\alpha\sim 1/19$, which yields $\lambda=0.28$, clearly of the same order of magnitude as the Cabibbo angle. Given the many uncertainties in this type of theory, the consistency of these results with nature is remarkable. We note that the numerical value of the expansion parameter clearly depends on the contribution of the vectorlike matter to C_{weak}, about which we have no direct experimental information.

In addition, the values of the mixed gravitational anomaly is also determined through the relation

$$C_g = 12 \frac{C_{\text{weak}}}{k_{\text{weak}}}. \tag{8.5}$$

For integer k_{weak} and $\mathbf{n}=3$, this implies that $C_{\text{grav}}=-216$, $-108,-72,\ldots$ for $k_{\text{weak}}=1,2,3,\ldots$, to be compared with the *visible* matter contribution to $C_{\text{grav}}=-80$. Thus additional fields are required, and $k_{\text{weak}} \leq 2$, to avoid fields with positive X charges that spoil the DSW vacuum. Another argument for new fields is that not all anomalies are cancelled, since we have, from the θ sector,

$$XXY^{(2)}=1, \quad Y^{(1)}Y^{(1)}Y^{(2)}=Y^{(1)}Y^{(2)}Y^{(2)}=-1, \tag{8.6}$$

and, from all *visible* matter,

$$XY^{(1)}Y^{(2)}=-18. \tag{8.7}$$

The construction of a hidden sector theory that cancels these anomalies, and provides the requisite that C_{grav} is rather arbitrary, since we have few guidelines: anomaly cancellation and the absence of flat directions which indicates that the X charges of the hidden matter should be negative.

If we use as a theoretical guide the $E_8 \times E_8$ heterotic theory, we expect an exceptional gauge theory in the hidden sector. In particular, Binétruy and Dudas [12] considered a hidden gauge group G with a pair of matter fields with the same X charge, but vectorlike with respect to all other symmetries, causing supersymmetry breaking. This theory contributes to few anomalies, only in C_{grav}, $(XY^{(1)}Y^{(2)})$, and the anomaly associated with the hidden gauge group G, related by the Green-Schwarz relation

$$C_G = -18 \frac{k_G}{k_{\text{weak}}}, \tag{8.8}$$

where k_G is the Kac-Moody integer level (k_G integer heavily constrains possible theories of this type). It must be augmented by other fields, since it does not cancel the remaining anomalies $(XXY^{(2)})$, $(Y^{(1)}Y^{(1)}Y^{(2)})$, and $(Y^{(1)}Y^{(2)}Y^{(2)})$. These will be accounted for by singlet fields.

There is a simple set of four singlet fields, Σ_a, which absorb many of the remaining anomalies, without creating unwanted flat directions. Their charges are given in the following:

	Σ_1	Σ_2	Σ_3	Σ_4
X	$-1/2$	$-1/2$	0	0
$Y^{(1)}$	0	0	$1/2$	$-1/2$
$Y^{(2)}$	$-9/4$	$-7/4$	$9/4$	$7/4$

They cancel the anomalies from the θ sector, since, over the Σ fields,

$$XY^{(1)}Y^{(2)}=0, \quad XXY^{(2)}=-1,$$

$$Y^{(1)}Y^{(1)}Y^{(2)}=Y^{(1)}Y^{(2)}Y^{(2)}=1, \tag{8.9}$$

as well as $C_{\text{grav}}=-1$.

The remaining anomalies can be accounted for by a simple gauge theory based on $G=E_6$; it has two matter fields with $X=x_1$, transforming as the **78** and one $\mathbf{27},\overline{\mathbf{27}}$ pair, each with $X=x_2$. For $k_{\text{weak}}=1$, we find that

$$C_{\text{grav}}=-216 \quad \rightarrow \quad 2(27x_2+78x_1)=-135. \tag{8.10}$$

The gauge anomaly condition is given by

$$C_G=2(6x_2+24x_1)=-18k_G. \tag{8.11}$$

For $k_G=1$, one of the charges is positive, leading to undesirable flat directions, while for $k_G=2$, we find $x_1=-9/20$ and $x_2=-6/5$. The adjoint fields have no $Y^{(1,2)}$ charges, and the pair of **27**-$\overline{\mathbf{27}}$ have vectorlike charges with respect to $Y^{(1,2)}$, with charges $5/9$ and $1/2$, respectively. This sector breaks supersymmetry, but it cannot be the main agent for supersymmetry breaking, since it produces nondegenerate squark masses, and our model does not have alignment.

The singlet fields have little effect on low-energy phenomenology. Computation of the powers of the θ fields in the mass invariants $\Sigma_a \Sigma_b$ yields in the DSW vacuum the mass matrix of the Σ fields before SUSY breaking:

$$M_\Sigma = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M\lambda_c^8 \\ 0 & 0 & M\lambda_c^8 & 0 \end{pmatrix}; \tag{8.12}$$

it has two zero eigenvalues. The Giudice-Masiero mechanism can fill in the 12 (and 21) entries after SUSY breaking, yielding

$$M_\Sigma = \begin{pmatrix} 0 & m\lambda_c^7 & 0 & 0 \\ m\lambda_c^7 & 0 & 0 & 0 \\ 0 & 0 & 0 & M\lambda_c^8 \\ 0 & 0 & M\lambda_c^8 & 0 \end{pmatrix}, \tag{8.13}$$

where m is of order of the SUSY breaking scale. The above matrix has now two large ($\sim 10^{11}$ GeV) and two small ($1-100$ MeV) eigenvalues. The two heavy states get diluted during inflation. The two light states are stable since their lowest-order coupling to the light fields is quartic, dominated by terms like $\Sigma_1\Sigma_2 H_u H_d$. Although stable and undiluted by inflation, their contribution to the energy density of the universe is negligible.

Finally we note that it is difficult to produce models for the hidden sector; for example we could take $G=E_7$ with $k_G=2$, two matter fields transforming as the **133** (adjoint) representation, but there does not seem to be any simple set of singlet fields with the requisite anomalies.

IX. *R* PARITY

The invariants of the minimal standard model and their associated flat directions have been analyzed in detail in the literature [31]. In models with an anomalous U(1), these invariants carry in general X charges, which, as we have seen, determines their suppression in the effective Lagrangian. Just as there is a basis of invariants, proved long ago by Hilbert, the charges of these invariants are not all independent; they can in fact be expressed in terms of the charges of the lowest-order invariants built out of the fields of the minimal standard model and some anomaly coefficients.

The X charges of the three types of cubic standard-model invariants that violate R parity as well as baryon and/or lepton numbers can be expressed in terms of the X charges of the MSSM invariants and the R-parity-violating invariant

$$X^{[\cancel{R}]} \equiv X(LH_u), \qquad (9.1)$$

through the relations

$$X_{LQ\bar{d}} = X^{[d]} - X^{[\mu]} + X^{[\cancel{R}]}, \qquad (9.2)$$

$$X_{LL\bar{e}} = X^{[e]} - X^{[\mu]} + X^{[\cancel{R}]}, \qquad (9.3)$$

$$X_{\overline{udd}} = X^{[d]} + X^{[\cancel{R}]} + \frac{1}{3}(C_{color} - C_{weak}) - \frac{2}{3}X^{[\mu]}. \quad (9.4)$$

Although they vanish in our model, we still display $X^{[u]}$ and $X^{[\mu]} = 0$, since these sum rules are more general.

In the analysis of the flat directions, we have seen how the seesaw mechanism forces the X charge of $\bar{N}$ to be a half-odd integer. Also, the Froggatt-Nielsen [32] suppression of the minimal standard-model invariants and the holomorphy of the superpotential require $X^{[u,d,e]}$ to be zero or negative integers, and the equality of the Kác-Moody levels of SU(2) and SU(3) forces $C_{color} = C_{weak}$, through the Green-Schwarz mechanism. Thus we conclude that the X charges of these operators are half-odd integers, and thus they cannot appear in the superpotential unless multiplied by at least one $\bar{N}$. This reasoning can be applied to the higher-order $\cancel{R}$ operators since their charges are given by

$$X_{QQQH_d} = X^{[u]} + X^{[d]} - \frac{1}{3}X^{[\mu]} - X^{[\cancel{R}]}, \qquad (9.5)$$

$$X_{\overline{ddd}LL} = 2X^{[d]} - X^{[u]} - \frac{5}{3}X^{[\mu]} + 3X^{[\cancel{R}]}, \qquad (9.6)$$

$$X_{QQQQ\bar{u}} = 2X^{[u]} + X^{[d]} - \frac{4}{3}X^{[\mu]} - X^{[\cancel{R}]}, \qquad (9.7)$$

$$X_{\overline{uuu}\bar{e}e} = 2X^{[u]} - X^{[d]} + 2X^{[e]} - \frac{2}{3}X^{[\mu]} - X^{[\cancel{R}]}, \qquad (9.8)$$

It follows that *there are no R-parity-violating operators, whatever their dimensions*: through the right-handed neutri-

nos, R parity is linked to half-odd integer charges, so that charge invariance results in R-parity invariance. Thus *none* of the operators that violate R parity can appear in holomorphic invariants: even after breaking of the anomalous X symmetry, the remaining interactions all respect R parity, leading to an *absolutely stable superpartner*. This is a general result deduced from the uniqueness of the DSW vacuum, the Green-Schwarz anomaly cancellations, and the seesaw mechanism.

X. PROTON DECAY

In the presence of the extra discrete symmetry we introduced before, the operators that mix MSSM fields and vectorlike matter and trigger proton decay are excluded. Since R parity is exactly conserved, the dangerous dimension-3 operators $LQ\bar{d}$ and $\overline{udd}$ that usually induce fast proton decay are also excluded. This leaves for the dominant sources of proton decay the dimension-5 operators that appear in the effective Lagrangian as

$$W = \frac{1}{M}[\kappa_{112i}Q_1Q_1Q_2L_i + \bar{\kappa}_{1jkl}\bar{u}_i\bar{u}_j\bar{d}_k\bar{e}_l], \quad (10.1)$$

where for the first operator the flavor index $i = 1,2$ if there is a charged lepton in the final state and $i = 1,2,3$ if there is a neutrino and $j = 2,3$, $k,l = 1,2$. We have denoted the suppression factors in the DSW vacuum in front of the operators by κ and $\bar{\kappa}$. These operators could, for example, give rise the proton decay modes $p \rightarrow \pi^+ \bar{\nu}_i$ and $p \rightarrow \pi^0 l_i^+$ or to $p \rightarrow K^+ \bar{\nu}_i$ and $p \rightarrow K^0 l_i^+$. In [29], the phenomenological limits on these suppression factors were computed to be

$$\kappa_{112i} \lesssim \lambda_c^{11} \qquad (10.2)$$

and

$$\bar{\kappa}_{1jkl}(K_{RR}^u)_{1j} \lesssim \lambda_c^{12}, \qquad (10.3)$$

where $K_{RR1j}^u = V_R^u \tilde{V}_R^\dagger$. V_R are the matrices that diagonalize on the right the quark and the squark matrices, respectively. We can easily calculate it in this model:

$$K_{RR1j}^u = \begin{pmatrix} 1 & \lambda_c^3 & \lambda_c^5 \\ \lambda_c^3 & 1 & \lambda_c^2 \\ \lambda_c^5 & \lambda_c^2 & 1 \end{pmatrix}. \qquad (10.4)$$

In Table III we give in the first column a list of the dangerous operators $QQQL$ ($\overline{uude}$) and in the second column the suppression κ_{ijkl} ($\bar{\kappa}_{ijkl}K_{RR}^u$) that we computed in our model.

Even though all operators in Table III seem naively sufficiently suppressed so that proton decay is within the experimental bound, it is interesting to examine them more closely from the phenomenological point of view. Consider the operator $Q_1Q_1Q_2L_2$. This operator can lead to proton decay via a W-ino, gluino, Z-ino, photino, or Higgsino exchange. The contribution via gluino exchange could be the dominant due

TABLE III. Operators inducing proton decay and their suppression.

Operator	Supression
$\mathbf{Q_1 Q_1 Q_2 L_1}$	λ_c^{14}
$\mathbf{Q_1 Q_1 Q_2 L_{2,3}}$	λ_c^{11}
$\mathbf{\bar{u}_1 \bar{u}_2 \bar{d}_1 \bar{e}_1}$	λ_c^{15}
$\mathbf{\bar{u}_1 \bar{u}_2 \bar{d}_1 \bar{e}_2}$	λ_c^{16}
$\mathbf{\bar{u}_1 \bar{u}_2 \bar{d}_2 \bar{e}_1}$	λ_c^{14}
$\mathbf{\bar{u}_1 \bar{u}_2 \bar{d}_2 \bar{e}_2}$	λ_c^{15}
$\mathbf{\bar{u}_1 \bar{u}_3 \bar{d}_1 \bar{e}_1}$	λ_c^{13}
$\mathbf{\bar{u}_1 \bar{u}_3 \bar{d}_1 \bar{e}_2}$	λ_c^{14}
$\mathbf{\bar{u}_1 \bar{u}_3 \bar{d}_2 \bar{e}_1}$	λ_c^{12}
$\mathbf{\bar{u}_1 \bar{u}_3 \bar{d}_2 \bar{e}_2}$	λ_c^{13}

to the strong coupling of the gluino. Here let us recall that experimental data strongly suggest a near degeneracy between squark masses in order to avoid large contributions to flavor-changing neutral currents (FCNCs). One mechanism that has been suggested [33] is where alignment between quarks and squarks takes place and therefore FCNCs are suppressed irrespectively of the SUSY breaking mechanism. One can calculate in the model the extent of such an alignment. We find that there is no sufficient quark-squark alignment and therefore FCNCs are not sufficiently suppressed. To agree with experimental data we have to assume that the squark masses that result from SUSY breaking are approximately degenerate, a fact that does not seem to be unlikely in the context of realistic superstring models [15]. In such a case, the contribution due to gluino exchange is negligible.

Generically, a careful calculation of a proton decay process not only involves uncertainties due to our ignorance of superpartner masses but also due to large uncertainties in hadronic matrix elements. Assuming nearly degenerate squarks, the dominant decay mode is via W-ino exchange and the decay rate for the process $p \to K^0 \mu^+$ is given by [34]

$$\Gamma(p \to K^0 \mu^+)$$
$$= \left(\frac{10.5 b \alpha_2 \cos \theta_c}{\pi M} \right)^2 \frac{(m_p^2 - m_K^2)^2}{8 \pi m_p^3 f_\pi^2} |0.7 \kappa_{1122} f(m_{\tilde{w}}, m_{\tilde{q}})|^2,$$

$$(10.5)$$

where here $b = (0.003 - 0.03)$ GeV^3 is an unknown strong matrix element, $\alpha_2 = \alpha / \sin^2 \theta_W$, and from our earlier estimates of the cutoff, $M \sim 3 \times 10^{16}$ GeV. We have two regimes to consider:

$$m_{\tilde{w}} \ll m_{\tilde{q}}: \quad f(m_{\tilde{w}}, m_{\tilde{q}}) = \frac{m_{\tilde{w}}}{m_{\tilde{q}}^2} \qquad (10.6)$$

and

$$m_{\tilde{w}} \gg m_{\tilde{q}}: \quad f(m_{\tilde{w}}, m_{\tilde{q}}) = \frac{1}{m_{\tilde{w}}} \ln \frac{m_{\tilde{w}}^2}{m_{\tilde{q}}^2}. \qquad (10.7)$$

The experimental bound on the decay $p \to K^0 + \mu^+$, which is the dominant one in our theory, is [36]

$$\Gamma(p \to K^0 \mu^+) < 10^{32} \text{ yr}^{-1}. \qquad (10.8)$$

For W-ino masses much larger than squark masses, this decay rate is several orders of magnitude lower than the experimental limit. For W-ino masses much lower than squark masses, the rate is near the experimental limit. For example, with $m_{\tilde{w}} \sim 100$ GeV, $m_{\tilde{q}} \sim 800$ GeV, and $b = 0.003$, we get the lifetime $\sim 10^{31}$ yr, near the experimental bound. Unfortunately our model cannot be more precise, because of the unknown prefactors of order-1 terms in the effective interactions; still it predicts that the proton decays preferentially into a neutral K and an antimuon with a lifetime at or near the present experimental limit. Finally we note that if we use the expansion parameter determined through the Green-Schwarz relation, and not the Cabibbo angle, our estimates get worse and our model implies a proton lifetime slightly shorter than the experimental bound. As we remarked earlier, this value of the expansion parameter depends on the contribution of the vectorlike matter to C_{weak}.

XI. FLAT DIRECTION ANALYSIS

Our model is now completely specified, except for the supersymmetry-breaking sector. We can study its flat directions and check whether the DSW vacuum is unique, using the techniques introduced in Ref. [18]. We shall only sketch the main points, and refer the interested reader to this reference for more details and the discussion of some subtleties.

In the presence of an anomalous U(1), the well-known correspondence between the zeros of the D terms and the holomorphic gauge invariants [35] breaks down. However, the existence of the DSW vacuum $|\langle \theta_1 \rangle|^2 = |\langle \theta_2 \rangle|^2 = |\langle \theta_3 \rangle|^2 = \xi^2$ allows us to rewrite the Abelian D-term constraints as

$$\begin{pmatrix} |\langle \theta_1 \rangle|^2 - \xi^2 \\ |\langle \theta_2 \rangle|^2 - \xi^2 \\ |\langle \theta_3 \rangle|^2 - \xi^2 \end{pmatrix} = \sum_a v_a^2 \begin{pmatrix} n_1^a \\ n_2^a \\ n_3^a \end{pmatrix} + \sum_i |\langle \chi_i \rangle|^2 \begin{pmatrix} n_1^i \\ n_2^i \\ n_3^i \end{pmatrix},$$

$$(11.1)$$

where the $\{\chi_i\}$ are standard-model singlets other than the θ fields, the v_a^2 are vacuum expectation values (VEVs) associated with a basis of standard-model invariants $\{S_a\}$, and the numbers n_α^a and n_α^i are associated with the invariant S_a and singlet χ_i, respectively, by Eq. (4.2). In the present model, the χ fields are the three right-handed neutrinos $\bar{N}_1, \bar{N}_2,$ and

TABLE IV. Flat directions (FDs) of MSSM and $\bar{N}_i$ fields.

Basis invariant	(n_1, n_2, n_3)	Flat direction	FD lifted by
$\bar{N}_3$	$(3/2, -1/2, 1/2)$	$\langle \bar{N}_3, \theta_1, \theta_3 \rangle$	$\bar{N}_2 \bar{N}_3^3 \theta_1^6 \theta_3^4$
$\mathbf{L_1 H_u}$	$(-3/2, 1/2, 5/2)$	$\langle \mathbf{L_1}, \mathbf{H_u}, \theta_2, \theta_3 \rangle$	$\mathbf{L_1} \bar{N}_3 \mathbf{H_u} \theta_3^3$
$\mathbf{L_2 H_u}$	$(-3/2, 1/2, -1/2)$	$\langle \mathbf{L_2}, \mathbf{H_u}, \theta_2, \theta_3 \rangle$	$\mathbf{L_2} \bar{N}_3 \mathbf{H_u}$
$\mathbf{L_3 H_u}$	$(-3/2, 1/2, -1/2)$	$\langle \mathbf{L_3}, \mathbf{H_u}, \theta_2, \theta_3 \rangle$	$\mathbf{L_3} \bar{N}_3 \mathbf{H_u}$
$\mathbf{L_2} \bar{e}_1 \mathbf{H_d}$	$(3, 2, -1)$	$\langle \mathbf{L_2}, \bar{e}_1, \mathbf{H_d}, \theta_1, \theta_2 \rangle$	$\mathbf{L_2} \bar{e}_3 \mathbf{H_d} \theta_1^3$
$\mathbf{L_3} \bar{e}_1 \mathbf{H_d}$	$(3, 2, -1)$	$\langle \mathbf{L_3}, \bar{e}_1, \mathbf{H_d}, \theta_1, \theta_2 \rangle$	$\mathbf{L_3} \bar{e}_3 \mathbf{H_d} \theta_1^3$
$\mathbf{L_2 L_3} \bar{e}_1$	$(3/2, 5/2, -3/2)$	$\langle \mathbf{L_2}, \mathbf{L_3}, \bar{e}_1, \theta_1, \theta_2 \rangle$	$\mathbf{L_2 L_3} \bar{e}_1 \bar{N}_1 \theta_1^3 \theta_2^6$
$\mathbf{L_2 L_3} \bar{e}_3$	$(3/2, 1/2, -1/2)$	$\langle \mathbf{L_2}, \mathbf{L_3}, \bar{e}_3, \theta_1, \theta_2 \rangle$	$\mathbf{L_2 L_3} \bar{e}_3 \bar{N}_3 \theta_1^3$
$\mathbf{L_2 Q_3} \bar{\mathbf{d}}_2$	$(3/2, 1/2, -1/2)$	$\langle \mathbf{L_2}, \mathbf{Q_3}, \bar{\mathbf{d}}_2, \theta_1, \theta_2 \rangle$	$\mathbf{L_2 Q_3} \bar{\mathbf{d}}_2 \bar{N}_3 \theta_1^3$
$\mathbf{L_3 Q_3} \bar{\mathbf{d}}_2$	$(3/2, 1/2, -1/2)$	$\langle \mathbf{L_3}, \mathbf{Q_3}, \bar{\mathbf{d}}_2, \theta_1, \theta_2 \rangle$	$\mathbf{L_3 Q_3} \bar{\mathbf{d}}_2 \bar{N}_3 \theta_1^3$
$\mathbf{L_2 Q_3} \bar{\mathbf{d}}_3$	$(3/2, 1/2, -1/2)$	$\langle \mathbf{L_2}, \mathbf{Q_3}, \bar{\mathbf{d}}_3, \theta_1, \theta_2 \rangle$	$\mathbf{L_2 Q_3} \bar{\mathbf{d}}_3 \bar{N}_3 \theta_1^3$
$\mathbf{L_3 Q_3} \bar{\mathbf{d}}_3$	$(3/2, 1/2, -1/2)$	$\langle \mathbf{L_3}, \mathbf{Q_3}, \bar{\mathbf{d}}_3, \theta_1, \theta_2 \rangle$	$\mathbf{L_3 Q_3} \bar{\mathbf{d}}_3 \bar{N}_3 \theta_1^3$
$\bar{\mathbf{u}}_3 \bar{\mathbf{d}}_2 \bar{\mathbf{d}}_3$	$(3/2, 1/2, -1/2)$	$\langle \bar{\mathbf{u}}_3, \bar{\mathbf{d}}_2, \bar{\mathbf{d}}_3, \theta_1, \theta_2 \rangle$	$\bar{\mathbf{u}}_3 \bar{\mathbf{d}}_2 \bar{\mathbf{d}}_3 \bar{N}_3 \theta_1^3$
$\mathbf{Q_3} \bar{\mathbf{u}}_3 \bar{e}_1 \mathbf{H_d}$	$(9/2, 3/2, -1/2)$	$\langle \mathbf{Q_3}, \bar{\mathbf{u}}_3, \bar{e}_1, \mathbf{H_d}, \theta_1, \theta_2 \rangle$	$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{H_u}$
$\mathbf{Q_3} \bar{\mathbf{u}}_3 \bar{e}_3 \mathbf{H_d}$	$(9/2, -1/2, 1/2)$	$\langle \mathbf{Q_3}, \bar{\mathbf{u}}_3, \bar{e}_3, \mathbf{H_d}, \theta_1, \theta_3 \rangle$	$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{H_u}$
$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{L_2} \bar{e}_1$	$(3, 2, -1)$	$\langle \mathbf{Q_3}, \bar{\mathbf{u}}_3, \mathbf{L_2}, \bar{e}_1, \theta_1, \theta_2 \rangle$	$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{H_u}$
$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{L_3} \bar{e}_1$	$(3, 2, -1)$	$\langle \mathbf{Q_3}, \bar{\mathbf{u}}_3, \mathbf{L_3}, \bar{e}_1, \theta_1, \theta_2 \rangle$	$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{H_u}$
$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{Q_3} \bar{\mathbf{u}}_3 \bar{e}_1$	$(9/2, 3/2, -1/2)$	$\langle \mathbf{Q_3}, \bar{\mathbf{u}}_3, \bar{e}_1, \theta_1, \theta_2 \rangle$	$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{H_u}$
$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{Q_3} \bar{\mathbf{u}}_3 \bar{e}_3$	$(9/2, -1/2, 1/2)$	$\langle \mathbf{Q_3}, \bar{\mathbf{u}}_3, \bar{e}_3, \theta_1, \theta_3 \rangle$	$\mathbf{Q_3} \bar{\mathbf{u}}_3 \mathbf{H_u}$
$\bar{\mathbf{d}}_1 \bar{\mathbf{d}}_2 \bar{\mathbf{d}}_3 \mathbf{L_2 L_3}$	$(3/2, 3/2, -1/2)$	$\langle \bar{\mathbf{d}}_1, \bar{\mathbf{d}}_2, \bar{\mathbf{d}}_3, \mathbf{L_2}, \mathbf{L_3}, \theta_1, \theta_2 \rangle$	$\bar{\mathbf{d}}_1 \bar{\mathbf{d}}_2 \bar{\mathbf{d}}_3 \mathbf{L_2 L_3} \bar{N}_3 \theta_1^3 \theta_2$

$\bar{N}_3$ and the Σ fields needed to ensure anomaly cancellation.[3] The basis of standard-model invariants includes the MSSM basis of Ref. [31] as well as invariants containing the vector-like fields, such as the ones discussed in Sec. VII. Equation (11.1) tells us that D-flat directions are parametrized by the vacuum expectation values of both the standard-model invariants and the χ fields. The generic effect of F-term constraints and supersymmetry breaking is to fix these VEVs, resulting in a particular low-energy vacuum. As stressed in Ref. [18], the computation of the n_α simplifies a lot the discussion of D and F flatness.

Consider first the flat directions involving only standard-model singlets. Assuming for simplicity that only one χ field acquires a VEV, we must distinguish between two cases.

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[3] As alluded to earlier, we have not included the SO(10) singlets S_1, S_2, and S_3 necessary to make up three complete families in the **27** of E_6; otherwise the superpotential would contain an invariant $S_1 \theta_2^3 \theta_3^3$ linear in S_1, which would spoil the DSW vacuum.

TABLE V. Flat directions involving vectorlike matter (up to quartic operators) in the discrete symmetry scenario.

Basis invariant	(n_1, n_2, n_3)	Flat direction	FD lifted by
$\bar{E}_1 H_d$	$(3, -1, -1)$	$\langle \bar{E}_1, H_d, \theta_1 \rangle$	$\bar{E}_1 E_1 \; \theta_1^3$
$\mathbf{D}_1 \bar{\mathbf{d}}_1$	$(3/2, -1/2, 3/2)$	$\langle \mathbf{D}_1, \bar{\mathbf{d}}_1, \theta_1, \theta_3 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\mathbf{D}_1 \bar{\mathbf{d}}_{2,3}$	$(3/2, -1/2, 1/2)$	$\langle \mathbf{D}_1, \bar{\mathbf{d}}_{2,3}, \theta_1, \theta_3 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\mathbf{L}_1 \bar{E}_1$	$(3/2, -1/2, 3/2)$	$\langle \mathbf{L}_1, \bar{E}_1, \theta_1, \theta_3 \rangle$	$\bar{E}_1 E_1 \; \theta_1^3$
$\mathbf{L}_{2,3} \bar{E}_1$	$(3/2, -1/2, -3/2)$	$\langle \mathbf{L}_{2,3}, \bar{E}_1, \theta_1, \theta_2 \rangle$	$\bar{E}_1 E_1 \; \theta_1^3$
$\mathbf{Q}_3 \bar{\mathbf{D}}_1 H_d$	$(9/2, 1/2, -1/2)$	$\langle \mathbf{Q}_3, \bar{\mathbf{D}}_1, H_d, \theta_1, \theta_2 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\mathbf{Q}_3 \bar{\mathbf{u}}_3 \bar{E}_1$	$(3, -1, -1)$	$\langle \mathbf{Q}_3, \bar{\mathbf{u}}_3, \bar{E}_1, \theta_1 \rangle$	$\bar{E}_1 E_1 \; \theta_1^3$
$\mathbf{L}_{2,3} \mathbf{Q}_3 \bar{\mathbf{D}}_1$	$(3, 1, -1)$	$\langle \mathbf{L}_{2,3}, \mathbf{Q}_3, \bar{D}_1, \theta_1, \theta_2 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\mathbf{D}_1 \bar{\mathbf{u}}_3 \bar{e}_3$	$(3, -1, 1)$	$\langle \mathbf{D}_1, \bar{\mathbf{u}}_3, \bar{e}_3, \theta_1, \theta_3 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\bar{\mathbf{u}}_3 \bar{\mathbf{d}}_{2,3} \bar{\mathbf{D}}_1$	$(3, 1, -1)$	$\langle \bar{\mathbf{u}}_3, \bar{\mathbf{d}}_{2,3}, \bar{\mathbf{D}}_1, \theta_1, \theta_2 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\mathbf{Q}_3 \bar{\mathbf{u}}_3 \mathbf{Q}_3 \bar{\mathbf{D}}_1$	$(9/2, 1/2, -1/2)$	$\langle \mathbf{Q}_3, \bar{\mathbf{u}}_3, \mathbf{Q}_3, \bar{\mathbf{D}}_1, \theta_1, \theta_2 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\bar{\mathbf{D}}_1 \bar{\mathbf{u}}_2 \bar{\mathbf{u}}_3 \bar{e}_1$	$(9/2, 7/2, -1/2)$	$\langle \bar{\mathbf{D}}_1, \bar{\mathbf{u}}_2, \bar{\mathbf{u}}_3, \bar{e}_1, \theta_1, \theta_2 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$
$\mathbf{Q}_3 \mathbf{D}_1 \mathbf{D}_2 \bar{\mathbf{E}}_1$	$(9/2, -1/2, 1/2)$	$\langle \mathbf{Q}_3, \mathbf{D}_1, \mathbf{D}_2, \bar{\mathbf{E}}_1, \theta_1, \theta_3 \rangle$	$\bar{D}_1 D_1 \; \theta_1^3$

All n_α are positive. Then $|\langle \theta_\alpha \rangle|^2 \geq \xi^2$ for $\alpha = 1,2,3$, whatever $\langle \chi \rangle$ may be. In addition, the superpotential contains an invariant of the form $\chi^m \theta_1^{m_1} \theta_2^{m_2} \theta_3^{m_3}$, with $m_\alpha = m \, n_\alpha$ (as discussed in Sec. IV, $m \geq 2$ is required in order not to spoil the DSW vacuum). The F-term constraints then impose $\langle \chi \rangle = 0$: the flat direction is lifted down to the DSW vacuum.

Some of the n_α are negative. The relations $|\langle \theta_\alpha \rangle|^2 \geq \xi^2$ no longer hold, and the low-energy vacuum may be different from the DSW vacuum. In our model, this happens only for $\bar{N}_3$, for which $(n_1, n_2, n_3) = (3/2, -1/2, 1/2)$. One can then see from Eq. (11.1) that the vacuum $\langle \bar{N}_3, \theta_1, \theta_3 \rangle$ with $|\langle \bar{N}_3 \rangle|^2 = 2 \, \xi^2$, $|\langle \theta_1 \rangle|^2 = 4 \, \xi^2$ and $|\langle \theta_3 \rangle|^2 = 2 \, \xi^2$ is perfectly allowed by D-term constraints. This is a rather unwelcome feature, because most Yukawa couplings vanish in this vacuum. Fortunately, the superpotential contains an invariant $\bar{N}_3^3 \bar{N}_2 \; \theta_1^6 \theta_3^4$, with no power of θ_2, which lifts the undesired vacuum.

This discussion can be generalized to flat directions involving several χ fields; we conclude that the model does not possess any other stable vacuum of singlets than the DSW vacuum. Thus, the low-energy mass hierarchies are completely determined by the symmetries at high energy.

Flat directions involving fields charged under $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ can be analyzed in a similar way. For each element S of the basis of invariants, we compute the numbers (n_1, n_2, n_3). If one of the n_α is negative, we must check that the superpotential contains a term of the form $S' \; \theta_1^{n'_1} \theta_2^{n'_2} \theta_3^{n'_3}$ (with S' a combination of basis G-invariants and χ fields), where either one of the following two conditions is fulfilled: (i) S' contains no other field than the ones appearing in S, and $n'_\alpha = 0$ or 1, if $n_\alpha < 0$ (with the additional constraint that no more than one such n'_α should be equal to 1); (ii) S' contains only one field that does not appear in S, and $n'_\alpha = 0$, if $n_\alpha < 0$. This ensures that there is no flat direction associated with the single invariant S.

Remarkably enough, those conditions are always fulfilled in our model, despite the great number of standard-model invariants. In Table IV, we list the MSSM basis invariants for which some of the n_α are negative. For each of these invariants (first column), we give the corresponding numbers n_1, n_2, and n_3 (second column), the associated flat direction that breaks the standard model symmetries (third column), and an invariant that lifts it (fourth column).

The case of flat directions involving vectorlike matter is slightly different. Since we have assumed the existence of a discrete symmetry that prevents numerous invariants from appearing in the superpotential, there could be flat directions associated with these invariants. But this is not the case, as

long as the vectorlike fields are massive. Their F terms take indeed the following form (gauge indices are not shown, and powers of the θ fields have been absorbed in the mass matrices for simplicity):

$$F_{\bar{E}_i} = M_{\bar{E}_i E_j} E_j + \cdots, \quad F_{\bar{\mathbf{D}}_i} = M_{\bar{\mathbf{D}}_i \mathbf{D}_j} \mathbf{D}_j + \cdots, \quad (11.2)$$

$$F_{E_j} = M_{\bar{E}_i E_j} \bar{E}_i + \cdots, \quad F_{\mathbf{D}_j} = M_{\bar{\mathbf{D}}_i \mathbf{D}_j} \bar{\mathbf{D}}_i + \cdots, \quad (11.3)$$

where the ellipses stand for possible higher-order contributions. Since the matrices $M_{\bar{E}E}$ and $M_{\bar{\mathbf{D}}\mathbf{D}}$ are invertible, one concludes that the vanishing of Eqs. (11.2) and (11.3) forbids any flat direction involving vectorlike fields, provided that it is associated with an invariant for which all n_α are positive. That this is true also for invariants with one or several negative n_α is less obvious. It is due to the following features of the model: the (1,1) entry of the vectorlike mass matrices is generated from the superpotential terms $\bar{E}_1 E_1 \, \theta_1^3$ and $\bar{\mathbf{D}}_1 \mathbf{D}_1 \, \theta_1^3$, and all invariants that have one or several negative n_α both satisfy $n_1 \geq 0$ and contain at least one vectorlike field of the first family. Therefore, condition (ii) is always fulfilled. This can be checked in Table V (where only operators up to superfield dimension 4 have been displayed).

We have thus checked that the superpotential contains terms that lift all flat directions associated with a single standard-model invariant. This is not sufficient, however, to ensure that the standard-model symmetries are not broken at the scale ξ. Other invariants than those of Tables IV and V are in general necessary to lift completely the flat directions associated with several standard-model invariants and singlets. While we did not perform a complete analysis — which would be rather tedious — it is clear that most, if not all, flat directions are forbidden by the F term constraints.

We conclude that the vacuum structure of our model is satisfactory: the only stable vacuum of singlets allowed by D- and F-term constraints is the DSW vacuum, and flat directions associated with a single $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ invariant are lifted by the F terms. The only expected effects of supersymmetry breaking are to lift the possible remaining flat directions and to shift slightly the DSW vacuum by giving a small or intermediate VEV to other singlets or to fields with standard-model quantum numbers.

XII. CONCLUSION

We have presented a simple model that extends the standard-model gauge group by three phase symmetries, one of which is anomalous. The extra symmetries are broken in the DSW vacuum, thereby providing a small *computable* expansion parameter, in terms of which the Yukawa couplings of the standard model can be expanded. The model has a natural cutoff characterized by the scale at which the anomalies are absorbed by the Green-Schwarz terms, which is the gauge unification scale. The expansion parameter, which depends on the contribution of the standard-model vectorlike matter to the weak anomaly, turns out to be close to the Cabibbo angle. All Yukawa hierarchies as well as the Weinberg angle are reproduced if the expansion parameter is taken to be the Cabibbo angle. The model is predictive in the neutrino sector, yielding three massive neutrinos with small mixings between the electron neutrino and the muon and tau neutrinos, and mixings of order 1 between the muon and tau neutrinos. With the cutoff near the unification scale, the solar neutrino deficit is explained in terms of the nonadiabatic MSW effect, and the atmospheric neutrino imbalance is reproduced. With the Cabibbo angle as an expansion parameter, our model is compatible with proton decay bounds.

Many of the uncertainties of the model are associated with the nature of its vectorlike matter, which determines gauge unification and the value of the expansion parameter. In addition, it must contain matter with no standard-model charges, to cancel anomalies. Although we made a definite proposal for those fields, our lack of experimental guidelines should be kept in mind. Our model shows the way in which many of the generic features encountered in the compactification of theories in higher dimensions can be used to deduce phenomenological constraints. Finally we note that the value of the cutoff is the gauge unification, underlining the well-known possible conflict with compactified string theories.

[1] M. Green and J. Schwarz, Phys. Lett. **149B**, 117 (1984).

[2] M. Dine, N. Seiberg, and E. Witten, Nucl. Phys. **B289**, 589 (1987); J. Atick, L. Dixon, and A. Sen, *ibid.* **B292**, 109 (1987).

[3] L. Ibáñez, Phys. Lett. B **303**, 55 (1993).

[4] L. Ibáñez and G. G. Ross, Phys. Lett. B **332**, 100 (1994).

[5] P. Binétruy and P. Ramond, Phys. Lett. B **350**, 49 (1995); P. Binétruy, S. Lavignac, and P. Ramond, Nucl. Phys. **B477**, 353 (1996).

[6] Y. Nir, Phys. Lett. B **354**, 107 (1995).

[7] V. Jain and R. Shrock, Phys. Lett. B **352**, 83 (1995).

[8] P. Ramond, in ''Kikkawa Proceedings,'' hep-ph/9604251, 1996.

[9] M. Gell-Mann, P. Ramond, and R. Slansky, in ''Sanibel Talk,'' Report No. CALT-68-709, 1979; in *Supergravity*, Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe*, Tsukuba, Japan, 1979, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979).

[10] P. Binétruy, S. Lavignac, Nikolaos Irges, and P. Ramond, Phys. Lett. B **403**, 38 (1997).

[11] John K. Elwood, Nikolaos Irges, and P. Ramond, Phys. Lett. B **413**, 322 (1997).

[12] P. Binetruy and E. Dudas, Phys. Lett. B **389**, 503 (1996); G. Dvali and A. Pomarol, Phys. Rev. Lett. **77**, 3728 (1996).

[13] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B398**, 319 (1993); **B420**, 468 (1994).

[14] A. Faraggi, Nucl. Phys. **B387**, 239 (1992); **B403**, 101 (1993), **B407**, 57 (1993).

[15] A. E. Faraggi and J. C. Pati, Report No. UFIFT-HEP-97-29, hep-ph/9712516.

[16] F. Gürsey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1976).

[17] G. Giudice and A. Masiero, Phys. Lett. B **206**, 480 (1988); V.S. Kaplunovsky and J. Louis, *ibid.* **306**, 269 (1993).

[18] N. Irges and S. Lavignac, Phys. Lett. B (to be published), Report No. UFIFT-HEP-97-34, hep-ph/9712239.

[19] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 247 (1962).

[20] S.P. Mikheyev and A.Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento C **9**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979).

[21] E. Kearns, presented at the ITP conference on Solar Neutrinos: News about SNUs, 1997.

[22] S.M. Kasahara *et al.*, Phys. Rev. D **55**, 5282 (1997).

[23] P. Binétruy, S. Lavignac, S. Petcov, and P. Ramond, Nucl. Phys. **B496**, 3 (1997).

[24] N. Hata and P. Langacker, Phys. Rev. D **56**, 6107 (1997).

[25] M.C. Gonzalez-Garcia, H. Nunokawa, O.L.G. Peres, T. Stanev, and J.W.F. Valle, hep-ph/9801368.

[26] C. Athanassopoulos *et al.*, Phys. Rev. Lett. **75**, 2650 (1995); **77**, 3082 (1996); nucl-ex/9706006.

[27] P.H. Frampton and M. Harada, Report No. IFP-748A-UNC, hep-ph/9711448.

[28] V. Barger, M.S. Berger, and R.J.N. Phillips, Phys. Rev. D **52**, 1663 (1995).

[29] V. Ben-Hamo and Y. Nir, Phys. Lett. B **339**, 77 (1994).

[30] A. Faraggi, Nucl. Phys. **B477**, 65 (1996).

[31] T. Gherghetta, C. Kolda, and S.P. Martin, Nucl. Phys. **B468**, 37 (1996).

[32] C. Froggatt and H.B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[33] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993).

[34] H. Murayama and D.B. Kaplan, Phys. Lett. B **336**, 221 (1994).

[35] F. Buccella, J.-P. Derendinger, S. Ferrara, and C.A. Savoy, Phys. Lett. **115B**, 375 (1982).

[36] Particle Data Group, R.M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996), p. 83