Strongly coupled U(1) lattice gauge theory as a microscopic model of Yukawa theory

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Dynamical chiral symmetry breaking in a strongly coupled U(1) lattice gauge model with charged fermions and a scalar is investigated by numerical simulation. Several composite neutral states are observed, in particular a massive fermion. In the vicinity of the tricritical point of this model we study the effective Yukawa coupling between this fermion and the Goldstone boson. The perturbative triviality bound of Yukawa models is nearly saturated. The theory is quite similar to strongly coupled Yukawa models for sufficiently large coupling except for the occurrence of an additional state—a gauge ball of mass about half the mass of the fermion. [S0556-2821(98)05115-7]

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The question of whether the Higgs-Yukawa mechanism of symmetry breaking and particle mass generation in contemporary high energy physics might be an effective theory to some more fundamental renormalizable quantum field theory with dynamical symmetry breaking has been raised many times. It is of particular interest with respect to the large top quark mass, stimulating, e.g., the idea of top color. The pursuit of this question leads frequently to devising strongly coupled gauge theories beyond the standard model with speculative nonperturbative dynamics. It might be inspiring to have prototypes of such "microscopic" gauge models of Yukawa theory with a solid and accessible dynamical framework. Numerical lattice methods are justified for this purpose provided a lattice model can be found with the required basic dynamical properties, though neglecting various phenomenological aspects. In particular, these properties should be preserved when approaching the continuum limit; i.e., the model should be nonperturbatively renormalizable.

A promising lattice model has been suggested in Ref. [1]. This "$\chi U\phi$ model" consists of a charged fermion field χ with strong vectorlike coupling to a compact U(1) gauge field U, and a scalar field ϕ of the same charge. Here we summarize the results of our extensive systematic investigations of this model in four dimensions by means of numerical simulations with dynamical fermions. Though we cannot decide by currently available means whether the model is renormalizable, we find some encouraging properties. A detailed account is given in a parallel paper [2] and in some previous works [3–6].

The same model has been investigated also in lower dimensions. There is little doubt that the $\chi U\phi$ model defines a renormalizable quantum field theory in two dimensions, belonging to the universality class of the chiral Gross-Neveu model [7]. Also first results in three dimensions suggest renormalizability [8].

Finding a "promising" lattice model meets two major difficulties. Strongly coupled lattice gauge theories with fermions exhibit frequently the required dynamical chiral symmetry breaking. However, this phenomenon comes mostly along with undesirable confinement of fermions acquiring mass. Furthermore, at strong coupling, it is difficult to find a second order phase transition required for an approach to continuum.

In the $\chi U\phi$ model, the scalar field ϕ helps to solve both problems. First, it shields the fermion charge and gives rise to an unconfined, i.e., physical massive, fermion $F=\phi^\dagger\chi$ in the phase with chiral symmetry broken dynamically by the gauge interaction (Nambu phase). Second, the scalar *suppresses* this symmetry breaking and at sufficiently strong gauge coupling induces a second order transition to a chiral symmetric (Higgs) phase, thus opening a way to continuum.

We find that one particular point of the phase transition line, the tricritical point E, represents a new kind (new universality class) of dynamical symmetry breaking mechanism in strongly coupled gauge theories with fermions in four dimensions. We present some evidence that a strongly coupled effective Yukawa theory results. It describes the interaction between the composite fermion F and some $\bar{\chi}\chi$ "mesons." At this point the model could serve as a microscopic model of Yukawa theory. But in addition, the mechanism produces also a scalar gauge ball.

To explain these findings, we now summarize the most relevant features of the model (more details can be found in [2]). The model is defined by the action

$$S=S_\chi+S_U+S_\phi,$$

$$S_\chi=\frac{1}{2}\sum_x \bar{\chi}_x\sum_{\mu=1}^{4}\eta_{x\mu}(U_{x,\mu}\chi_{x+\mu}-U^\dagger_{x-\mu,\mu}\chi_{x-\mu})+am_0\sum_x\bar{\chi}_x\chi_x,$$

$$S_U=-\beta\sum_P\cos(\Theta_P),$$

$$S_\phi=-\kappa\sum_x\sum_{\mu=1}^{4}(\phi^\dagger_x U_{x,\mu}\phi_{x+\mu}+\text{H.c.}).$$

Here $\Theta_P\in[0,2\pi)$ is the plaquette angle, i.e., the argument of the product of U(1) gauge field link variables $U_{x,\mu}$ along a plaquette P. Taking $\Theta_P=a^2gF_{\mu\nu}$, where a is the lattice spacing, and $\beta=1/g^2$, one obtains for weak coupling g the continuum gauge action $S_U=\frac{1}{4}\int d^4xF^2_{\mu\nu}$. The staggered fermion field χ has (real) bare mass am_0 in lattice units. It leads

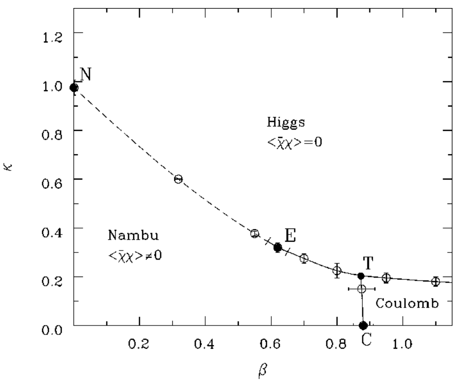

FIG. 1. Phase diagram of the $\chi U \phi$ model in the chiral limit, $m_0 = 0$. The $\beta = 0$ limit corresponds to the Nambu–Jona-Lasinio (NJL) model. The NE line is a line of second order phase transitions. Other lines are lines of first order transitions. In the Nambu phase, chiral symmetry is broken dynamically and the fermion $F = \phi^\dagger \chi$ is massive. In the Higgs phase, F is massless. E is the tricritical point where the scaling behavior is different from the NJL model. The Coulomb and Higgs phases extend until $\beta = \infty$. The data points are positions of the phase transitions extrapolated to the infinite volume.

to four fermion species in the continuum limit. Sign factors $\eta_{x\mu}$ are standard for staggered fermions. The complex scalar field is constrained, $|\phi| = 1$. As its ''hopping parameter'' κ increases, it drives the model into the usual Higgs phase.

We note that there is no Yukawa coupling between χ and ϕ, as both fields have the same charge. The model has U(1) global chiral symmetry in the limit of vanishing fermion bare mass in physical units, $m_0 = 0$. This is the case we are really interested in. The numerical simulations have to be carried out at nonvanishing m_0, however, and an extrapolation to the chiral limit performed. For $am_0 \neq 0$, the chiral transformation relates the model at $\pm am_0$.

The phase diagram at $m_0 = 0$ is shown in Fig. 1. One limit case is $\kappa = 0$, corresponding to the massless compact lattice QED. Another important limit case is $\beta = 0$. Here the model can be rewritten exactly as the lattice NJL model [9] with the critical point N. The strong four-fermion coupling of that model corresponds to small κ. The Nambu phase thus connects the broken chiral symmetry phases of both these limit models. Only a part of its boundary, the NE line, represents second order phase transitions.

Point E is far away from any limit case and it does not appear to be accessible by any reliable analytic method, neither on the lattice nor in continuum. It is ''tricritical'' because in the full parameter space (including am_0) there are, apart from NE, two further second order ''wing'' lines entering E from the positive and negative am_0 directions. The existence of a common point E of these three second order lines is neither predicted nor understood. The evidence is purely numerical, but quite strong [2]. Its position is $\beta_E = 0.62(3), \kappa_E = 0.32(2)$.

The importance of point E is rooted in the experience from statistical mechanics that a tricritical point belongs to a universality class different from that of any of the second order lines entering into it [10]. (The basic properties of tricritical points relevant in our context are summarized, e.g., in [11].) The usual universality of second order lines suggests—and this is supported by our earlier investigations [3]—that the whole NE line except point E corresponds to the same continuum model as point N, the NJL model. The gauge field is presumably auxiliary and the model is therefore of limited interest there. However, point E is expected to be different, the gauge field playing an important role.

To verify this expectation, we have investigated the critical exponents and spectrum of the model in the vicinity of E. The study of the exponents uses advanced techniques of statistical physics and is described in [2]. Here we only mention the found value $\nu_t \simeq 1/3$ of the correlation length tricritical exponent. It is different from $\nu \simeq 1/2$ along the NE line, confirming the particularity of E. It also differs from the prediction of the classical theory of tricritical points usually believed to hold in four dimensions [10] and predicting $\nu_t = 1/2$. This indicates that point E is a tricritical point with the important role of quantum fluctuations.

Some insight into the continuum physics, which might be obtained at point E, is provided by the spectrum and its scaling behavior. The masses am_Q are in lattice units and thus only their ratios are physical. They are determined in the Nambu phase without any gauge fixing from the correlation functions of various gauge invariant composite operators Q. In this sense the massive physical fermion F, as well as other physical states, is composite. The interaction between them is due to the van der Waals remnant of the fundamental interactions.

The fermion mass $am_F > 0$ decreases when the NE line is approached from the Nambu phase and there it is rather insensitive to the lattice size. It is small in the Higgs phase on finite lattices, decreasing with increasing lattice size. The data are consistent with the expected vanishing of am_F in the Higgs phase in the infinite volume limit.

We find several $\bar{\chi}, \chi$ bound states and borrow their names from QCD. One of them is the obligatory pseudoscalar Goldstone boson π with the dependence on am_0 as required by current algebra. The π meson is massless in the chiral limit in the whole Nambu phase. It would get ''eaten'' if the global chiral symmetry of the $\chi U \phi$ model were gauged. This process would be identical with the standard Higgs mechanism. Thus we do not need to discuss it and mention it only for completeness.

We obtain some results for the pion decay constant. The data suggest a large ratio f_π / m_F increasing when point E is approached. However, the value is sensitive to the lattice volume and we cannot yet extrapolate it to the infinite volume and continuum limit. If the ratio diverges, it would indicate the triviality of the model [12]. Its current value $f_\pi / m_F \simeq 1/3$ for $am_F \simeq 0.4$ can be considered as a lower bound.

Also the σ meson in the antifermion-fermion channel is observed. The σ mass is quite dependent both on the lattice size and bare fermion mass, preventing its prediction in the continuum limit with $m_0 = 0$. A strong lattice size dependence of f_π and σ mass has been observed and explained by

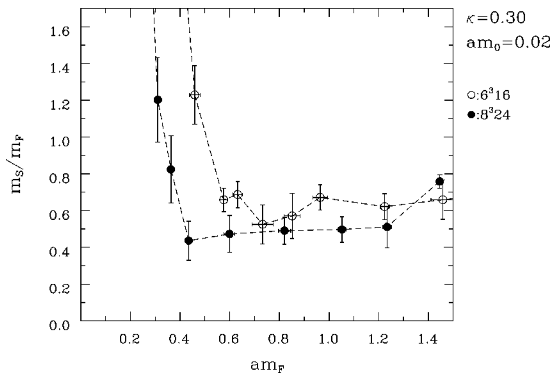

FIG. 2. Mass ratio m_S/m_F as a function of the fermion mass when point E is approached. The data have been obtained on $6^3 16$ (open circles) and $8^3 24$ (solid circles) lattices.

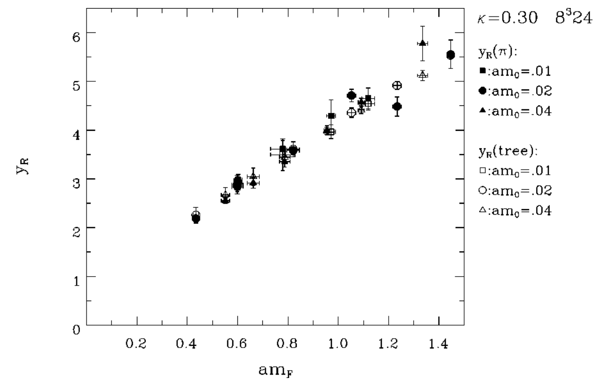

FIG. 3. Effective Yukawa coupling y_{R} of the π meson to F (solid symbols) and the tree level relation (1) (open symbols) as a function of am_F for different am_0 at $\kappa=0.30$ on a $8^3 24$ lattice.

means of the Schwinger-Dyson equations in the limit case $\beta=0$, the NJL model, and holds apparently also in the vicinity of E.

The ρ mass is insensitive to the lattice size and scales like the fermion mass, with the approximate value $m_\rho \simeq 2m_F$. We cannot distinguish between a bound state and a resonance.

Also a neutral scalar (S boson) is seen, appearing as a composite of $\phi^\dagger$ and ϕ and as a state of pure gauge field. In the Higgs phase it corresponds to the Higgs boson associated with the perturbative Higgs mechanism occurring in that phase at large β. In the Nambu phase it is more natural to interpret the S boson as a scalar gauge ball. The transition between both interpretations is smooth, however, and the corresponding channels appreciably mix in the vicinity of E.

The mass am_S is nonzero in all phases and goes to zero when the two wing critical lines are approached for any am_0. For $am_0=0$, the mass am_S goes to zero only at point E. This illustrates the particular character of the tricritical point E. Whereas only one of the masses am_F or am_S vanishes on each of the second order lines, they both vanish at the tricritical point. As their ratio remains finite, both corresponding states are present in the continuum limit taken at this point. This is the best evidence that the physical content (universality class) of point E differs from that of any of the adjacent second order lines and thus does not correspond to the NJL model obtained at N.

As am_S is only moderately dependent on the lattice size, we can estimate its scaling behavior when point E is approached. This is shown in Fig. 2 for $\kappa=0.30$, which is approximately the κ coordinate of E. The ratio m_S/m_F remains constant when am_F decreases, as long as finite size effects are negligible. The sudden rise of the data at small am_F is due to the strong finite size dependence of am_F in the Higgs phase and shifts correspondingly to smaller am_F when the lattice volume increases. These observations suggest that the value $m_S/m_F \simeq 1/2$ would be obtained in the continuum limit at E.

In the rest of this paper we concentrate on the renormalized Yukawa coupling y_{R} between F and π. It is obtained from the three-point function of the corresponding composite operators and thus can be interpreted as an effective Yukawa coupling, which would describe the interaction between F and π in the regime where their composite structure can be neglected. Our measurement, performed in a similar way as in [13], is described in detail in [2,6]. In spite of the complexity of the corresponding expressions, y_{R} is measurable with good precision. The reason is the strict locality of the used operators. An attempt to measure also an analogous coupling of the S boson failed because S is described by extended operators.

The renormalized Yukawa coupling y_{R} is presented in Fig. 3 (solid symbols) for three different am_0 at $\kappa=0.30$ in the Nambu phase close to E. We show it in dependence on am_F and compare it with the tree level relation

$$y_{\mathrm{R}}^{(\mathrm{tree})}=\frac{am_F}{\langle\overline{\chi}\chi\rangle}\sqrt{Z_\pi}. \tag{1}$$

The agreement is so good that the open symbols representing Eq. (1) in Fig. 3 are nearly invisible.

The values for y_{R} are not significantly dependent on am_0. Therefore we expect that the value for y_{R} in the chiral limit

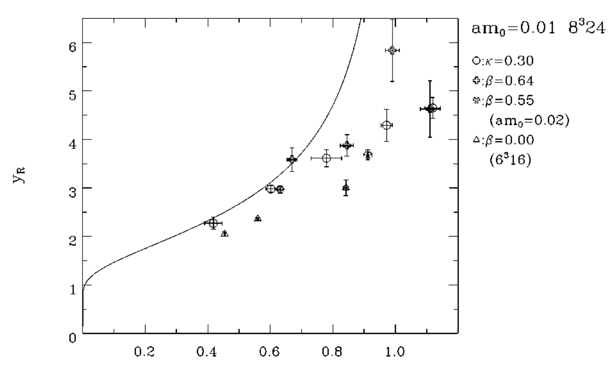

FIG. 4. y_{R} of the π meson for different couplings along line NE. For $\beta=0.55$ the tree level definition was used. Most data have been taken on a $8^3 24$ lattice (at $\beta=0$ on $6^3 16$) with $am_0=0.01$ (at $\beta=0.55$ is $am_0=0.02$). Shown is also the curve of maximal y_{R} as it results from the first order perturbative expansion in a corresponding Yukawa model. This curve goes to 0 for $am_F \to 0$, as expected from triviality.

is rather well represented by our results at $am_0=0.01$. In Fig. 4 we show these results for substantially different gauge couplings. The results close to point E, for $\beta=0.64$, $\kappa=0.30$, and $\beta=0.55$, are very similar. The values of y_R at $\beta=0$ (NJL model) are significantly below the other data, however. This implies that the Yukawa coupling gets stronger as one moves from the NJL model to the vicinity of E by increasing β at fixed am_F.

In the interval $y_R \simeq 2-5$, where the Yukawa coupling is determined reliably, we compare our result with the curve of maximal renormalized coupling, resulting from the first order perturbative calculation in the Yukawa model (triviality bound):

$$y_{\mathrm{R\,max}}=\frac{1}{\sqrt{2\beta_0|\ln(am_F)|}}, \quad \beta_0=\frac{N_F}{4\pi^2}, \tag{2}$$

with $N_F=4$ the number of fermions including lattice doublers. In the lattice Yukawa models this upper bound is nearly saturated by the values of y_R obtained at the maximal possible bare Yukawa coupling [14,15]. As seen in Fig. 4, also in our model close to E, the data are only slightly below the curve (2). Thus, in this interval, y_R has the same form as that in the strongly coupled Yukawa models.

For $y_R<2$, the data are significantly influenced by the finite size effects, and we cannot make any conclusion. It might be that some deviation from Yukawa theory occurs there. The fact that y_R is stronger in the vicinity of E than in the NJL model suggests the question of whether it vanishes in the continuum limit, $am_F=0$, taken there. Is it zero as in Yukawa theory, or could triviality be avoided at the tricritical point? A reliable extrapolation of y_R to $am_F=0$ at the tricritical point of the $\chi U\phi$ model is a challenge for future investigations.

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[1] C. Frick and J. Jersák, Phys. Rev. D **52**, 340 (1995).
[2] W. Franzki and J. Jersák, this issue, Phys. Rev. D **58**, 034508 (1998).
[3] W. Franzki, C. Frick, J. Jersák, and X. Q. Luo, Nucl. Phys. **B453**, 355 (1995).
[4] W. Franzki, C. Frick, J. Jersák, and X. Q. Luo, Prog. Theor. Phys. Suppl. **122**, 171 (1996).
[5] W. Franzki, Nucl. Phys. B (Proc. Suppl.) **53**, 702 (1997).
[6] W. Franzki, Ph.D. thesis, RWTH Aachen, 1997.
[7] W. Franzki, J. Jersák, and R. Welters, Phys. Rev. D **54**, 7741 (1996).
[8] I. M. Barbour, E. Focht, W. Franzki, J. Jersák, and N. Psycharis, Phys. Rev. D (to be published), hep-lat/9804032.
[9] I.-H. Lee and R. E. Shrock, Phys. Rev. Lett. **59**, 14 (1987).
[10] I. D. Lawrie and S. Sarbach, in *Phase Transitions and Critical Phenomena*, edited by C. Domb and J. Lebowitz (Academic, New York, 1984), Vol. 9, p. 1.
[11] H. G. Evertz, J. Jersák, T. Neuhaus, and P. M. Zerwas, Nucl. Phys. **B251** [FS13], 279 (1985).
[12] S. Hands and J. B. Kogut, ''Logarithmic corrections to the equation of state in the SU(2)×SU(2) NJL model,'' hep-lat/9705015.
[13] M. Göckeler, R. Horsley, P. E. L. Rakow, and G. Schierholz, Phys. Lett. B **353**, 100 (1995).
[14] A. K. De and J. Jersák, in *Heavy Flavors*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1992), p. 732.
[15] W. Bock, C. Frick, J. Smit, and J. C. Vink, Nucl. Phys. **B400**, 309 (1993).