Inclusive B-meson production in e^+e^- and $p\bar{p}$ collisions

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We provide nonperturbative fragmentation functions for B mesons, both at leading and next-to-leading order in the $\overline{\text{MS}}$ factorization scheme with five massless quark flavors. They are determined by fitting the fractional energy distribution of B mesons inclusively produced in e^+e^- annihilation at CERN LEP1. Theoretical predictions for the inclusive production of B mesons with high transverse momenta in $p\bar{p}$ scattering obtained with these fragmentation functions nicely agree, both in shape and normalization, with data recently taken at the Fermilab Tevatron. [S0556-2821(98)00117-9]

I. INTRODUCTION

The study of b-quark production in high-energy hadronic interactions offers the opportunity to test perturbative quantum chromodynamics (QCD) [1]. Hadron-collider experiments usually consider the cross section integrated over a fixed range in rapidity η and over all values of transverse momentum p_T above a variable threshold $p_T^{\min}$. First measurements of such cross sections were performed by the UA1 Collaboration at the CERN $p\bar{p}$ collider with center-of-mass (c.m.) energy $\sqrt{s}=630$ GeV [2]. More recent experimental results at $\sqrt{s}=1.8$ TeV were presented by the Collider Detector at Fermilab (CDF) [3] and D0 [4] Collaborations at the Fermilab Tevatron. On the theoretical side, such cross sections were calculated up to next-to-leading order (NLO) in the strong coupling constant α_s [1,5,6]. The shapes of the theoretical curves agree well with the data of all three experiments, UA1, CDF, and D0. However, independent of the beam energy, the absolute normalizations of the experimental cross sections exceed, by about a factor of two, the respective predictions obtained with the conventional scale $\mu = \sqrt{p_T^2+m_b^2}$ and a typical b-quark mass of $m_b=4.75$ GeV. The experimental cross sections could only be reproduced by the theoretical predictions if μ and m_b were reduced to $\mu = \sqrt{p_T^2+m_b^2}/2$ and $m_b=4.5$ GeV and parton density functions (PDF's) with particularly large values of the asymptotic scale parameter Λ were chosen.

In the experimental studies, the b-quark production cross section is not actually measured directly as a function $p_T^{\min}$. The original measurements refer to the production of B mesons, which decay either semileptonically, or exclusively or inclusively into J/ψ mesons. The cross sections for the production of bare b quarks were then obtained by correcting for the fragmentation of b quarks into B mesons with the help of various Monte Carlo (MC) models. Since this is a model-dependent procedure, it remains unclear whether the

disagreement between the experimental data and the NLO predictions is actually real. In order to extract the b-quark production cross section, one needs an independent measurement of the fragmentation of b quarks into B mesons. In fact, two years ago the CDF Collaboration published data on their first measurement of the B-meson differential cross section $d\sigma/dp_T$ for the exclusive decays $B^+\to J/\psi K^+$ and $B^0\to J/\psi K^{*0}$ based on the 1992–1993 run (run 1A) [7]. Similarly to the case of b-quark production, the measured cross section was found to exceed the NLO prediction by approximately a factor of two, while there was good agreement in the shape of the p_T distribution. Agreement in the normalization could only be achieved by choosing extreme values for the input parameters of the NLO calculation, i.e., by reducing μ and m_b and by increasing Λ [6].

In Ref. [8], the CDF Collaboration extended their analysis [7] by incorporating the data taken during the 1993–1996 run (run 1B), which yielded an integrated luminosity of 54.4 pb^{-1} to be compared with 19.3 pb^{-1} collected during run 1A [8], and presented the cross section $d\sigma/dp_T$ for the inclusive production of B^+ and B^0 mesons with $p_T>6$ GeV in the central rapidity region $|\eta|<1$. Again, the NLO prediction with input parameters similar to those used for the integrated b-quark cross section in Refs. [1,5,6] ($\mu = \sqrt{p_T^2+m_b^2}$ and $m_b=4.75$ GeV) was found to agree with the data in the shape, while its normalization came out significantly too low, by a factor of 2.1 ± 0.4. Here, it was assumed that the fragmentations of b quarks into B mesons can be described by a Peterson fragmentation function (FF) [9] with $\epsilon=0.006$. This value for the ϵ parameter was extracted more than ten years ago from a global analysis of data on B-meson production in e^+e^- annihilation at the SLAC e^+e^- storage ring PEP and the DESY e^+e^- collider PETRA [10], based on MC models which were then up-to-date. The result of this comparison is to be taken with a grain of salt, since the underlying description of $b\to B$ fragmentation is *ad hoc* and not backed up by model-independent data. It is the purpose

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of this work to improve on this situation. The fragmentation of b quarks into B mesons has been measured by the OPAL Collaboration at the CERN e^+e^- collider LEP1 [11]. The produced B^+ and B^0 mesons were identified via their semileptonic decays containing a fully reconstructed charmed meson. This resulted in the measurement of the distribution of the B mesons in the scaling variable $x=2E(B)/\sqrt{s}$, where $E(B)$ is the measured energy of the B^+/B^0 candidate and $\sqrt{s}=M_Z$ is the LEP1 c.m. energy. Earlier measurements of the $b\to B$ FF were reported by the L3 Collaboration at LEP1 [12]. In the following, we shall base our analysis on the OPAL data, which have higher statistics and contain more x bins than the L3 data.

At LEP1, B mesons were dominantly produced by $Z \to b\bar{b}$ decays with subsequent fragmentation of the b quarks and antiquarks into B mesons, which decay weakly. In the reaction $e^+e^-\to b\bar{b}\to B+X$ at the Z-boson resonance, the b quarks and antiquarks typically have large momenta. A large-momentum b quark essentially behaves like a massless particle, radiating a large amount of its energy in the form of hard, collinear gluons. This leads to the presence of logarithms of the form $\alpha_s \ln(M_Z^2/m_b^2)$ originating from collinear singularities in a scheme, where m_b is taken to be finite. These terms appear in all orders of perturbation theory and must be resummed. This can be done by absorbing the m_b-dependent logarithms into the FF of the b quark at some factorization scale of order M_Z. Alternatively, one can start with $m_b=0$ and factorize the collinear final-state singularities into the FF's according to the modified minimal subtraction ($\overline{\text{MS}}$) scheme, as is usually done in connection with the fragmentation of light quarks into light mesons. This is the so-called massless scheme [13], in which m_b is neglected, except in the initial conditions for the FF's. This scheme was used for NLO calculations of charm and bottom production in e^+e^- [14], $p\bar{p}$ [15], γp [16,17], and $\gamma\gamma$ [18] collisions, with the additional feature that the massless c and b quarks were transformed into their massive counterparts by the use of perturbative FF's [14]. These perturbative FF's enter as a theoretical input at a low initial scale μ_0 of order m_c or m_b, respectively, and are subject to evolution to higher scales μ with the usual Altarelli-Parisi (AP) equations [19]. Following Ref. [20], this theory was extended by including nonperturbative FF's, which describe the transition from heavy quarks to heavy mesons [17,21,22].

In this work, we describe the fragmentation of massless b quarks into B mesons by a one-step process characterized entirely in terms of a nonperturbative FF, as is usually done for the fragmentation of u, d, and s quarks into light mesons. We assume simple parametrizations of the b-quark FF at the starting scale. We determine the parameters appearing therein through fits to the OPAL data [11] at lowest order (LO) and NLO. These b-quark FF's are then used to predict the differential cross section $d\sigma/dp_T$ of B-meson production in $p\bar{p}$ scattering at $\sqrt{s}=1.8$ TeV, which can be directly compared with recent data from the CDF Collaboration [8].

This paper is organized as follows. In Sec. II, we recall the theoretical framework for the extraction of FF's from e^+e^- data, which was previously used for c-quark fragmentation into $D^{*\pm}$ mesons [23], and present our results for the b-quark FF's at LO and NLO in the $\overline{\text{MS}}$ factorization scheme with five massless flavors. We assume three different forms for the FF's at the starting scale, which enables us to assess the resulting theoretical uncertainty in other kinds of high-energy processes, such as $p\bar{p}$ scattering. In Sec. III, we apply the nonperturbative FF's thus obtained to predict the cross section of B-meson production in $p\bar{p}$ collisions at the Tevatron and compare the result with recent data from CDF [8]. Our conclusions are summarized in Sec. IV.

II. B-MESON PRODUCTION IN e^+e^- COLLISIONS

Our procedure to construct LO and NLO sets of FF's for B mesons is very similar to the case of $D^{*\pm}$ mesons treated in Refs. [22,23]. Here we only give those details which differ from Refs. [22,23].

The OPAL data on the inclusive production of B^+ and B^0 mesons in e^+e^- annihilation at the Z-boson resonance serve as our experimental input [11]. These data are presented as differential distributions in $x=2E(B)/\sqrt{s}$, where $E(B)$ is the measured energy of the B^+ or B^0 candidate. This function peaks at fairly large x. For the fitting procedure we use the experimental x bins, with width $\Delta x=0.08$, in the interval $0.28<x<1$ and integrate the theoretical functions over Δx, which is equivalent to the experimental binning procedure. There is a total of nine data points.

When we talk about the $b\to B$ FF, we have in mind the four fragmentation processes $\bar{b}\to B^+$, $\bar{b}\to B^0$, $b\to B^-$, and $b\to\bar{B}^0$. In Ref. [24], the respective branching fractions are all assumed to be equal. If we neglect the influence of the electroweak interactions, this follows from the $u\leftrightarrow d$ flavor symmetry and the charge-conjugation invariance of QCD. We thus make the stronger assumption that the FF's of these four processes all coincide. We take the starting scales for the FF's of the gluon and the u, d, s, c, and b quarks and antiquarks into B mesons to be $\mu_0=2m_b$, with $m_b=5$ GeV. The FF's of the gluon and the first four quark flavors are assumed to be zero at the starting scale. These FF's are generated through the μ evolution. For the parametrization of the b-quark FF at the starting scale μ_0, we employ three different forms. The first one is usually adopted for the FF's of light hadrons, namely

$$D_b(x,\mu_0)=Nx^\alpha(1-x)^\beta. \qquad (2.1)$$

This form has been used in Ref. [20] to describe the nonperturbative effects of b-quark fragmentation, in addition to a perturbative contribution. The standard (S) parametrization (2.1) depends on three free parameters, N, α, and β, which are determined by fits to the OPAL data [11] after evolution to the factorization scale $M_f=M_Z$. As our second parametrization, we use the Peterson (P) distribution [9],

$$D_b(x,\mu_0)=N\frac{x(1-x)^2}{[(1-x)^2+\epsilon x]^2}. \qquad (2.2)$$

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This choice is particularly suited to describe a FF that peaks at large x. It has been frequently used in connection with the fragmentation of heavy quarks, such as c or b quarks, into their mesons. It depends only on two parameters, N and ϵ.

The third parametrization is theoretically motivated. There exists a particular class of FF's which are calculable in perturbative QCD, namely those of gluons and heavy quarks into heavy-heavy bound states, such as $c\bar{c}$, $b\bar{b}$ [25], and $c\bar{b}$ mesons [26]. These perturbative FF's can also be applied to describe the fragmentation of b quarks into bound states of b and light quarks, in the sense of a model assumption rather than a formula derived in perturbative QCD. The formula for the $\bar{b}\to B_c$ transition was derived by Braaten (B) et al. [26] and reads

$$
\begin{aligned}
D_b(x,\mu_0) = N \frac{rx(1-x)^2}{[1-(1-r)x]^6} [&6 - 18(1-2r)x \\
&+ (21-74r+68r^2)x^2 - 2(1-r) \\
&\times(6-19r+18r^2)x^3 \\
&+ 3(1-r)^2(1-2r+2r^2)x^4],
\end{aligned}
\tag{2.3}
$$

where $r = m_c/(m_b+m_c)$ and N is given in terms of α_s, m_c, and the B_c-meson wave function at the origin. Similar formulas also exist for $\bar{b}\to B_c^*, B_c^{**}$ [26]. Naively applying this formula for r to the fragmentation process $b\to B$ would yield a rather small number, which is not well determined. Thus, our philosophy is to treat N and r as free parameters if one of the quarks in the bound state is light. In Ref. [26], the branching fraction of $c\to B_c$ was found to be two orders of magnitude smaller than the one of $\bar{b}\to B_c$. Extrapolating to the case of B mesons, it hence follows that our assumption $D_q(x,\mu_0)=0$, where q denotes a light quark, should be well founded even if q is the light constituent of the B meson.

We calculate the cross section $(1/\sigma_{\mathrm{tot}})d\sigma/dx$ for $e^+e^- \to \gamma$, $Z\to B^+/B^0+X$ to LO and NLO in the $\overline{\mathrm{MS}}$ scheme with five massless quark flavors as described in Ref. [27], where all relevant formulas and references may be found. In particular, we choose the renormalization and factorization scales to be $\mu = M_f = \sqrt{s}$. As for the asymptotic scale parameter appropriate for five active quark flavors, we adopt the LO (NLO) value $\Lambda_{\overline{\mathrm{MS}}}^{(5)} = 108$ MeV (227 MeV) from Ref. [27]. As in Ref. [22], we solve the AP equations in x space by iteration of the corresponding integral equations. In the Appendix of Ref. [22], the timelike splitting functions are listed in a convenient form, i.e., with the coefficients of the delta functions and plus distributions explicitly displayed. As in Ref. [22], we take the b-quark mass to be $m_b = 5$ GeV. Since m_b only enters via the definition of the starting scale μ_0, its precise value is immaterial for our fit.

The OPAL data are presented in Fig. 3 of Ref. [11] as the distribution dN/dx normalized to the bin width $\Delta x = 0.08$. In order to convert these data to the inclusive cross section $(1/\sigma_{\mathrm{tot}})d\sigma/dx$, we need to multiply them by the overall factor $2R_b f(b\to B)/\Delta x = 2.198$, where $R_b = \Gamma(Z\to b\bar{b})/\Gamma(Z \to \mathrm{hadrons})$, $f(b\to B)$ is the measured $b\to B$ branching frac-

TABLE I. Fit parameters for the $b\to B$ FF's according to sets S, P, and B at LO and NLO and respective values of χ^2 per degree of freedom. All other FF's are taken to be zero at the starting scale $\mu_0 = 2m_b = 10$ GeV.

Set	N	α	β	ϵ	r	χ^2_{DF}
LO S	56.4	8.39	1.16	–	–	0.80
NLO S	79.4	8.06	1.45	–	–	1.21
LO P	0.0952	–	–	0.0126	–	0.67
NLO P	0.116	–	–	0.0198	–	0.27
LO B	0.308	–	–	–	0.121	2.50
NLO B	0.280	–	–	–	0.156	1.66

tion, and the factor of two accounts for the fact that our cross-section formula [27] includes the fragmentation of both b and $\bar{b}$. Following Ref. [24], we identify $f(b\to B) = f(\bar{b}\to B^+) = f(\bar{b}\to B^0)$. For consistency, we adopt the OPAL results $R_b = 0.2171 \pm 0.0021 \pm 0.0021$ [28] and $f(b\to B) = 0.405 \pm 0.035 \pm 0.045$ [29], where the first (second) error is statistical (systematic).

The values for the input parameters in Eqs. (2.1), (2.2), and (2.3) which result from our LO and NLO fits to the OPAL data are summarized in Table I. In the following, we refer to these FF's as sets LO S, NLO S, LO P, NLO P, LO B, and NLO B, respectively. The corresponding χ^2 values per degree of freedom (χ^2_{DF}) are listed in the last column of Table I; there is a total of nine degrees of freedom. Except for the sets of type S, the χ^2_{DF} values for the NLO fits are slightly lower than those for the LO fits. The Peterson ansatz (2.2) yields the best fits. This is surprising, since it has only two free parameters, one less than the standard form (2.1). The sets of type B have the largest χ^2_{DF} values. Since the b-quark FF is peaked at $x\gtrsim 0.5$, we have $\alpha \gg \beta$ in the case of sets LO S and NLO S. The ϵ parameters of sets LO P and NLO P are larger than the standard value $\epsilon = 0.006$ [10] usually quoted in the literature. It is important to note that the values of ϵ obtained in the various analyses depend on the underlying theory for the description of the fragmentation process $b\to B$, in particular, on the choice of the starting scale μ_0, on whether the analysis is done in LO or NLO (as may be seen from Table I), and on how the final-state collinear singularities are factorized in NLO. We emphasize that our results for ϵ in Table I refer to the pure $\overline{\mathrm{MS}}$ factorization scheme with five massless flavors and $\mu_0 = 2m_b = 10$ GeV. If we were to interpret the values for r in Table I with the formula $r = m_q/(m_b+m_q)$, which is naively adapted from the analogous definition for $c\bar{b}$ bound states [26], then we would find $m_q = 688$ MeV and 924 MeV at LO and NLO, respectively. These values are a factor of 2–3 larger than the generally assumed constituent-quark masses of the u and d quarks. This just illustrates the model character of using ansatz (2.3) in connection with heavy-light bound states.

In Figs. 1(a)–1(c), we compare the OPAL data [11] with the theoretical results evaluated with sets S, P, and B, respectively. Except at low x, the LO and NLO results are very similar. At low x, we observe significant differences between LO and NLO. In this region, the perturbative treatment
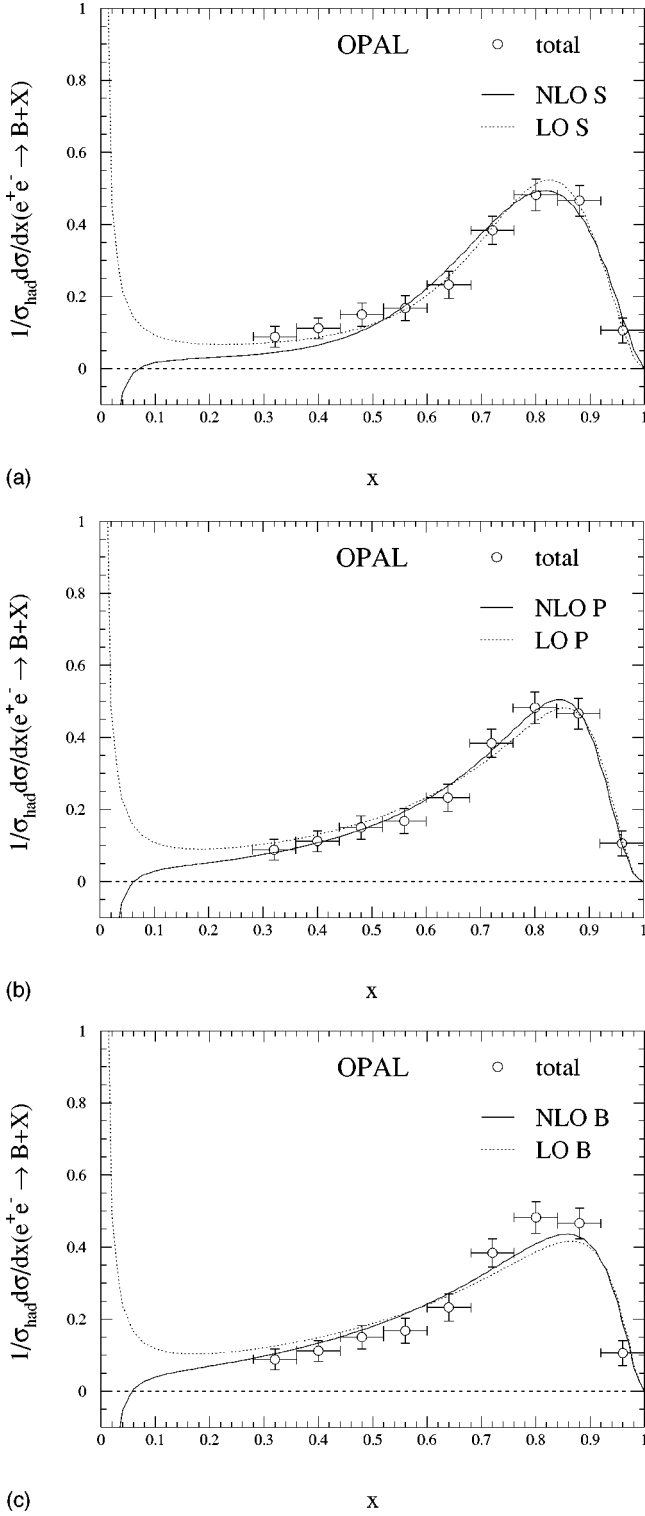
FIG. 1. The cross sections of inclusive B^+/B^0-meson production in e^+e^- annihilation at $\sqrt{s}=M_Z$ evaluated with sets (a) LO S and NLO S, (b) LO P and NLO P, and (c) LO B and NLO B are compared with the OPAL data [11].

TABLE II. $b \to B$ branching fractions and mean B to b momentum fractions evaluated from Eqs. (2.4) and (2.5), respectively, at the starting scale and at the Z-boson resonance using the various FF sets.

Set	$B_b(2m_b)$	$B_b(M_Z)$	$\langle x \rangle_b(2m_b)$	$\langle x \rangle_b(M_Z)$
LO S	0.425	0.411	0.813	0.697
NLO S	0.384	0.370	0.787	0.672
LO P	0.448	0.431	0.787	0.677
NLO P	0.405	0.388	0.758	0.650
LO B	0.460	0.442	0.768	0.663
NLO B	0.416	0.398	0.739	0.635

and strongly increases as $x \to 0$. Therefore, our results should only be considered meaningful for $x \gtrsim x_{cut}$ with $x_{cut}=0.15$, say. As already observed in connection with the χ^2_{DF} values, sets LO P and NLO P give the best description of the data. The contribution due to gluon fragmentation, which only enters at NLO, is insignificant, below 1%. The contribution due to the first four quark flavors is mostly concentrated at low x and is also very small. For $x > x_{cut}$, it makes up less than 1% of the total integrated cross section.

It is interesting to study the $b \to B$ branching fraction,

$$B_b(\mu) = \int_{x_{cut}}^1 dx \, D_b(x,\mu),   \quad (2.4)$$

where, for reasons explained above, we have introduced a lower cutoff at $x_{cut}=0.15$. In Table II, we present the values of $B_b(\mu)$ at threshold $\mu=2m_b$ and at the Z-boson resonance $\mu=M_Z$ for the various FF sets. As expected, $B_b(\mu)$ is rather stable under the evolution from $2m_b$ to M_Z. The values of $B_b(M_Z)$ are consistent with the input $f(b \to B)=0.405\pm0.035\pm0.045$ [29] which was used to scale the experimental data points [11] so as to obtain the fully normalized cross section.

Another quantity of interest is the mean B to b momentum fraction,

$$\langle x \rangle_b(\mu) = \frac{1}{B_b(\mu)} \int_{x_{cut}}^1 dx \, x D_b(x,\mu).   \quad (2.5)$$

Table II also contains the values of $\langle x \rangle_b(\mu)$ at $\mu=2m_b$ and M_Z evaluated with the various FF sets. The differences between sets S, P, and B on the one side and between LO and NLO on the other side are small. As μ runs from $2m_b$ to M_Z, $\langle x \rangle_b(\mu)$ decreases from approximately 0.8 down to below 0.7. This is a typical feature of the μ evolution, which generally softens the FF's. Our values of $\langle x \rangle_b(M_Z)$ can be compared with the experimental result reported by OPAL [11]:

$$\langle x \rangle_b(M_Z) = 0.695 \pm 0.006 \pm 0.003 \pm 0.007,   \quad (2.6)$$

where the errors are statistical, systematic, and due to model dependence, respectively. Our results in Table II are in reasonable agreement with Eq. (2.6). In connection to this, we remark that Eq. (2.6) is not directly obtained from the mea-

ceases to be valid. Here, the massless approximation also loses its validity. Since B mesons have mass, $m(B)=5.28$ GeV, they can only be produced for $x > x_{min}=2m(B)/M_Z = 0.12$. The LO result has a minimum in the vicinity of x_{min}

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sured distribution, which would be difficult to do, since there are no data points below $x = 0.2$. To extrapolate to the unmeasured region, OPAL uses four different models which describe the primordial fragmentation of *b* quarks inside their MC simulation. Equation (2.6) is actually determined from the MC fits to the measured data points. Obviously, the quoted error for the model dependence can only account for the specific model dependence inside their particular MC approach, and need not be characteristic of the absolute model dependence. A rather model-independent fit to the *x* distribution, including a MC estimate for the region $x < 0.2$, leads to $\langle x \rangle_b(M_Z) = 0.72 \pm 0.05$ [11], where the error is only statistical and does not account for the uncertainty due to the extrapolation. Our results in Table II are somewhat smaller than this value and are barely consistent with the experimental error given above. Nevertheless, we believe that our results in Table II are in reasonable agreement with the independent determinations of $\langle x \rangle_b(M_Z)$ quoted in Ref. [11].
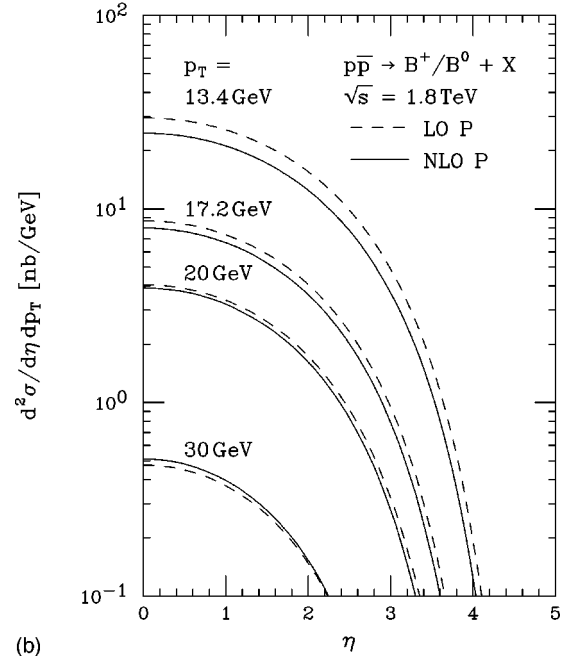
III. *B*-MESON PRODUCTION IN $p\bar{p}$ COLLISIONS

In this section, we compare our LO and NLO predictions for the cross section of inclusive B^+/B^0 production in $p\bar{p}$ collisions at the Fermilab Tevatron ($\sqrt{s} = 1.8$ TeV) with recent data from the CDF Collaboration [8]. These data come as the p_T distribution $d\sigma/dp_T$ integrated over the central rapidity region $|\eta| < 1$ for p_T values between 7.4 and 20 GeV. They are normalized in such a way that they refer to the single channel $p\bar{p} \to B^+ + X$. In the case of run 1A, where both B^+ and B^0 mesons were detected, the respective cross sections were averaged; i.e., their sum was divided by a factor of two.

Our formalism is very similar to Ref. [30], where inclusive light-meson production in $p\bar{p}$ collisions was studied in the QCD-improved parton model. The relevant formulas and references may be found in Ref. [30], and we refrain from repeating them here. We work at NLO in the $\overline{\text{MS}}$ scheme with $n_f = 5$ massless flavors. For the proton and antiproton parton distribution functions (PDF's), we use set CTEQ4M [31] with $\Lambda_{\overline{\text{MS}}}^{(5)} = 202$ MeV. We evaluate α_s from the two-loop formula with this value of $\Lambda_{\overline{\text{MS}}}^{(5)}$. We recall that the evolution of the FF sets NLO S, NLO P, and NLO B is performed with $\Lambda_{\overline{\text{MS}}}^{(5)} = 227$ MeV, which is very close to the above value. We identify the factorization scales associated with the proton, antiproton, and the *B* meson and collectively denote them by M_f. We choose renormalization and factorization scales to be $\mu = M_f = 2m_T$, where $m_T = \sqrt{p_T^2 + m_b^2}$ is the *B*-meson transverse mass. Whenever we present LO results, they are consistently computed using set CTEQ4L [31] of the proton and antiproton PDF's, our LO sets of *B*-meson FF's, the one-loop formula for α_s with $\Lambda_{\overline{\text{MS}}}^{(5)} = 181$ MeV [31], and the LO hard-scattering cross sections. We adopt the kinematic conditions from Ref. [8]. Since we employ $D_b(x,\mu)$ both for the *b* and $\bar{b}$ quarks, the resulting cross section corresponds to the sum of the B^+ and B^- yields. Thus, it needs to multiplied by a factor of 1/2, in order to match the cross section quoted in Ref. [8].
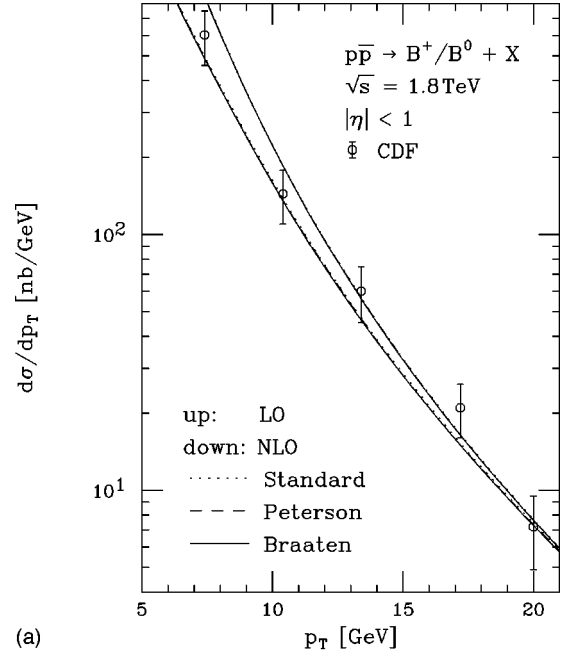
(a)

(b)

FIG. 2. (a) The cross section $d\sigma/dp_T$ of inclusive B^+/B^0-meson production in $p\bar{p}$ collisions with $\sqrt{s} = 1.8$ TeV, integrated over $|\eta| < 1$, is compared with the CDF data [8]. The predictions are calculated at LO and NLO with sets S, P, and B. (b) The cross section $d\sigma/d\eta\, dp_T$ at fixed values of p_T evaluated with sets LO P and NLO P.

First, we consider the p_T distribution $d\sigma/dp_T$ integrated over the rapidity region $|\eta| < 1$ as in the CDF analysis [8]. In Fig. 2(a), we compare the CDF data [8] with the LO and NLO predictions evaluated with our various sets of *B*-meson FF's. The NLO distributions fall off slightly less strongly with increasing p_T than the LO ones. The results for sets LO S, LO P, and LO B almost coincide. The same is true of the results for sets NLO S, NLO P, and NLO B. This means that
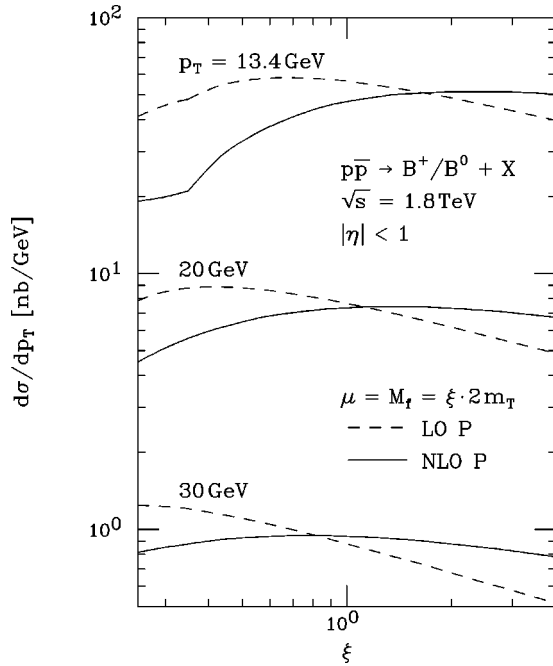
FIG. 3. Scale dependence of the cross section $d\sigma/dp_T$, integrated over $|\eta|<1$, at fixed values of p_T. The predictions are calculated at LO and NLO with set P.

the details of the $b \to B$ fragmentation is tightly constrained by the LEP data, and that the considered variation in the functional form of the b-quark FF at the starting scale has very little influence on the p_T distribution. Henceforth, we shall only employ sets LO P and NLO P, which yielded the best fits to the OPAL data [11]. We observe that our prediction agrees very well with the CDF data, within their errors. This is even true for the data point with smallest p_T, $p_T = 7.4$ GeV, where the massless approach is presumably not valid any more. It should be emphasized that the NLO prediction reproduces both the shape and the absolute normalization of the measured p_T distribution, while the previous investigations mentioned in the Introduction fell short of the data by roughly a factor of two.

The CDF Collaboration has not yet presented results on the η distribution of the produced B mesons, which would allow for another meaningful test of the QCD-improved parton model endowed with B-meson FF's. Anticipating that such a measurement will be done in the future, we show in Fig. 2(b) the η dependence of $d^2\sigma/d\eta dp_T$ evaluated with sets LO P and NLO P at $p_T = 13.4$, 17.2, 20, and 30 GeV. The first three of these p_T values are among those for which CDF performed measurements of $d\sigma/dp_T$ [8]. Since the η spectrum is symmetric around $\eta = 0$, we only consider $\eta \geqslant 0$ in Fig. 2(b). As expected, the cross section falls off with η increasing from zero up to the kinematic limit, which depends on p_T.

In order to assess the reliability of our predictions, at least to some extent, we now investigate the scale dependence of the cross section considered in Fig. 2(a). To this end, we introduce the scale factor ξ such that $\mu = M_f = 2\xi m_T$. In Fig. 3, the ξ dependence of $d\sigma/dp_T$ is displayed for $p_T = 13.4$, 20, and 30 GeV. The calculation is performed with sets LO P

and NLO P. For the two highest p_T values, $p_T = 20$ and 30 GeV, we observe the expected pattern. The LO results for $d\sigma/dp_T$ essentially decrease with ξ increasing, whereas the NLO results are rather ξ independent and exhibit points of horizontal tangent close to $\xi = 1$. Furthermore, the LO and NLO curves intersect near these points. Thus the scale choice $\xi = 1$ is favored both from the principles of minimal sensitivity [32] and fastest apparent convergence [33]. These observations reassure us of the perturbative stability and the theoretical soundness of our calculation in the upper p_T range. For $p_T = 13.4$ GeV, the NLO prediction of $d\sigma/dp_T$ shows a stronger scale dependence, in particular, when the scale is drastically reduced. If we limit the scale variation to the interval $1/2 < \xi < 2$, which is frequently considered in the literature, the NLO cross section still varies by a factor of 1.56, to be compared with 1.15 at $p_T = 20$ GeV. We hence conclude that, below $p_T = 13.4$ GeV, our NLO predictions should be taken with a grain of salt. The dents in the curves for $p_T = 13.4$ GeV appear at the value of ξ where $M_f = \mu_0$. This is because we identify $D_b(x, M_f) = D_b(x, \mu_0)$ if $M_f < \mu_0$, i.e., the FF's are frozen below their threshold.

We must also remember that, for p_T values comparable to m_b, the massless-quark approximation ceases to be valid, since terms of order m_b^2/p_T^2 are then not negligible anymore. For $p_T = 13.4$ GeV and 20 GeV, we have $m_b^2/p_T^2 = 0.14$ and 0.063, respectively, so that the massless approximation should certainly be valid for $p_T = 20$ GeV. On the other hand, for $p_T = 20$ GeV, we have $\alpha_s \ln(p_T^2/m_b^2) = 0.42$, assuming that $\alpha_s = 0.15$, so that the NLO calculation in the massive scheme, where these logarithmic terms are not resummed, should then already be inadequate. From these considerations, we conclude that our predictions should be fairly reliable for $p_T \gtrsim 15$ GeV.

IV. CONCLUSIONS

In this paper, we considered the inclusive production of single B mesons in the QCD-improved parton model endowed with nonperturbative FF's. We chose to work at NLO in the pure $\overline{\text{MS}}$ factorization scheme with five massless quark flavors. This theoretical framework is known to lead to an excellent description of a wealth of experimental information on inclusive light-hadron production in different types of high-energy reactions [27,34]. It is thus expected to also work well in the case of B mesons provided that the characteristic mass scale M of the process by which they are produced is large compared to the b-quark mass. Then, the large logarithms of the type $\alpha_s \ln(M^2/m_b^2)$ which are bound to arise in any scheme where bottom is treated as a massive flavor get properly resummed by the AP evolution, while the omission of the terms suppressed by powers of m_b^2/M^2 is a useful approximation. The criterion $M \gg m_b$ is certainly satisfied for e^+e^- annihilation on the Z-boson resonance, and for hadroproduction of B mesons with $p_T \gg m_b$. Owing to the factorization theorem, the FF's are universal in the sense that they just depend on the produced hadrons and the partons from which they sprang, but not on the processes by which the latter were produced. Thus, the theoretical framework

adopted here is particularly suited for a consistent description of LEP1 and high-p_T Tevatron data of inclusive B-meson production. By the same token, a massive calculation at fixed order would be inappropriate for this purpose.

Our procedure was as follows. We determined LO and NLO B-meson FF's by fitting the fractional energy distribution of the B-meson sample collected by the OPAL Collaboration at LEP1 [11]. In order to get some handle on the theoretical uncertainty, we adopted three different functional forms for the $b \to B$ FF at the starting scale, which we took to be $\mu_0 = 2m_b = 10$ GeV. The ansatz proposed by Peterson *et al.* [9] yielded the best LO and NLO fits, with $\chi^2_{\mathrm{DF}} = 0.67$ and 0.27, respectively. The ϵ parameter, which measures the smearing of the Peterson distribution, came out as 0.0126 and 0.0198, respectively, i.e., more than twice as large as the standard value $\epsilon = 0.006$ determined by Chrin [10] more than a decade ago, before the LEP1 era. In this connection, we should emphasize that the results for the fit parameters, including the value of ϵ, are highly scheme dependent at NLO, and must not be naively compared disregarding the theoretical framework to which they refer. The $b \to B$ branching fraction and the mean B to b momentum fraction evaluated from the resulting FF's after the evolution to $\mu = M_Z$ turned out to be in reasonable agreement with the model-dependent determinations by OPAL [11]. Using our FF's, we made theoretical predictions for the inclusive hadroproduction of single B mesons with large p_T. We found good agreement, both in shape and normalization, with the p_T distribution recently measured in the central rapidity region by the CDF Collaboration at Fermilab [8]. From the study of the scale dependence of the LO and NLO calculations, we concluded that our results should be reliable for $p_T \gtrsim 15$ GeV. To our surprise, the central prediction, with scales $\mu = M_f = 2m_T$, also nicely agreed with the CDF data in the low-p_T range, where the massless scheme is expected to break down. We recall that the massive NLO calculation with traditional Peterson fragmentation [10] was found to fall short of these data by a factor of two. It would be interesting to also test the predicted η distribution against experimental data.

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