Effective chiral meson-baryon Lagrangian from quark-diquark flavor dynamics

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The approach of path integral hadronization is applied to an SU(2) model of quark-diquark flavor dynamics. Within such a scheme we derive an effective chiral meson-baryon Lagrangian, where the Goldberger-Treiman relation, found earlier at the quark-meson level, is now reestablished at the composite hadron level. Masses and coupling constants of composite hadrons are then calculable by the parameters of the underlying microscopic quark-diquark picture. [S0556-2821(98)00115-5]

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I. INTRODUCTION

It is well known that the low-energy properties of mesons and baryons can be suitably described by phenomenological effective Lagrangians [1–3] which embody the global (chiral) flavor symmetries of quantum chromodynamics [SU(2)×SU(2) or SU(3)×SU(3)]. The appearance of an approximate chiral symmetry and its dynamical breakdown ensures both the generation of Goldstone pions as well as the existence of current algebra relations and low energy theorems. Concerning meson physics, the approach of path integral bosonization applied to QCD-motivated Nambu–Jona-Lasinio– (NJL-) type models [4] turned out to be a powerful method for deriving effective chiral meson Lagrangians [3,5] (for further references see the recent review [6]).

Considering baryons, we see from the point of view of baryon spectroscopy that baryons can be treated analogously to mesons by noticing that in a color-singlet baryon any two quarks must be in a $\bar{3}_C$ color state, which then transforms under the color group as an antiquark. Moreover, the gluon exchange between two quarks in the $\bar{3}_C$ state turns out to be attractive, leading to the formation of bound states referred to as diquarks. Diquarks then serve as effective (colored) degrees of freedom and are expected to play a significant dynamical role in the structure of baryons.

The important concept of diquarks has a long history (for the first papers see [7]). It is worth remarking that there is now mounting experimental evidence for diquark correlations in the baryon arising from various fields in hadronic physics, such as baryon spectroscopy, deep-inelastic lepton-hadron scattering, hard proton-proton scattering, or weak decays [8].

Altogether, this then suggests that colorless baryons might be understood as bound diquark-quark states. Clearly, the analytical and numerical calculation of such a resulting two-body system is much easier to handle than a three-body calculation. In particular, by applying path integral methods to quark-diquark models it was possible to derive Faddeev-type equations determining the spectrum of composite baryons [9–11].

Given the successes of the chiral phenomenological theories in the description of low-energy phenomena, it is now a major challenge to derive also the effective chiral meson-baryon Lagrangians, including the hadronic Goldberger-Treiman relation, directly from an underlying microscopic quark-diquark interaction. We choose in this paper a simple SU(2) model, containing a local interaction of quarks with (scalar) diquarks in order to demonstrate the powerfulness of the method of path integral techniques in collective fields referred to as ‘‘path integral hadronization.’’

II. EFFECTIVE CHIRAL MESON LAGRANGIAN AND FIELD TRANSFORMATIONS

In order to fix our notation required for the derivation of the nonlinear meson-baryon Lagrangian, it is useful to first recapitulate some results of the bosonization of the NJL model [5,6], emphasizing here nonlinear field transformations [3]. Thus, we consider the NJL Lagrangian with scalar and pseudoscalar couplings possessing a global flavor and color $SU(2)_A \times SU(2)_V \times SU(3)_C$ symmetry:

$$\mathcal{L}_{\mathrm{NJL}} = \bar{q}(i\hat{\partial} - m_0)q + \frac{G}{2}[(\bar{q}q)^2 + (\bar{q}_i\gamma_5\vec{\tau}q)^2], \tag{1}$$

where $\hat{\partial} = \partial^\mu\gamma_\mu$, G is a universal coupling constant of dimension $(\mathrm{mass})^{-2}$, $\vec{\tau}$ are the Pauli matrices of the flavor group SU(2), and m_0 is an explicit chiral symmetry-breaking quark mass (summation over repeated indices is always understood). Introducing collective meson fields σ, π_i ($i = 1,2,3$) by the Gauss trick

$$\exp\left(i\int d^4x\,\frac{G}{2}[(\bar{q}q)^2 + (\bar{q}_i\gamma_5\vec{\tau}q)^2]\right) \equiv \mathcal{N}_1 \int \mathcal{D}\sigma\mathcal{D}\vec{\pi}\exp\left[i\int d^4x\left(-\frac{1}{2G}(\sigma^2+\vec{\pi}^2) - \bar{q}(\sigma + i\gamma_5\vec{\tau}\vec{\pi})q\right)\right],$$

one gets a semibosonized Lagrangian

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$$\mathcal{L}_{\rm NJL}^{qM}=\overline{q}(i\hat{\partial}-\sigma-i\gamma_5\vec{\tau}\vec{\pi})q-\frac{1}{2G}[(\sigma-m_0)^2+\vec{\pi}^2], \quad (2)$$

where the bare quark mass m_0 has been absorbed into the field σ.

It is convenient to define new scalar and pseudoscalar fields σ' and $\vec{\Phi}$ via the exponential parametrization

$$\sigma+i\gamma_5\vec{\tau}\vec{\pi}=(m+\sigma')\exp\left(-\frac{i}{F_\pi}\gamma_5\vec{\tau}\vec{\Phi}\right)$$

(F_π is the pion decay constant and the constituent quark mass $m\equiv\langle\sigma\rangle_0$ is fixed by the gap equation [5]).

Performing an appropriate chiral rotation of quark fields,[1]

$$q=\exp\left(\frac{i}{F_\pi}\gamma_5\frac{\vec{\tau}}{2}\vec{\Phi}\right)\chi,$$

provides us a Lagrange density in terms of redefined fields:

$$\begin{aligned}\mathcal{L}_{\rm NJL}^{qM}&(\chi,\vec{\Phi},\sigma')\\ &=-\frac{1}{2G}(m+\sigma')^2+\frac{m+\sigma'}{16G}m_0{\rm Tr}_{F,D}\\ &\quad\times\left[\exp\left(-\frac{i}{F_\pi}\gamma_5\vec{\tau}\vec{\Phi}\right)+{\rm H.c.}\right]\\ &\quad+\overline{\chi}\left\{i\gamma_\mu\left[\partial^\mu+\exp\left(-\frac{i}{F_\pi}\gamma_5\frac{\vec{\tau}}{2}\vec{\Phi}\right)\partial^\mu\right.\right.\\ &\quad\left.\left.\times\exp\left(\frac{i}{F_\pi}\gamma_5\frac{\vec{\tau}}{2}\vec{\Phi}\right)\right]-m-\sigma'\right\}\chi, \quad (3)\end{aligned}$$

where the trace is taken over flavor and Dirac indices.

Note the nonlinear transformation law of the meson field $\vec{\xi}\equiv\vec{\Phi}/F_\pi$ under global chiral transformations $g=\exp[(i/2)\gamma_5\vec{a}\vec{\tau}]\exp[(i/2)\vec{v}\vec{\tau}]\in SU(2)_A\times SU(2)_V$ [2,3] (a_i and v_i are real parameters):

$$g\cdot\exp\left(\frac{i}{2}\gamma_5\vec{\tau}\vec{\xi}(x)\right)=\exp\left(\frac{i}{2}\gamma_5\vec{\tau}\vec{\xi}'(x)\right)\cdot h(x), \quad (4)$$

where

$$h(x)=\exp\left(\frac{i}{2}\vec{\tau}\vec{u}'(\vec{\xi}(x),g)\right)\in SU(2)_{V,\rm loc}$$

is an element of the local vector group.

For later use it is convenient to rewrite the Dirac operator of the χ field in Eq. (3) by employing the Cartan decomposition [3]

$$\begin{aligned}\exp&\left(-\frac{i}{2}\gamma_5\vec{\tau}\vec{\xi}\right)\partial^\mu\exp\left(\frac{i}{2}\gamma_5\vec{\tau}\vec{\xi}\right)\\ &=\frac{i}{2}\gamma_5\vec{\tau}\vec{\mathcal{A}}^\mu(\xi)+\frac{i}{2}\vec{\tau}\vec{\mathcal{V}}^\mu(\xi). \quad (5)\end{aligned}$$

The fields $\chi,\vec{\mathcal{A}}_\mu,\vec{\mathcal{V}}_\mu$ have the following simple transformation law under Eq. (4) $[\mathcal{V}_\mu\equiv(\vec{\tau}/2)\vec{\mathcal{V}}_\mu$, etc]:

$$\begin{aligned}\chi&\mapsto\chi'=h(x)\chi,\\ \mathcal{V}_\mu&\mapsto\mathcal{V}'_\mu=h(x)\mathcal{V}_\mu h^\dagger(x)-h(x)i\partial_\mu h^\dagger(x),\\ \mathcal{A}_\mu&\mapsto\mathcal{A}'_\mu=h(x)\mathcal{A}_\mu h^\dagger(x).\end{aligned}$$

As we see, $\mathcal{V}_\mu$ transforms as a gauge field with respect to $SU(2)_{V,\rm loc}$. This allows one to define the following chiral-covariant derivative of the rotated quark field χ:

$$D_\mu\chi=(\partial_\mu+i\mathcal{V}_\mu)\chi. \quad (6)$$

Thus, using Eqs. (3), (5), and (6), the inverse propagator of the χ field takes the form

$$S^{-1}=i\hat{D}-m-\sigma'-\hat{\mathcal{A}}\gamma_5. \quad (7)$$

Finally, by integrating over the χ field in the generating functional of Eq. (3) and freezing the σ' field, one arrives at a nonlinear pion Lagrangian. Indeed, performing the loop expansion of the resulting quark determinant ($\det S^{-1}$) and choosing a gauge-invariant regularization, we arrive at a masslike term of the $\mathcal{A}_\mu(\xi)$ field contributing to $\mathcal{L}_{\rm eff}^M$:[2]

$$\mathcal{L}_{\rm eff}^M=\frac{m^2}{g_{\pi qq}^2}{\rm Tr}_F\mathcal{A}_\mu^2+\Delta\mathcal{L}_{\rm sb}.$$

Here $g_{\pi qq}$ is the induced meson-quark coupling constant and $\Delta\mathcal{L}_{\rm sb}=\mathcal{O}(m_0)$ is the symmetry-breaking mass term given by the second term in Eq. (3). As discussed in [3] the Cartan form $\mathcal{A}_\mu(\xi)$ is just the chiral covariant derivative of the ξ field, admitting the expansion

$$\vec{\mathcal{A}}_\mu(\xi)\equiv D_\mu\vec{\xi}=\partial_\mu\vec{\xi}+\mathcal{O}(\xi^3)=\frac{1}{F_\pi}\partial_\mu\vec{\Phi}+\mathcal{O}(\Phi^3).$$

Thus, using the Goldberger-Treiman relation

$$F_\pi=\frac{m}{g_{\pi qq}},$$

at the quark level [5], one obtains the effective chiral meson Lagrangian

$$\mathcal{L}_{\rm eff}^M=\frac{F_\pi^2}{2}D_\mu\vec{\xi}D^\mu\vec{\xi}+\Delta\mathcal{L}_{\rm sb}. \quad (8)$$

[1]In general, i.e., for more than two flavors, this transformation leads to a chiral Wess-Zumino anomaly.

[2]Note that possibly arising field strength terms of the $\mathcal{V}_\mu$ and $\mathcal{A}_\mu$ fields are vanishing. A mass term $\sim\mathcal{V}_\mu^2$ does not appear due to gauge-invariant regularization.

III. EFFECTIVE CHIRAL MESON-BARYON LAGRANGIAN

In order to derive an effective chiral meson-baryon Lagrangian from a microscopic quark-diquark model, the semi-bosonized Lagrangian (3) has to be supplemented by the kinetic part of the scalar isoscalar diquark D, which here will be treated as an elementary field, and by a quark-diquark interaction term. For illustration, let us consider a simple quark-diquark model defined by the following extended SU(2)$\times$SU(2) chiral-invariant Lagrangian:

$$\mathcal{L}^{qMD}=\mathcal{L}_{\mathrm{NJL}}^{qM}+D^{\dagger}\Delta^{-1}D+\tilde{G}\bar{\chi}D^{\dagger}D\chi, \tag{9}$$

$$\Delta^{-1}=-\partial_{\mu}\partial^{\mu}-M_D^2, \tag{10}$$

with diquark mass M_D and a local effective quark-diquark interaction with coupling constant $\tilde{G}$. For motivating our choice of a *local* quark-diquark interaction term, notice that it is close in spirit to the static approximation used in Ref. [12] for the quark-exchange potential in the quark-diquark Faddeev bound state equation.[3] Such an exchange potential arises quite naturally from quark-exchange diagrams emerging in the bosonization of an NJL model with two-body $(q\tilde{q})$ and (qq) forces [9–11]. Note that in the latter approach diquarks are treated as composite particles. Thus our simple model with a local interaction of quarks and elementary diquarks represents in the sense discussed above a ‘‘certain approximation’’ to an NJL model with quark-exchange diagrams and composite diquarks, but is much easier to handle.

As in the NJL model, we are now in the position to introduce collective baryon fields B using a Gauss trick. Here we have

$$\exp\left(i\int d^4x\tilde{G}\bar{\chi}D^{\dagger}D\chi\right)$$
$$=\mathcal{N}'\int\mathcal{D}B\mathcal{D}\bar{B}\exp\left[i\int d^4x\left(-\frac{1}{\tilde{G}}\bar{B}B-\bar{\chi}D^{\dagger}B-\bar{B}D\chi\right)\right]. \tag{11}$$

Let us next perform the ‘‘hadronization’’ of the Lagrangian (9) by integrating step by step over quark and diquark fields in the respective generating functional:

$$Z=\mathcal{N}_1\int\mathcal{D}\sigma'\mathcal{D}\Phi_i\mathcal{D}B\mathcal{D}\bar{B}\mathcal{D}D\mathcal{D}D^{\dagger}$$
$$\times\exp\left[i\int d^4x\left(-i\mathrm{Tr}_{F,D}\ln S^{-1}-\frac{1}{\tilde{G}}\bar{B}B\right)\right]$$
$$\times\exp\left(i\int\int d^4x\,d^4y[D^{\dagger}(x)(\Delta^{-1}-\bar{B}SB)_{(x,y)}D(y)]\right),$$

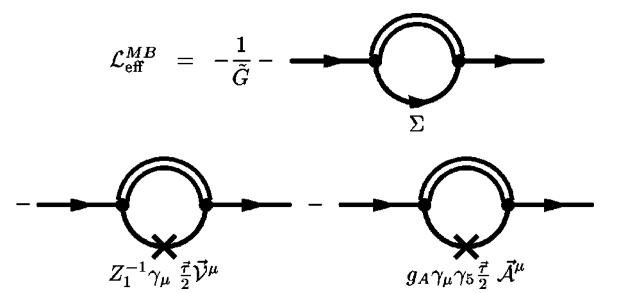

FIG. 1. Self-energy contribution of the nucleon, vector vertex diagram, and axial-vector vertex diagram resulting from expanding the second logarithm in Eq. (12) of the text.

$$Z=\mathcal{N}_2\int\mathcal{D}\sigma'\mathcal{D}\Phi_i\mathcal{D}B\mathcal{D}\bar{B}$$
$$\times\exp\left[i\int d^4x\left(-i\mathrm{Tr}_{F,D}\ln S^{-1}\right.\right.$$
$$\left.\left.-\frac{1}{\tilde{G}}\bar{B}B+i\ln(1-\bar{B}S\Delta B)_{(x,x)}\right)\right], \tag{12}$$

with the quark and diquark propagators S and Δ defined in Eqs. (7) and (10), respectively.

Expanding the logarithms in power series at the one-loop level (see Fig. 1) and taking into account only lowest-order derivative terms, one describes both the generation of kinetic and mass terms for the composite baryon fields as well as the meson-baryon interaction. This yields the expression

$$\mathcal{L}_{\mathrm{eff}}^{MB}=\int d^4x\int d^4y\,\bar{B}(x)$$
$$\times\left[\left(-\frac{1}{\tilde{G}}-Z_1^{-1}\gamma_{\mu}\frac{\vec{\tau}}{2}\vec{\mathcal{V}}^{\mu}(x)-g_A\gamma_{\mu}\gamma_5\frac{\vec{\tau}}{2}\vec{\mathcal{A}}^{\mu}(x)\right)\right.$$
$$\left.\times\delta^4(x-y)-\Sigma(x-y)\right]B(y), \tag{13}$$

where the baryon self-energy Σ admits in momentum space the decomposition $\Sigma(p)=\hat{p}\Sigma_V(p^2)+\Sigma_S(p^2)$, whose low-momentum expansion will be quoted below; Z_1^{-1} and g_A are the vector vertex renormalization and axial coupling constant defined by suitably regularized loop integrals.

Performing the low-momentum expansion of Σ and introducing renormalized fields, we have

$$\mathcal{L}_{\mathrm{eff}}^{MB}=\bar{B}_{\mathrm{ren}}(i\hat{D}-M_B)B_{\mathrm{ren}}-g_A^{\mathrm{ren}}\bar{B}_{\mathrm{ren}}\gamma_{\mu}\gamma_5\frac{\vec{\tau}}{2}B_{\mathrm{ren}}\vec{\mathcal{A}}^{\mu}, \tag{14}$$

with $B=Z^{1/2}B_{\mathrm{ren}}$, $g_A^{\mathrm{ren}}=Z\,g_A$, and Z being the Z factor derived from the baryon propagator which satisfies the QED-type Ward identity $Z=Z_1$.

As seen from Eq. (13), the nucleon mass is estimated by

[3]As shown in [13], the solution based on the static approximation of [12] qualitatively reproduces the trend of the calculation with the exact potential reasonably well.

$$\frac{1}{\tilde{G}}+[M_B\Sigma_V(M_B^2)+\Sigma_S(M_B^2)]=0.$$

Recalling $\mathcal{A}_i^{\mu}=\partial^{\mu}\xi_i+\cdots$, we obtained an axial-vector derivative coupling of pions to baryons. To get rid of the derivative, let us redefine the baryon fields via

$$B_{\rm ren}=\exp\left(-\frac{i}{2}\,g_A^{\rm ren}\gamma_5\vec{\tau}\vec{\xi}\right)\tilde{B}.$$

Then the expansion in a power series in $\vec{\Phi}$ provides us the expression

$$\mathcal{L}_{\rm eff}^{MB}=\overline{\tilde{B}}(i\hat{\partial}-M_B)\tilde{B}+g_A^{\rm ren}\,\frac{M_B}{F_\pi}\,\overline{\tilde{B}}i\gamma_5\vec{\tau}\tilde{B}\vec{\Phi}+\mathcal{O}(\Phi^2). \tag{15}$$

Obviously, we have to identify the constant, associated with the Yukawa interaction, with the pion-nucleon coupling constant as

$$g_{\bar{\tilde{B}}\tilde{B}\Phi}=\frac{M_B}{F_\pi}\,g_A^{\rm ren}, \tag{16}$$

which is nothing but the Goldberger-Treiman relation at the composite hadron level. Combining Eqs. (8) and (15), the total meson-baryon Lagrangian reads

$$\mathcal{L}_{\rm eff,tot}^{MB}=\mathcal{L}_{\rm eff}^{M}+\mathcal{L}_{\rm eff}^{MB}.$$

As a simple application, let us calculate the renormalized axial coupling constant from the matrix element associated with the third Feynman diagram of Fig. 1,

$$iM_{\mu}^{g_A}=-3i\int iS(p-k)\gamma_\mu\gamma_5 iS(p-k)i\Delta(k)\,\frac{d^4k}{(2\pi)^4}.$$

For typical quark and diquark constituent masses (we suppose an exact isospin symmetry $m_u=m_d=m$) used in such type of approaches,

$$m=0.450\ \text{GeV},$$

$$M_D=0.650\ \text{GeV},$$

and a cutoff

$$\Lambda=0.750\ \text{GeV},$$

the model leads to a value

$$g_A^{\rm ren}=1.07,$$

which is in accordance with other related studies (see, e.g., [14]), but somewhat lower than the experimental value $g_A=1.26$.

IV. SUMMARY

The primary aim of the present paper was to present ideas and path integral hadronization techniques applied to quark-diquark flavor dynamics. Starting from a simple local chiral SU(2)×SU(2) model of quarks and diquarks, we have in particular derived an effective composite meson-baryon Lagrangian. This Lagrangian describes the essential features of low-energy meson-baryon physics including as an important result the Goldberger-Treiman relation which is now reestablished at the composite hadron level. Moreover, masses and coupling constants of composite hadrons are calculable and expressed by the parameters of the underlying quark-diquark model (quark and diquark masses, interaction strength $\tilde{G}$, and loop momentum cutoff Λ).

In order to get from such kind of models more extensive predictions for low-energy hadron characteristics (such as masses, coupling constants, electromagnetic radia, anomalous magnetic moments, etc.) exceeding the number of input parameters, we have to further generalize the above approach to chiral SU(3) symmetry, including in addition axial-vector diquarks as well as electromagnetic interactions.

Axial-vector diquarks are expected to be important in order to get the SU(3) F/D ratio of meson-baryon coupling constants and to reproduce anomalous magnetic moments of the octet of SU(3) baryons. Especially, they are necessary in order to describe the SU(3) decuplet of $3^+/2$ resonances.

These issues will be the subject of further investigations which will be considered elsewhere.

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