Magnetic and axial vector form factors as probes of orbital angular momentum in the proton

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We have recently examined the static properties of the baryon octet (magnetic moments and axial vector coupling constants) in a generalized quark model in which the angular momentum of a polarized nucleon is partly spin $\langle S_z\rangle$ and partly orbital $\langle L_z\rangle$. The orbital momentum was represented by the rotation of a flux tube connecting the three constituent quarks. The best fit is obtained with $\langle S_z\rangle=0.08\pm0.15$, $\langle L_z\rangle=0.42\pm0.14$. We now consider the consequences of this idea for the q^2 dependence of the magnetic and axial vector form factors. It is found that the isovector magnetic form factor $G_M^{\mathrm{isovec}}(q^2)$ differs in shape from the axial form factor $F_A(q^2)$ by an amount that depends on the spatial distribution of orbital angular momentum. The model of a rigidly rotating flux tube leads to a relation between the magnetic, axial vector and matter radii, $\langle r^2\rangle_{\mathrm{mag}}=f_{\mathrm{spin}}\langle r^2\rangle_{\mathrm{axial}}+\frac{5}{2}f_{\mathrm{orb}}\langle r^2\rangle_{\mathrm{matt}}$, where $f_{\mathrm{orb}}/f_{\mathrm{spin}}=\frac{1}{3}\langle L_z\rangle/G_A$, $f_{\mathrm{spin}}+f_{\mathrm{orb}}=1$. The shape of $F_A(q^2)$ is found to be close to a dipole with $M_A=0.92\pm0.06$ GeV. [S0556-2821(98)03415-8]

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I. INTRODUCTION

In a recent paper [1] we performed a fit to the magnetic moments of the baryon octet in a model in which these quantities are determined partly by the quark spins Δu, Δd, Δs, and partly by an orbital angular momentum $\langle L_z\rangle$, shared between the constituent quarks. The model is exemplified by the following ansatz for the proton and neutron magnetic moments:

$$\mu_p=\mu_u\Delta u+\mu_d\Delta d+\mu_s\Delta s+\left[\frac{2}{3}\mu_u+\frac{1}{3}\mu_d\right]\langle L_z\rangle,$$
$$\mu_n=\mu_u\Delta d+\mu_d\Delta u+\mu_s\Delta s+\left[\frac{1}{3}\mu_u+\frac{2}{3}\mu_d\right]\langle L_z\rangle. \tag{1}$$

The part containing Δu, Δd, Δs, is the ''spin'' contribution to the magnetic moments, arising from the polarization of quarks and antiquarks in a polarized proton:

$$\Delta q=(q_+-q_-)+(\overline{q}_+-\overline{q}_-). \tag{2}$$

The part proportional to $\langle L_z\rangle$ is the ''orbital'' (or convective) contribution, determined by the prescription of dividing the orbital angular momentum in proportion to the constituent quark masses. The complete set of baryon magnetic moments obtained by this prescription is shown in Table I. These expressions, without the orbital part, were written down in Refs. [2,3].

There are two essential elements that go into the above equations for the magnetic moments:

(1) It is assumed that one may use the quark spins Δq in place of the quantities

$$\delta q=(q_+-q_-)-(\overline{q}_+-\overline{q}_-) \tag{3}$$

that are appropriate to an expression for the magnetic moment. This approximation is justified if antiquarks in a proton carry little polarization. An example of such a situation is the chiral quark model [4], in which antiquarks are embedded in a cloud of spin-zero mesons.

(2) The partition of $\langle L_z\rangle$ in proportion to the masses of the constituent quarks is based on the picture of a baryon as a symmetric three-pronged flux tube of equal segments (Fig. 1), rotating collectively around the spin axis [1]. The appearence of the same magnetons μ_u, μ_d, μ_s, in the orbital as in the spin part means, in particular, that the orbital g-factor has been taken to be $g_l=1$. (In a more general description, one could interpret $\langle L_z\rangle$ as $\langle g_lL_z\rangle$.)

TABLE I. Parametrization of magnetic moments in the rotating flux-tube model [model (A) of Ref. [1]]. The fits are based on $\lambda=m_d/m_s=0.6$, $\mu_u=-2\mu_d$, $\mu_s=0.6\mu_d$ (for these values, the orbital contribution to the neutral baryons n, Ξ^0 and Λ^0 vanishes).

$$\mu(p)=\mu_u\Delta u+\mu_d\Delta d+\mu_s\Delta s+[\tfrac{2}{3}\mu_u+\tfrac{1}{3}\mu_d]\langle L_z\rangle$$
$$\mu(n)=\mu_u\Delta d+\mu_d\Delta u+\mu_s\Delta s+[\tfrac{1}{3}\mu_u+\tfrac{2}{3}\mu_d]\langle L_z\rangle$$
$$\mu(\Sigma^+)=\mu_u\Delta u+\mu_d\Delta s+\mu_s\Delta d+\left[\frac{2\lambda}{1+2\lambda}\mu_u+\frac{1}{1+2\lambda}\mu_s\right]\langle L_z\rangle$$
$$\mu(\Sigma^-)=\mu_u\Delta s+\mu_d\Delta u+\mu_s\Delta d+\left[\frac{2\lambda}{1+2\lambda}\mu_d+\frac{1}{1+2\lambda}\mu_s\right]\langle L_z\rangle$$
$$\mu(\Xi^-)=\mu_u\Delta s+\mu_d\Delta d+\mu_s\Delta u+\left[\frac{\lambda}{2+\lambda}\mu_d+\frac{2}{2+\lambda}\mu_s\right]\langle L_z\rangle$$
$$\mu(\Xi^0)=\mu_u\Delta d+\mu_d\Delta s+\mu_s\Delta u+\left[\frac{\lambda}{2+\lambda}\mu_u+\frac{2}{2+\lambda}\mu_s\right]\langle L_z\rangle$$
$$\mu(\Lambda^0)=\tfrac{1}{6}(\Delta u+4\Delta d+\Delta s)(\mu_u+\mu_d)+\tfrac{1}{3}(2\Delta u-\Delta d+2\Delta s)\mu_s+\left[\frac{\lambda}{1+2\lambda}\mu_u+\frac{\lambda}{1+2\lambda}\mu_d+\frac{1}{1+2\lambda}\mu_s\right]\langle L_z\rangle$$

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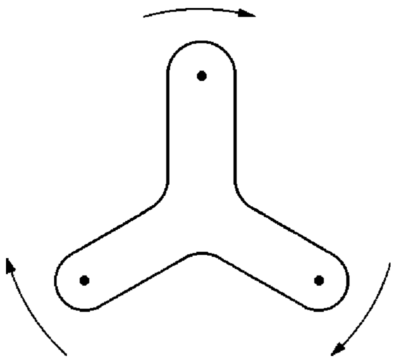

FIG. 1. Flux tube connecting three constituent quarks, rotating collectively around proton spin axis.

II. S_z AND L_z FROM STATIC PROPERTIES

The fit to the empirical values of the magnetic moments in [1] was carried out under the following constraints:

(i) The quark magnetic moments were assumed to satisfy $\mu_u = -2\mu_d$, $\mu_s = 0.6\mu_d$.

(ii) The quark spins Δu, Δd and Δs were constrained to satisfy the measured values of the axial vector couplings $a^{(3)}$ and $a^{(8)}$:

$$a^{(3)} = \Delta u - \Delta d = 1.26$$
$$a^{(8)} = \Delta u + \Delta d - 2\Delta s = 0.58. \qquad (4)$$

These conditions are equivalent to the statement $F = 0.46$, $D = 0.80$, in terms of which $a^{(3)} = F + D$ ($\equiv G_A$, the axial vector coupling constant of neutron decay) and $a^{(8)} = 3F - D$.

(iii) Each magnetic moment was assigned a theoretical uncertainty of $\pm 0.1\mu_N$ (as in Ref. [2]). This ensured that all of the baryons were given essentially the same weight in the fit and the χ^2 per degree of freedom was about unity.

In this manner, the magnetic moments are reduced to functions of three variables, which we choose to be μ_u, $\langle L_z\rangle$ and $\langle S_z\rangle$, the last being defined as

$$\langle S_z\rangle = \frac{1}{2}(\Delta u + \Delta d + \Delta s) \equiv \frac{1}{2}\Delta\Sigma. \qquad (5)$$

The result of the fit is

$$\mu_u = 2.16 \pm 0.08, \quad \langle S_z\rangle = 0.076 \pm 0.13, \quad \langle L_z\rangle = 0.42 \pm 0.10 \qquad (6)$$

with $\chi^2/\mathrm{DOF} = 1.1$. For the central value of μ_u, the allowed domain of $\langle S_z\rangle$ and $\langle L_z\rangle$ is given by the ellipse shown in Fig. 2. Allowing μ_u to vary over the interval given in Eq. (6), we obtain the domain shown in Fig. 3, from which we infer a final estimate

$$\langle S_z\rangle = 0.08 \pm 0.15, \quad \langle L_z\rangle = 0.42 \pm 0.14. \qquad (7)$$

It is remarkable that the domain of $\langle S_z\rangle$ and $\langle L_z\rangle$ determined by the static properties of the baryons satisfies rather closely the condition $\langle S_z\rangle + \langle L_z\rangle = \frac{1}{2}$. That is, the spin and orbital momenta of the quarks and antiquarks saturate the angular momentum of the proton, *without imposing this as an external requirement*. This may be regarded as *a posteriori* justification for the assumption $g_l = 1$. The fact that $\langle S_z\rangle + \langle L_z\rangle \approx \frac{1}{2}$ supports the idea, that the spin and orbital angular momentum are linked together by a transition of the form $q_+ \to q'_- + M$ ($L = 1$), M being a spin-zero meson [4]. This in turn provides support to the assumption $\delta q \approx \Delta q$, based on negligible antiquark polarization.

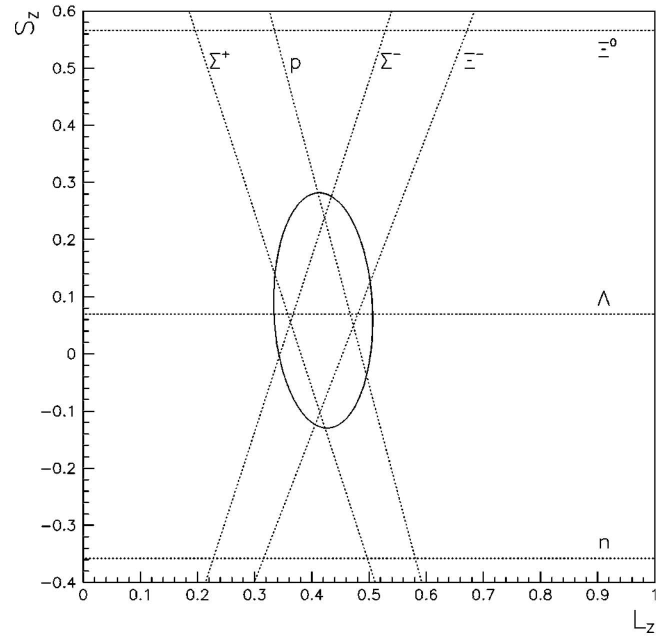

FIG. 2. The fitted domain of $\langle S_z\rangle$ and $\langle L_z\rangle$ for the central value of μ_u. The dotted lines represent the baryon magnetic moments of Table I.

Also indicated in Fig. 3 is the location of two ‘‘sign-

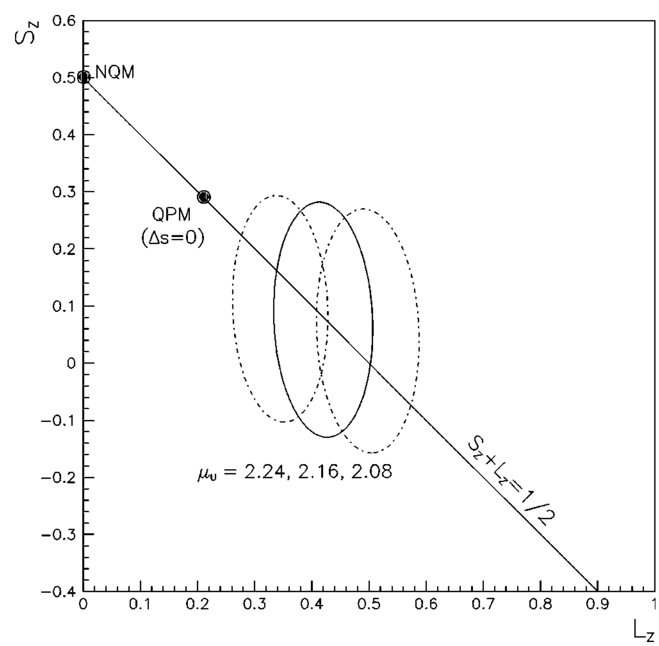

FIG. 3. Allowed domain of $\langle S_z\rangle$ and $\langle L_z\rangle$ for the full interval of $\mu_u = 2.16 \pm 0.08$.

posts,'' that serve as reference points in the angular momentum structure:

(i) NQM: This is the ''naive quark model,'' which describes the nucleon as 3 independent quarks in $1s$ orbits, corresponding to $\langle S_z\rangle=\frac{1}{2}$, $\langle L_z\rangle=0$. The SU(6) symmetry of the model leads to the prediction $\Delta u=\frac{4}{3}$, $\Delta d=-\frac{1}{3}$, $\Delta s=0$, axial vector couplings $a^{(3)}=\frac{5}{3}$, $a^{(8)}=1$, and the magnetic moment ratio $\mu_p/\mu_n=-\frac{3}{2}$.

(ii) QPM ($\Delta s=0$): This is the special case of the quark parton model discussed in Ref. [5], in which Δu and Δd were allowed to be free, but Δs was neglected. The characteristic prediction of this model is $a^{(8)}=a^{(0)}$, where $a^{(0)}=\Delta u+\Delta d+\Delta s$, implying $\langle S_z\rangle=\frac{1}{2}a^{(0)}=\frac{1}{2}(3F-D)=0.29$, the remaining angular momentum being attributed to $\langle L_z\rangle=\frac{1}{2}-\langle S_z\rangle=0.21$. This version of the QPM leads to the Ellis-Jaffe sum rules [6] for polarized structure functions:

$$\int g_1^p(x)dx=\frac{1}{2}\left(F-\frac{1}{9}D\right),\quad \int g_1^n(x)dx=\frac{1}{3}\left(F-\frac{2}{3}D\right). \tag{8}$$

In what follows, we consider a test for the presence of orbital angular momentum $\langle L_z\rangle$ and its specific association with the collective rotation of the constituent quarks.

III. TESTS FOR L_z IN MAGNETIC AND AXIAL VECTOR FORM FACTORS

We focus on the isovector magnetic moment of the nucleon, obtained by taking the difference of μ_p and μ_n in Eq. (1):

$$\mu_p-\mu_n=(\mu_u-\mu_d)\left[G_A+\frac{1}{3}\langle L_z\rangle\right]. \tag{9}$$

Note that the terms containing Δs cancel in the difference. We regard this equation as a decomposition of the isovector magnetic moment into a part depending on the axial vector charge and a part depending on orbital angular momentum. Introducing the abbreviation

$$f_{\rm spin}\equiv\frac{\mu_u-\mu_d}{\mu_p-\mu_n}G_A,\quad f_{\rm orb}\equiv\frac{\mu_u-\mu_d}{\mu_p-\mu_n}\frac{1}{3}\langle L_z\rangle, \tag{10}$$

Eq. (9) amounts to

$$1=f_{\rm spin}+f_{\rm orb}. \tag{11}$$

Returning to the three-parameter fit given by Eq. (6), we can regard the fitted parameters as being $\langle S_z\rangle$, $f_{\rm spin}$ and $f_{\rm orb}$ (in place of $\langle S_z\rangle$, μ_u and $\langle L_z\rangle$). For the central value of $\langle S_z\rangle$, the domain of $f_{\rm spin}$ and $f_{\rm orb}$ determined by the various magnetic moments is shown in Fig. 4. The fitted values, taking into account the spread of $\langle S_z\rangle$, are $f_{\rm spin}=0.87\pm0.03$, $f_{\rm orb}=0.096\pm0.03$. Considering that these values nearly satisfy the isovector magnetic moment relation, $f_{\rm spin}+f_{\rm orb}=1$, we use the following approximate values, which satisfy Eq. (11) exactly

$$f_{\rm spin}=0.90\pm0.03,\quad f_{\rm orb}=0.10\pm0.03. \tag{12}$$

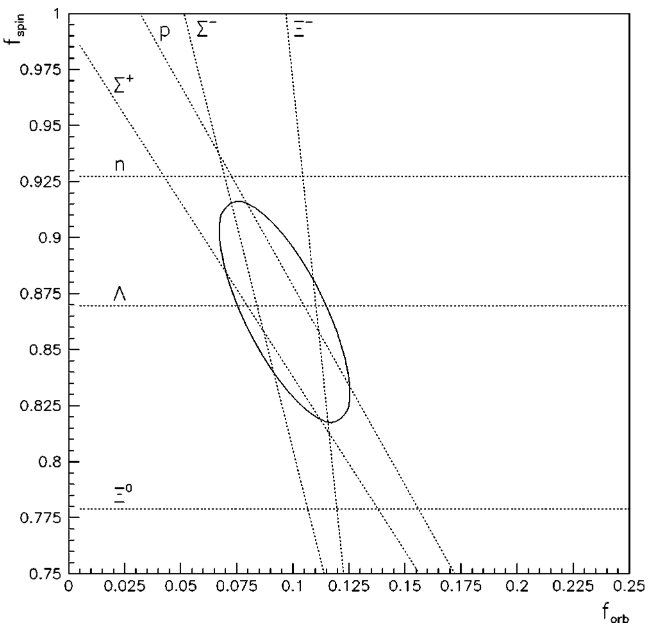

FIG. 4. The fitted domain of $f_{\rm spin}$ and $f_{\rm orb}$ for the central value of $\langle S_z\rangle$. The dotted lines represent the baryon magnetic moments of Table I.

Note that the ratio $f_{\rm orb}/f_{\rm spin}=\langle L_z\rangle/3G_A=\frac{1}{9}$ implies $\langle L_z\rangle=0.42$, as given in Eq. (6). Equation (12) amounts to the statement that the isovector magnetic moment is 90% due to quark spin polarization and 10% due to quark rotation.

We now define spatial distributions $\rho_{\rm mag}(r)$, $\rho_{\rm axial}(r)$ and $\rho_{\rm orb}(r)$ whose volume integrals yield the quantities $(\mu_p-\mu_n)$, G_A and $\langle L_z\rangle$ appearing in Eq. (9):

$$\mu_p-\mu_n=\int d^3x\,\rho_{\rm mag}(r),$$
$$G_A=\int d^3x\,\rho_{\rm axial}(r), \tag{13}$$
$$\langle L_z\rangle=\int d^3x\,\rho_{\rm orb}(r).$$

The local form of Eq. (9) then reads

$$\rho_{\rm mag}(r)=(\mu_u-\mu_d)\left[\rho_{\rm axial}(r)+\frac{1}{3}\rho_{\rm orb}(r)\right]. \tag{14}$$

Introducing, for convenience, ''normalized'' densities

$$\tilde\rho_{\rm mag}(r)\equiv\rho_{\rm mag}(r)/(\mu_p-\mu_n),$$
$$\tilde\rho_{\rm axial}(r)\equiv\rho_{\rm axial}(r)/G_A, \tag{15}$$
$$\tilde\rho_{\rm orb}(r)\equiv\rho_{\rm orb}(r)/\langle L_z\rangle,$$

Eq. (14) assumes the form

$$\tilde\rho_{\rm mag}(r)=f_{\rm spin}\tilde\rho_{\rm axial}(r)+f_{\rm orb}\tilde\rho_{\rm orb}(r). \tag{16}$$

The functions $\tilde{\rho}_{\mathrm{i}}(r)$ all satisfy $\int\tilde{\rho}_{\mathrm{i}}(r)d^3x=1$, so that the integrated form of Eq. (16) is simply the relation (11). Fourier transforming Eq. (16), we get a relation between the isovector magnetic, axial vector and ''orbital'' form factors of the nucleon:

$$H_{\mathrm{mag}}^{\mathrm{isovec}}(Q^2)=f_{\mathrm{spin}}H_{\mathrm{axial}}(Q^2)+f_{\mathrm{orb}}H_{\mathrm{orb}}(Q^2) \tag{17}$$

where

$$H_{\mathrm{i}}(Q^2=\vec{Q}^2)=\int \tilde{\rho}_{\mathrm{i}}(r)e^{i\vec{Q}\vec{x}}d^3x \tag{18}$$

with $H_{\mathrm{i}}(0)=1$.

The form factor $H_{\mathrm{mag}}^{\mathrm{isovec}}(Q^2)$ is an experimentally measured quantity, related to the magnetic (Sachs) form factors of the proton and the neutron by

$$H_{\mathrm{mag}}^{\mathrm{isovec}}(Q^2)=\frac{G_M^p(Q^2)-G_M^n(Q^2)}{\mu_p-\mu_n}. \tag{19}$$

To the extent that $G_M^p(Q^2)$ and $G_M^n(Q^2)$ are both proportional to $(1+Q^2/0.71\ \mathrm{GeV}^2)^{-2}$, we have

$$H_{\mathrm{mag}}^{\mathrm{isovec}}(Q^2)=\frac{1}{\left(1+\frac{Q^2}{M_V^2}\right)^2},\quad M_V=0.84\ \mathrm{GeV}. \tag{20}$$

The (normalized) axial vector form factor is likewise usually parametrized as a dipole

$$H_{\mathrm{axial}}(Q^2)=\frac{1}{\left(1+\frac{Q^2}{M_A^2}\right)^2}. \tag{21}$$

It is clear from Eq. (17) that the difference between $H_{\mathrm{mag}}^{\mathrm{isovec}}(Q^2)$ and $H_{\mathrm{axial}}(Q^2)$ is a measure of the orbital contribution proportional to f_{orb}: in the limit $f_{\mathrm{orb}}=0$, $f_{\mathrm{spin}}=1$, these two form factors would be identical and we would have $M_A\equiv M_V$.

The orbital form factor $H_{\mathrm{orb}}(Q^2)$ is a calculable feature of our model, which ascribes the orbital angular momentum to the rigid rotation of a flux tube. Assuming matter in the proton to be distributed as $\rho_{\mathrm{matt}}\propto e^{-r/a}$, the density of orbital angular momentum $\tilde{\rho}_{\mathrm{orb}}$ is proportional to $r^2e^{-r/a}$. The resulting orbital form factor is

$$H_{\mathrm{orb}}(Q^2=\vec{Q}^2)=\frac{\int e^{i\vec{Q}\vec{x}}r^2e^{-r/a}d^3x}{\int r^2e^{-r/a}d^3x}=\frac{1-Q^2a^2}{(1+Q^2a^2)^4}. \tag{22}$$

In particular, the rms radius associated with the orbital form factor is

$$\langle r^2\rangle_{\mathrm{orb}}=\frac{30}{a^2}. \tag{23}$$

This is to be compared with the rms radius of the matter distribution

$$\langle r^2\rangle_{\mathrm{matt}}=\frac{12}{a^2},\quad \text{i.e.,}\quad \langle r^2\rangle_{\mathrm{orb}}=\frac{5}{2}\langle r^2\rangle_{\mathrm{matt}}. \tag{24}$$

Equation (17) thus implies a relation between the mean square radii of the various form factors:

$$\begin{aligned}\langle r^2\rangle_{\mathrm{mag}}^{\mathrm{isovec}}&=f_{\mathrm{spin}}\langle r^2\rangle_{\mathrm{axial}}+f_{\mathrm{orb}}\langle r^2\rangle_{\mathrm{orb}}\\&=f_{\mathrm{spin}}\langle r^2\rangle_{\mathrm{axial}}+\frac{5}{2}f_{\mathrm{orb}}\langle r^2\rangle_{\mathrm{matt}}.\end{aligned} \tag{25}$$

To the extent that the matter radius of the proton is assumed to be the same as the magnetic radius, we have the prediction

$$\langle r^2\rangle_{\mathrm{axial}}=\frac{1-\frac{5}{2}f_{\mathrm{orb}}}{f_{\mathrm{spin}}}\langle r^2\rangle_{\mathrm{mag}}. \tag{26}$$

Using the values $f_{\mathrm{orb}}=0.10\pm0.03$, $f_{\mathrm{spin}}=0.90\pm0.03$ obtained from the fits to the magnetic moments, and the dipole parametrization given in Eqs. (20) and (21), the above relation yields

$$M_A=(1.10\pm0.07)M_V=0.92\pm0.06\ \mathrm{GeV} \tag{27}$$

in quite reasonable agreement with the value $M_A\approx1.0\ \mathrm{GeV}$ deduced from elastic neutrino-nucleon scattering [7]. It may be remarked here that measurements of elastic pp and $\bar{p}p$ scattering, when interpreted in a geometrical model [8] tend to give a matter radius slightly larger than the charge radius, namely $\sqrt{\langle r^2\rangle_{\mathrm{matt}}}\approx0.89$ fm, as compared to $\sqrt{\langle r^2\rangle_{\mathrm{charge}}}\approx0.84$ fm. If this difference is taken into account, the prediction for M_A obtained from Eq. (26) increases by about one percent.

Finally, we can also obtain from Eq. (17) a more detailed prediction for the shape of the axial vector factor $H_{\mathrm{axial}}(Q^2)$, in terms of the empirically known magnetic form factor $H_{\mathrm{mag}}^{\mathrm{isovec}}=(1+Q^2/0.71\ \mathrm{GeV}^2)^{-2}$ and the calculated orbital form factor $H_{\mathrm{orb}}(Q^2)$ given in Eq. (22). The result is plotted in Fig. 5, and is close to a dipole with $M_A\approx0.92$ GeV.

IV. CONCLUDING REMARKS

We presented in Ref. [1] a model of the proton as a collectively rotating system of quarks, with an orbital angular momentum determined by the baryon magnetic moments and the axial vector couplings to be $\langle L_z\rangle=0.42\pm0.14$. We have now shown that the same assumption of a rigidly rotating structure leads to a difference between the normalized axial vector and isovector magnetic form factors, which is dependent on the spatial distribution of orbital angular momentum. The model of rigid rotation leads to an axial vector form

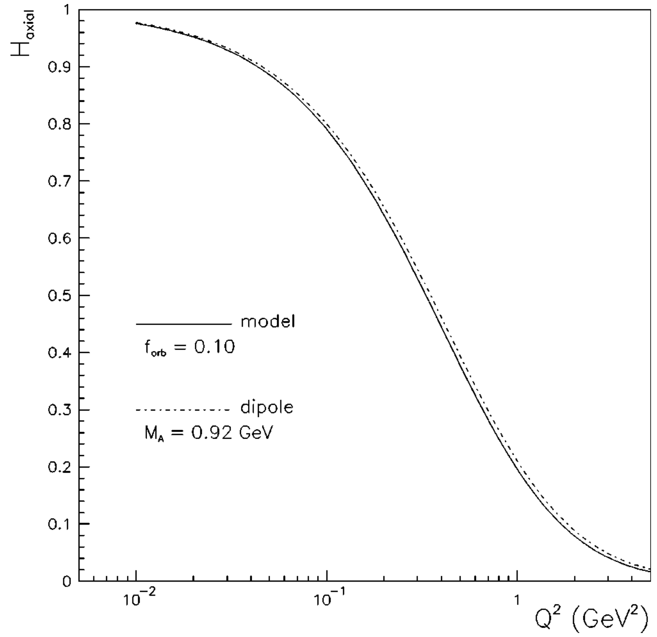

FIG. 5. Predicted shape of axial vector form factor with $f_{\rm spin}=0.90$, $f_{\rm orb}=0.10$, compared to a dipole with $M_A=0.92$ GeV.

factor which is close to a dipole with $M_A=0.92\pm0.06$ GeV. Our model of a rotating matter distribution has some similarity to that discussed by Chou and Yang [9], who proposed a test for the velocity profile of a polarized proton in hadronic interactions.

It is of interest to ask how our results for $\langle S_z\rangle$ and $\langle L_z\rangle$, namely

$$\langle S_z\rangle=0.08\pm0.15,\quad \langle L_z\rangle=0.42\pm0.14, \tag{28}$$

compare with those obtained from other considerations. Our fit indicates a dominance of orbital over spin angular momentum. This feature is opposite to that in the nonrelativistic quark model,

$$\langle S_z\rangle=\frac{1}{2},\quad \langle L_z\rangle=0\ \ (\text{NQM}), \tag{29}$$

and closer to the soliton picture of the proton represented by the Skyrme model [10]

$$\langle S_z\rangle=0,\quad \langle L_z\rangle=\frac{1}{2}\ \ (\text{Skyrme}). \tag{30}$$

An interesting version of the soliton model, that interpolates between the NQM and Skyrme limits, is the chiral quark soliton picture [11], which predicts

$$\langle S_z\rangle=\frac{9}{2}\,\frac{G_A}{1+F/D}\left[\frac{F}{D}-\frac{5}{9}\right]\quad (\chi\,\text{QSM}). \tag{31}$$

In the limit $F/D=5/9$, which is the Skyrme model value, one has $\langle S_z\rangle=0$, while in the NQM limit $G_A=\frac{5}{3}$, $F/D=\frac{2}{3}$ one has $\langle S_z\rangle=\frac{1}{2}$. For the measured values $F=0.46$, $D=0.80$, this model yields $\langle S_z\rangle=0.07$, which is very close to the estimate in Eq. (28).

Information about $\langle S_z\rangle$ has also been derived from the analysis of structure functions $g_1^{p,n}$ measured in polarized deep inelastic scattering [12,13]. The integrals of these structure functions can be written as

$$\int g_1^{p,n}(x,Q^2)dx=\frac{C_1^{NS}(Q^2)}{12}\left[\pm a^{(3)}+\frac{1}{3}a^{(8)}\right]+\frac{C_1^S(Q^2)}{9}a^{(0)}(Q^2) \tag{32}$$

where C_1^{NS} and C_1^S are perturbatively calculable coefficients. The singlet axial coupling $a^{(0)}(Q^2)$ differs from $\Delta\Sigma=\Delta u+\Delta d+\Delta s$ as a consequence of the gluon anomaly. In the Adler-Bardeen factorization scheme, $a^{(0)}(Q^2)$ is related to $\Delta\Sigma$ by

$$a^{(0)}(Q^2)=\Delta\Sigma-n_f\frac{\alpha_s(Q^2)}{2\pi}\Delta G(Q^2) \tag{33}$$

where $\Delta G(Q^2)$ is the net polarization of gluons in a polarized nucleon. A determination of $\Delta\Sigma$ from the measured quantity $a^{(0)}(Q^2)$ is only possible by invoking a model for the polarized gluon density, and fitting it to the observed Q^2 dependence of the structure functions. The result of one such fit [14] is

$$\langle S_z\rangle=\frac{1}{2}\Delta\Sigma=0.22\pm0.045\ \ (\text{polarized structure functions}). \tag{34}$$

Other analyses [13,12,15] obtain values of $\langle S_z\rangle$ between 0.1 and 0.3. Within errors, the result for $\langle S_z\rangle$ obtained from high energy experiments is compatible with the result (28) obtained from a fit to the static properties.

It remains to be seen whether a specific test of rotational angular momentum $\langle L_z\rangle$ and its radial distribution can be devised. We have argued that the difference in shapes of the axial vector and isovector magnetic form factors is a probe of orbital angular momentum. A precise determination of $F_A(Q^2)$, which does not presume a dipole behavior from the outset, would be of great interest in this respect.

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