Equivalence of renormalized covariant and light-front perturbation theory. II. Transverse divergences in the Yukawa model

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Light-front dynamics can only become a viable alternative to the covariant approach if doubts about its covariance can be taken away. As a minimal requirement we take that the physical quantities calculated with light-front perturbation theory are the same as those obtained using covariant perturbation theory. If this situation occurs, we use the word equivalent to characterize it. For quantities that involve the calculation of superficially convergent diagrams, proofs of equivalence exist. For some types of divergent diagrams the proof of equivalence is complicated. Here we deal with diagrams with transverse divergences. Our method is based on minus regularization, which is inspired on BPHZ regularization. In a calculation using numerical methods we show how to obtain a rotationally invariant amplitude for two triangle diagrams contributing to the decay of a scalar boson in the Yukawa model. It concludes our proof of equivalence of covariant and light-front perturbation theory. [S0556-2821(98)02014-1]

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I. INTRODUCTION

Covariant field theory is the formalism of choice to describe situations where creation and annihilation of particles are important and where typical velocities are comparable to the velocity of light. If the interactions are sufficiently weak, perturbation theory is usually applied and gives in many cases extremely accurate answers. However, in the case of strong interactions, or when bound states are considered, nonperturbative methods must be developed. Light-front quantization [1] is a Hamiltonian method in which the light-like variable $x^+=(x^0+x^3)/\sqrt{2}$ plays the role of time, and is therefore referred to as light-front time. This method has found many applications since it was conceived. Still, some problems of a fundamental nature remained. One that we are particularly interested in is the question of whether full covariance can be maintained in the Hamiltonian formulation, which is of course not manifestly covariant. A partial answer can be obtained in perturbation theory. Then the problem can be reformulated as follows: can one prove that light-front perturbation theory produces the same values of the S-matrix elements as covariant perturbation theory? If the answer to this question is affirmative, then we use the word equivalent to describe the situation.

The present paper is concerned with one aspect of this problem, viz. the treatment of transverse divergences in a simple model: the Yukawa model with spin-1/2 fermions, spin-0 bosons and a scalar coupling.

A. k^--integration and equivalence

In the work we did before, we used the method of Kogut and Soper [2] to define light-front perturbation theory. This method defines light-front time-ordered (x^+-ordered) amplitudes by integration of the integrand of a covariant diagram, say

$$F(q)=\int \mathrm{d}^4k I(q;k), \tag{1}$$

over the light-front energy variable $k^-=(k^0-k^3)/\sqrt{2}$. In this paper, q always denotes the external momenta and k the loop momentum. We can also write Eq. (1) using light-front coordinates:

$$F(q)=\int \mathrm{d}k^+\mathrm{d}^2k^\perp\int \mathrm{d}k^- I(q^-,q^+,q^\perp;k^-,k^+,k^\perp). \tag{2}$$

Next, one expresses the integral over k^-, using Cauchy's formula, as a sum of residues. One arrives in this way at an expression that can be interpreted, possibly after recombination of the terms in this sum, as the splitting of the covariant amplitude $F(q)$ into a sum of noncovariant but light-front time-ordered amplitudes.

This procedure, sometimes called naive light-cone quantization, has been in principle known since the early work of Kogut and Soper [2]. For convergent diagrams, it is nicely pictured in Fig. 1.

The covariant diagram in Fig. 1 is an ill-defined object and needs some prescription to give it a definite meaning. For example, the measure of the Minkowskian integration is not positive definite. The covariant prescription involves the introduction of Feynman parameters to complete the squares in the denominator, the removal of terms odd in the loop momentum k and Wick rotation to obtain a Euclidian integral.

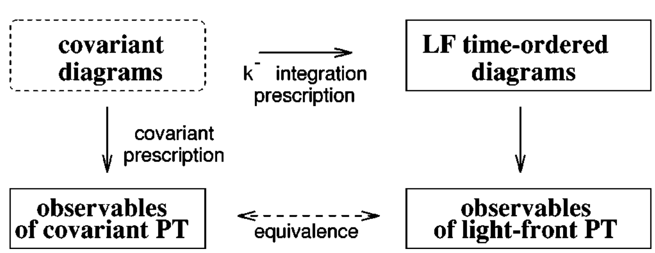


FIG. 1. The ''ideal'' case: Outline of our proof of equivalence of light-front (LF) and covariant perturbation theory (PT) for convergent diagrams. The dashed box indicates an ill-defined object.

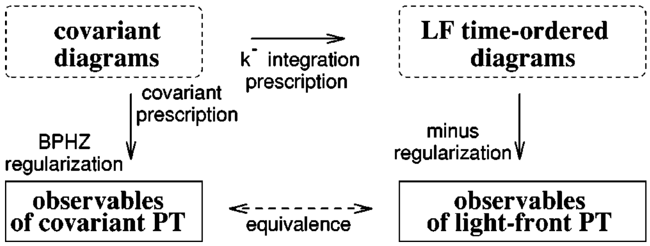


FIG. 2. Outline of our proof of equivalence for diagrams with ultraviolet divergences. Dashed boxes indicate ambiguously defined objects.

It has been the work of Ligterink and Bakker [3] that proves in detail that the rules for constructing light-front time-ordered diagrams, explained in many articles [2,4], are correct upon using the k^--integration prescription. They were the first authors to give a systematic derivation of all the different time-ordered diagrams corresponding to a given covariant amplitude, for any number of particles involved. If the k^--integral is convergent and the corresponding covariant diagram is also superficially convergent, then what remains can be written in terms of well-defined, convergent Euclidian integrals.

When the k^--integration is divergent, the prescription must be altered. Naive light-front quantization fails in this case and one must first find a way to regulate the k^--integrals. We proposed in a previous paper [5] a regularization that maintains covariance. There we showed that the longitudinal divergences give rise to so-called forced instantaneous loops (FILs) and we showed how to deal with them such that covariance is maintained. This method was also applied to the Yukawa model containing spin-1/2 and spin-0 particles. We were able to regularize the k^--integrals for the diagrams with one loop. However, in order to show full equivalence to the covariant calculation one needs to compute the full integral including the integrations over k^+ and $k^\perp$.

B. Ultraviolet and transverse divergences

Even after the usual procedure has been followed, the covariant integral can still be ultraviolet divergent. Ligterink and Bakker did not only discuss diagrams that are superficially convergent, but also what to do in cases where the covariant diagram is divergent. Their method of regularizing divergent diagrams, minus regularization [6], is also used in the present paper. A scheme for the equivalence of ultraviolet divergent diagrams is given in Fig. 2.

Several techniques are available to remove the ultraviolet divergences, not involving the k^-- integration. They remain in the light-front time-ordered diagrams as divergences of the integrals over the transverse momenta. Therefore these diagrams are also ill-defined, as indicated by the dashed box in Fig. 2. A problem is that many of the techniques which are used to regularize covariant diagrams have limited use for light-front time-ordered diagrams. For example, one cannot use dimensional regularization for the longitudinal divergences. Still, it is common to apply it to the transverse divergences. The strength of the regularization scheme we use, minus regularization, is that it does not discriminate between transverse and longitudinal divergences. Minus regularization is based on the Bogoliubov-Parasiuk-Hepp-Zimmerman (BPHZ) method of regularization [7–11]. In their paper, Ligterink and Bakker applied minus regularization to three self-energy diagrams. Our contribution is to extend their method to more complicated diagrams and prove that there is a one-to-one relation between minus and BPHZ regularization, such that the physical observables found using light-front perturbation theory exactly match those found in covariant perturbation theory.

In the Yukawa model there are five covariant diagrams with ultraviolet divergences. The boson and the fermion self-energy were discussed in our previous article on longitudinal divergences. Minus regularization was applied and simultaneously removed the longitudinal and the transverse divergences. Equivalence was established.

In two cases we were not able to either find an answer in the literature or produce ourselves full analytic results for the integrals involved; so we had to resort to numerical integration. In this paper we discuss these two diagrams: the one-boson exchange correction to the boson-fermion-fermion vertex and the fermion loop with three external boson lines. The first one was considered by Burkardt and Langnau [12], who stated that naive light-cone quantization leads to a violation of rotational invariance of the corresponding S-matrix elements and found that invariant results can be obtained using noncovariant counterterms. Here we show that no violation of rotational invariance occurs if our method of regularization is applied. Furthermore, our results for the light-front time-ordered diagrams sum up to the covariant amplitude, calculated using conventional methods.

C. Light-front structure functions

The two triangle diagrams can be written in the form of a sum of tensors in the external momenta, multiplied by scalar functions, which we call (covariant) structure functions. After splitting a covariant diagram in light-front time-ordered ones, these can be written again in terms of tensors multiplied by functions of the external momenta. The latter are called light-front structure functions. They are not invariant as they are not defined by four-dimensional invariant integrals, but rather by three-dimensional integrals. The different structure functions have different divergences and they must be treated according to their types of divergence, which we enumerate.

(1) *Light-front structure functions without transverse divergences*. Neither the covariant nor the light-front formulation contains any divergences. Integration over k^- suffices to prove equivalence. Minus regularization is not allowed.

(2) *Light-front structure functions with cancelling transverse divergences.* The individual light-front time-ordered diagrams contain divergences not present in the covariant amplitude. Application of minus regularization to the time-ordered diagrams is not allowed. We show that the divergences cancel if all the time-ordered diagrams are added, and that their sum equals the corresponding covariant amplitude.

(3) *Light-front structure functions with overall transverse divergences*. Divergences appear in the covariant amplitude as well as in the light-front time-ordered diagrams. We apply BPHZ regularization to the covariant amplitude and minus regularization to the time-ordered diagrams.

For the first two cases one can prove equivalence using analytic methods alone. The proof of equivalence can be found in Refs. [3,5]. For the structure functions with overall transverse divergences we have to use numerical techniques. We show that for the decay of a boson at rest, for both triangle diagrams, one obtains a rotational invariant amplitude, identical to the covariant calculation using BPHZ regularization. The fifth diagram with transverse divergences, the fermion box, will not be discussed.

D. Outline

The setup for this article is as follows. In Sec. II we introduce minus regularization. In Sec. III and Sec. IV we discuss the equivalence of covariant and light-front perturbation theory for the fermion triangle and the one-boson exchange correction. In both cases we start with the covariant calculation and do the BPHZ regularization if necessary. Then we calculate the light-front time-ordered diagrams and apply the method mentioned above. In both cases, we conclude by giving a numerical example of rotational invariance.

II. MINUS REGULARIZATION

Minus regularization is inspired by the BPHZ method of regularization, which gives finite and covariant results. By construction, we ensure that minus regularization does the same. First we sketch the method in the case of one-loop diagrams with one independent external momentum (self-energies), and next when two independent external momenta (triangle diagrams) are present. We conclude by generalizing this to a one-loop diagram with n external momenta. For convenience, we shall assume in the latter case that only logarithmic and linear divergences are present, such that only the first term of the Taylor expansion around the renormalization point needs to be subtracted.

Wherever we use the word ‘‘amplitude’’ in this section, we refer to an invariant function of the external momenta. It is understood that the integrals defining the invariant functions are formally written down in terms of four-dimensional integrals, which are split into time-ordered pieces by integration over k^-.

A. One external momentum

First we discuss the simple case of one external momentum, which can be applied for self-energy diagrams.

1. BPHZ regularization

We start with the BPHZ regularization method, which can be applied to covariant diagrams. The amplitude has the following form:

$$F(q^2)=\int d^4k I_{\rm cov}(q^2;k)$$
$$=F(0)+q^2F'(0)+\ldots \tag{3}$$

where $I_{\rm cov}(q^2,k)$ is the covariant integrand generated by applying standard Feynman rules. BPHZ regularization renders the amplitude finite by subtracting the infinite parts. We choose the point $q^2=0$ as the renormalization point, around which we expand the amplitude in a Taylor series. The higher orders in the expansion (3) are denoted by the ellipsis. The regularized amplitude is then

$$F^{\rm R}(q^2)=F(q^2)-F(0). \tag{4}$$

However, this is a purely formal operation, since we are subtracting two infinite quantities. It is better to write

$$F^{\rm R}(q^2)=\int d^4k[I_{\rm cov}(q^2;k)-I_{\rm cov}(0;k)]$$
$$=\int_0^{q^2}dq'^2\int d^4k\frac{\partial}{\partial q'^2}I_{\rm cov}(q'^2;k). \tag{5}$$

This guarantees that the amplitude becomes finite.

2. Minus regularization

Typical for minus regularization is that one writes the amplitude, as well as the renormalization point, in light-front coordinates. The covariant choice $q^2=0$ corresponds to $q^-=q^{\perp 2}/(2q^+)$. A time-ordered amplitude corresponding to the covariant form (3) can be written in light-front coordinates as follows:

$$F(q^-,q^+,q^\perp)=\int d^3k I_{\rm lfto}(q^-,q^+,q^\perp;k)$$
$$=F\left(\frac{q^{\perp 2}}{2q^+},q^+,q^\perp\right)$$
$$+2q^+\left(q^- -\frac{q^{\perp 2}}{2q^+}\right)F'\left(\frac{q^{\perp 2}}{2q^+},q^+,q^\perp\right)+\cdots \tag{6}$$

where $I_{\rm lfto}$ is the integrand of the light-front time-ordered diagram, which was generated by integrating the covariant integrand $I_{\rm cov}$ over k^- as is explained in Ref. [3]. The prime denotes differentiation with respect to q^-. Similar to Eq. (5) we can write the regularized amplitude as

$$F^{\rm MR}(q^-,q^+,q^\perp)$$
$$=\int_{q^{\perp 2}/2q^+}^{q^-}dq'^-\int d^3k\frac{\partial}{\partial q'^-}I_{\rm lfto}(q'^-,q^+,q^\perp;k). \tag{7}$$

So far we have described the minus regularization method introduced by Ligterink and Bakker [6].

B. Two external momenta

In Ref. [6] three self-energy diagrams were discussed. For the triangle diagram the minus regularization technique needs to be extended.[1] We will tune the technique by comparing it to BHPZ regularization.

1. BPHZ regularization

The amplitude has the following covariant form:

$$F(q_1^2,q_2^2,q_1\cdot q_2)=\int d^4k I_{\rm cov}(q_1^2,q_2^2,q_1\cdot q_2;k)$$
$$=F(\tilde{0})+q_1^2F_1'(\tilde{0})+q_2^2F_2'(\tilde{0})$$
$$+q_1\cdot q_2F_3'(\tilde{0})+\cdots \tag{8}$$

where $\tilde{0}$ is the renormalization point $q_1^2=q_2^2=q_1\cdot q_2=0$ and F_i' is the derivative of F with respect to the ith argument.

$$F^{\rm R}(q_1^2,q_2^2,q_1\cdot q_2)=F(q_1^2,q_2^2,q_1\cdot q_2)-F(\tilde{0}). \tag{9}$$

Again, this is a purely formal operation, since we are subtracting two infinite quantities. We write

$$F^{\rm R}(q_1^2,q_2^2,q_1\cdot q_2)=\int d^4k[I_{\rm cov}(q_1^2,q_2^2,q_1\cdot q_2;k)$$
$$-I_{\rm cov}(\tilde{0};k)]. \tag{10}$$

We cannot, as in the previous section, differentiate with respect to all external momenta. We would then subtract finite parts from the Taylor series, containing physical information. This can be circumvented by introducing a dummy variable λ, which parametrizes a straight line in the space of the invariants between the actual external momenta $q_1^2,q_2^2,q_1\cdot q_2$ and the renormalization point:

$$F^{\rm R}(q_1^2,q_2^2,q_1\cdot q_2)$$
$$=\int_0^1 d\lambda\int d^4k\,\frac{\partial}{\partial\lambda}I_{\rm cov}(\lambda q_1^2,\lambda q_2^2,\lambda q_1\cdot q_2;k). \tag{11}$$

We have verified that the λ-method gives the correct result for the case where one independent external momentum occurs.

2. Minus regularization

Again, we write the amplitude in the light-front time-ordered case as a three-dimensional integral:

TABLE I. The light-front parametrization of the renormalization point r^μ for two equivalent choices of minus regularization, MR0 and MR1.

	MR0	MR1
$(r_1^-,r_1^+,r_1^\perp)$	$(0,q_1^+,0^\perp)$	$(q^{\perp 2}_1/(2q_1^+),q_1^+,q_1^\perp)$
$(r_2^-,r_2^+,r_2^\perp)$	$\chi(0,q_1^+,0^\perp)$	$\chi(q^{\perp 2}_1/(2q_1^+),q_1^+,q_1^\perp)$

$$F(q_i^-,q_i^+,q_i^\perp)=\int d^3k I_{\rm lfto}(q_i^-,q_i^+,q_i^\perp;k). \tag{12}$$

The regularized amplitude is

$$F^{\rm R}(q_i^-,q_i^+,q_i^\perp)=F(q_i^-,q_i^+,q_i^\perp)-F(r_i^-,r_i^+,r_i^\perp), \tag{13}$$

where r defines the renormalization surface. It is a hypersurface determined by the following conditions:

$$r_1^2=2r_1^-r_1^+-r_1^{\perp 2}=0,$$
$$r_2^2=2r_2^-r_2^+-r_2^{\perp 2}=0, \tag{14}$$
$$r_1\cdot r_2=r_1^-r_2^++r_1^+r_2^--r_1^\perp\cdot r_2^\perp=0.$$

This set of equations is equivalent to

$$r_1^2=0,\quad r_2=\chi r_1. \tag{15}$$

The r_i^+ enter in the integration boundaries; therefore we would like them to remain unaffected by regularization ($r_i^+=q_i^+$). This implies that χ can be found from

$$\chi=\frac{q_2^+}{q_1^+}. \tag{16}$$

The only freedom that remains is the choice for $r_1^\perp$. Two choices come easily to mind: $r_1^\perp=0$ (method MR0) and $r_1^\perp=q_1^\perp$ (method MR1):

$$({\rm MR0})\quad r_1^\perp=0^\perp\Rightarrow r_2^\perp=0^\perp, \tag{17}$$

$$({\rm MR1})\quad r_1^\perp=q_1^\perp\Rightarrow r_2^\perp=\chi q_1^\perp. \tag{18}$$

The details are worked out in Table I.

The light-front coordinates of the renormalization point are used in the following way to find the regularized light-front amplitude:

$$F^{\rm MR}(q_i^-,q_i^+,q_i^\perp)=\int_0^1 d\lambda\int d^3k\frac{\partial}{\partial\lambda}I_{\rm lfto}(\lambda(q_i^--r_i^-)+r_i^-,q_i^+,\lambda(q_i^\perp-r_i^\perp)+r_i^\perp;k). \tag{19}$$

[1] We suggest the name MR$^+$.

In this formula we recognize our choice $r_i^+ = q_i^+$.

C. Several external momenta

The method just described can be generalized to the case of a loop with an arbitrary number of external lines. The procedure is almost the same as for two external momenta. The renormalization surface is given by

$$r_i^2 = 2r_i^- r_i^+ - r_i^{\perp 2} = 0, \tag{20}$$

$$r_i \cdot r_j = r_i^- r_j^+ + r_i^+ r_j^- - r_i^\perp \cdot r_j^\perp = 0 \quad (i \neq j). \tag{21}$$

These equations are equivalent to

$$r_1^2 = 0, \quad r_i = \chi_i r_1. \tag{22}$$

Again, we make the choice to leave the plus components of the momenta unaffected by regularization: $r_i^+ = q_i^+$. This implies that the χ_i are fractional longitudinal light-front momenta:

$$\chi_i = \frac{q_i^+}{q_1^+}. \tag{23}$$

Two choices for $r_1^\perp$ are listed below. This then determines all other $r_i^\perp$:

$$(\mathrm{MR0}) \quad r_1^\perp = 0^\perp \Rightarrow r_i^\perp = 0^\perp, \tag{24}$$

$$(\mathrm{MR1}) \quad r_1^\perp = q_1^\perp \Rightarrow r_i^\perp = \chi_i q_1^\perp. \tag{25}$$

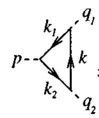
D. Summary

The way we set up minus regularization does not rely on the structure of the covariant or the time-ordered diagrams, but works on the level of the external momenta only. If an amplitude has a covariant structure before regularization, minus regularization guarantees that it remains covariant. In our implementation of BPHZ regularization, the renormalization point corresponds to all invariants connected to the external momenta being equal to zero. These conditions allow minus regularization to take on a number of forms. Of these, we shall apply MR0 and MR1. The main difference between them is that MR0 does not choose one of the momenta as a preferred direction, and therefore it explicitly maintains all symmetries of the external momenta. Furthermore, MR0 gives rise to shorter formulas for the regularized integrands.

In the next two sections both methods are being applied to the parts of two light-front time-ordered triangle diagrams in the Yukawa model containing transverse divergences, viz. the fermion triangle and the one-boson exchange correction.

III. EQUIVALENCE FOR THE FERMION TRIANGLE

In the Yukawa model there is an effective three boson interaction, because for a fermion loop with a scalar coupling Furry's theorem does not apply. The leading order contribution to this process is the fermion triangle. A scalar boson of mass μ and momentum p comes in and decays into two bosons of momentum q_1 and q_2 respectively. The fermions in the triangle have mass m. The covariant expression for the amplitude is



$$= \int_{\mathrm{Min}} \frac{\mathrm{d}^4k \ \mathrm{Tr}\,[(\not{k}_1 + m)(\not{k}_2 + m)(\not{k} + m)]}{(k_1^2 - m^2)(k_2^2 - m^2)(k^2 - m^2)}. \tag{26}$$

The subscript "Min" denotes that the integration is over Minkowski space. The usual imaginary parts of the Feynman propagators have been dropped. We have omitted numerical factors and have set the coupling constant to unity. The momenta k_1 and k_2 indicated in the diagram are given by

$$k_1 = k - q_1, \quad k_2 = k + q_2. \tag{27}$$

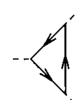
Of course, by momentum conservation we have

$$p = q_1 + q_2. \tag{28}$$

We evaluate the integral (26) first in the usual covariant way, and subsequently carry out k^--integration to produce the light-front time-ordered diagrams. Note that integral (26) is an ill-defined formula. In both methods mentioned we have to define what we mean by this integral.

A. Covariant calculation

The following method is usually applied to calculate the fermion triangle in a covariant way. First, one introduces Feynman parameters x_1 and x_2, and then one shifts the loop variable k to complete the squares in the denominator. The result is



$$= 8 \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \int_{\mathrm{Min}} \mathrm{d}^4k \times \frac{m^3 + m\,(3k^2 + \mathcal{P}^2) + \text{terms odd in } k}{(k^2 - m^2 + \mathcal{Q}^2)^3}, \tag{29}$$

with

$$\mathcal{Q}^2 = x_1(1-x_1)q_1^2 + x_2(1-x_2)q_2^2 + 2x_1x_2 q_1 \cdot q_2, \tag{30}$$

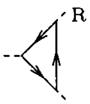
$$\mathcal{P}^2 = x_1(3x_1-2)q_1^2 + x_2(3x_2-2)q_2^2 + (2(x_1+x_2) - 6x_1x_2 - 1)q_1 \cdot q_2. \tag{31}$$

As a last step, we remove the terms odd in k.

B. BPHZ regularization

The regularized fermion triangle can be found by applying the BPHZ regularization scheme (11) to the covariant formula (29). The integral is now finite; so we can do the

Wick rotation and perform the k integrations. The result is



$$^{\mathrm{R}} = -4\pi^2 i \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \int_0^1 \mathrm{d}\lambda \times \frac{m\left(m^2(5\mathcal{Q}^2-\mathcal{P}^2)-6\lambda\mathcal{Q}^4\right)}{\left(m^2-\lambda\mathcal{Q}^2\right)^2}. \quad (32)$$

The superscript R indicates an integral regularized according to the BPHZ method.

C. Light-front calculation

Using the method given in Ref. [3] we proceed as follows. The k^- dependence of a spin projection in the numerator is removed by separating it into an on-shell spin projection and an instantaneous part:

$$\not k_i + m = (\not k_{i\,\mathrm{on}} + m) + (k^- - k^-_{i\mathrm{on}})\gamma^+, \quad (33)$$

where the vector $k^\mu_{i\,\mathrm{on}}$ is given by

$$(k_i^-, k_i^+, k_i^\perp)_{\mathrm{on}} = \left(\frac{k_i^{\perp 2}+m^2}{2k_i^+}, k_i^+, k_i^\perp \right). \quad (34)$$

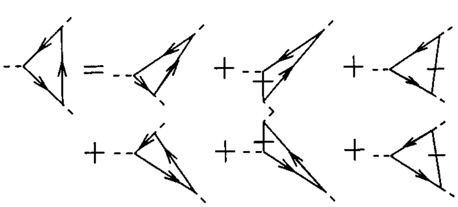
Factors like $(k^- - k^-_{i\,\mathrm{on}})$ can be divided out against propagators and this cancellation gives rise to instantaneous fermions. The integration over k^- is performed by contour integration. The poles of the propagators are given by

$$H^- = \frac{k^{\perp 2}+m^2}{2k^+}, \quad (35)$$

$$H_1^- = q_1^- - \frac{k_1^{\perp 2}+m^2}{2k_1^+}, \quad (36)$$

$$H_2^- = -q_2^- + \frac{k_2^{\perp 2}+m^2}{2k_2^+}. \quad (37)$$

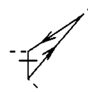
This integration gives rise to the different time-ordered diagrams, as explained in more detail in [3,5]. The result is



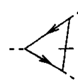
(38)

The diagrams on the right-hand side are light-front time-ordered diagrams. Time goes from left to right. The pictures can be recognized as time-ordered diagrams because of the time-ordering of the vertices and the occurrence of instantaneous fermions, indicated by a horizontal tag. Explicitly,

$$= 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \times \frac{\mathrm{Tr}\left[(\not k_{1\mathrm{on}}+m)(\not k_{2\mathrm{on}}+m)(\not k_{\mathrm{on}}+m)\right]}{(H_1^- - H_2^-)(H_1^- - H^-)}, \quad (39)$$



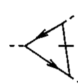
$$= 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \times \frac{\mathrm{Tr}\left[(\not k_{1\mathrm{on}}+m)\gamma^+(\not k_{\mathrm{on}}+m)\right]}{H_1^- - H^-}, \quad (40)$$



$$= 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \times \frac{\mathrm{Tr}\left[(\not k_{1\mathrm{on}}+m)(\not k_{2\mathrm{on}}+m)\gamma^+\right]}{H_1^- - H_2^-}, \quad (41)$$

$$= -2\pi i \int \mathrm{d}^2 k^\perp \int_{-q_2^+}^{0} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \times \frac{\mathrm{Tr}\left[(\not k_{1\mathrm{on}}+m)(\not k_{2\mathrm{on}}+m)(\not k_{\mathrm{on}}+m)\right]}{(H_1^- - H_2^-)(H^- - H_2^-)}, \quad (42)$$

$$= -2\pi i \int \mathrm{d}^2 k^\perp \int_{-q_2^+}^{0} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \times \frac{\mathrm{Tr}\left[\gamma^+(\not k_{2\mathrm{on}}+m)(\not k_{\mathrm{on}}+m)\right]}{H^- - H_2^-}, \quad (43)$$



$$= -2\pi i \int \mathrm{d}^2 k^\perp \int_{-q_2^+}^{0} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \times \frac{\mathrm{Tr}\left[(\not k_{1\mathrm{on}}+m)(\not k_{2\mathrm{on}}+m)\gamma^+\right]}{H_1^- - H_2^-}. \quad (44)$$

Note that the diagrams (41) and (44) with the instantaneous exchanged fermions have the same integrand. However, the longitudinal momentum k^+ has a different sign.

Although we could have expected diagrams with two instantaneous fermions, we see that they are not present. This is so because we use a scalar coupling and therefore two γ^+ matrices becoming neighbors give 0. No so-called forced instantaneous loops are present. These FILs obscure the equivalence of light-front and covariant perturbation theory and have been analyzed in Ref. [5]. They will not be discussed in this paper, since they are related to longitudinal divergences.

The traces can be calculated. We obtain

$$\mathrm{Tr}[(\not k_{1\mathrm{on}}+m)(\not k_{2\mathrm{on}}+m)(\not k_{\mathrm{on}}+m)] = 4m(m^2 + k_{1\mathrm{on}}\cdot k_{\mathrm{on}} + k_{2\mathrm{on}}\cdot k_{\mathrm{on}} + k_{1\mathrm{on}}\cdot k_{2\mathrm{on}}), \quad (45)$$

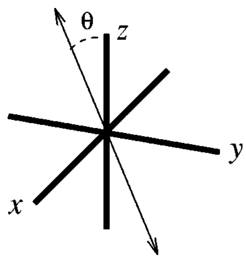


FIG. 3. A boson is at rest and decays into two particles flying off in opposite directions. The angle θ is the angle between the momentum of one of the fermions and the z-axis.

$$\mathrm{Tr}[(\not{k}_{1\mathrm{on}}+m)(\not{k}_{2\mathrm{on}}+m)\gamma^{+}]=4m(2k^{+}-q_1^{+}+q_2^{+}), \tag{46}$$

$$\mathrm{Tr}[(\not{k}_{1\mathrm{on}}+m)\gamma^{+}(\not{k}_{\mathrm{on}}+m)]=4m(2k^{+}-q_1^{+}), \tag{47}$$

$$\mathrm{Tr}[\gamma^{+}(\not{k}_{2\mathrm{on}}+m)(\not{k}_{\mathrm{on}}+m)]=4m(2k^{+}+q_2^{+}). \tag{48}$$

We see that the high orders in $k^{\perp}$ have disappeared in the traces. However, logarithmic divergences remain in all light-front time-ordered diagrams (39)–(44). We tackle them with minus regularization, as introduced in the previous subsection.

D. Equivalence

As the fermion triangle is a scalar amplitude, there is only one structure function present. It belongs to the first category we mentioned in the Introduction: it is logarithmically divergent, but has no longitudinal divergences.

We applied minus regularization to the integrands of the six light-front time-ordered diagrams, using both the MR0 and MR1 methods. We used MATHEMATICA to do the substitution and the differentiation with respect to λ, given by Eq. (19). However, MATHEMATICA was not able to do the integration, neither analytically nor numerically. Therefore the integrand was implemented in FORTRAN which was well capable of doing the four-dimensional integration using IMSL routines based on Gaussian integration.

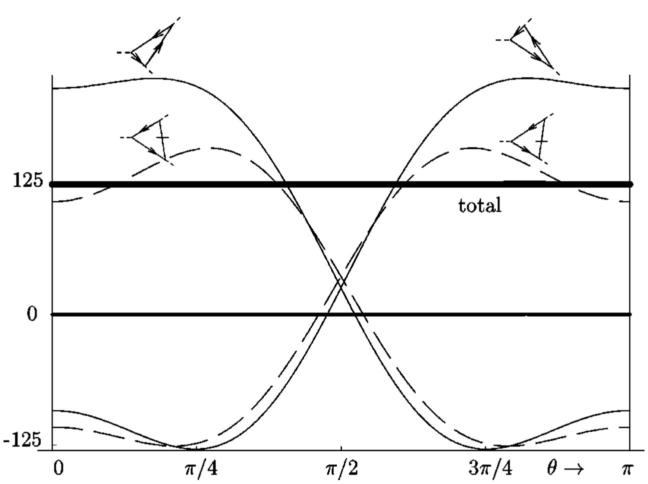


FIG. 4. The thick line at a value of 125 represents the sum of the six light-front time-ordered amplitudes. It is independent of the angle θ, defined in the previous figure. The four largest contributions come from the diagrams without instantaneous parts (solid lines) and the diagrams with an instantaneous exchanged fermion (dashed lines), as indicated by the diagrams.

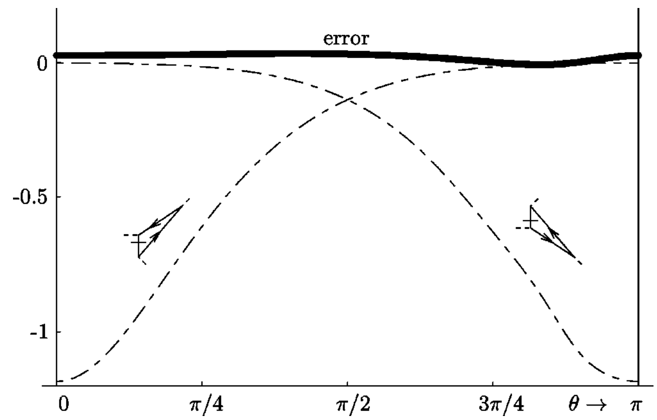


FIG. 5. The amplitudes of the two small contributions (double-dashed lines) and the difference between the sum of the six light-front time-ordered diagrams and the covariant amplitude (thick solid line).

Because the integrations cannot be done exactly, we saw no possibility of giving a rigorous proof of the equivalence of light-front and covariant perturbation theory. Instead we make a choice for the parameters, such as the masses and the external momenta, and show that our method gives the same result as the covariant calculation with BPHZ regularization. We calculated the decay amplitude of a scalar boson at rest, as is pictured in Fig. 3.

From a physical point of view, there is no preferred direction, and therefore we demand that our choice of the coordinates of the light-front have no influence on the outcome of the calculation. The decay amplitude, which is a scalar quantity, should give the same result for each possible direction in which the bosons can fly off.

There are six minus-regularized light-front time-ordered fermion triangle diagrams contributing to the boson decay. Each individual light-front time-ordered diagram has a manifest rotational invariance in the x-y-plane, and therefore we expect the same for the sum. However, since light-front perturbation theory discriminates between the z-direction and the other space-like directions, the light-front time-ordered diagrams can (and should) differ as a function of the angle, θ, between the momentum of one of the particles flying off and the z-axis. The absolute value of the momentum was fixed. It is not immediately clear that the sum should be invariant. This investigation becomes more interesting since it is believed [12] that rotational invariance is broken in naive light-cone quantization of the Yukawa model. However, the results shown in Figs. 4–6 demonstrate that rotational invariance is not broken. Note that we have dropped the factor $-i$ common to all diagrams.

Two light-front time-ordered diagrams (40),(43) contributing to the boson decay and indicated by double-dashed lines are so small they can hardly be identified in Fig. 4. In Fig. 5 we depict these two on a scale that is a factor of 100 larger. In the same figure we show the difference of the sum of the six light-front time-ordered diagrams (using MR1 and

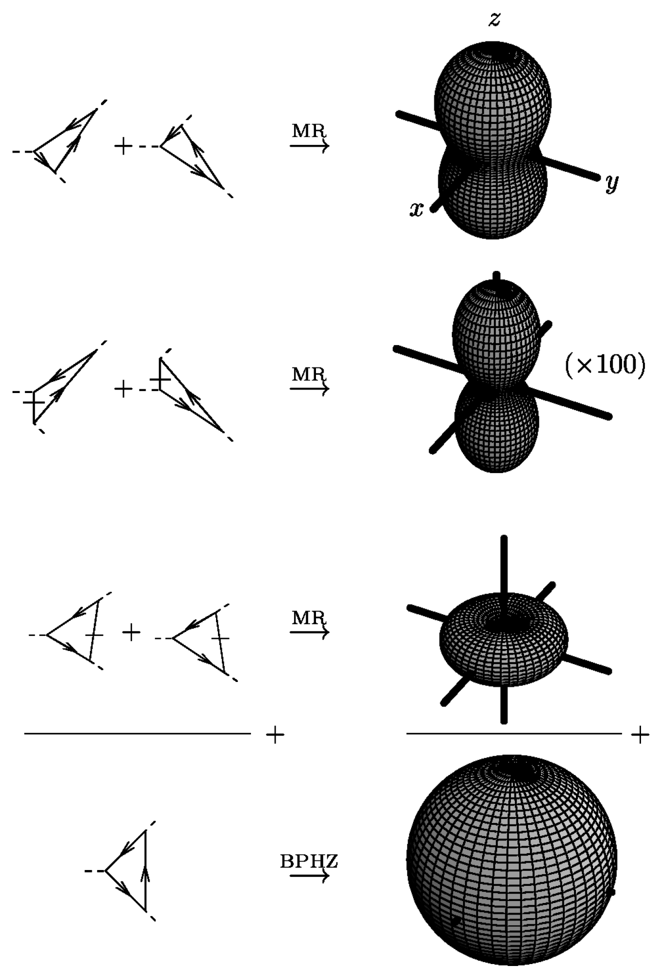


FIG. 6. Commutative diagram of the boson decay amplitude. The boson is at rest in the origin and decays. The outgoing bosons fly off in opposite directions. Points on the surfaces have polar coordinates (A,θ,ϕ), where A is the magnitude of the amplitude and θ and ϕ are the polar angles of the momentum of one of the outgoing particles, as defined in Fig. 3. Because the diagrams on the second line are very small, the scale has been enlarged by a factor of 100. For the light-front time-ordered diagrams on the first three lines minus regularization (both MR0 and MR1) is used, for the covariant diagram on the last line we used BPHZ regularization.

128 points in every integration variable) and the covariant result. It has a maximum of 0.03%.

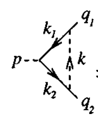
In Figs. 4 and 5 we see that interchanging the outgoing bosons is the same as replacing θ by $\pi-\theta$.

We verified that the individual diagrams are rotational invariant around the z-axis. We illustrate this in Fig 6.

Summing up, we find that the sum of the minus regularized light-front time-ordered diagrams is rotational invariant. The deviation from the covariant result is smaller than 0.03%. It is illustrated in Fig. 5. We checked, by varying the number of integration points, that the deviations are due to numerical inaccuracies only. We conclude that, for the fermion triangle, the covariant calculation in combination with the BPHZ regularization scheme gives the same result as the light-front calculation in combination with minus regularization.

IV. EQUIVALENCE FOR THE ONE-BOSON EXCHANGE CORRECTION

The second process under investigation was studied before by Burkardt and Langnau [12]. A scalar boson of mass μ and momentum p decays into two fermions of mass m and momentum q_1 and q_2 respectively. The lowest order correction to this process is the one-boson exchange correction. The amplitude is given by the integral

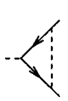


$$= \int_{\mathrm{Min}} \frac{\mathrm{d}^4k \;\; (\not{k}_1+m)(\not{k}_2+m)}{(k_1^2-m^2)(k_2^2-m^2)(k^2-\mu^2)}. \tag{49}$$

Again, this equation is undefined as it stands. First we have to make it a well-defined object. In Sec. IV A we apply the covariant method and in Sec. IV C we use light-front coordinates.

A. Covariant calculation

Using Feynman parametrization the one-boson exchange correction can be rewritten as



$$= 2\int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \int_{\mathrm{Min}} \mathrm{d}^4k \; \frac{\not{k}^2+[(1-x_1)\not{q}_1+x_2)\not{q}_2+m][-x_1\not{q}_1-(1-x_2)\not{q}_2+m]+\mathrm{odd}}{(k^2-\mathcal{M}^2+\mathcal{Q}^2)^3} \tag{50}$$

with

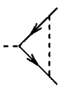
$$\mathcal{M}^2=(x_1+x_2)m^2+(1-x_1-x_2)\mu^2, \tag{51}$$

$$\mathcal{Q}^2=x_1(1-x_1)q_1^2+x_2(1-x_2)q_2^2+2x_1x_2q_1\cdot q_2, \tag{52}$$

and where terms odd in k in the numerator are not specified, since they will be removed according to the covariant prescription. We also define

$$\mathcal{P}^2=\mathcal{Q}^2+(1-x_1-x_2)q_1\cdot q_2. \tag{53}$$

From Eq. (50) we can infer that the Dirac structure of the diagram is



$$= F^1 + F^{2\mu}\gamma_\mu + F^{3\mu\nu}\frac{1}{2}[\gamma_\mu,\gamma_\nu] \tag{54}$$

where the vector part contains a symmetric and an antisymmetric part,

$$F^{2\mu}=F^{2s}(q_1^\mu+q_2^\mu)+F^{2a}(q_1^\mu-q_2^\mu), \tag{55}$$

and the tensor part has the form

$$F^{3\mu\nu}=(q_1^\mu q_2^\nu-q_1^\nu q_2^\mu)F^3. \tag{56}$$

The functions F^i depend on the masses and the external momenta q_1^2, q_2^2 and $q_1\cdot q_2$. If we define the integral operator

$$I[f]=2\int_0^1 \mathrm{d}x_1\int_0^{1-x_1}\mathrm{d}x_2\int_{\mathrm{Min}}\mathrm{d}^4k(k^2-\mathcal{M}^2+\mathcal{Q}^2)^{-3}f, \tag{57}$$

then we have, using $\not{q}_1^2=q_1^2$, etc.,

$$F^1=I[k^2+m^2-\mathcal{P}^2], \tag{58}$$

$$F^{2a}=2mI[1-x_1-x_2], \tag{59}$$

$$F^{2s}=2mI[-x_1+x_2], \tag{60}$$

$$F^3=I[1-x_1-x_2]. \tag{61}$$

We see that the only function which needs to be regularized is F^1. The functions F^2 and F^3 are convergent and do not require regularization in a covariant calculation.

B. BPHZ regularization

The regularized structure function F^{1R} can be found by applying the BPHZ regularization scheme (11) to the structure function (58). The integral is now finite; so we can do the Wick rotation and perform the k integrations:

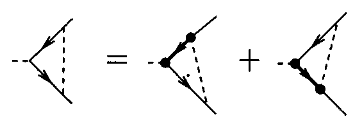
$$F^{1R}(q_1^2,q_2^2,q_1\cdot q_2)=-2\pi^2 i\int_0^1\mathrm{d}x_1\int_0^{1-x_1}\mathrm{d}x_2\int_0^1\mathrm{d}\lambda \times\left(\frac{\mathcal{Q}^2(\lambda\mathcal{P}^2-m^2)}{2(\mathcal{M}^2-\lambda\mathcal{Q}^2)^2}+\frac{\mathcal{Q}^2+\frac{1}{2}\mathcal{P}^2}{\mathcal{M}^2-\lambda\mathcal{Q}^2}\right). \tag{62}$$

We have not been able to do all three integrations exactly. The λ integration and one of the x integrations can be done analytically, and the remaining integration numerically. As $F^{2\mu}$ and F^3 do not need to be regularized, this concludes the covariant calculation of the one-boson exchange correction.

C. Light-front calculation

In our previous paper [5] it was shown how to derive the light-front time-ordered diagrams corresponding tothe covariant diagram (49) using k^--integration. One can write the time-ordered diagrams individually, or one can combine propagating and instantaneous parts into so-called blinks. Blinks, introduced by Ligterink and Bakker [3], have the advantage that the $1/k^+$-singularities cancel and the number of diagrams is reduced.

In the two triangle diagrams studied here it makes no difference whether blinks are used or not. In the case of the fermion triangle we calculated light-front time-ordered diagrams. Here we use blinks, to demonstrate that our technique also works in this case. The one-boson exchange correction has two blinks:

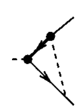


(63)

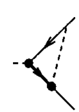
The poles of the two fermion propagators in the triangle are given by Eqs. (36) and (37). The pole of the boson propagator is given by

$$H^-=\frac{k^{\perp 2}+\mu^2}{2k^+}. \tag{64}$$

The amplitudes including blinks are



$$= -2\pi i\int \mathrm{d}^2k^\perp\int_{-q_2^+}^0\frac{\mathrm{d}k^+}{8k_1^+k_2^+k^+} \times\frac{(\not{k}_{2\mathrm{on}}-\not{p}+m)(\not{k}_{2\mathrm{on}}+m)}{(H_1^- - H_2^-)(H^- - H_2^-)}, \tag{65}$$

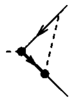


$$= 2\pi i\int \mathrm{d}^2k^\perp\int_0^{q_1^+}\frac{\mathrm{d}k^+}{8k_1^+k_2^+k^+} \times\frac{(\not{k}_{1\mathrm{on}}+m)(\not{k}_{1\mathrm{on}}+\not{p}+m)}{(H_1^- - H_2^-)(H_1^- - H^-)}. \tag{66}$$

We will now focus on the blink in Eq. (66). It simplifies because we can use

$$\not{k}_{1\mathrm{on}}\not{k}_{1\mathrm{on}} = k_{1\mathrm{on}} \cdot k_{1\mathrm{on}} = m^2. \tag{67}$$

Therefore we obtain



$$= 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \times \frac{2m^2 + \not{k}_{1\mathrm{on}}(\not{p} + 2m)}{(H_1^- - H_2^-)(H_1^- - H^-)}. \tag{68}$$

In the same way as we did for the covariant amplitude we can identify the different Dirac structures

$$= F_1^1 + F_1^{2\mu}\gamma_\mu + F_1^{3\mu\nu}\frac{1}{2}[\gamma_\mu, \gamma_\nu]. \tag{69}$$

Although at first sight it looks as if the diagram in Eq. (68) has a covariant structure, covariance is spoiled by the integration boundaries for k^+. Therefore these functions are not covariant objects. We have to investigate equivalence for the structure functions separately.

The light-front structure function F_1^1 can be found by taking the trace of Eq. (68), since all the other structures are traceless. Carrying out the traces one finds

$$F_1^1 = 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \frac{2m^2 + k_{1\mathrm{on}} \cdot p}{(H_1^- - H_2^-)(H_1^- - H^-)}. \tag{70}$$

The other structures of the blink diagram (68) are

$$F_1^{2\mu} = 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \frac{2m(k_{1\mathrm{on}})^\mu}{(H_1^- - H_2^-)(H_1^- - H^-)}, \tag{71}$$

$$F_1^{3\mu\nu} = 2\pi i \int \mathrm{d}^2 k^\perp \int_0^{q_1^+} \frac{\mathrm{d}k^+}{8k_1^+ k_2^+ k^+} \frac{(k_{1\mathrm{on}})^\mu p^\nu}{(H_1^- - H_2^-)(H_1^- - H^-)}. \tag{72}$$

In a similar way we can derive the structure functions corresponding to the other blink diagram.

D. Equivalence

We can identify the different types of divergences, as explained in the Introduction.

1. Light-front structure functions without transverse divergences

The parts of the blinks without any ultraviolet divergences are $F_i^{2\mu}$ and $F_i^{3\mu\nu}$, except for μ being $-$. No cancellations need to be found and no regularization is necessary.

2. Light-front structure functions with cancelling transverse divergences

In the last two structure functions we see something odd happening. Both $F_i^{2\mu}$ and $F_i^{3\mu\nu}$ are divergent for μ being $-$. However, these divergences are not present in the covariant structure functions $F^{2\mu}$ and $F^{3\mu\nu}$. It would be illegal to apply minus regularization, since the covariant amplitude does not need to be regularized. We found that the divergences corresponding to the first blink cancel exactly against those of the second blink. To simplify the calculation we use internal variables x' and $k^\perp$ and external variables χ, q_i^- and $q_i^\perp$. These are introduced in the Appendix.

We have to verify the following relation of equivalence:

$$F^{2-} = F_1^{2-} + F_2^{2-}. \tag{73}$$

According to the reasons mentioned above we have to demand that the divergent parts in the right-hand side cancel. We find that only the highest order contribution in $k^\perp$ contributes to a divergent integral, because we can write

$$F_i^{2-} = \int \mathrm{d}^2 k^\perp \left(\frac{f_i^{2-}}{k^{\perp 2}} + g_i^{2-}(k^\perp) \right), \tag{74}$$

where $g_i^{2-}(k^\perp)$ is the part of the integrand without ultraviolet divergences, and the term with f_i^{2-} gives rise to a logarithmically divergent integral. We have to check if we have

$$f_1^{2-} + f_2^{2-} = 0. \tag{75}$$

In the Appendix the full formulas for the functions f_i^{2-} are given, from which it follows that condition (75) holds. For μ being $-$ in the structure function $F_1^{3\mu\nu}$ one can apply the same method.

3. Light-front structure functions with transverse divergences

The structure function F^1 in the covariant calculation contains an ultraviolet divergence. In the light-front structure functions F_i^1 these appear as divergences in the transverse direction. The equation under investigation is the following:

$$F_1^{1\mathrm{MR}} + F_2^{1\mathrm{MR}} = F^{1\mathrm{R}}. \tag{76}$$

For the same reason as for the fermion triangle, an analytic proof of this equation is not possible. We investigated rotational invariance of the left-hand side of this equation, and furthermore we checked if it gives the same result as the covariant calculation on the right-hand side. A boson is at rest and decays into two fermions as indicated in Fig. 3. The fermion mass is taken to be the same as the boson mass. Therefore there can be no on-shell singularities of intermediate states. Also, we dropped the common factor $-i$. The contributions of the two blink diagrams are given in the com-

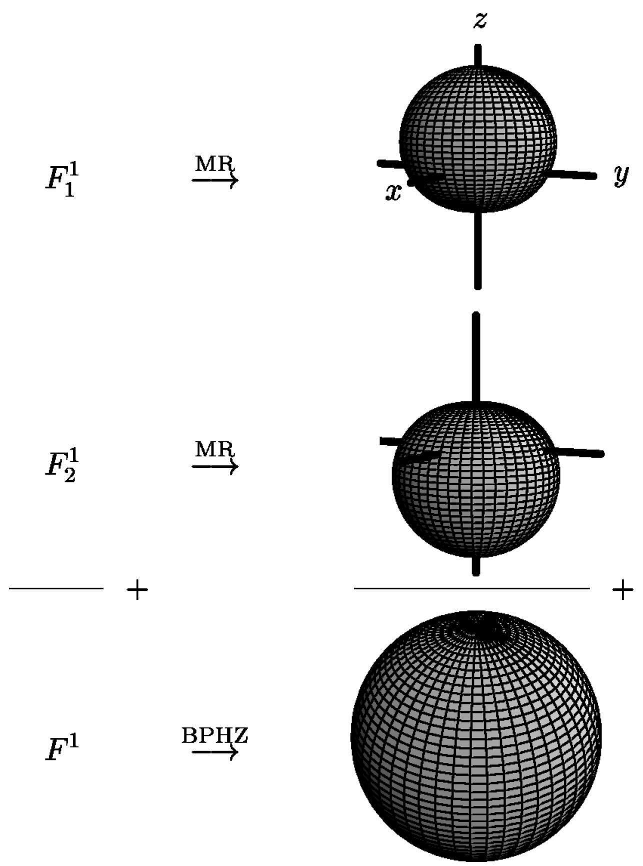


FIG. 7. Commutative diagram of the one-boson exchange correction. A boson decays at rest. The outgoing fermions fly off in opposite directions. The distance from the origin gives the amplitude of the regularized diagram for the fermion flying off in this direction. For the light-front structure functions on the first two lines, minus regularization (MR1) is used; for the covariant structure function on the last line, we used BPHZ regularization.

mutative diagram of Fig. 7. We made the arbitrary choice of applying minus regularization MR1, and used 128 points in integration variable.

The error, i.e., the difference between the covariant calculation with BPHZ regularization and the sum of minus regularized blinks, has a maximum of 0.02%. This deviation results from numerical inaccuracies, as was checked by varying the number of integration points.

We conclude that no significant deviation from a rotational invariant amplitude is found. Moreover, we found that the sum of the light-front time-ordered diagrams is the same as the covariant amplitude for the one-boson exchange correction. Again, the procedure of k^--integration and minus regularization proved to be a valid method.

V. CONCLUSIONS

In the Yukawa model with a scalar coupling there are five single-loop diagrams with transverse divergences, of which two also contain longitudinal divergences. For all other one-loop diagrams and all multiple-loop diagrams that do not contain subdivergences, the proof of the equivalence of covariant and light-front perturbation theory was given by Ligterink and Bakker [3] upon using the k^--integration prescription. For the two single-loop diagrams with longitudinal divergences this integration is ill-defined. This problem was dealt with in a previous paper [5].

Of the three remaining diagrams two are thoroughly analyzed in this paper. For the parts of these diagrams without transverse divergences the k^--integration recipe of Ligterink and Bakker applies. For the parts with transverse divergences a proof of equivalence is complicated by the fact that the amplitudes depend on three independent scalar products of the external momenta. We applied an extended version of the method of minus regularization invented by Ligterink and Bakker. It is on a friendly footing with the light-front, because it can be applied to both longitudinal and transverse divergences. Moreover, it has strong similarities to BPHZ regularization, which is suitable for covariant perturbation theory. We were able to tune the regularization in such a way that minus regularization is analogous to BPHZ regularization. Therefore, we expect an exact equality between the covariant and the light-front amplitudes. We showed that rotational invariance is maintained and we expect that other nonmanifest symmetries on the light-front, such as boosts in the x-y-plane, are also conserved.

The final formulas obtained did not yield to analytic integration. Therefore we had to resort to multidimensional numerical integration. As rotational invariance was shown previously to be violated in naive light-cone quantization [12], we investigated rotational invariance, which is one of the nonmanifest symmetries on the light-front. Our results demonstrate, within the errors due to the numerical methods used, that covariant and light-front time-ordered perturbation theories give the same physical matrix elements.

One diagram with transverse divergences has not been discussed in our two papers on equivalence, namely the fermion box with four external boson lines. It is a scalar object, similar to the fermion triangle. The results obtained for the latter convinced us that upon minus regularization we shall find a covariant result. As there are more time-orderings, and because one cannot test for rotational invariance as easily as for the triangle diagrams, we did not investigate this much more complicated situation.

We trust that with our elaborate discussion of divergent diagrams in the Yukawa model we have illustrated the power of minus regularization and taken away doubts about the covariance of light-front perturbation theory.

ACKNOWLEDGMENTS

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APPENDIX: INTERNAL AND EXTERNAL VARIABLES

We get more insight into the properties of the structure functions if we rewrite them in terms of internal and external variables. This can be done by defining

$$x' = \frac{k^+}{q_1^+} = (x-1)\chi, \tag{A1}$$

$$x = \frac{k^+ + q_2^+}{q_2^+} = \frac{x'+\chi}{\chi}. \tag{A2}$$

Or, equivalently,

$$k^+ = x' q_1^+ = (x-1) q_2^+ ,$$

$$k_1^+ = (x'-1) q_1^+ ,$$

$$k_2^+ = x q_2^+ .$$

In the numerator of the integrals defining light-front structure functions we encounter on-shell spin projections. They can be rewritten in terms of internal variables using

$$k_{1\text{on}}^- = \frac{k_1^{\perp 2} + m^2}{2(x'-1) q_1^+}, \tag{A3}$$

$$k_{2\text{on}}^- = \frac{k_2^{\perp 2} + m^2}{2 x q_2^+}. \tag{A4}$$

The energy denominators can also be written in terms of internal and external variables. The poles are given by Eqs. (36), (37), and (64):

$$2q_1^+(H_1^- - H_2^-) = 2q_1^+\left(p^- + \frac{k_1^{\perp 2}+m^2}{2k_1^+} - \frac{k_2^{\perp 2}+m^2}{2k_2^+}\right)$$
$$= (p^2 + p^{\perp 2})\frac{1+\chi}{\chi} - \frac{k_1^{\perp 2}+m^2}{1-x'} - \frac{k_2^{\perp 2}+m^2}{x\chi}, \tag{A5}$$

$$2q_1^+(H_1^- - H^-) = 2q_1^+\left(q_1^- - \frac{k^{\perp 2}+\mu^2}{2k^+} + \frac{k_1^{\perp 2}+m^2}{2k_1^+}\right)$$
$$= q_1^2 + q_1^{\perp 2} - \frac{k^{\perp 2}+\mu^2}{x'} - \frac{k_1^{\perp 2}+m^2}{1-x'}, \tag{A6}$$

$$2q_2^+(H^- - H_2^-) = 2q_2^+\left(q_2^- + \frac{k^{\perp 2}+\mu^2}{2k^+} - \frac{k_2^{\perp 2}+m^2}{2k_2^+}\right)$$
$$= q_2^2 + q_2^{\perp 2} - \frac{k^{\perp 2}+\mu^2}{1-x} - \frac{k_2^{\perp 2}+m^2}{x}. \tag{A7}$$

The integration measures can be rewritten as follows:

$$2\pi i \int_0^{q_1^+} \frac{dk^+ 4 q_1^+ q_2^+}{8 k_1^+ k_2^+ k^+} = -\pi i \int_0^1 \frac{dx'}{(1-x') x x'}, \tag{A8}$$

$$-2\pi i \int_{-q_2^+}^0 \frac{dk^+ 4 q_1^+ q_2^+}{8 k_1^+ k_2^+ k^+} = -\pi i \int_0^1 \frac{dx}{(1-x') x (1-x)}. \tag{A9}$$

We conclude that it is possible to write the structure functions in terms of the external variables q_1^-, q_2^-, $q_1^\perp$, $q_2^\perp$ and χ and integrals over the internal variables x or x' and $k^\perp$. The divergent part of the structure functions F_i^2 can now be written as

$$f_1^{2-} = -\pi i \int_0^1 \frac{dx'}{(1-x') x x'} \frac{m}{(x'-1) q_1^+} \frac{q_1^+}{q_2^+}$$
$$\times \left(\frac{1}{1-x'} + \frac{1}{x\chi}\right)^{-1} \left(\frac{1}{x'} + \frac{1}{1-x'}\right)^{-1}, \tag{A10}$$

$$f_2^{2-} = -\pi i \int_0^1 \frac{dx}{(1-x') x (1-x)} \frac{m}{x q_2^+}$$
$$\times \left(\frac{1}{1-x'} + \frac{1}{x\chi}\right)^{-1} \left(\frac{1}{x} + \frac{1}{1-x}\right)^{-1}. \tag{A11}$$

Upon cancelling common factors, and using Eq. (A2), we can evaluate the integrals and obtain

$$f_1^{2-} = -f_2^{2-} = \pi i \frac{\chi}{1+\chi} \frac{m}{q_2^+} = \pi i \frac{m}{p^+}. \tag{A12}$$

Therefore condition (75) is verified.

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