Late-time evolution of charged gravitational collapse and decay of charged scalar hair. I

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We study *analytically* the asymptotic evolution of *charged* fields around a Reissner-Nordström black hole. Following the no-hair theorem we focus attention on the *dynamical* mechanism by which the charged hair is radiated away. We find an inverse power-law relaxation of the charged fields at future timelike infinity, along future null infinity, and an oscillatory inverse power-law relaxation along the future outer horizon. We show that charged hair is shed *slower* than neutral hair. Our results are also of importance to the study of mass inflation and the stability of Cauchy horizons during a dynamical gravitational collapse of charged matter to form a charged black hole. [S0556-2821(98)01614-2]

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I. INTRODUCTION

The statement that black holes have no hair was introduced by Wheeler in the early 1970s [1]. The various no-hair theorems state that the external field of a black hole relaxes to a Kerr-Newman field described solely by three parameters: the black hole's mass, charge, and angular momentum. The mechanism responsible for the relaxation of *neutral* external perturbations was first studied by Price [2]. He has found that the late-time behavior of these neutral perturbations for a fixed position r is dominated by $t^{-(2l+3)}$ tails (if there is no initial static field), where l is the multipole moment of the mode and t is the standard Schwarzschild time coordinate. The behavior of *neutral* perturbations along null infinity and along the future event horizon was further studied by Gundlach, Price, and Pullin [3]. They have found that the neutral perturbations along null infinity decay according to an inverse power law $u^{-(l+2)}$, where u is the outgoing Eddington-Finkelstein null coordinate. Along the event horizon the perturbations decay according to $v^{-(2l+3)}$, where v is the ingoing Eddington-Finkelstein null coordinate.

In this work we study the gravitational collapse of a *charged* matter to form a charged black hole. In such a collapse one should expect that *charged* perturbations will develop outside the collapsing star. In particular we focus attention on the late-time behavior of such *charged* scalar perturbations along these three asymptotic regions.

Our results are of importance for two major areas of black-hole physics.

(1) The no-hair theorem of Mayo and Bekenstein [4] states that black holes cannot have a charged scalar hair. However, it was never before studied how a charged black hole, which is formed during a gravitational collapse of a *charged* matter, dynamically sheds its charged scalar hair during the collapse. We study, here, the mechanism by which the *charged* hair is radiated away.

(2) The mass-inflation scenario and the stability of Cauchy horizons were studied under the assumption of the existence of inverse power-law (neutral) perturbations along the outer horizon of a Reissner-Nordström black hole. However, these models did not take into account the existence of *charged* perturbations which are expected to appear in the dynamical collapse of a *charged* star. Here, we study the asymptotic behavior of such perturbations.

The plan of the paper is as follows. In Sec. II we describe our physical system and formulate the evolution equation. In Sec. III we study the late-time evolution of charged scalar perturbations for a collapse that leads to the formation of a (charged) black hole. Here we generalize the formalism of Refs. [2,3] to the charged situation. We study the case $|eQ|\ll 1$, which simplifies things enough to allow us *analytical* derivations of our results. In paper II in this series we will examine the problem for a general value of $|eQ|$. We find an inverse power-law behavior of the charged perturbations along the three asymptotic regions. However, the exponents differ for those of neutral perturbations. Additionally along the outer horizon there are periodic *oscillations* on top of this power-law decay (which do not exist for neutral perturbations).

In Sec. IV we study the behavior of charged perturbations in the noncollapsing case (imploding and exploding shells). Qualitatively, we find the same late-time behavior as in the collapsing situation. In Sec. V we compare the late-time behavior of charged perturbations with the late-time behavior of neutral perturbations. We find that the dynamical process of shedding hair is different for neutral and charged hair, both quantitatively and qualitatively. We show that a black hole which is formed from the gravitational collapse of *charged* matter becomes ''bald'' *slower* than a neutral black hole due to the existence of charged perturbations. Furthermore, while the late-time behavior of neutral perturbations is determined by the space-time curvature, the late-time behavior of charged fields is dominated by *flat* space-time effects (scattering due to the *electromagnetic* interaction in *flat* space-time). We conclude in Sec. VI with a brief summary of our results.

II. DESCRIPTION OF THE SYSTEM

The external gravitational field of a spherically symmetric collapsing star of mass M and charge Q is given by the Reissner-Nordström metric

$$ds^2=-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2+r^2d\Omega^2. \tag{1}$$

We will also use the tortoise radial coordinate y, defined by $dy=dr/(1-2M/r+Q^2/r^2)$, in terms of which the metric becomes

$$ds^2=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)(-dt^2+dy^2)+r^2d\Omega^2, \tag{2}$$

where $r=r(y)$.

We consider the evolution of massless *charged* scalar perturbation fields outside a charged collapsing star. The wave equation for the complex scalar field is [5]

$$\phi_{;ab}g^{ab}-ieA_ag^{ab}(2\phi_{;b}-ieA_b\phi)-ieA_{a;b}g^{ab}\phi=0, \tag{3}$$

where e is a constant.

Resolving the charged scalar field into spherical harmonics $\phi=\Sigma_{l,m}\eta^l_m(t,r)Y^m_l(\theta,\varphi)/r$, we obtain a wave equation for each multipole moment:

$$\eta_{,tt}-2ieA_t\eta_{,t}-\eta_{,yy}+\tilde{V}\eta=0, \tag{4}$$

where

$$\tilde{V}=\tilde{V}_{M,Q,l,e}(r)=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\left[\frac{l(l+1)}{r^2}+\frac{2M}{r^3}-\frac{2Q^2}{r^4}\right]-e^2A_t^2. \tag{5}$$

Here we have suppressed the indices l,m on η.

The electromagnetic potential satisfies the relation

$$A_t=\Phi-\frac{Q}{r}, \tag{6}$$

where Φ is a constant.

In order to get rid of the physically unimportant quantity Φ, we introduce the auxiliary field $\psi=e^{-ie\Phi t}\eta$, in terms of which the equation of motion (4) becomes

$$\psi_{,tt}+2ie\frac{Q}{r}\psi_{,t}-\psi_{,yy}+V\psi=0, \tag{7}$$

where

$$V=V_{M,Q,l,e}(r)=\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\left[\frac{l(l+1)}{r^2}+\frac{2M}{r^3}-\frac{2Q^2}{r^4}\right]-e^2\frac{Q^2}{r^2}. \tag{8}$$

It is well known that a gauge transformation of the form $\eta\to e^{-i\alpha t}\eta$ (where α is a real constant) merely adds a constant to A_t, i.e., $A_t\to A_t-(1/e)\alpha$.

III. EVOLUTION OF CHARGED PERTURBATIONS IN THE COLLAPSING CASE (BLACK-HOLE FORMATION)

The general solution to the wave equation (7) can be written as

$$\psi=\sum_{k=0}^{l}A_kr^{-k}[e^{-ieQ\ln r}G^{(l-k)}(u)+(-1)^ke^{ieQ\ln r}F^{(l-k)}(v)]+\sum_{k=0}^{\infty}[B_k(r)G^{(l-k-1)}(u)+C_k(r)F^{(l-k-1)}(v)], \tag{9}$$

where G and F are arbitrary functions. The coefficients $A_k=A_k(l)=(l+k)!/[2^kk!(l-k)!]$ are equal to those that arise in the neutral case [2] and $B_k(r)=B_k(r;eQ,l,M)$, $C_k(r)=C_k(r;eQ,l,M)$. Here $u\equiv t-y$ is a retarded time coordinate and $v\equiv t+y$ is an advanced time coordinate. For any function H, $H^{(k)}$ is the kth derivative of $H^{(0)}$; negative-order derivatives are to be interpreted as integrals. The first sum in Eq. (9) represents the zeroth-order solution, i.e., neglecting terms of order $O(eQ)$, $O(M/r)$, $O(Q/r)$, and higher.

The functions $B_k(r)$ satisfy the recursion relation

$$\begin{aligned}&2\lambda^2B_k'+2ieQB_kr^{-1}-\lambda^2(B_{k-1}'\lambda^2)'\\&-\lambda^2[A_k(-k-ieQ)r^{-k-1-ieQ}\lambda^2]'\\&-2A_{k+1}r^{-k-2-ieQ}[ieQ(\lambda^2-1)+\lambda^2(k+1)]\\&+V(r)[A_kr^{-k-ieQ}+B_{k-1}]=0,\end{aligned} \tag{10}$$

where $\lambda^2\equiv1-2M/r+Q^2/r^2$ and $B'\equiv dB/dr$.

The functions $C_k(r)$ satisfy an analogous recursion relation; however, we will not need them as the late-time behavior of the charged scalar field does not depend on C_k. We can now expand $B_k(r)$ in the form

$$B_k(r)=a_kr^{-(k+1)-ieQ}+b_kr^{-(k+2)-ieQ}+\cdots, \tag{11}$$

where $a_k\equiv a_k(l,eQ), b_k\equiv b_k(l,eQ)\cdots$.

Substituting Eq. (11) into Eq. (10), one finds, for the lowest-order coefficients,

$$a_l=-ieQA_l\frac{2l+1}{2(l+1)}[1+O(eQ)]. \tag{12}$$

The star begins to collapse at retarded time $u=u_0$. The world line of the stellar surface is asymptotic to an ingoing null line $v=v_0$, while the variation of the field on the stellar surface is asymptotically infinitely redshifted [2,6]. This effect is caused by the time dilation between static frames and infalling frames. A static external observer sees all processes on the stellar surface become ''frozen'' as the star approaches the horizon. Thus, he sees all physical quantities approach a constant. Using the above effect, we make the explicit assumption that after some retarded time $u=u_1$, $D_u\phi=0$ on $v=v_0$, where $D_\mu=\partial_\mu-ieA_\mu$ is the gauge-covariant derivative. This assumption has been proven to be very successful for the neutral scalar field [2,3].

We start with the first stage of the evolution, i.e., the scattering of the charged field in the region $u_0\leqslant u\leqslant u_1$.

The first sum in Eq. (9) represents the primary waves in the wave front, while the second sum represents backscattered waves. The interpretation of these integral terms as backscatter comes from the fact that they depend on data spread out over a *section* of the past light cone, while outgoing waves depend only on data at a fixed u [2]. It should be noted that this physical distinction between the primary waves and the backscattered waves is valid in the region $r \ll |Q| e^{1/|eQ|}$.

After the passage of the primary waves there is no outgoing radiation for $u > u_1$, aside from backscattered waves. This means that $G(u_1)=0$. Thus, for a large r at $u=u_1$, the dominant term in Eq. (9) is

$$\psi(u=u_1,r)=a_l r^{-(l+1)} G^{(-1)}(u_1). \tag{13}$$

This is the dominant backscatter of the primary waves.

After we had determined the dominant backscatter of the charged scalar field, we shall next consider the asymptotic evolution of the field. We confine our attention to the region $y \gg M, |Q|, u > u_1$. In this region (and for $|eQ| \ll 1$) the evolution of the field is dominated by *neutral flat* space-time terms, i.e.,

$$\psi_{,tt} - \psi_{,rr} + \frac{l(l+1)}{r^2}\psi = 0. \tag{14}$$

[It should be noted that for $r \gg M, |Q|$, we have $\psi_{,yy} \simeq \psi_{,rr} + (2M/r^2)\psi_{,r}$. However, in this region $O(\psi_{,rr}) = O(r^{-2}\psi) \gg O(Mr^{-2}\psi_{,r})$. So, in this region, we may replace r by y.]

Thus, the solution for ψ can be written as

$$\psi = \sum_{k=0}^{l} A_k y^{-k} [g^{(l-k)}(u) + (-1)^k f^{(l-k)}(v)]. \tag{15}$$

Comparing Eq. (15) with the initial data (13) on $u=u_1$, one finds

$$f(v) = F_0 v^{-1}, \tag{16}$$

where

$$F_0 = ieQG^{(-1)}(u_1)(-1)^{l+1}(2l+1)!!/(l+1)! + O[(eQ)^2]. \tag{17}$$

For late times $t \gg y$ we can expand

$$g(u) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} g^{(n)}(t) y^n$$

and

$$f(v) = \sum_{n=0}^{\infty} \frac{1}{n!} f^{(n)}(t) y^n.$$

Using these expansions we can rewrite Eq. (15) as

$$\psi = \sum_{n=-l}^{\infty} K_n y^n [f^{(l+n)}(t) + (-1)^n g^{(l+n)}(t)], \tag{18}$$

where the coefficients K_n are those given in the neutral case [2].

Using the boundary conditions for small r (regularity as $y \to -\infty$, at the horizon of a black hole, or at $r=0$, as in the next section), one finds that at late times the terms $h(t) \equiv f(t) + (-1)^l g(t)$ and $f^{(2l+1)}(t)$ must be of the same order (see [3] for additional details). Thus, we conclude that

$$f(t) \simeq F_0 t^{-1}, \tag{19}$$

$$g(t) \simeq (-1)^{l+1} F_0 t^{-1}, \tag{20}$$

and

$$h(t) = O[t^{-2(l+1)}]. \tag{21}$$

Hence, we find that the late-time behavior of the charged scalar field for $|Q| e^{1/|eQ|} \gg t \gg y \gg M, |Q|$ is

$$\begin{aligned}\psi &\simeq 2K_{l+1} y^{l+1} f^{(2l+1)}(t) \\ &= -2K_{l+1} F_0 (2l+1)! t^{-2(l+1)} y^{l+1} + O[(eQ)^2].\end{aligned} \tag{22}$$

This is the late-time behavior of the charged scalar field at timelike infinity i_+.

From Eqs. (15), (16), and (20) one finds that the behavior of the charged scalar field at future null infinity $scri_+$ (i.e., at $u \ll v \ll |Q| e^{1/|eQ|}$) is

$$\psi(v \gg u, u) \simeq A_0 g^{(l)}(u) \simeq -F_0 l! u^{-(l+1)}. \tag{23}$$

Finally, we go on to consider the behavior of the charged scalar field at the black-hole outer horizon r_+.

As $y \to -\infty$ the wave equation (7) can be approximated by the equation

$$\psi_{,tt} + 2ie\frac{Q}{r_+}\psi_{,t} - \psi_{,yy} - \frac{e^2Q^2}{r_+^2}\psi = 0, \tag{24}$$

whose general solution is

$$\psi = e^{-ieQ/r_+ t}[\alpha(u) + \gamma(v)]. \tag{25}$$

On $v = v_0$ we take $D_u \phi = 0$ (for $u \to \infty$). Thus, $\alpha(u)$ must be a constant, and with no loss of generality we can choose it to be zero. Next, we expand $\gamma(v)$ for $t \gg |y|$ as

$$\psi = e^{-ieQ/r_+ t}\gamma(v) = e^{-ieQ/r_+ t}\sum_{n=0}^{\infty}\frac{1}{n!}\gamma^{(n)}(t) y^n. \tag{26}$$

In order to match the $y \ll -M$ solution (26) with the $y \gg M$ solution (22), we make the ansatz $\psi \simeq \psi_{stat}(r) t^{-2(l+1)}$ for the solution in the region $y \ll -M$ and $t \gg |y|$. In other words, we assume that the solution in the $y \ll -M$ region has the same late-time t dependence as the $y \gg M$ solution. Using this assumption, one finds $\psi_{stat} = \Gamma_0 e^{ieQ/r_+ y}$ and $\gamma(t) = \Gamma_0 e^{ieQ/r_+ t} t^{-2(l+1)}$ (where Γ_0 is a constant). Thus, the late-time behavior of the charged scalar field at the horizon is (for $v \ll |Q| e^{1/|eQ|}$)

$$\psi(u \to \infty, v) = \Gamma_0 e^{ieQ/r_+ y} v^{-2(l+1)}. \tag{27}$$

IV. EVOLUTION OF CHARGED PERTURBATIONS IN THE NONCOLLAPSING CASE (IMPLODING AND EXPLODING SHELLS)

We now consider the case of imploding and exploding shells of a charged scalar field. In this situation, the charge of the space-time falls to zero as the evolution proceeds (the charged scalar field, which is the source of that charge, escapes to infinity). One might think that this dissipation of the charge of the spacetime would lead to a late-time evolution which is identical with the neutral case. However, this is *not* the case; as we have seen, the late-time behavior of the charged scalar field at constant r and at $scri_+$ depends on the *backscattering* of the initially outgoing waves. This backscattering is different for the charged scalar field compared with the neutral one, and hence would lead to a different late-time behavior. It should be noted that the backscattering is taking place in an early stage ($u<u_1$), when the space-time still contains a considerable amount of charge. Thus, the initial data on $u=u_1$ are still given by Eq. (13). Furthermore, the late-time behavior of the charged scalar field at constant r and at $scri_+$ is independent of the small-r nature of the background (this is the situation in the neutral case as well [2,3]). Thus, we expect that the noncollapsing charged field would produce a similar late-time behavior compared with the collapsing one.

Finally, it should be noted that in this situation the space-time has no internal ‘‘infinity’’ (no event horizon) and thus $r_+=\infty$.

V. CHARGED SCALAR HAIR VS NEUTRAL SCALAR HAIR

The no-hair theorems for black holes state that a black hole can have neither neutral scalar hair [7,8] nor a charged scalar-field hair [4]. Price [2] has investigated the mechanism which leads to the relaxation of such external neutral scalar hair. However, it was never before investigated how a charged black hole, which is formed during a gravitational collapse of a charged matter, dynamically sheds its *charged* scalar hair during the process of the collapse.

Here, we have shown that the two mechanisms are quite different, both quantitatively and qualitatively. While the late-time behavior of a neutral scalar field outside a black hole is dominated by $\sim t^{-(3+2l)}$ tails, the relaxation of the *charged* scalar hair outside a charged black hole is dominated by a $\sim t^{-(2+2l)}$ behavior. Therefore, we conclude that charged perturbations die *slower* than neutral ones; i.e., a charged black hole, which is formed during a gravitational collapse of a charged matter, is expected to lose its *charged* scalar hair and relax to its final state *slower* than a neutral one. In a more pictorial way, a black hole, which is formed from the gravitational collapse of a *charged* matter, becomes ‘‘bald’’ slower than a neutral one.

Mathematically, it is the relation of r to y which determines the dominant initial backscattering (and therefore the behavior of the late-time tails) for *neutral* perturbations [2]. This means that to a leading order in M the evolution of neutral perturbations depends on the space-time curvature (on M) in the first step ($u_0<u<u_1$), but not in the second step.

On the other hand, the *flat* space-time *charged* terms in the evolution equation for the charged scalar field are of critical importance in determining the dominant initial backscattering for *charged* perturbations. Indeed, the flat space-time evolution equation is

$$\psi_{,tt}+2ie\frac{Q}{r}\psi_{,t}-\psi_{,rr}+\frac{l(l+1)-e^2Q^2}{r^2}\psi=0, \quad (28)$$

whose general solution can be written as

$$\psi=\sum_{k=0}^{\infty} r^{-k}[e^{-ieQ\ln r}C_k p^{(-k)}(u)+e^{ieQ\ln r}D_k q^{(-k)}(v)], \quad (29)$$

where

$$C_k=C_k(l,eQ)$$
$$=\frac{1}{2^k k!}\prod_{n=0}^{k-1}[l(l+1)-n(n+1)-ieQ(2n+1)],$$
$$D_k=D_k(l,eQ)=(-1)^k C_k^*, \quad (30)$$

for $k\geqslant 1$ and $C_0=D_0\equiv 1$.

For $eQ=0$ this infinite series is cut off at $k=l+1$, i.e.,

$$\frac{C_{l+1}}{C_l}=\frac{D_{l+1}}{D_l}=-ieQ\frac{2l+1}{2(l+1)}. \quad (31)$$

In other words, for $eQ=0$ there is *no* backscatter of the waves.

For $|eQ|\ll 1$, we may rewrite Eq. (29) as

$$\psi\simeq\sum_{k=0}^{l} A_k r^{-k}[e^{-ieQ\ln r}G^{(l-k)}(u)$$
$$+(-1)^k e^{ieQ\ln r}F^{(l-k)}(v)]$$
$$+\sum_{k=l+1}^{\infty} r^{-k}[e^{-ieQ\ln r}C_k G^{(l-k)}(u)$$
$$+e^{ieQ\ln r}D_k F^{(l-k)}(v)], \quad (32)$$

where A_k are the coefficients given in Sec. III and $C_k=A_k+O(eQ)$ for $0\leqslant k\leqslant l$. Thus, we conclude that the dominant backscatter of the primary waves, for $|Q|\ll r\ll |Q|e^{1/|eQ|}$, is given by $C_{l+1}r^{-(l+1)}G^{(-1)}(u_1)$ where

$$C_{l+1}=-ieQA_l\frac{2l+1}{2(l+1)}[1+O(eQ)].$$

This is just the result obtained earlier; see Eq. (12).

The physical significance of this result is the conclusion that unlike neutral perturbations the late-time behavior of a *charged* scalar field is entirely determined by *flat* space-time effects. In other words, the scattering is caused by the *electromagnetic* interaction in *flat* space-time.

Our results show that mass inflation in a gravitational collapse of a charged scalar field will be stronger than in the collapse of a neutral field. Mass-inflation models have relied heavily on the existence of inverse power-law tails along the outer horizon of a Reissner-Nordström black hole. However, these models did not take into account the existence of *charged* perturbations outside the collapsing star (during the gravitational collapse of a charged star to form a Reissner-Nordström (RN) black hole we, of course, do expect to find *charged* perturbations outside the star). We have investigated the behavior of these charged perturbations on the *outer* horizon of a RN black hole. We do find a rather similar behavior of the charged field along the outer horizon [see Eq. (27)], although with a *different* exponent and with periodic *oscillations* (which do not exist for neutral perturbations). The *power-law* falloff (times periodic oscillations) of the *charged* perturbations suggests that mass inflation should occur in the gravitational collapse of *charged* matter in which a charged black hole forms. Moreover, since *charged* perturbations have *smaller* dumping exponents compared with neutral ones, they will dominate the influx through the outer horizon and hence will be the *main* cause for the mass-inflation phenomena. One should remember that the mass function diverges like $m(v) \approx v^{-p} e^{\kappa_0 v}$ (for $v \to \infty$, near the Cauchy horizon), where $\frac{1}{2}p$ is the dumping exponent of the field [9].

VI. SUMMARY AND CONCLUSIONS

We have studied the gravitational collapse of *charged* matter to form a charged black hole. The main issue considered is the late-time behavior of *charged* scalar perturbations outside the collapsing star. We have shown that *power-law* tails develop at timelike infinity (at a fixed radius at late times) and along null infinity. Along the outer horizon there is an *oscillatory power-law* tail. The period of these oscillations is determined by the quantity eQ/r_+. The exponents of these inverse power-law tails are all *smaller* compared with neutral perturbations. Thus, we conclude that a black hole which is formed from the gravitational collapse of a *charged* matter becomes ‘‘bald’’ *slower* than a neutral one due to the presence of charged perturbations.

While the late-time behavior of neutral perturbations is determined by the space-time curvature (mathematically, the relation between y and r), the asymptotic behavior of charged fields is dominated by *flat* space-time effects.

Our work reveals the *dynamical* mechanism by which the *charged* scalar hair is radiated away, leaving behind a Reissner-Nordström black hole. We have shown that this mechanism differs from the neutral one both *quantitatively* (different power-law exponents and oscillatory behavior along the black-hole outer horizon) and *qualitatively* (the initial backscattering and thus the late-time behavior are dominated by *flat* space-time terms, namely, by the *electromagnetic interaction*, rather than by curvature effects).

Furthermore, we have shown that the late-time behavior of charged fields in the noncollapsing situation (imploding and exploding shells) is dominated by a similar inverse power-law behavior both at a fixed radius (and late times) and along null infinity.

Finally, our results are of importance for the mass-inflation scenario and stability of Cauchy horizons. Here, we have shown that the asymptotic behavior of charged perturbations along the outer horizon of a RN black hole is characterized by an *oscillatory inverse power-law* behavior (with smaller exponents compared with neutral tails). Thus, one should expect that these inverse power-law *charged* perturbations will cause a mass-inflation singularity during the gravitational collapse of the charged matter that forms a *charged* black hole. Moreover, the *slower* relaxation of charged perturbations makes them the *dominant* cause for the divergence of the mass function.

The most significant shortfall in our analysis is the limitation to the case $|eQ| \ll 1$. In an accompanying paper (paper II) we extend our *analytical* results to include *general* values of eQ (using a spectral decomposition) and we confirm them *numerically*. On the other hand, the main advantage of *this* approach is the fact that it gives a *clear* picture of the *physical* mechanism responsible for the late-time behavior of charged perturbations; namely, the tail arises because of backscattering of the *charged* field off the *electromagnetic* potential far away from the black hole. The physical picture that arises from this paper is clear—dealing with charged (massless) perturbations, one may *neglect* any curvature effects.

In accompanying papers we study the *fully nonlinear* gravitational collapse of a charged scalar field to form a *charged* black hole. In order to numerically confirm our *analytical* predictions we will first focus attention on the asymptotic behavior of the *charged* field outside the *dynamically* formed charged black hole.

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[1] R. Ruffini and J. A. Wheeler, Phys. Today **24**, 30 (1971); C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[2] R. H. Price, Phys. Rev. D **5**, 2419 (1972).

[3] C. Gundlach, R. H. Price, and J. Pullin, Phys. Rev. D **49**, 883 (1994).

[4] A. E. Mayo and J. D. Bekenstein, Phys. Rev. D **54**, 5059 (1996).

[5] S. W. Hawking and G. F. R. Ellis, *The Large Scale Structure of Space Time* (Cambridge University Press, Cambridge, England, 1973).
[6] J. Bičák, Gen. Relativ. Gravit. **3**, 331 (1972).
[7] J. E. Chase, Commun. Math. Phys. **19**, 276 (1970).
[8] J. D. Bekenstein, Phys. Rev. Lett. **28**, 452 (1972); Phys. Rev. D **5**, 1239 (1972); **5**, 2403 (1972).
[9] E. Poisson and W. Israel, Phys. Rev. D **41**, 1796 (1990).