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Geometric reheating after inflation

Bruce A. Bassett* and Stefano Liberati†
International School for Advanced Studies, Via Beirut 2-4, 34014, Trieste, Italy

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Inflationary reheating via resonant production of non-minimally coupled scalar particles with only gravitational coupling is shown to be extremely strong, exhibiting a negative coupling instability for $\xi<0$ and a wide resonance decay for $\xi\gg 1$. Since non-minimal fields are generic after renormalization in curved spacetime, this offers a new paradigm in reheating—one which naturally allows for efficient production of the massive bosons needed for grand unified theory baryogenesis. We also show that both vector and tensor fields are produced resonantly during reheating, extending the previously known correspondences between bosonic fields of different spin during preheating. [S0556-2821(98)50214-7]

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I. INTRODUCTION

Typical modern incarnations of inflation arise within supergravity, string or grand unified theories (GUT) where the inflaton, ϕ, is only one of many fields. Studies of inflation including couplings to these other fields, as required to reheat the universe after inflation, yield extremely complex dynamics [1] and are little investigated beyond hybrid models [2]. Here we turn to a minimalist view in which preheating [3–7] occurs with the inflaton coupled only gravitationally to other fields. We call this *geometric reheating* to emphasize its gravitational origin. The most powerful example of this new mechanism is provided by non-minimally coupled fields (Sec. II B), where the strength of the effect is due to the congruence of two facts: (1) the Ricci curvature oscillates during preheating and (2) the non-minimal coupling, ξ, is a free parameter. The first ensures that there is resonance, the second that the effect is non-perturbative.

The possible importance of this effect is motivated by renormalization group studies in curved-spacetime [8–10], which have shown that even if the bare coupling, ξ_0, is minimal, after renormalization $\xi\neq 0$ generically. Since we are particularly interested in the preheating realm which occurs when inflation ends near the Planck scale, we are near the ultraviolet (UV) fixed points of the renormalization group equations. While the UV fixed points may correspond to a conformally invariant field ($m=0$, $\xi=\frac{1}{6}$), in different GUT models the coupling may also diverge, $|\xi|\to\infty$, in the UV limit [8,11]. In both cases the nature of geometric reheating is very different from the standard models based on explicit self-interactions or particle-physics couplings between fields (see, e.g. [3]).

Further we shall study the gravitational production of spin 0, 1 and 2 particles due to the expansion of the universe during preheating, and will show that a unified treatment in terms of parametric resonance exists. This is shown by reducing the evolution equations to generalized Mathieu form:

$$x''+[A(k)-2q\cos 2z]x=0, \tag{1}$$

with time-dependent parameters.[1]

The Mathieu equation has rapidly growing solutions controlled by the Floquet index μ_k. In the case that $1\gg q>0$, the Floquet index in the *first* resonance band of the Mathieu equation is given by $\mu_k=[(q/2)^2-(2k/m-1)^2]^{1/2}$ [12,3]. This can be extended to give μ_k^N in the Nth resonance band [13] as long as $A>0$ and $2N^{3/2}\gg q$:

$$\mu_k^N=-\frac{1}{2N}\frac{\sin 2\delta}{[2^{N-1}(N-1)]^2}q^N, \tag{2}$$

where δ varies in the interval $[-\pi/2,0]$ and $\mu_k^N\ll 1$.

When $A(k)<0$ a qualitative change occurs and the dominant effect comes from the fact that one effectively has an inverted harmonic oscillator yielding the *negative coupling instability* [14]. In this case the Floquet index can be as large as $\mu_k\sim|q|^{1/2}$, there are no stability bands to speak of and the typical variances are larger by a factor $|q|^{1/2}$ than in the $A(k)>0$ case.

To be concrete, consider the case of a scalar field in a Freidmann-Lamaitre-Robertson-Walker (FLRW) universe [$g_{\mu\nu}=\mathrm{diag}(-1,a^2(t)/(1-Kr^2),a^2(t)r^2,a^2(t)r^2\sin^2\theta)$, $K=\pm 1,0$ is the curvature constant]. We shall restrict[2] ourselves to the quadratic potential,

$$V(\phi)=\frac{m_\phi^2}{2}\phi^2. \tag{3}$$

For $K=0$, the latter potential gives an oscillatory behavior of the field, $\phi=\Phi\sin(m_\phi t)$, with $\Phi\sim 1/m_\phi t$. In what follows we shall try to preserve maximal generality; we denote with

[1]While it is known that the Mathieu formulation is insufficient [6] in some respects, and has lead to the introduction of other approximations—principally that of stochastic resonance [3]—the Mathieu equation remains a powerful diagnostic test for the strength of particle production.

[2]We note that application of stochastic resonance methods to the vector, tensor, and non-minimal scalar fields of this paper for the potential $V=(\lambda/4)\phi^4$ requires an extension of the existing theory to scattering in a quartic potential as opposed to the standard quadratic potential [3,7].

*Email address: bruce@stardust.sissa.it
†Email address: liberati@sissa.it

$\doteq$ results which are derived specifically for the potential (3). We use natural units with $\kappa=8\pi$, $G=1$.

The energy density and pressure for a minimally coupled scalar field, treated as a perfect fluid, are $\mu=\kappa(\frac{1}{2}\dot{\phi}^2+V(\phi))$, $p=\kappa(\frac{1}{2}\dot{\phi}^2-V(\phi))$. This breaks down if the field is non-minimally coupled (an imperfect fluid treatment must be used), if the effective potential is not adequate [6], or if large density gradients exist. The FLRW Ricci tensor is [15]

$$R^0{}_0=3\frac{\ddot{a}}{a}$$

$$R^i{}_j=\left[\frac{\ddot{a}}{a}+2\left(\frac{\dot{a}}{a}\right)^2+\frac{2K}{a^2}\right]\delta^i{}_j, \tag{4}$$

where $i,j=1,\ldots,3$. The Ricci scalar is

$$R=6\left[\frac{\ddot{a}}{a}+\left(\frac{\dot{a}}{a}\right)^2+\frac{K}{a^2}\right]. \tag{5}$$

The Raychaudhuri equation for the evolution of the expansion $\Theta=3\dot{a}/a$ is[3] given by

$$\dot{\Theta}=-\frac{3\kappa}{2}\dot{\phi}^2+\frac{3K}{a^2}, \tag{6}$$

while the Friedmann equation is

$$\Theta^2+\frac{9K}{a^2}=3\kappa\mu=3\kappa\left(\frac{1}{2}\dot{\phi}^2+V(\phi)\right). \tag{7}$$

As an example, when $K=0$ and $\dot{a}/(am_\phi)\ll 1$, one may solve Eq. (7) perturbatively [17], to obtain

$$\Theta\doteq\frac{2}{t}\left(1-\frac{\sin 2m_\phi t}{2m_\phi t}\right), \tag{8}$$

to first order in $\dot{a}/(am_\phi)$. This is only valid after preheating when $\Phi\ll 1$, but shows that the expansion oscillates about the mean Einstein–de Sitter (EDS) pressure-free solution. Equation (8) can be integrated to give the scale factor:

$$a(t)\doteq\bar{a}\exp\left(\frac{\sin 2m_\phi t}{3m_\phi t}-\frac{2\,\mathrm{ci}(2m_\phi t)}{3}\right) \tag{9}$$

where $\bar{a}=t^{2/3}$ is the background EDS evolution, and $\mathrm{ci}(m_\phi t)=-\int_t^\infty \cos(m_\phi z)/z dz$. This example explicitly demonstrates how temporal averaging (which yields $\bar{a}$) removes the resonance.

Via Eqs. (6),(7) one can systematically replace all factors of $\dot{a},\ddot{a}$ with factors of $\dot{\phi}$ and $V(\phi)$ terms.[4] In this way one can show that the vector and tensor wave equations take the form of Mathieu equations during reheating [18].

[3]The expansion is generally defined as $\Theta\equiv u^a{}_{;a}$ where u^a is the 4-velocity and ; denotes covariant derivative [16].

[4]Indeed, a useful combination is $2\dot{\Theta}+\Theta^2=-3\kappa p\doteq\frac{3}{2}\kappa m_\phi^2\Phi^2\cos(2m_\phi t)$.

II. SCALAR FIELDS

Consider now the effective potential

$$V(\phi,\chi_\nu)=V(\phi)+\frac{1}{2}\sum_\nu^N m_\nu^2\chi_\nu^2+\frac{1}{2}\sum_\nu^N \xi_\nu\chi_\nu^2 R, \tag{10}$$

describing the inflaton with potential $V(\phi)$ coupled only via gravity to N scalar fields, χ_ν, which have no self-interactions, masses m_ν and non-minimal couplings ξ_ν. The equation of motion for modes of the νth field is

$$\ddot{\chi}_k^\nu+\Theta\dot{\chi}_k^\nu+\left(\frac{k^2}{a^2}+m_\nu^2+\xi_\nu R\right)\chi_k^\nu=0. \tag{11}$$

From Eqs. (5),(6),(7) the Ricci scalar is given by[5]

$$R=-\kappa\dot{\phi}^2+4\kappa V(\phi)\quad (K=0). \tag{12}$$

A. The minimally coupled case

Consider $\xi_\nu=0$. Then Eq. (11) reduces, on using Eqs. (6), (7), to

$$\frac{d^2(a^{3/2}\chi_k)}{dt^2}+\left(\frac{k^2}{a(t)^2}+m_\nu^2+\kappa\frac{3}{8}\dot{\phi}^2-\kappa\frac{3}{4}V(\phi)+\frac{3}{4}\frac{K}{a^2}\right)(a^{3/2}\chi_k)=0. \tag{13}$$

There exists parametric resonance because the expansion Θ oscillates. The potential (3) yields Eq. (1) $(K=0)$ with time-dependent parameters:

$$A(k,t)\doteq\frac{k^2}{a^2m_\phi^2}+\frac{m_\nu^2}{m_\phi^2},\quad q\doteq\frac{3}{16}\kappa\Phi^2. \tag{14}$$

From this we see that the production of particles is reduced as m_ν increases. Indeed, since $A\to m_\nu^2/m_\phi^2$, $q\to 0$ due to the expansion, production of minimally coupled bosons is rather weak and shuts off quickly due to horizontal motion on the stability chart. We stress that the production is, however, much stronger than that obtained in previous studies, where the scalar factor evolves monotonically [19]. This mild situation changes dramatically, when a non-minimal coupling is introduced.

B. Non-minimal preheating

Now include the *arbitrary* non-minimal coupling ξ_ν. Using Eq. (12) one can reduce Eq. (11) to $(K=0)$:

[5]Assuming that at the start of reheating the inflaton is the dominant contributor to the energy density of the universe.

$$\frac{d^2(a^{3/2}\chi_k)}{dt^2} + \left[\frac{k^2}{a(t)^2} + m_\nu^2 + \kappa\left(\frac{3}{8}-\xi\right)\dot{\phi}^2 - \kappa\left(\frac{3}{4}-4\xi\right)V(\phi)\right] \times (a^{3/2}\chi_k) = 0. \tag{15}$$

Defining a new variable $z = m_\phi t + \pi/2$, Eq. (15) takes the form of Eq. (1) with time-dependent parameters:

$$A(k,t) \doteq \frac{k^2}{a^2 m_\phi^2} + \frac{m_\nu^2}{m_\phi^2} + \frac{\kappa\xi}{2}\Phi^2$$

$$q(t) \doteq \frac{3}{4}\kappa\left(\frac{1}{4}-\xi\right)\Phi^2. \tag{16}$$

The crucial observation is that since ξ is initially free to take on any value,[6] $A(k)$ is neither restricted to be positive nor small.

From Eq. (16) it is clear that $A(k) < 0$ for sufficiently negative ξ. The possibility of negative A was the thesis of the work by Greene *et al.* [14]. However, in their model, this powerful negative coupling instability was only partially effective due to the non-zero vacuum expectation value acquired by the χ field due to its coupling, g, with the inflaton. Here we only have gravitational couplings and the same constraint is removed.

Negative A (induced when $q<0$) implies that the physical region of the (A,q) plane is altered. Instead of $A \geqslant 2|q|$, we have $A \geqslant \pi\Phi^2 - 2|q|/3$. Now when $2|q|/3 > |A| \geqslant 1$, we have $\mu_k \sim |q|^{1/2} \simeq (6\pi|\xi|)^{1/2}\Phi$ along the physical separatrix $A = \pi\Phi^2 - 2|q|/3$. Since the renormalized $|\xi|$ may have very large values, this opens the way to exceptionally efficient reheating—see Figs. (1),(2)—via resonant production of highly non-minimally coupled fields with important consequences for GUT baryogenesis [14] and non-thermal symmetry restoration.

For example, let us consider $m_\phi \simeq 2\times 10^{13}$ GeV as required to match cosmic microwave background (CMB) anisotropies $\Delta T/T \sim 10^{-5}$. Then GUT baryogenesis with massive bosons χ with $m_\chi > 10^{14}$ GeV simply requires $\xi < -(\pi\Phi^2)^{-1}$, with Φ in units of the Planck energy. Instead if one requires the production of GUT-scale gauge bosons with masses $m_{gb} \sim 10^{16}$ GeV, this is still possible if the associated non-minimal coupling is of order $\xi \sim -10^3$. Such coupling values have been considered in, e.g. [20]. The massive bosons with $m_\chi \sim 10^{14}$ GeV can be produced in the usual manner via parametric resonance if $\xi>0$, but this process is weaker (cf. [21]).

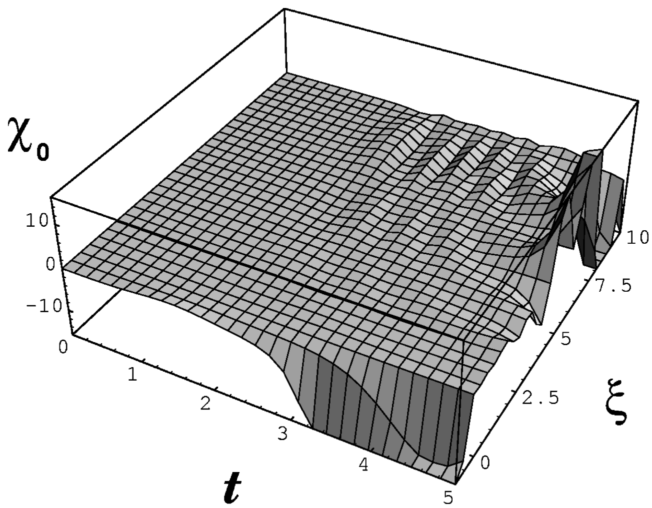

FIG. 1. The evolution of the $k=0$ mode $(m_\nu^2/m_\phi^2 \simeq 1)$, as a function of time and the non-minimal coupling parameter ξ. For positive ξ the evolution is qualitatively that of the standard preheating with resonance bands. However, for negative A (negative ξ) the solution changes qualitatively and there is a negative coupling instability. There are generically no stable bands and the Floquet index corresponding to $-|\xi|$ is much larger, scaling as $\mu_k \sim |\xi|^{1/2}$.

Since the coupling between ϕ and χ is purely gravitational, back reaction effects in the standard sense (see [3,6]) cannot shut off the resonance. The inflaton continues to oscillate and produce non-minimally coupled particles, receiving no corrections to $m^2_{\phi,eff}$ from $\langle \delta\chi^2 \rangle$.

To estimate the maximum variance $\langle \delta\chi^2 \rangle$ is therefore rather difficult. The standard method is to establish the time, when the resonance is shut off by the growth of $A(k)$ which pushes the $k=0$ mode out of the dominant first resonance band. For this we must understand how $A(k)$ changes as the χ-field gains energy and alters the Ricci curvature. If we assume that most of the energy goes into the χ_0 mode, jus-

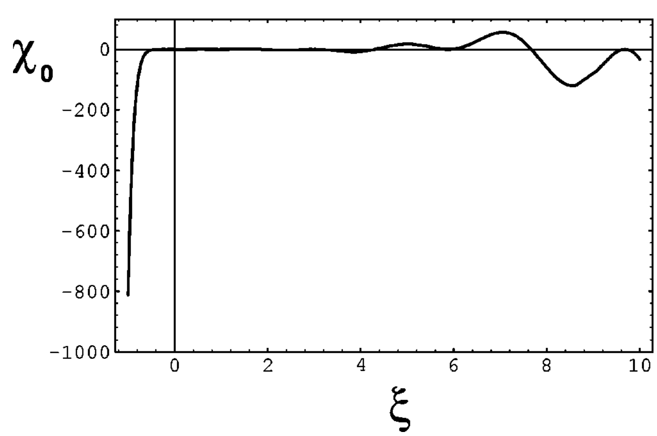

FIG. 2. A slice of the spectrum in Fig. (1) at $t=5$ as a function of the non-minimal coupling ξ. The qualitative differences between $\xi<0$ and $\xi>0$ are clear.

[6]The only constraints that one might impose are that the effective potential should be bounded from below and that the strong energy condition, $R_{ab}u^a u^b > 0 \Leftrightarrow T_{ab}u^a u^b > -T/2$, be satisfied. The first is difficult to impose since R oscillates and the second since one should use the renormalized stress-tensor, $\langle T_{ab} \rangle$.

tified in the $\xi<0$ case,[7] then the change to the Ricci curvature is $\delta R_\chi = 8\pi(E-S)$, where [22]

$$E = G_{eff}\left(\frac{\dot{\chi}_0^2}{2} + \frac{m_\chi^2 \chi_0^2}{2} - 12\xi\chi_0\dot{\chi}_0\frac{\dot{a}}{a}\right) \tag{17}$$

is the T^{00} component of the χ stress tensor,

$$S = \frac{3G_{eff}}{1+192\pi G_{eff}\xi^2\chi_0^2}\left[\frac{\dot{\chi}_0^2}{2} - \frac{m_\chi^2\chi_0^2}{2} + 4\xi\left(\frac{\dot{a}^2}{a^2} - \chi_0\dot{\chi}_0\frac{\dot{a}}{a} - m_\chi^2\chi_0^2\right) + 64\pi\xi^2\chi_0^2 E\right], \tag{18}$$

is the spatial trace of the stress tensor $T^i{}_i$ corresponding to $3p$ in the perfect fluid case and $G_{eff} = (1+16\pi\xi\chi^2)^{-1}$ is the effective gravitational constant. Since χ_0 is rapidly growing, the major contribution of δR_χ will be to $A(k)$, causing a rapid vertical movement on the instability chart. Once $\delta A + A > 2|q| + |q|^{1/2}$, the resonance is shut-off. If $\xi<0$, most of the decaying ϕ energy is pumped into the small k modes (see Fig. 2). Subsequently, we expect the oscillations in χ_0 to produce a secondary resonance due to the self-interaction and non-linearity of Eqs. (17),(18).

The case of a $\lambda_\chi \chi^4/4$ self-interaction provides another mechanism that may be dominant in ending the resonance: namely $m_{\phi,eff}^2$, and hence $A(k)$, gains corrections proportional to $\lambda_\chi\langle\delta\chi^2\rangle$ which shuts off the resonance [14] leaving a peak variance of order $\langle\delta\chi^2\rangle \simeq m_\phi^2(4|q| - m_\chi^2)/\lambda_\chi$ (assuming that $\xi<0$). If $\xi>0$ the variance is smaller by a factor $|q|^{1/2}$.

III. THE VECTOR CASE

Until now, reheating studies have been limited to minimally coupled scalar fields, fermions and gauge bosons [23]. In the case of vector fields the minimum one can do to preserve gauge-invariance is to couple to a complex scalar field via the current since real scalar fields carry no quantum numbers. We consider here only vacuum vector resonances, however.

A massive spin-1 vector field in curved spacetime satisfies the equations

$$(-\nabla_a\nabla^a + m_{\mathcal{A}}^2)\mathcal{A}^b + R^b{}_a\mathcal{A}^a = 0. \tag{19}$$

These equations are equivalent to the Maxwell-Proca equations for the vector potential $\mathcal{A}_a$ only after an appropriate gauge choice which removes one unphysical polarizations. In our case we shall use the so-called tridimensional transversal gauge condition:

$$\mathcal{A}_0 = 0, \quad \nabla^i\mathcal{A}_i = 0. \tag{20}$$

This set is equivalent to the Lorentz gauge, although it does not conserve the covariant form of the latter. Nonetheless, in either case, gauge-invariant quantities such as the radiation energy density, are unaffected.

In a FLRW background, the Ricci tensor is diagonal, which together with the gauge choice (20) and expansion over eigenfunctions, ensures the decoupling of the set of Eqs. (19). We can reduce the system to a set of decoupled Mathieu equations. The Ricci tensor is [see Eq. (4)]:

$$R^a{}_b = \kappa V(\phi)\delta^a{}_b - \kappa\dot{\phi}^2\delta^a{}_0\delta^0{}_b, \tag{21}$$

which leads to the Mathieu parameters for the spatial components ($a^{3/2}\mathcal{A}^i$):

$$A(k) \doteq \frac{k^2}{a^2 m_\phi^2} + \frac{m_{\mathcal{A}}^2}{m_\phi^2} + 2q, \quad q \doteq \frac{\kappa\Phi^2}{8} \tag{22}$$

showing that vector fields are also parametrically amplified, albeit weakly, during reheating as in the scalar case.

IV. THE GRAVITON CASE

It has been shown using the electric and magnetic parts of the Weyl tensor [18] that there exists a formal analogy between the scalar field and graviton cases during resonant reheating. Here we will show that the correspondence also holds in the Bardeen formalism. The gauge-invariant (at first order) transverse-traceless (TT) metric perturbations h_{ij} describe gravitational waves in the classical limit. In the Heisenberg picture one expands over eigenfunctions, Y_{ab} of the *tensor* Laplace-Beltrami operator with scalar mode functions h_k, which satisfy the equation of motion:

$$\ddot{h}_k + \Theta\dot{h}_k + \left(\frac{k^2+2K}{a^2}\right)h_k = 0, \tag{23}$$

or equivalently

$$(a^{3/2}h_k)^{\cdot\cdot} + \left(\frac{k^2+2K}{a^2} + \frac{3}{4}p\right)(a^{3/2}h_k) = 0, \tag{24}$$

where $p = \kappa(\dot{\phi}^2/2 - V)$ is the pressure. This gives a time-dependent Mathieu equation [cf. Eq. (14)] with parameters:

$$A(k) \doteq \frac{k^2}{a^2 m_\phi^2}, \quad q \doteq -\frac{3\kappa\Phi^2}{16}. \tag{25}$$

In this case, a negative coupling instability is impossible and only for $\Phi \sim M_{pl}$ is there significant graviton production. Note, however, that if temporal averaging is used, the average equation of state is that of dust, $\overline{p} = 0$. Equation (24) then predicts (falsely) that there is no resonant amplification of gravitational waves since the value of q corresponding to the temporarily averaged evolution vanishes.

V. CONCLUSIONS

We have described a new—geometric—reheating channel after inflation, one which occurs solely due to gravitational

[7] In the case $\xi \gg 1$ one needs to use $\langle\delta\chi^2\rangle$ instead.

couplings. While this is not very strong in the gravitational wave and minimally coupled scalar field cases, it can be very powerful in the non-minimally coupled case, either due to broad-resonance ($\xi \gg 1$) or negative coupling ($\xi < 0$) instabilities. Particularly in the latter case, it is possible to produce large numbers of bosons which are significantly more massive than the inflaton, as required for GUT baryogenesis. It further gives rise to the possibility that the post-inflationary universe may be dominated by non-minimally coupled fields. These must be treated as imperfect fluids which would thus alter both density perturbation and background spacetime evolution, which are known to be significantly different [24] than in the simple perfect fluid case. We have further presented a unified approach to resonant production of vector and tensor fields during reheating in analogy to the scalar case.

Future work should examine in greater detail back reaction issues in the non-minimal case, and the situation in potentials with self-interaction relevant to symmetry restoration.

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