χ_{cJ} production in e^+e^- annihilation

G. A. Schuler
Theory Division, CERN, CH-1211 Geneva 23, Switzerland

M. Vänttinen*
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen Ø, Denmark
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Inclusive production of χ_{cJ} in e^+e^- annihilations is an excellent probe of the role of higher Fock states in the production of heavy quarkonia. Within the non-relativistic QCD approach, contributions from the short-distance production of color-octet $c\bar{c}$ pairs are significantly larger than those from color-singlet production. At the same time, χ_{cJ} production rates are significantly smaller than expected in the color evaporation approach. Measurements of the χ_{cJ} production at CLEO and future B-factories will thus constitute a major test of theoretical approaches to the production of heavy quarkonia. [S0556-2821(98)04813-9]

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Calculations of heavy-quark production in e^+e^- collisions are now approaching a level of high accuracy. Fifteen years after the calculation of the next-to-leading-order (NLO) perturbative QCD corrections to the total *open* heavy-quark cross section [1], we have now seen the completion of the NLO corrections to three-jet cross sections with massive quarks [2]. Despite continuous efforts over nearly 20 years, such a precision has not yet been reached in the calculation of the cross sections of heavy *bound states* [3–12]. Predictive power depends on the understanding of the long-distance bound-state formation process, which is therefore at the heart of current quarkonium physics. In this Brief Report we investigate the inclusive production of P-wave quarkonium states in e^+e^- collisions. This is the first study of these processes since essentially [13] all previous work was concerned with the S-wave J/ψ or Υ mesons.

For a large quark mass m, a quarkonium bound state of a heavy quark Q and its antiquark $\bar{Q}$ is a non-relativistic system. The spectroscopy of both charmonia and bottomonia is well described in non-relativistic potential models, where the quarkonia are considered to be pure $Q\bar{Q}$ states bound by an instantaneous potential. The question arises as to whether the production of quarkonia is also dominated by this leading Fock state. Is the $Q\bar{Q}$ pair produced, already at short distances, in a configuration that corresponds to the asymptotic valence Fock component of the quarkonium? If it is, then all non-perturbative information in the theoretical prediction reduces to a single number, namely the coordinate-space wave function at the origin, which can be extracted, e.g. from a potential-model calculation.

It is widely believed that higher Fock states are of decisive importance. Yet there exist different approaches when it comes to relating observable cross sections to short-distance $Q\bar{Q}$ production amplitudes. In the color evaporation model (CEM) [3], the quarkonium production cross section is prescribed to be a (process-independent) fraction of the $Q\bar{Q}$ production cross section, below the physical threshold for the production of a pair of heavy-light mesons. In the non-relativistic QCD (NRQCD) approach [14], the transition from $Q\bar{Q}$ to quarkonium is represented in terms of a multipole expansion, which leads to scaling rules for transition probabilities in terms of v, the mean heavy-quark velocity in the meson. Any given cross section is a linear combination of several non-perturbative factors, multiplied by process-dependent hard factors. To the leading order in $\Lambda_{\mathrm{QCD}}/\mu$ ($\mu \gtrsim mv$) the former are well-defined process-independent NRQCD matrix elements (MEs).

We have calculated the $e^+e^- \to \chi_{cJ} + X$ cross sections within the NRQCD approach, using values of MEs determined from measurements of other reactions. On the one hand, the results strongly violate the process-independence of cross-section ratios assumed in the CEM. Thus an experimental upper limit on χ_{cJ} production in e^+e^- annihilations can establish that non-perturbative effects play a decisive role in bound-state formation. On the other hand, the NRQCD results are dominated by color-octet production channels. Observation of the predicted shapes and normalizations of the cross sections will therefore provide striking evidence of the importance of higher Fock states and the scaling of their contributions with v.

In NRQCD, any cross section is given as a series expansion in both $\alpha_s(m)$ and v, to the leading order in $\Lambda_{\mathrm{QCD}}/\mu$. Let us first consider the case of S-wave quarkonia, in particular the J/ψ ($J^{PC}=1^{--}$). The leading contribution in v is given by processes where the $c\bar{c}$ pair is produced in the leading, color-singlet Fock state [15], $|c\bar{c}_1(^3S_1)\rangle$:

$$e^+e^- \to c\bar{c}_1(^3S_1) + c\bar{c}, \tag{1}$$

$$e^+e^- \to c\bar{c}_1(^3S_1) + gg. \tag{2}$$

The order v^2 correction to J/ψ production still involves only the leading Fock state and can be considered as a relativistic correction to the wave function. Subleading Fock states first contribute at the relative order v^4, where the short-distance

*Present address: Department of Physics, Technical University of Munich, 85747 Garching, Germany.

production of a color-octet $c\bar{c}$ pair is followed by dipole transitions to the final quarkonium. The chromo-electric (E1) and chromo-magnetic (M1) dipole transition scale as v^2 (E1) and v^4 (M1), at least for $\alpha_s(m_c v)/\pi \ll 1$ [11]. Dominant color-octet J/ψ production processes at e^+e^- colliders are

$$e^+e^- \to c\bar{c}_8(^1S_0)+g, \tag{3}$$

$$e^+e^- \to c\bar{c}_8(^3P_J)+g, \tag{4}$$

$$e^+e^- \to c\bar{c}_8(^3S_1)+q\bar{q} \quad (q=u,d,s). \tag{5}$$

Processes (3), (4) are enhanced by $1/\alpha_s(m_c)$ relative to processes (1), (2), and (5) is enhanced, at large energies, by a logarithm $\ln(s/m_c^2)$ with respect to process (1) (which, in turn, dominates process (2) by the power s/m_c^2 at large s). Current estimates of the long-distance MEs $\langle \mathcal{O}_{1,8}^{J/\psi}(^{2S+1}L_J)\rangle$ [14], that parametrize the transition from a $c\bar{c}_{1,8}(^{2S+1}L_J)$ state to the J/ψ, are listed, for example, in [11]. At the center-of-mass energy studied by the CLEO experiment at the Cornell Electron Storage Ring, $\sqrt{s}=10.6$ GeV, the cross sections for the color-singlet processes (1) and (2) are 0.20 pb and 0.35 pb, respectively, about one third of the experimental cross section $\sigma=1.65\pm0.25\pm0.33$ pb [16]. Hence there is room for color-octet contributions, which we estimate as 0.50 pb for process (3), 0.60 pb for process (4), and 0.01 pb for process (5).

Conclusions on the presence of color-octet mechanisms would, however, be premature because of large theoretical uncertainties. First, the color-singlet MEs are known to 50% at best, and an uncertainty factor of 2 is certainly not exaggerated for the color-octet MEs. Secondly, there could be large perturbative corrections. And finally, truly relativistic corrections of order v^2 may also be large, cf. direct J/ψ production in hadronic collisions [17].

Therefore it has been suggested [9] to study the energy distribution $d\sigma/dz$ of the J/ψ, where $z=2E_{J/\psi}/\sqrt{s}$. The predicted large color-octet processes (3) and (4) are concentrated near $z=1$ and should thus dominate the upper end point of the z spectrum. A dramatic change should be visible, notably in the polar angular distribution of the J/ψ with respect to the beam axis. Unfortunately, measurements at large z are plagued by a large background from radiative ψ' production [16]. Another potentially useful observable is the J/ψ polarization, which is measurable via the angular distributions of its decay leptons. Partial calculations already exist [6,12], but the measurement is definitely not easy.

We propose the study of the χ_{cJ} production as a means of investigating the importance of color-octet contributions in quarkonium production. Contrary to S-wave quarkonia, the J^{++} χ_{cJ} mesons receive contributions from one of their higher Fock states $|c\bar{c}_8(^3S_1)g\rangle$ already at the leading order in v: the associated E1 transition suppresses the cross section by v^2, but this suppression is compensated because an S-wave operator scales as $1/v^2$ relative to a P-wave operator.

Color-singlet production of χ_{cJ} proceeds through

$$e^+e^- \to c\bar{c}_1(^3P_J)+c\bar{c}. \tag{6}$$

Note that the contribution from

$$e^+e^- \to c\bar{c}_1(^3P_J)+gg \tag{7}$$

is zero for pure photon exchanges in the s channel. Also the Z-exchange contribution is negligible at low energies because it is proportional to s/M_Z^2, and at high energies because process (7) is suppressed by m_c^2/s relative to process (6).

Color-octet contributions to the energy distribution of χ_{cJ} away from $z=1$ arise from Eq. (5) and the processes

$$e^+e^- \to c\bar{c}_8(^3S_1)+c\bar{c}, \tag{8}$$

$$e^+e^- \to c\bar{c}_8(^3S_1)+gg. \tag{9}$$

All three processes possess the same scaling in v and $\alpha_s(m_c)$ as the color-singlet process (6). Two further color-octet processes dominate near the $z=1$ end-point, namely (3) and (4). The process

$$e^+e^- \to c\bar{c}_8(^3S_1)+g \tag{10}$$

is negligible for similar reasons as process (7).

The calculation of processes (5), (6), (8), (9) is standard but tedious. We calculate the distributions in both the energy and the polar angle θ of the χ_{cJ};

$$d^2\sigma/dz\ d\cos\theta = S(z)[1+\alpha(z)]. \tag{11}$$

Details will be presented elsewhere. Here we comment on the relation of our results and previous calculations.

At high energies, $m_c^2/s\to 0$, the cross section for process (6) reduces to

$$\frac{d\sigma}{dz}=2\sigma(e^+e^-\to c\bar{c})\,\frac{1}{m_c^5}\langle \mathcal{O}_1^{\chi_{cJ}}(^3P_J)\rangle D^{(1)}_{c\to c\bar{c}(^3P_J)}(z), \tag{12}$$

where $D^{(1)}_{c\to c\bar{c}(^3P_J)}$ are the partonic color-singlet charm fragmentation functions for $c\to c\bar{c}_1(^3P_J)+c$. Our expressions for these functions agree with [18,19].

The fragmentation limit of process (8) is more involved. Writing it as

$$\frac{d\sigma}{dz}=2\sigma(e^+e^-\to c\bar{c})\,\frac{1}{m_c^3}\langle \mathcal{O}_8^{\chi_{cJ}}(^3S_1)\rangle \hat{D}_8(z), \tag{13}$$

we observe that $\hat{D}_8(z)$ agrees with (3.6) in [19] if we take $\mu^2=z^2(1-z)s$ in the result of [19]. Note that Eq. (13) is not the sum of two fragmentation processes, $e^+e^-\to c\bar{c}$ followed by $c\to\chi_J$ or $\bar{c}\to\chi_J$ and $e^+e^-\to c\bar{c}g$ followed by $g\to\chi_J$, as is the high-energy limit of process (5). We can decompose Eq. (13) as

$$\frac{d\sigma}{dz}=\frac{1}{m_c^3}\langle O_8^{\chi_J}(^3S_1)\rangle\Bigg\{2\sigma_{c\bar{c}}[D^{(8)}_{c\to c\bar{c}(^3S_1)}(z)+R_8(z)]+\int_z^1\frac{d\hat{z}}{\hat{z}}\,\frac{d\sigma_{c\bar{c}g}}{dy}\bigg|_{y=z/\hat{z}}D^{(8)}_{g\to c\bar{c}(^3S_1)}(\hat{z})\Bigg\}, \qquad (14)$$

where $\sigma_{c\bar{c},c\bar{c}g}$ is the cross section for $e^+e^-\to c\bar{c},c\bar{c}g$; the gluon fragmentation function is simply $D^{(8)}_{g\to c\bar{c}(^3S_1)}(z)=\pi\alpha_s\delta(1-z)/24$, and $D^{(8)}_{c\to c\bar{c}(^3S_1)}$ is obtained by appropriately removing the NRQCD ME and changing the color factor in the color-singlet $c\to J/\psi$ fragmentation function given in [5]. Also, $D^{(8)}_{c\to c\bar{c}(^3S_1)}$ coincides with the equal-flavor limit of the $\bar{b}\to\bar{b}c_8(^3S_1)+\bar{c}$ fragmentation function of [18]. However, from Eq. (14) it is clear that $D^{(8)}_{c\to c\bar{c}(^3S_1)}$ is not the color-octet part of the $c\to\chi_{cJ}$ fragmentation function, as conjectured in [18]. Interference terms present in the $c\bar{c}_8(^3S_1)+c\bar{c}$ diagrams for the equal-flavor case (the nonzero remainder term R_8) forbid a simple rescaling of the color-singlet result.

Our cross section for process (9) agrees with [7] if we appropriately replace the color-singlet NRQCD ME and overall color factors by their color-octet counterparts. Finally, our cross section for process (5) reduces to a form that is analogous to the θ-integrated expression in [10].

For our numerical estimates we use $m_c=1.48$ GeV, $\alpha_s=0.28$, and $\alpha_{\mathrm{em}}=0.0075$. At $z<1$ only the rather well-known NRQCD MEs [11] $\langle\mathcal{O}_1^{\chi_{cJ}}(^3P_J)\rangle=0.32\ \mathrm{GeV}^5$ and $\langle\mathcal{O}_8^{\chi_{c1}}(^3S_1)\rangle=9.8\times10^{-3}\ \mathrm{GeV}^3$ are needed. Near $z=1$, two up-to-now unknown MEs enter in the combination

$$M_a^{\chi_{cJ}}=\langle\mathcal{O}_8^{\chi_{cJ}}(^1S_0)\rangle+\frac{a(r,\cos\,\theta)}{m_c^2}\langle\mathcal{O}_8^{\chi_{cJ}}(^3P_0)\rangle, \qquad (15)$$

where $3.6<a<4.7$ at the CLEO c.m. energy. $M_a^{\chi_{cJ}}$ is related to a combination of J/ψ MEs by a velocity scaling: $M_a^{\chi_{cJ}}\sim v^2M_a^{J/\psi}$ (not displaying spin counting factors). Because of large uncertainties in experimental fits of $M_a^{J/\psi}$ [20], we also consider alternative estimates. A velocity scaling requires that $\langle\mathcal{O}_8^{\chi_{cJ}}(^1S_0,{}^3P_0)\rangle\sim v^4\langle\mathcal{O}_8^{\chi_{cJ}}(^3S_1)\rangle$. Moreover, $\langle\mathcal{O}_8^{\chi_{cJ}}(^3P_0)\rangle$ can be estimated from $\langle\mathcal{O}_8^{J/\psi}(^3S_1)\rangle$ and color-singlet MEs by assuming a universal strength for double E1 transitions, without referring to a velocity scaling. As a value consistent with these various estimates we take $M_4^{\chi_{c1}}=0.003\ \mathrm{GeV}^3$. This is at the lower side of the range suggested by hadroproduction data, but, on the other hand, on the high side concerning consistency with B-decay data.

The ratio of the two MEs in Eq. (15) is more difficult to estimate, but given the limited a range, it is of minor importance. Below, we take $\langle\mathcal{O}_8^{\chi_{cJ}}(^1S_0)\rangle\sim(1/6)\langle\mathcal{O}_8^{\chi_{cJ}}(^3P_0)\rangle\ \mathrm{GeV}^{-2}$. We shall in fact find such a clear hierarchy of contributions that conclusions can be drawn despite sizeable uncertainties in the MEs of Eq. (15). In any case, our findings for $z<1$ are not affected at all.

TABLE I. Integrated cross sections and angular coefficient α for $e^+e^-\to\chi_{c1,2}+X$ at $\sqrt{s}=10.58$ GeV. The color-octet χ_{c2} cross sections are a factor 5/3 larger than the corresponding χ_{c1} ones. The cut $z>0.693$ excludes χ_{cJ}'s originating from B-meson decays.

	All z		$z>0.693$	
	σ [fb]	α	σ [fb]	α
$c\bar{c}+c\bar{c}_1(^3P_1)\to\chi_{c1}$	18.1	0.44	15.3	0.49
$c\bar{c}+c\bar{c}_1(^3P_2)\to\chi_{c2}$	8.4	0.10	7.5	0.09
$c\bar{c}+c\bar{c}_8(^3S_1)\to\chi_{c1}$	6.1	0.24	4.0	0.35
$q\bar{q}+c\bar{c}_8(^3S_1)\to\chi_{c1}$	15.6	0.26	11.7	0.34
$gg+c\bar{c}_8(^3S_1)\to\chi_{c1}$	5.5	-0.03	4.2	-0.05
$g+c\bar{c}_8(^1S_0)\to\chi_{c1}$	4.1	1.00	4.1	1.00
$g+c\bar{c}_8(^3P_J)\to\chi_{c1}$	45.6	0.64	45.6	0.64
Total χ_{c1}	95.0	0.47	84.8	0.53
Total χ_{c2}	136.5	0.46	123.4	0.51

The total $\chi_{c1,2}$ cross sections are dominated by color-octet processes, as shown in Table I. The color-singlet parts are two orders of magnitude smaller than the J/ψ cross sections. The color-octet contributions lead to a marked increase. Still, the rates are only about 1/20–1/10 of the J/ψ production rate. This confirms that the χ_{cJ} production in e^+e^- annihilations is indeed very sensitive to the power counting of long-distance effects in quarkonium formation. The nonobservation of χ_c's at CLEO [16] (for integrated luminosities of about 3.1 fb^{-1} on the $\Upsilon(4S)$ and about 1.6 fb^{-1} off the resonance) already causes problems [21] for the CEM, where the $\chi_{cJ}:J/\psi$ ratio is predicted to be the same as was measured in the fixed-target [17] and collider [22] hadroproduction, i.e. about 1 (5/3) for $J=1$ $(J=2)$.

The rates in Table I are for direct production, i.e. excluding the production of χ_{cJ} in the decay of other hadrons. B-meson decay contributions can be removed experimentally, e.g. by a cut on z at CLEO. There is also a contribution from $\psi(2S)$ decays, $\psi(2S)\to\gamma+\chi_{cJ}$. We estimate the χ_{c1} cross sections to be 11 fb for process (1), 19 fb for process (2), 8.3 fb for process (3), 19 fb for process (4), and 0.64 fb for process (5). In total we hence expect about 60 fb (10% less for χ_{c2}), with about one half due to color-octet mechanisms. We conclude that the direct χ_{cJ} production dominates over the indirect one.

Figure 1 shows the energy distribution of the direct χ_{cJ} production. A steep rise at large z signals the importance of the $e^+e^-\to c\bar{c}_8(^3P_J)+g$ process. Signatures for the other color-octet mechanisms are also clearly visible, e.g. the $\chi_{c1}:\chi_{c2}$ ratio close to 1 at $z<1$, as opposed to the ratio <0.5 obtained from color-singlet processes alone.

With a separate measurement of $\chi_{c1,2}$ and with sufficient statistics to determine the double-differential cross section $d^2\sigma/dz\ d\ \cos\,\theta$, the angular distribution parameter $\alpha(z)$ will also serve to identify NRQCD production channels. As shown in Fig. 2, the color-octet value of $\alpha(z)$ for χ_{c1} is significantly lower than the color-singlet value, whereas for χ_{c2} the color-octet value (at large z) is significantly higher than the color-singlet one.

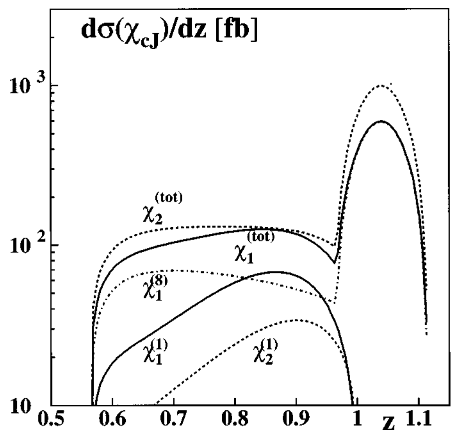

FIG. 1. Energy distribution of $\sigma(e^+e^- \to \chi_{c1,2}+X)$ at $\sqrt{s}=10.58$ GeV: total, color-singlet, and color-octet χ_{c1} contributions, and total and color-singlet χ_{c2} contributions. The peak from the $e^+e^- \to c\bar{c}_8(^1S_0,{}^3P_J)+g$ processes at large z has been smeared over a range of width $\sim v^2$. In a physical process such a smearing follows from the energy transfer in the non-perturbative transition.

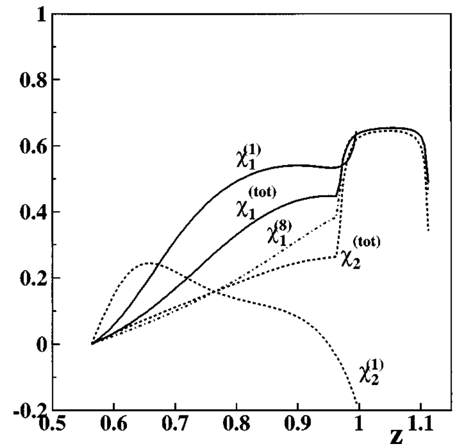

FIG. 2. Energy dependence of the angular coefficient α in $e^+e^- \to \chi_{c1,2}+X$ at $\sqrt{s}=10.58$ GeV: total, color-singlet, and color-octet χ_{c1} contributions, and total and color-singlet χ_{c2} contributions.

In summary, inclusive χ_{cJ} production is a powerful tool to establish the size of higher Fock state contributions in the formation of heavy bound states. Different theoretical approaches predict markedly different cross-section ratios. For example, the $\chi_{c2}:J/\psi$ ratio is as low as 1/100 if only $c\bar{c}$ pairs in the leading Fock state contribute, while it is of order 1 if the bound-state formation proceeds dominantly through (color-singlet or -octet) S-wave $c\bar{c}$ pairs, as it does in the CEM. At all values of z, the velocity-scaling rules of NRQCD yield a ratio of about 1/10, since the color-octet processes induced by higher Fock states are much more pronounced for the χ_{cJ} production than for the J/ψ production. This is partly because color-octet processes enter the χ_{cJ} production at the same order as the color-singlet ones, and partly because the C-parity suppresses the process $e^+e^- \to c\bar{c}gg$, which dominates the J/ψ production.

Signatures for color-octet contributions are also clearly visible in the energy and angular distributions. The expected rates could already be visible at CLEO with current statistics, and will definitely be measured in the near future at CLEO and at B-factories.

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