Effective action of $N=1$ supersymmetric Yang-Mills theory

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(Received 5 January 1998; published 8 June 1998)

We propose a generalization of the Veneziano-Yankielowicz effective low-energy action for $N=1$ SUSY Yang-Mills theory which includes composite operators interpolating pure gluonic bound states. The chiral supermultiplet of anomalies is embedded in a larger three-form multiplet and an extra term in the effective action is introduced. The mass spectrum and mixing of the lowest-spin bound states are studied within the effective Lagrangian approach. The physical mass eigenstates form two multiplets, each containing a scalar, pseudoscalar, and Weyl fermion. The multiplet containing the states which are most closely related to glueballs is the lighter one. [S0556-2821(98)00215-X]

PACS number(s): 11.30.Pb, 11.15.Tk, 12.60.Jv

I. INTRODUCTION

Non-Abelian supersymmetric gauge theories have attracted much attention since tremendous progress was made in understanding the ground state structure of some of those models [1,2]. And yet some puzzles remain. In the present paper we consider the simplest non-Abelian supersymmetric gauge model, $N=1$ SUSY Yang-Mills theory (SYM) with the $SU(N_c)$ gauge group. This model describes interactions of gluons and gluinos. In analogy with QCD, one expects that the spectrum of the model consists of colorless bound states of those fundamental excitations, namely glueballs (gg), gluinoball mesons $(\tilde{g}\tilde{g})$, and ''glueballinos'' $(g\tilde{g})$.

Description of the color singlet bound states in terms of non-Abelian gauge fields is a complicated task. However, one can use the effective field theory technique. Knowing all global symmetries and anomalies of the model, one constructs an effective action in terms of colorless variables. For the case of $N=1$ SUSY Yang-Mills theory, the effective action[1] was constructed by Veneziano and Yankielowicz (VY) [3]. The VY action [3] involves interpolating operators for gluino-gluino and gluino-gluon bound states. However, it does not include composite operators corresponding to pure gluonic composites (glueballs) and attempts to generalize the VY action to include them have failed up to now. This is perplexing since QCD is closely related to SYM. If there were some fundamental impediment to constructing a supersymmetric effective action representing the full set of expected colorless fields (glueballs as well as gluinoball mesons), it could have important implications for our understanding of the QCD spectrum.

In this paper we report that it is possible to extend the VY action in a way that allows interpolating operators corresponding to the gluon-gluon bound states to be included as dynamical variables. In Sec. I we briefly discuss the VY approach and indicate why the interpolating operators of the gluon-gluon bound states are not included in that action. We show, in Sec. II, that the problem can be cured by embedding the chiral multiplet, which is used in the VY construction, into a larger tensor multiplet. Rewriting the VY action in terms of that tensor multiplet, and adding one extra term to the action, one discovers that the low-energy theory includes the interpolating operators of all the lowest-spin bound states which are expected on the basis of the naive ''valence'' construction including $l=0$ and $l=1$ states. The spectrum is of course consistent with the $N=1$ SUSY algebra. In Sec. III we study the mass spectrum and mixing of gluino-gluino, gluon-gluino, and gluon-gluon composites. We briefly compare our results with recent lattice gauge theory calculations of the spectrum of $N=1$ SUSY Yang-Mills theory.

A. The VY effective action

The classical action of $N=1$ SYM theory is invariant under $U(1)_R$, scale, and superconformal transformations. In the quantum theory these symmetries are broken by the chiral, scale, and superconformal anomalies, respectively. Composite operators that appear in the expressions for the anomalies can be thought of as component fields of a chiral supermultiplet S [4]

$$S \equiv A(y)+\sqrt{2}\,\theta\Psi(y)+\theta^2 F(y),$$

where the following notation for the composite operators is introduced:[2]

$$A \equiv \frac{\beta(g)}{2g}\lambda^{\alpha}\lambda_{\alpha}, \quad \sqrt{2}\Psi_{\alpha} \equiv -\frac{\beta(g)}{2g}\{-i\lambda_{\alpha}D + (\sigma^{\mu\nu}\lambda)_{\alpha}G_{\mu\nu}\},$$

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[1]To what extent that action can be thought of as describing particles and can be called the effective action will be discussed in the next section.

[2]We follow conventions of Wess and Bagger [5].

0556-2821/98/58(1)/015009(8)/$15.00

$$F \equiv -\frac{\beta(g)}{g}\left\{-\frac{1}{4}G_{\mu\nu}^2 - \frac{i}{2}\overline{\lambda\sigma\nabla}\lambda + \frac{1}{2}D^2 - \frac{i}{4}G_{\mu\nu}\tilde{G}_{\mu\nu} + \frac{i}{2}\partial_\mu J_\mu^5\right\}. \quad (1)$$

In these expressions $\beta(g)$ stands for the SYM beta function for which the exact expression is known [6], λ_α denotes a two-component gluino field (Weyl spinor), D stands for the D component of the non-Abelian vector superfield, and the normalization of the dual stress-tensor is given by $\tilde{G}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\tau}G^{\lambda\tau}$. All the composite operators in Eq. (1) have zero anomalous dimensions. Thus, they can be treated as interpolating operators for the lowest-spin colorless bound states present in the spectrum of the model. Furthermore, one might argue that the effective action for those bound states can be specified in terms of the S field [3].

Following Ref. [3] one constructs an effective superpotential for the theory. The superpotential which reproduces correctly all the three anomalies of SYM theory is given by the expression [3]

$$W(S) \propto \left(S\log\frac{S}{\mu^3} - S\right) + \text{H.c.} \quad (2)$$

Here μ is the dynamically generated scale of SYM theory: $\mu = \mu_0 e^{-8\pi^2/3N_c g^2}$ where the running coupling g is defined at some scale μ_0.

In order to fix a complete effective Lagrangian description of the lowest-spin states of the theory, one needs to define also an effective Kähler potential in terms of the S field. Since all the anomalies are already taken into account by the superpotential (2), the Kähler potential should in its turn respect the $U(1)_R$, scale and superconformal symmetries. As a simplest expression satisfying those requirements one finds [3]

$$K_{\text{VY}}(S^+,S) \equiv (S^+S)^{1/3}. \quad (3)$$

The most general form of the Kähler potential as a function of S, S^+, and derivatives of these superfields was determined in Ref. [7]. It was shown [7] that the only expression that satisfies chiral, scale, and superconformal Ward identities can be written in the following form:

$$K(S^+,S) \equiv (S^+S)^{1/3} f(\chi,\chi^+),$$

$$\text{where} \quad \chi \equiv S^{1/3}(\bar{D}^2 S^{+1/3})^{-1/2}.$$

Here f stands for an arbitrary function of two variables satisfying the reality condition $f^*(x,y) = f(y,x)$ [7]. However, the Kähler potential K, being combined with the superpotential (2), yields a meaningful theory with a bounded-from-below potential if and only if $f=1$ [7]. Thus, the VY anzatz (3), being the simplest one, turns out to be the only physically acceptable expression which could be combined with the superpotential (2) to define an effective action of the model. Let us stress again that the validity of assertion assumes that the effective action of the theory is defined as a functional of the S (and S^+) field and its higher derivatives only.

Bringing Eqs. (2) and (3) together, one writes down the VY effective action in the following form

$$\text{Action} = \int d^4x\, \frac{1}{\alpha}(S^+S)^{1/3}|_D + \gamma\left[\left(S\log\frac{S}{\mu^3} - S\right)\bigg|_F + \text{H.c.}\right], \quad (4)$$

where the positive constants α and γ are introduced. The value of γ can be fixed explicitly [8]. In our notation $\gamma = -[N_c g/16\pi^2\beta(g)] > 0$.

The axial $U(1)_R$ symmetry is broken by the anomaly to a discrete $\mathbf{Z}_{2N_c}$ symmetry in the $N=1$ $SU(N)$ SYM theory. That $\mathbf{Z}_{2N_c}$ symmetry group is itself broken down to $\mathbf{Z}_2$ due to the nonzero gluino condensate [10]:

$$\langle\lambda\lambda\rangle \propto \mu^3 \; e^{2\pi i k/N_c}, \quad k=0,1,\ldots,N_c-1. \quad (5)$$

Hence we have N_c physically inequivalent vacua, each characterized by its own phase of the gaugino condensate (5). The VY effective action of Eq. (4) describes the theory around one of them (here $k=0$). It was conjectured in Ref. [9] that the theory might contain a new vacuum which is in a $\mathbf{Z}_{2N_c}$ chirally symmetric superconformal phase with zero gaugino condensate. A number of arguments have been given against the existence of such a phase [11,12]. In any case, we concentrate our attention on the conventional phase of the theory with nonzero gluino condensate [10].

At this stage we would like to comment on the physical meaning of the effective action (4). This is not an effective action in the Wilsonian sense (see discussions in Refs. [7,13]). In Ref. [7] the action (4) was constructed as a generating functional of one-particle-irreducible (1PI) Green's functions, an object first introduced in Quantum Field Theory in Ref. [14]. That means that the action (4), being written in terms of composite colorless fields of SYM theory, can be used to calculate various Green's functions of those composite variables.[3] Performing those calculations, however, one is not supposed to take into account diagrams with composite fields being propagating in virtual loops. Loop effects are already included in effective vertices and propagators occurring in the action (4). Thus, all calculations with the expression (4) are to be carried out in the tree level approximation.[4]

The simplest kind of Green's function one might be interested in is a two point correlator. As we mentioned above, the composite operators entering the expression (4) are the

[3]One should keep in mind that actual variables in this action are VEV's of the composite operators calculated in a theory with nonzero external sources for those operators (see for instance discussions in Ref. [7]). For the sake of simplicity of presentation, we denote those VEV's by the corresponding composite fields.

[4]For nonsupersymmetric gluodynamics, analogous actions were constructed in Refs. [15,16] for the CP even sector of the theory, and in Ref. [17] for the CP odd sector of the model.

interpolating fields for the bound states of $N=1$ SYM theory. Thus, a two point correlator (or simply a propagator) of those fields can be used to determine the mass of the corresponding bound state. Hence, the effective action (4) (or more exactly the generating functional of 1PI diagrams) can readily be used to deduce masses of composite bound states of the theory. In what follows we concentrate our attention on this aspect of the effective action approach to $N=1$ SUSY YM theory.

A great advantage of SUSY gauge theories is the fact that all physical states of the model are components of SUSY multiplets. That can be deduced directly from the corresponding SUSY algebra. Based on the $N=1$ SUSY algebra written in terms of generators with a definite space-time parity [18], one expects the following lowest-spin multiplets of spin-parity eigenstates: $[0^{-+}, \frac{1}{2}^{i+}, 0^{++}]$ and $[0^{++}, \frac{1}{2}^{(-i)+}, 0^{-+}]$. In SYM theory one expects these states to be realized as the following composites:

Ia A pseudoscalar, 0^{-+}, $l=0$, $s=0$ gluino-gluino bound state;

Ib A spinor, $\frac{1}{2}^{i+}$, $l=1$, $s=1/2$ gluon-gluino bound state;

Ic A scalar, 0^{++}, $l=1$, $s=1$ gluino-gluino bound state;

IIa A scalar, 0^{++}, $l=0$, $s=0$ gluon-gluon bound state;

IIb A spinor, $\frac{1}{2}^{(-i)+}$, $l=0$, $s=1/2$ gluon-gluino bound state;

IIc A pseudoscalar, 0^{-+}, $l=1$, $s=1$ gluon-gluon bound state.[5]

In general, these states would be assigned to two different supermultiplets. Note that the complex fields A, Ψ, and F, introduced in Eq. (13), form linear combinations of the interpolating operators for the states listed above. So, at least formally, the above bound states are present in the supermultiplet S and, consequently, in the action (4). Hovewer, the gluon-gluon bound states, referred to hereafter as "glueballs," enter the action (4) through the F term of the chiral multiplet. F terms generically appear as auxiliary fields of a model and are usually integrated out. For instance in the VY approach, there is no kinetic term for the F component of the chiral superfield so the F field can be integrated out by means of equations of motion. Having eliminated the F field, one is left with the effective Lagrangian description of the gluino-containing bound states only (the first three states in the list above). Thus no glueballs are present in the VY action.[6]

In general, there is no reason to believe that in SYM theory, the spin-zero glueball states are heavier than the gluino containing mesons. Thus, at least *a priori*, spin-zero glueballs have every right to be considered in the effective action of the model.

One can attempt to construct a new chiral superfield which would include the interpolating operators for glueballs in a lowest supercomponent [20]. An appropriate superfield is proportional to $D^2 S$ [20]. However, R-symmetry arguments do not allow one to introduce a nontrivial coupling of that chiral multiplet to the VY supermultiplet [20]. Another approach is needed.

B. Generalization of the VY effective action

In order to determine how glueballs can be included in the action (4), let us concentrate our attention on the expression for the F field. Using the equation of motion for the gluino field and for the D component one gets[7]

$$F \equiv \frac{\beta(g)}{4g}[G_{\mu\nu}^2 + iG_{\mu\nu}\tilde{G}_{\mu\nu}].$$

Let us introduce the following notation:

$$\Sigma \equiv \frac{\beta(g)}{4g}G_{\mu\nu}^2, \quad Q \equiv \frac{\beta(g)}{4g}G_{\mu\nu}\tilde{G}_{\mu\nu}.$$

Thus, the decomposition of the F field into its real and imaginary parts is $F=\Sigma+iQ$. As we have already mentioned, the F field appears in the VY action without a kinetic term. The term bilinear in the F field is proportional to

$$F^+F=\Sigma^2+Q^2.$$

Besides that, there are terms linear in the F field in the expression for the effective action, thus, the F field can easily be integrated out by means of its algebraic equations of motion [3].

In order to reveal subtleties of this procedure, let us write down the following relation

$$Q=\frac{1}{4!}\varepsilon_{\mu\nu\alpha\beta}H^{\mu\nu\alpha\beta}, \tag{6}$$

where $H^{\mu\nu\alpha\beta}$ is a field strength for a three-form potential $C_{\nu\alpha\beta}$, $H_{\mu\nu\alpha\beta}=\partial_\mu C_{\nu\alpha\beta}-\partial_\nu C_{\mu\alpha\beta}-\partial_\alpha C_{\nu\mu\beta}-\partial_\beta C_{\nu\alpha\mu}$. The $C_{\mu\nu\alpha}$ field itself is defined as a composite operator of colored gluon fields A_μ^a, $C_{\mu\nu\alpha}=[\beta(g)/64g\pi^2](A_\mu^a\overline{\partial_\nu}A_\alpha^a-A_\nu^a\overline{\partial_\mu}A_\alpha^a-A_\alpha^a\overline{\partial_\nu}A_\mu^a+2f_{abc}A_\mu^aA_\nu^bA_\alpha^c)$, with f_{abc} being structure con-

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[5]Though the spin-orbital decomposition is an intrinsically nonrelativistic notion, it can be applicable to relativistic cases (see the discussion in Ref. [19]). A "spin" in that case is defined as the rank of the spinor which describes the corresponding wave function, and an "orbital momentum" is set by the coordinate dependence of the relativistic wave function [19].

[6]When the equations of motion are used the number of degrees of freedom of the fermionic field Ψ also reduces. The off-mass-shell spinor Ψ has four real degrees of freedom, while it propagates only two independent degrees of freedom, when the on-shell condition is imposed. Those two degrees of freedom describe only one (out of two) gluino-gluon bound states given in the list above.

[7]In general, one is not allowed to use the equation of motion if the vacuum expectation value (VEV) of the F field is considered.

stants of the corresponding $SU(N_c)$ gauge group. The right-left derivative in this expression acts as $A\overleftrightarrow{\partial}B \equiv A(\partial B) - (\partial A)B$.[8]

Using these definitions, one finds that the expression bilinear in the F field acquires the following form

$$F^+F = \Sigma^2 - \frac{1}{4!}H^2_{\mu\nu\alpha\beta}.$$

The second term in this expression is a kinetic term for the three-form potential $C_{\mu\nu\alpha}$. As before, the Σ field can be integrated out, however, one should be careful in dealing with the $C_{\mu\nu\alpha}$ field.

In Ref. [17] it was argued that the three-form field $C_{\mu\nu\alpha}$ plays an important role in the description of the pseudoscalar glueball. That glueball can be coupled to the QCD η' meson by means of the $C_{\mu\nu\alpha}$ field [21]. In the case of SYM theory, the analog of the η' meson is the gluino-gluino bound state which acquires mass due to the anomaly in the $U(1)_R$ current within the VY approach. Thus, it is natural to attempt to couple the pseudoscalar glueball to the pseudoscalar gluino-gluino bound state within the VY action using the three-form potential $C_{\mu\nu\alpha}$.

To elaborate this approach, let us rewrite the SUSY transformations for the components of the S superfield in terms of Σ and $C_{\mu\nu\alpha}$ (instead of F and F^+) [22,23]:

$$\delta_\zeta A = \sqrt{2}\zeta\Psi,$$

$$\delta_\zeta \Psi = i\sqrt{2}\sigma^\mu\bar{\zeta}\partial_\mu A + \sqrt{2}\zeta\left(\Sigma + \frac{i}{6}\varepsilon_{\mu\nu\alpha\beta}\partial^\mu C^{\nu\alpha\beta}\right),$$

$$\delta_\zeta \Sigma = \frac{i}{\sqrt{2}}(\overline{\zeta\sigma^\mu}\partial_\mu\Psi + \zeta\sigma^\mu\partial_\mu\bar{\Psi}),$$

$$\delta_\zeta C_{\nu\alpha\beta} = \frac{1}{\sqrt{2}}\varepsilon_{\nu\alpha\beta\mu}(\overline{\zeta\sigma^\mu}\Psi - \zeta\sigma^\mu\bar{\Psi}).$$

The set of fields given above forms an irreducible representation of supersymmetry algebra. All these fields can be assigned to a supermultiplet introduced in Ref. [22]. That supermultiplet is called a constrained three-form supermultiplet [22,24]. The easiest way to present this multiplet is to introduce the following real tensor superfield[9] U

$$\begin{aligned} U = {} & B + i\theta\chi - i\overline{\theta\chi} + \frac{1}{16}\theta^2 A^* \\ & + \frac{1}{16}\bar{\theta}^2 A + \frac{1}{48}\theta\sigma^\mu\bar{\theta}\varepsilon_{\mu\nu\alpha\beta}C^{\nu\alpha\beta} \\ & + \frac{1}{2}\theta^2\bar{\theta}\left(\frac{\sqrt{2}}{8}\bar{\Psi} + \bar{\sigma}^\mu\partial_\mu\chi\right) \\ & + \frac{1}{2}\bar{\theta}^2\theta\left(\frac{\sqrt{2}}{8}\Psi - \sigma^\mu\partial_\mu\bar{\chi}\right) + \frac{1}{4}\theta^2\bar{\theta}^2\left(\frac{1}{4}\Sigma - \partial^2 B\right). \end{aligned} \tag{7}$$

It is a matter of a straightforward calculation to check that the real superfield U satisfies the relation[10]

$$\bar{D}^2 U = -\tfrac{1}{4}S. \tag{8}$$

Thus, the real tensor multiplet U, defined by the expression (8), includes all the components of the chiral supermultiplet S. Besides that the multiplet has also an additional scalar B and fermion χ. Thus, using the relation (8) the VY action can be rewritten in terms of the bigger multiplet U. We will show below that this allows one to include glueball operators in the effective action.[11]

First, let us notice some features of SUSY transformations of the components of the U field. The components which are shared by the tensor multiplet U and the chiral multiplet S (namely A, Ψ, Σ, and C) transform among themselves, while other fields (B and χ) are connected by SUSY rotations to the other four components. Furthermore, one can define a ''gauge'' transformation of the U field as the following shift $U \to U + Y$, where the superfield Y satisfies the relation $\bar{D}^2 Y = 0$. It is important to notice that by means of this ''gauge'' transformation, one can get rid of the B and χ fields in the expression for the U multiplet. This is the analog of the Wess-Zumino gauge for the tensor multiplet U. Thus, any Lagrangian written in terms of the S field only, if reexpressed in terms of the U field, is necessarily invariant under the ''gauge'' transformation defined above. As a result, the B and χ fields can always be ''gauged'' away from that Lagrangian. Thus in order to be able to retain the B and χ fields

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[8]The quantity Q can also be expressed through the Chern-Simons current K_μ as $Q = \partial_\mu K_\mu$. Using this equation one can deduce the relation between the Chern-Simons current and the three-form potential $C_{\nu\alpha\beta}$, these two quantities are Hodge dual to each other: $K^\mu = (1/3!)\varepsilon^{\mu\nu\alpha\beta}C_{\nu\alpha\beta}$.

[9]In this discussion we follow the conventions of Ref. [23].

[10]Despite a seeming similarity, the tensor multiplet U should not be interpreted as a usual vector multiplet. The vector field which might be introduced in this approach as a Hodge dual of the three-form potential $C_{\mu\nu\alpha}$ would give mass terms with the wrong sign in our approach (see Sec. II), thus, the actual physical variable is the three-form potential $C_{\mu\nu\alpha}$ rather than its dual vector field (the Chern-Simons current).

[11]This representation of SUSY was earlier used in the context of SYM theory in Ref. [25] to study the phenomenon of gaugino condensation with a field dependent gauge coupling. We thank J.-P. Derendinger for bringing Ref. [25] to our attention. See also discussions in Ref. [26].

as dynamical variables, one must include terms in the Lagrangian which breaks this ''gauge'' invariance. The simplest term of this type is the quadratic term $(U^2)|_D$. Once such a term is included in the Lagrangian, the ''gauge'' symmetry becomes explicitly broken and the B and χ components of the superfield U survive as dynamical variables.

Let us now apply the U field formalism to the VY action. In the case at hand, the chiral symmetry is spontaneously broken by the gluino condensate. In terms of the chiral superfield, this corresponds to the existence of a nonzero VEV of the S field

$$\langle S\rangle=\mu^3.$$

With that in mind the appropriate relation between the U field and the chiral multiplet is

$$\bar{D}^2U=-\tfrac{1}{4}(S-\langle S\rangle). \tag{9}$$

The only result of this modification is that the field A in Eq. (7) gets replaced by the quantity $A-\langle A\rangle$.

Now use the relation (9) to rewrite the action (4) in terms of the U field. In order to break the ''gauge'' invariance of the VY action, we add a term proportional to U^2 to the VY Lagrangian. An appropriate term with zero R-charge and correct dimensionality is

$$\left(-\frac{U^2}{(S^+S)^{1/3}}\right)\Bigg|_D. \tag{10}$$

Below, we are going to show that once this term is added to the VY action (4), the following fields become dynamical.

The B field propagates and it represents one massive real scalar degree of freedom (identified later with the scalar glueball).

The three-form potential $C_{\mu\nu\alpha}$, which becomes massive, also propagates. It represents one physical degree of freedom (identified with the pseudoscalar glueball).

The complex field A, being decomposed into parity eigenstates, describes the massive gluino-gluino scalar and pseudoscalar mesons.

χ and Ψ describe the massive gluino-gluon fermionic bound states.

Relations between masses of these states will be given in the next section.

C. The mass spectrum

Based on the arguments given in the previous section, one can write down the effective Lagrangian for the lowest-spin multiplets of the $N=1$ SUSY YM theory in the following form

$$\mathcal{L}=\frac{1}{\alpha}(S^+S)^{1/3}|_D+\gamma\left[\left(S\log\frac{S}{\mu^3}-S\right)\Bigg|_F+\text{H.c.}\right]+\frac{1}{\delta}\left(-\frac{U^2}{(S^+S)^{1/3}}\right)\Bigg|_D, \tag{11}$$

where α and δ are arbitrary positive constants. One obtains the VY Lagrangian in the limit $\delta\to\infty$. In general, higher powers of U (and derivatives) can also be added to this Lagrangian. Those terms would introduce new quartic, quintic, and other higher interaction terms. However, the quadratic part of the action which defines masses will not be affected. In that respect, the effective Lagrangian (11) can be treated as the one describing small perturbations of fields about a vacuum state.[12]

Let us determine the SUSY vacuum state defined by the Lagrangian (11). The potential for the model is a complicated function of the variables present in the U superfield. After integration over the auxiliary Σ field, the bosonic part of the potential is

$$V=\frac{2}{\delta(16)^2}\frac{|\phi|^6+\mu^6-2\mu^3|\phi|^3\cos 3\rho}{|\phi|^2}+\frac{3}{\delta(48)^2}\frac{C^2_{\mu\nu\tau}}{|\phi|^2}+\frac{9\alpha|\phi|^4}{4}\frac{1}{1-(\alpha/\delta)(B^2/|\phi|)^4}\times\left[\frac{B}{24\delta|\phi|^2}\left(1+\frac{2\mu^3}{|\phi|^3}\cos 3\rho\right)-3\gamma\log\frac{|\phi|^2}{\mu^2}\right]^2, \tag{12}$$

where we introduced the notations $\phi\equiv A^{1/3}$ and $\rho\equiv\arg\phi$.

In order to find the vacuum state, one should find the absolute minimum of the potential (12). Since we are dealing with a supersymmetric model, the value of the potential in that minimum has to be zero. As a result of Lorentz invariance, the VEV of the three-form field is zero, i.e. $\langle C_{\mu\nu\tau}\rangle=0$. The VEV of Q is also zero due to the CP invariance of the model. Further calculations are tedious and we will not present them here. After some algebra one finds that the only global, CP invariant minimum of the potential (12) is given by: $\langle\phi\rangle=\mu$, $\langle B\rangle=\langle C\rangle=\langle\rho\rangle=0$. The effective Lagrangian (11) describes small perturbations of fields about the vacuum state defined by these VEV's.

We would like to make a comment here. It deals with the region of validity of the potential (12) [i.e., of the Lagrangian (11)]. The expression (12) can be a correct potential for a supersymmetric model if $\delta|\phi|^4>\alpha B^2$. Since the fields ϕ and B describe perturbations about the values $\phi=\mu$ and $B=0$, respectively, the inequality above is satisfied for small perturbations of both fields. The singularity in the potential at $\delta|\phi|^4=\alpha B^2$ indicates that for large perturbations, the higher dimensional terms omitted in Eq. (11) should become important. As we mentioned above, we are mainly concerned with the mass spectrum of the model which can be studied using small perturbations about the ground state, so that the approximation given in Eq. (11) is good enough for our goals.[13]

[12]Notice that one can write the Lagrangian (11) in terms of the U field alone. However, for clarity of presentation, we leave the VY part of the Lagrangian in the original form.

[13]The potential (12) describes a SUSY minimum in the field space. The potential well has infinitely high walls, at values of the fields satisfying $\delta|\phi|^4=\alpha B^2$.

The mass spectrum for the bosonic part of the model can be calculated from the potential (12). However, we find it more illuminating to write down the quadratic part of the Lagrangian (11). Then physical masses can be computed as eigenvalues of the corresponding quadratic forms.

Let us rewrite the expression for the Lagrangian (11) in terms of component fields keeping only the quadratic terms:

$$\mathcal{L}=-\frac{1}{2}\partial_\mu s\partial_\mu s-\frac{1}{2}\partial_\mu p\partial_\mu p+\frac{1}{6}C_{\mu\nu\tau}\partial^2 C^{\mu\nu\tau}$$
$$-\frac{1}{2}\partial_\mu B\partial_\mu B+i\partial_\mu\overline{\zeta\sigma}^\mu\zeta+i\partial_\mu\overline{\chi\sigma}^\mu\chi+\Sigma^2-\frac{3}{8}\sqrt{\frac{\alpha}{\delta}}\mu\Sigma B$$
$$-\frac{9\alpha}{\delta(16)^2}\mu^2 s^2-\frac{9\alpha}{\delta(16)^2}\mu^2 p^2-\frac{27\alpha}{(48)^2\delta}\mu^2 C^2_{\mu\nu\tau}$$
$$-\frac{9\alpha\gamma}{2}\mu\zeta^2-\frac{9\alpha\gamma}{2}\mu\overline{\zeta}^2$$
$$-\frac{i3\sqrt{2}}{16}\sqrt{\frac{\alpha}{\delta}}\mu\chi\zeta+\frac{i3\sqrt{2}}{16}\sqrt{\frac{\alpha}{\delta}}\mu\overline{\chi\zeta}$$
$$+9\sqrt{2}\alpha\gamma\mu s\Sigma-9\sqrt{2}\alpha\gamma\mu pQ+\cdots\ , \tag{13}$$

where dots stand for cubic and higher dimensional interaction terms. The fields which appear in Eq. (13) are related to the original fields of Eq. (7)

$$A^{1/3}=\phi\equiv\mu+\sqrt{\frac{\alpha}{2}}(s+ip),\qquad \zeta\equiv\frac{\Psi}{3\sqrt{\alpha}A^{2/3}}.$$

Besides that, in Eq. (13) we performed the following rescaling of the variables

$$\Sigma\to 3\sqrt{\alpha}\mu^2\Sigma,\qquad C\to 3\sqrt{\alpha}\mu^2 C,$$
$$\chi\to\sqrt{\delta}\mu\chi,\qquad B\to\sqrt{\delta}\mu B. \tag{14}$$

The s and p fields, as they stand in the Lagrangian (13), describe, respectively, the scalar and pseudoscalar gluino-gluino excitations. The C field, being a massive three-form potential that propagates one physical degree of freedom, describes the pseudoscalar glueball. The Σ field can be integrated out by means of the equation of motion:

$$\Sigma=\frac{3}{16}\sqrt{\frac{\alpha}{\delta}}\mu B-\frac{9\sqrt{2}}{2}\alpha\gamma\mu s. \tag{15}$$

In accordance with this expression, the B field which is left in the effective Lagrangian describes a mixed state of the scalar glueball (former Σ field) and the scalar gluino-gluino bound state s.

If the VY superpotential were neglected for some reason in the expression (11), then the two Weyl fermion fields χ and ζ would combine together to form one Dirac massive bispinor. However, the presence of the VY superpotential yields an additional contribution to the mass term of the ζ field. Thus χ and ζ cannot be treated as the components of one Dirac spinor. Instead one is left with two Weyl fermions describing two different spin 1/2 massive states. In general, the form of the bilinear terms in the Lagrangian (13) suggests that all the physical states of this theory should occur as mixed states of the initially pure bound states of the Lagrangian (13).

Below, we deduce the masses of these mixed physical states. Let us write down the mass and mixing terms of the Lagrangian (13) separately. Substituting the expression for the Σ field (15) into the Lagrangian (13), one finds the following pairs of variables being mixed with one another:

$$B-s\ \ \text{system:}\ \ \frac{9\alpha}{\delta(16)^2}\mu^2 s^2+\frac{81}{2}\alpha^2\gamma^2\mu^2 s^2+\frac{9\alpha}{\delta(16)^2}\mu^2 B^2$$
$$-\frac{27\sqrt{2}}{16}\sqrt{\frac{\alpha}{\delta}}\alpha\gamma\mu^2 Bs;$$
$$C-p\ \ \text{system:}\ \ \frac{9\alpha}{\delta(16)^2}\mu^2 p^2+\frac{27\alpha}{(48)^2\delta}\mu^2 C^2_{\mu\nu\tau}$$
$$+\frac{9\sqrt{2}}{6}\alpha\gamma\mu p\varepsilon_{\mu\nu\tau\sigma}\partial^\mu C^{\nu\tau\sigma};$$
$$\chi-\zeta\ \ \text{system:}\ \ \frac{9\alpha\gamma}{2}\mu\zeta^2+\frac{9\alpha\gamma}{2}\mu\overline{\zeta}^2+\frac{i3\sqrt{2}}{16}\sqrt{\frac{\alpha}{\delta}}\mu\chi\zeta$$
$$-i3\sqrt{\frac{2}{16}}\sqrt{\frac{\alpha}{\delta}}\mu\overline{\chi\zeta}. \tag{16}$$

In order to find the physical masses, one must diagonalize the corresponding mass matrices. Concentrate, for instance, on the first row of these expressions which describes the mixed state of scalar meson, s, and scalar meson-glueball, B. The former gets mass both from the superpotential and U^2-term, while the latter gets mass only from the U^2-term. When the mixing term is switched on, the initially heavier state (s) gets even heavier, and the initially lighter state (B) becomes even lighter than they were originally. Performing the diagonalization, one finds that the physical eigenstates are mixed states with the following mass eigenvalues:

$$\frac{1}{2}m^2_{\mathrm{H}}=\frac{9\alpha}{\delta(16)^2}\mu^2+\frac{81}{4}\alpha^2\gamma^2\mu^2$$
$$+\frac{81}{4}\alpha^2\gamma^2\mu^2\sqrt{1+\frac{1}{288}\frac{\alpha}{\delta}\frac{1}{(\alpha\gamma)^2}}, \tag{17}$$

and

$$\frac{1}{2}m_{\mathrm{L}}^2=\frac{9\alpha}{\delta(16)^2}\mu^2+\frac{81}{4}\alpha^2\gamma^2\mu^2 - \frac{81}{4}\alpha^2\gamma^2\mu^2\sqrt{1+\frac{1}{288}\frac{\alpha}{\delta}\frac{1}{(\alpha\gamma)^2}}. \quad (18)$$

Here, the subscript ''H'' refers to the heavier state $\tilde{s}$ which, without mixing, would have been a pure gluino-gluino bound state (the s particle). ''L'' refers to the lighter state $\tilde{B}$ (B in the absence of mixing).

Explicit calculation shows that the $C-p$ and $\chi-\zeta$ systems possess exactly the same properties. Namely, the heavier mass eigenstates $\tilde{p}$, $\tilde{\zeta}$ acquire the mass squared eigenvalues given by the expression for m_{H}^2, and the lighter eigenstates $\tilde{C}$, $\tilde{\chi}$ have mass squared eigenvalues equal to m_{L}^2. Thus, as expected, the physical states form two multiplets, one with mass m_{H}^2 and the other with mass m_{L}^2.

Let us discuss various limits of Eqs. (17) and (18). Suppose $\alpha\to 0$. In that limit the superpotential and the U^2 terms can be neglected in the Lagrangian (11). The Kähler potential, which would be the only term left in the expression (11), would yield only kinetic terms for the excitations. Thus all those states would be massless. This is in agreement with the expressions (17) and (18) which turn into zero as $\alpha\to 0$.

In the $\gamma\to 0$ limit the superpotential disappears. Thus, the mass terms for the physical states come only from the U^2 term in the Lagrangian (11). As a result, all the masses are expected to be equal and there is no mixing between pure gluonic and fermionic states described above. Also, as noted earlier, the χ and ζ Weyl fermions come together to form one massive Dirac bispinor. The limit $\gamma\to 0$ is physically equivalent to the limit $\delta\to 0$; in both cases the superpotential can be neglected in comparison with the U^2 term.

Finally, let us consider the $\delta\to\infty$ limit. One can neglect the U^2 term in that case. As a result one rederives the VY Lagrangian with the spectrum given by the second term in Eq. (17) [or Eq. (18)] multiplied by the factor of two. No glueballs are incorporated in the effective theory in that limit.

II. SUMMARY AND DISCUSSION

We have shown here how to generalize the Veneziano-Yankielowicz effective action for $N=1$ supersymmetric Yang Mills theory to include composite operators corresponding to pure gluonic, and not exclusively gluino-containing, bound states. We accomplish this by embedding the chiral multiplet of anomalies into a larger three-form tensor supermultiplet and adding an extra term to the VY effective action. The extra term is necessary in order to retain the variables corresponding to glueballs as dynamical fields in the effective Lagrangian.

Studying the potential of the model, we find that the physical eigenstates fall into the two different ''multiplets'' with masses given by Eqs. (17) and (18). Neither of them contain pure gluino-gluino, gluino-gluon, or gluon-gluon bound states. Instead, the physical excitations are mixed states of these composites. The heavier set of states contains a pseudoscalar meson, which without mixing reduces to the 0^{-+} gluino-gluino bound state (the analog of the QCD η' meson), a scalar meson that without mixing is an $l=1$ 0^{++} gluino-gluino excitation, and a mixed fermionic gluino-gluon bound state.

These heavier states become the chiral supermultiplet described by the VY action in the limit that the additional term we have added to the effective Lagrangian is removed. The new states which appear as a result of our generalization forms a lighter multiplet: a scalar meson, which for small mixing becomes a 0^{++} $(l=0)$ glueball; a pseudoscalar state, which for small mixing is identified as a 0^{-+} $(l=1)$ glueball; a mixed fermionic gluino-gluon bound state.

We call the reader's attention to an interesting feature of the effective action introduced here. Although the physical states fall into multiplets whose J^P quantum numbers correspond to two chiral supermultiplets, the action is not written in terms of two chiral supermultiplets. Another representation of SUSY is used, as explained in Sec. II. In particular, the pseudoscalar glueball in this approach is described by the only physical component of the massive three-form potential $C_{\mu\nu\alpha}$. The field strength of that potential couples to the pseudoscalar gluino-gluino bound state as it would couple to the η' meson in QCD.

Masses of the physical states depend on two independent mass parameters [$(\alpha/\delta)\mu$ and $\alpha\gamma\mu$] formed from constants occurring in the effective Lagrangian. However the result that the lighter multiplet contains the predominantly glueball excitation is independent of the values of these parameters. This prediction receives some support from recent lattice studies of SUSY YM theory. Although the lattice results for the mass spectrum are available only away from the SUSY point, the general tendency is that a state which is mostly a glueball is lighter than the state which is mostly a gluino-gluino composite[14] [27]. More detailed lattice results would in principle permit fixing the combinations α/δ and $\alpha\gamma$ which are undetermined within the effective Lagrangian approach. However determining the masses of the two multiplets is likely to be difficult: if mixing is small, they are nearly degenerate and if mixing is large, they are in general both excited by a given source.

ACKNOWLEDGMENTS

The authors are grateful to J. Gates for emphasizing the possible importance of nonstandard representations of SUSY, one of which proved essential in our construction. The work of M.S. was supported by the NSF grant PHY-94-23002.

[14]We refrain from using quenched approximation results [28] to obtain information on the supersymmetric theory because in SUSY Yang-Mills theory all composite masses are proportional to the gluino condensate, as can be seen explicitly from our results. In the absence of dynamical gluinos, masses can only be proportional to the SUSY breaking condensate $\langle G_{\mu\nu}G^{\mu\nu}\rangle$.

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