

Unified explanation of the solar and atmospheric neutrino puzzles in a supersymmetric SO(10) model

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It was recently suggested that in a class of supersymmetric SO(10) models with Higgs multiplets in **10**, and a single **126**+ $\overline{\mathbf{126}}$ representation, if the $\overline{\mathbf{126}}$ contributes both to the right handed neutrino masses as well as to the charged fermion masses, one can have a complete prediction of the neutrino masses and mixings. It turns out that if one chooses only one **10**, there are no regions in the parameter space where one can have a large ν_μ - ν_τ mixing angle necessary to solve the atmospheric neutrino deficit while at the same time solving the solar neutrino puzzle via the $\nu_e \leftrightarrow \nu_\mu$ oscillation. We show that this problem can be solved in a particular class of SO(10) models with a pair of **10** multiplets if we include the additional left-handed triplet contribution to the light neutrino mass matrix. This model cannot reproduce the mass and mixing parameters required to explain the Liquid Scintillation Neutrino Detector observations nor does it have neutrino hot dark matter. [S0556-2821(98)01713-5]

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Strong indications in favor of nonvanishing neutrino masses are emerging from several experiments: (i) the deficits of solar neutrino flux observed by the four solar neutrino experiments Homestake, Kamiokande, SuperKamiokande, SAGE, and GALLEX [1] compared to the standard solar model calculations [2] can be understood if neutrinos are massive and the electron neutrinos emitted by the Sun oscillate to another neutrino species; and (ii) the atmospheric muon neutrino deficits observed earlier by Kamiokande, IMB, and Soudan II [3] experiments and confirmed recently by Super Kamiokande can be understood if ν_μ oscillates similarly. The Liquid Scintillation Neutrino Detector (LSND) [4] results have provided the first laboratory indication of $\bar{\nu}_\mu \leftrightarrow \bar{\nu}_e$ oscillation and, if confirmed by KARMEN [5], would seal the case for nonzero neutrino masses, in an unequivocal manner.

As is well known, the solar neutrino deficit can be explained in terms of the matter induced resonant Mikheyev-Smirnov-Wolfenstein (MSW) oscillation [6] for two choices of masses and mixing angles [7]. Our interest here is in the so-called small angle solution for which $\Delta m_{e\mu}^2 \simeq (0.3-1.0) \times 10^{-5} \text{ eV}^2$ and $2 \times 10^{-3} \leq \sin^2 2\theta_{e\mu} \leq 2 \times 10^{-2}$; The atmospheric neutrino deficit could be due to either $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_\mu \leftrightarrow \nu_e$ oscillation. Preliminary indications from the electron energy distribution in SuperKamiokande favors $\nu_\mu \leftrightarrow \nu_\tau$ oscillation. Similarly a preliminary fit to all the atmospheric neutrino data (sub-GeV, multi-GeV including the zenith angle dependence) seems to require $2 \times 10^{-4} \leq m_{\mu\tau}^2 (\text{eV}^2) \leq 10^{-2}$ with $\sin^2 2\theta_{\mu\tau} \simeq 0.6-1.0$ [8]. Note the hierarchical pattern of mass differences. The LSND results require that $0.3 \text{ eV}^2 \leq \Delta m_{e\mu}^2 \leq 10 \text{ eV}^2$ with the mixing angle in the few percent range. If we accept the above results, it is clear that with only three neutrinos, it is not possible to explain the three results (i.e., solar, atmospheric, and LSND) simultaneously. Therefore within conventional grand unified theories with three generations, one may hope to understand only two of the above results. Furthermore, since hierarchical

mass patterns for neutrinos is a generic feature of theories that implement the seesaw mechanism [9], the solar and the atmospheric neutrino data appear more amenable to theoretical understanding in simple models.

It is the goal of this paper to present a simple grand unified theory (GUT) that leads to the supersymmetric standard model at low energies and predict Δm^2 and $\sin^2 2\theta$ values in the above range for $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$ sectors so that we have a theoretical understanding of the solar and the atmospheric neutrino data. We believe this result to be significant since we do not use any extra fermions nor any extra symmetries for the purpose.

The simplest GUT that leads naturally to small neutrino masses via the seesaw mechanism [9] is the SO(10) model where the local $B-L$ symmetry is broken by the **126**+ $\overline{\mathbf{126}}$ representation. It also has another attractive feature that it leads to automatic R -parity conservation so that unwanted (and uncontrolled) baryon violating interactions of the minimal supersymmetric standard model (MSSM) are forbidden and one obtains a stable lightest supersymmetric particle (LSP) which can act as the cold dark matter of the universe. The minimal set of Higgs multiplets needed to break all gauge symmetries of the theory while keeping supersymmetry unbroken down to the weak scale is **45**+**54** (denoted by A and S), **126**+ $\overline{\mathbf{126}}$ (denoted by Δ and $\overline{\Delta}$), and a single **10**, denoted by H .

It was shown, sometime ago [10,11] that in this minimal model, all Yukawa couplings and Higgs vevs responsible for fermion masses and mixings (a total of twelve parameters in all in the absence of CP violation) are completely determined by the quark and lepton masses and the quark Cabibbo-Kobayashi-Maskawa (CKM) angles. As a result the light and heavy Majorana mass matrices for the neutrinos are completely determined except for the overall scale ν_R , the scale of $B-L$ symmetry breaking, provided one assumes the simple seesaw formula (to be called type-I seesaw formula [9])

$$M_\nu = -M_\nu^D M_{N_R}^{-1} [M_\nu^D]^T. \quad (1)$$

This enables a complete prediction of neutrino mixing angles and any two neutrino mass ratios. Every choice for the signs of the various charged fermion masses lead to distinct scenarios and separate predictions. It was found that there were predictions that could accommodate only small angle MSW solution to the solar neutrino puzzle but not the atmospheric neutrino puzzle. The reason was that the maximum value for $\sin^2 2\theta_{\mu\tau}$ mixing angle predicted by this model was less than 0.3 or so, where as the present 99% confidence level fits seem to require $\sin^2 2\theta_{\mu\tau} \approx 0.60$ or higher [8].

One may try to take advantage of the fact that in most left-right and SO(10) models, a generalized seesaw formula for neutrino masses holds [12] (to be called the type-II seesaw formula):

$$M_\nu = f v_L - M_\nu^D M_{N_R}^{-1} [M_\nu^D]^T \quad (2)$$

(where $v_L \approx \lambda V_{wk}/v_R$ and is induced as long as there are **54**-dimensional Higgs multiplets in the theory) and see if it is possible to obtain larger values for $\sin^2 2\theta_{\mu\tau}$. Such models would have two free parameters, v_R and v_L . In supersymmetric (SUSY) GUT models, coupling constant unification including threshold corrections puts v_R from 10^{13} to 10^{15} GeV range leading to $v_L \approx 10^{-2}$ to 1 eV for $\lambda \approx 1$. The two terms then give comparable contributions and we have two parameters $\{v_L, v_R\}$ that determine the neutrino masses and mixings. We have made an extensive numerical analysis of the predictions of this model for neutrino masses and were unable to find any reasonable values of v_L and v_R which can accommodate both the small angle MSW solution to the solar neutrino problem as well as the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation solution to the atmospheric neutrino puzzle. Thus neutrino experiments may play the role similar to the role that proton decay experiments played in ruling out minimal nonsupersymmetric SU(5) model.

We therefore are led to consider a slight generalization of the above idea and consider an SO(10) model with two **10**-dimensional Higgs multiplets instead of one as in the minimal model. The rest of the field content is the same. The remainder of the paper will be devoted to studying the neutrino masses in this model. The low-energy theory in this model is the MSSM with the Higgs doublets in general being linear combinations of the doublets in the **10**'s (denoted $H_{1,2}$) and the **126**. We will assume the following specific form for them:

$$\begin{aligned} H_u &= \alpha_1 H_u(\mathbf{10}_1) + \alpha_2 H_u(\overline{\mathbf{126}}) + \alpha_3 H_u(\mathbf{126}), \\ H_d &= \beta_1 H_d(\mathbf{10}_2) + \beta_2 H_d(\overline{\mathbf{126}}) + \beta_3 H_d(\mathbf{126}). \end{aligned} \quad (3)$$

How the light doublets arise with this specific form is of course related to the difficult problem of doublet-triplet splitting in SO(10) models which is not addressed here.

Let us now discuss how the neutrino mixing angles can be extracted from this model. The first point is that the most general Yukawa superpotential of the model given by

$$W_Y = h_{i,ab} \psi_a \psi_b H_i + f_{ab} \psi_a \psi_b \overline{\Delta}, \quad (4)$$

where ψ_a ($a=1,3$) represent the **16**-dimensional spinors corresponding to the three family of fermions. Since SO(10) symmetry implies that h_i and f are symmetric matrices, (we ignore CP violation from Yukawa sector), we can diagonalize any one of them and we have fifteen free Yukawa coupling parameters in terms of which the fermion masses and mixings are expressed as follows:

$$\begin{aligned} M_u &= h_1 v_u + f \kappa_u, & M_d &= h_2 v_d + f \kappa_d, \\ M_{\nu^D} &= h v_u - 3f \kappa_u, & M_l &= h_2 v_d - 3f \kappa_d. \end{aligned} \quad (5)$$

Using these relations, we find that at the GUT scale, we have

$$M_{\nu^D} = r_1 (M_l - M_d) + M_u, \quad (6)$$

$$M_{\nu_L \nu_L} = r_2 (M_d - M_l), \quad (7)$$

$$M_{N_R} = r_3 (M_d - M_l), \quad (8)$$

where $r_1 = \kappa_u/\kappa_d$, $r_2 = v_L/4\kappa_d$, and $r_3 = v_R/4\kappa_d$ and $M_{\nu_L \nu_L}$ is assumed to denote the $f v_L$ contribution to the neutrino mass matrix. From the above equations it is clear that we need to supply six parameters to determine the neutrino masses and mixings and they are the three mixing angles in the charged lepton mass matrix and r_i ($i=1,2,3$). We demand that the three charged lepton mixing angles are zero. We then scan the parameter space for r_i to see if any desirable solution exists.

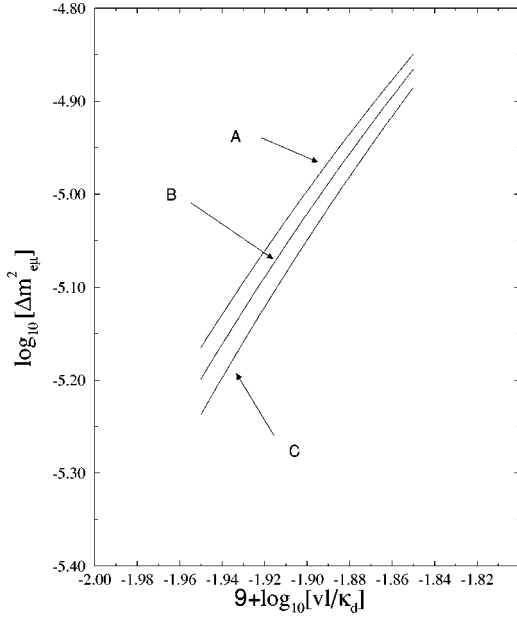
To proceed with this program, first note that the above relations between fermion masses hold at the GUT scale. So, we extrapolate the observed values of quark and lepton masses to the GUT scale, using simple analytic formulas given by Naculich [14]. We work in a basis where M_u is diagonal and $M_d = V_{ckm} D_d V_{ckm}^\dagger$. At the GUT scale the diagonalized values for the masses in GeV and the values of the angles are

$$\begin{aligned} m_u &= 0.0011, & m_c &= -0.3785, & m_t &= -112.34, \\ m_d &= 0.00131, & m_s &= 0.0148, & m_b &= -1.177, \\ m_e &= 0.0003, & m_\mu &= -0.0699, & m_\tau &= 1.183, \\ s_{12} &= -0.2210, & s_{13} &= 0.0040, & s_{23} &= 0.0310, \end{aligned}$$

where s_{12} is the Cabbibo angle, s_{13} and s_{23} are roughly the V_{ub} and V_{cb} elements of V_{ckm} .

In the basis we are working, M_l is diagonal. Furthermore, since the signs of the fermion masses are arbitrary, we choose a basis where the various fermion masses have the signs as given above. We then use Eqs. (6) and (8) for each of the cases, to obtain the neutrino masses and mixing angles.

Note that we still need to know $v_L \equiv 4r_2 \kappa_d$ and $v_R \equiv 4r_3 \kappa_d$. One can use theoretical arguments for the orders of magnitude of the parameters v_L and v_R that are plausible. Note, for instance that since the value of the induced vacuum


 FIG. 1. Predicted $\Delta m_{e\mu}^2$ for various v_L and v_R .

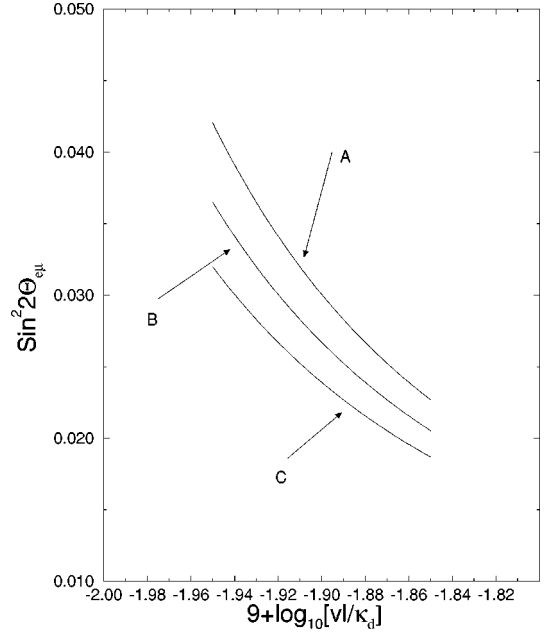
expectancy value (VEV) $v_L \approx \kappa^2/v_R$, for v_R in the range of $10^{13} - 10^{16}$ GeV, $v_L \approx 1 - 10^{-2}$ eV is quite reasonable. One way to determine v_R is to use the unification constraint as it applies to the minimal model. We assume that the theory below the v_R scale is the minimal supersymmetric standard model (MSSM). Since different choices of the particle spectrum above the intermediate scale give different values of v_R , we use another method to constrain this parameter.

Baryogenesis constraints on the scale v_R . A very simple mechanism for baryogenesis in SO(10) models is to generate a lepton asymmetry at a high temperature via the decays of the right-handed Majorana neutrinos and have this lepton asymmetry converted to a baryon asymmetry [15] by the sphaleron processes. An important necessary condition for this to happen is that at-least one of the right handed Majorana neutrinos must have a decay rate that is slower than the expansion rate of the universe when $T \approx M_N$. The general formulas are

$$\Gamma_{N_a} \approx \frac{\sum_b h_{i,ab}^2 + \alpha f_{ab}^2}{8\pi} M_{N_a} \leq 1.73(g^*)^{1/2} \frac{M_{N_a}^2}{M_{Pl}}. \quad (9)$$

Since in our model the Yukawa couplings are all predicted in terms of VEV's v_u , v_d , and κ_d , we can obtain a lower bound on M_{N_i} if we know the VEV's and using the predicted value for the f matrix, we can then deduce v_R . Since our analysis is independent of v_u and v_d , the only constraint on them is that $\sqrt{v_u^2 + v_d^2 + \kappa_u^2 + \kappa_d^2} = 246$ GeV. Using the fact that we have chosen $r_1 = 40.3$ (see below), we can get $\kappa_d \approx 1$ GeV. Using them, we find that $v_R \geq 10^{14}$ GeV.

Prediction for neutrino masses and mixings. Using MATHEMATICA we have scanned over all possible choices for the signs of the charged fermions to see if there is a prediction that fits the requirements of both the solar and the atmo-


 FIG. 2. Predicted $\sin^2 2\Theta_{e\mu}$ for various v_L and v_R .

spheric neutrino puzzles. In Figs. 1 and 2 we we plot the $\Delta m_{e\mu}^2$ and $\sin^2 2\Theta_{e\mu}$ as functions of v_L and v_R . In Figs. 3 and 4 we plot $\Delta m_{\mu\tau}^2$ and $\sin^2 2\Theta_{\mu\tau}$ as functions of v_L and v_R . Cases {A,B,C} have $r_3 = \{1.81, 1.89, 1.98\} \times 10^{13}$, respectively. We see that $\sin^2 2\Theta_{\mu\tau}$ is the most sensitive function of v_L and we find acceptable solutions, displayed in Fig. 4: $\{m_\nu, m_{\nu_\mu}, m_{\nu_\tau}\} = -\{0.063, 3.087, 10.88\} 10^{-3}$ in eV and

$$U_\nu = \begin{pmatrix} -0.989 & -0.081 & -0.123 \\ -0.147 & 0.539 & 0.829 \\ 0.001 & -0.838 & 0.545 \end{pmatrix}, \quad (10)$$

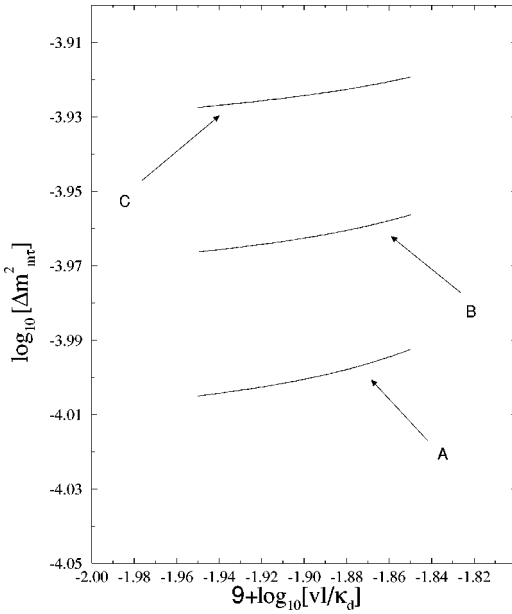
which gives $\sin^2 2\Theta_{e\mu} = 2.8 \times 10^{-2}$ and $\sin^2 2\Theta_{\mu\tau} = 0.84$ and the Yukawa couplings $h_{1/2}$ and f are

$$h_1 = \frac{1}{v_u} \begin{pmatrix} -0.064 & 0.123 & 0.181 \\ 0.123 & -3.721 & 1.488 \\ 0.181 & 1.488 & -17.285 \end{pmatrix}, \quad (11)$$

$$h_2 = \frac{1}{100v_d} \begin{pmatrix} 0.155 & -0.228 & -0.338 \\ -0.228 & -0.775 & -2.769 \\ -0.338 & -2.769 & -58.598 \end{pmatrix}, \quad (12)$$

$$f = \frac{1}{1000k_d} \begin{pmatrix} 0.405 & -0.761 & -1.126 \\ -0.761 & 20.735 & -9.230 \\ -1.126 & -9.230 & -589.692 \end{pmatrix}, \quad (13)$$

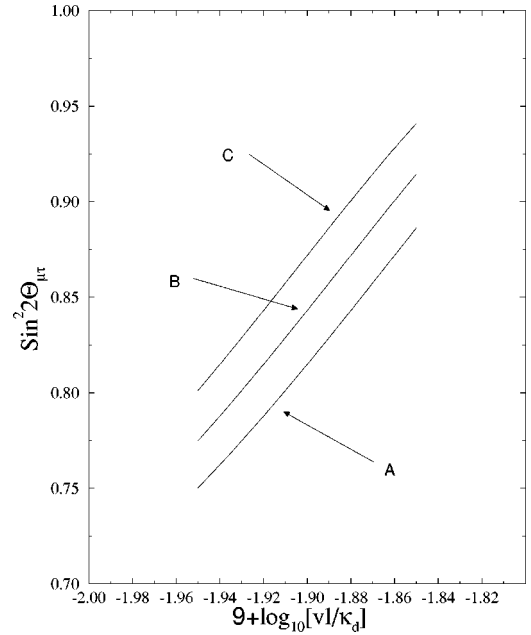
where we had $r_1 = 40.3$, $r_2 = 3.15 \times 10^{-12}$, $r_3 = 1.89 \times 10^{13}$. Using this explicit determination of $h_{1/2}$ in combination with baryogenesis constraint Eq. (9), we obtain the lower limit on

FIG. 3. Predicted $\Delta m_{\mu\tau}^2$ for various ν_L and ν_R .

$\nu_R \geq 10^{14}$ GeV as stated earlier. A few comments are in order on other aspects of the SUSY SO(10) model characterized by the superpotential in Eq. (4).

(i) The doublet-triplet splitting in this model has the non-trivial property that it leads to realistic fermion mass spectrum in contrast with the Dimopoulos-Wilczek (DW) mechanism. The point is that in the DW case the MSSM doublets arise from **10**-dimensional SO(10) multiplets thereby leading to incorrect mass relations $m_e/m_\mu = m_d/m_s$, which is off by a factor 10 or so. In contrast, in our model the low energy MSSM doublets are admixtures of doublets in **10** and **126** and is therefore free of such difficulties.

(ii) It is also worth emphasizing that, the near maximal mixing angle for $\nu_\mu \leftrightarrow \nu_\tau$ sectors needed to explain the atmospheric neutrino data is very hard to obtain with the type-I seesaw formula as has been clear in many studies [10,11,13]. One generally needs heavier vectorlike quarks [16] for this purpose. Thus our analysis would speak in favor of the type-II seesaw formula which puts constraints on the SO(10) model building.

FIG. 4. Predicted $\sin^2 2\Theta_{\mu\tau}$ for various ν_L and ν_R .

(iii) Strictly speaking the prediction of neutrino masses is sensitive to the renormalization of the seesaw formula [17]. However, in our case, the Yukawa couplings are so small that [as can be seen from Eqs. (11)–(13)] this extrapolation does not noticeably alter the above predictions at low energies.

In conclusion, we have shown that the use of a type-II seesaw formula in a next to minimal SUSY SO(10) model without extra matter multiplets or extra symmetries can explain both the solar and atmospheric neutrino deficits but not the LSND results. Thus if the LSND results are confirmed by KARMEN experiment, this class of SUSY SO(10) models (minimal and next to minimal) cannot accommodate it simultaneously with the solar and the atmospheric neutrino results and alternative theoretical frameworks must be investigated.

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