Two body nonleptonic Λ_b decays in the quark model with a factorization ansatz

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The two body nonleptonic Λ_b decays are analyzed in the factorization approximation, using the quark model, treating $\xi = 1/N_c$ as a free parameter. It is shown that the experimental branching ratio for $\Lambda_b \to \Lambda J/\psi$ restricts ξ and this ratio can be understood for a value of ξ which lies in the range $0 \leqslant \xi \leqslant 0.5$ suggested by two body B meson decays. The branching ratios for $\Lambda_b \to \Lambda_c D_s^*(D_s)$ are predicted to be larger than the previous estimates. Finally it is pointed that the CKM-Wolfenstein parameter $\rho^2 + \eta^2$, where η is the CP phase, can be determined from the ratio of widths of $\Lambda_b \to \Lambda \bar{D}$ and $\Lambda_b \to \Lambda J/\psi$ or that of $\Lambda_b \to p\, D_s$ and $\Lambda_b \to \Lambda_c\, D_s$, independent of the parameter ξ. [S0556-2821(98)05513-1]

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I. INTRODUCTION

Two body nonleptonic decays of bottom baryons provide useful information for QCD effects in weak decays and indirect CP asymmetries which involve Cabibbo-Kobayashi-Maskawa- (CKM-)Wolfenstein parameters ρ and η. The standard framework to study nonleptonic decays of bottom baryons is provided by an effective Hamiltonian approach, which allows a separation between short- and long-distance contributions in these decays. The latter involves the matrix elements $\langle MB'|O_i|B\rangle$ at a typical hadronic scale, where O_i is an operator in the effective Hamiltonian. These matrix elements cannot be calculated at present from first principles. Thus one has to resort to some approximate schemes. Such schemes are often complicated by competing mechanisms, such as factorization, baryon pole terms, and W-exchange terms, each of which has uncertainties of its own. The purpose of this paper is to study a class of two body bottom baryon nonleptonic decays in the framework of the factorization scheme, where, neglecting final state interactions, hadronic matrix elements are factorized into a product of two matrix elements of the form $\langle B'|J_\mu|B\rangle$ and $\langle 0|J'_\mu|M\rangle$ for which more information may be available.

Following the phenomenological success of factorization in the heavy to heavy nonleptonic B meson decays [1], this framework has been extended to the domain of heavy to light transitions [2]. The factorization ansatz here introduces one free parameter, called $\xi = 1/N_c$ (N_c being number of colors), which is introduced to compensate for the neglect of the color octet-octet contribution in evaluating the hadronic matrix elements in the heavy to light sectors. The range $0 \leqslant \xi \leqslant 0.5$ has been found [2] to be consistent with the data on a number of measured B meson decays. We apply the factorization to decays $\Lambda_b \to \Lambda J/\psi$, $\Lambda_b \to \Lambda_c D_s(D_s^*)$, $\Lambda_b \to \Lambda \bar{D}$, and $\Lambda_b \to p\, D_s$. In addition, we use the quark model to fix current coupling constants which appear in the matrix elements $\langle B'|J_\mu|B\rangle$. We show that the measured branching ratio for $\Lambda_b \to \Lambda J/\psi$ can be accounted for in this approach with the parameter ξ in the above mentioned range. Its previous estimates obtained either by extracting form factors at zero recoil from experiment and using flavor symmetry of heavy quark effective theory (HQET) [3] or by extending the Stech's approach for form factors to baryons [4] were of order 10^{-5}, much smaller than its measured value. Our estimates for the branching ratios for $\Lambda_b \to \Lambda_c D_s(D_s^*)$ are larger than their previous estimates [5,6]. The decays $\Lambda_b \to \Lambda \bar{D}$ and $\Lambda_b \to p\, D_s$ can give information on the CKM-Wolfenstein parameter $(\rho^2 + \eta^2)$ [7] or $|V_{ub}/V_{cb}|$ independent of ξ.

We write the effective Hamiltonian [8]

$$H_{\rm eff}(\Delta B = 1) = \frac{G_F}{\sqrt{2}}\left[\sum_{q=u,c} V_{cb}V_{qs}^*(C_1 O_1^c + C_2 O_2^c) + \sum_{q=u,c} V_{ub}V_{qs}^*(C_1 O_1^u + C_2 O_2^u)\right], \tag{1}$$

where C_i are the Wilson coefficients evaluated at the renormalization scale μ; the current-current operators $O_{1,2}$ are

$$O_1^c = (\bar{c}^\alpha b_\alpha)_{V-A}(\bar{s}^\beta q_\beta)_{V-A},$$
$$O_2^c = (\bar{c}^\alpha b_\beta)_{V-A}(\bar{s}^\beta q_\alpha)_{V-A}, \tag{2}$$

and O_i^u are obtained through replacing c by u. Here α and β are SU(3) color indices while $(\bar{c}^\alpha b_\beta)_{V-A} = \bar{c}^\alpha \gamma_\mu (1+\gamma_5) b_\beta$, etc. The related Wilson coefficients at $\mu = 2.5$ GeV in the next-to-leading logarithmic (NLL) precision are [2]

$$C_1 = 1.117,$$
$$C_2 = -0.257. \tag{3}$$

These are not very different from those at $\mu = 5$ GeV in the leading logarithmic approximation (LLA) [9]: $C_1(m_b) = 1.11$ and $C_2(m_b) = -0.26$.

In the factorization scheme we encounter matrix elements of the form

$$\langle B(p')|J_\mu|B_b(p)\rangle=\overline{u}(p')\Gamma_\mu u(p)=\overline{u}(p')i\{[g_V(s)-g_A(s)\gamma_5]\gamma_\mu+[f_V(s)+h_A(s)\gamma_5]\sigma_{\mu\nu}q_\nu+i[h_V(s)-f_A(s)\gamma_5]q_\mu\}u(p), \tag{4}$$

where B_b is a baryon, which contains b quark while B is any baryon not containing it. Here $s=-q^2=-(p-p')^2$. In the heavy quark spin symmetry limit [10], the vector and axial vector form factors are related [when B_b belongs to the triplet representation of flavor SU(3)] as follows:

$$g_V(s)=g_A(s)=f_1, \tag{5}$$

$$f_V=h_V=h_A=-f_A=\frac{1}{m_{B_b}}f_2. \tag{6}$$

For a decay of the type $B_b(p)\rightarrow B(p')+X(p_X)$, the matrix elements are of the form

$$T=i\frac{G'}{\sqrt{2}}\langle 0|J'_\mu|X(p_X)\rangle\overline{u}(p')\Gamma_\mu u(p)\frac{1}{(2\pi)^3}\sqrt{\frac{mm'}{p_o p'_o}}. \tag{7}$$

In the rest frame of B_b, the decay rate of B_b and its polarization are given by

$$\Gamma=\frac{G'^2}{2}\frac{1}{4\pi m^2}\int ds p'(s)\{\rho(s)\Gamma^\rho(s)+\sigma(s)\Gamma^\sigma(s)\}, \tag{8}$$

where $q=p_X=p-p'$, $s=-q^2$, and

$$\begin{aligned}\Gamma^\rho(s)=\{&Q(s)(g_V^2+g_A^2)-3mm's(g_V^2-g_A^2)+3s[(m+m')((m-m')^2-s)g_Vf_V-(m-m')((m+m')^2-s)g_Af_A]\\&+s[Q''(s)(f_V^2+h_A^2)-3mm's(f_V^2-h_A^2)]-2mp'(s)\mathbf{n}\cdot\mathbf{s}[((m^2-m'^2)-2s)g_Ag_V+s((m-3m')g_Vh_A\\&-(m+3m')g_Af_V)+sf_Vh_A(s-m^2-5m'^2)]\},\end{aligned} \tag{9}$$

$$\begin{aligned}\Gamma^\sigma(s)=\Big\{&Q'(s)(g_V^2+g_A^2)-s[(m-m')((m+m')^2-s)g_Vf_V+(m+m')((m-m')^2-s)g_Af_A]+\frac{1}{2}s^2[((m+m')^2-s)h_V^2\\&+((m-m')^2-s)f_A^2]-2mp'(s)\mathbf{n}\cdot\mathbf{s}[(m^2-m'^2)g_Vg_A-s((m+m')g_Ah_V+(m-m')g_Vg_A)+s^2h_Vf_A]\Big\}.\end{aligned} \tag{10}$$

Here it is understood that the form factors are functions of s and $\rho=\rho_V$, ρ_A, or 0 with X as 1^-, 1^+, or O^-, while correspondingly $\sigma=0$, σ_A or $\sigma_{p\cdot s}$ and

$$p'(s)=\frac{1}{2m}\{[(m^2+m'^2)-s]^2-4m^2m'^2\}^{1/2}, \tag{11}$$

$$Q(s)=\tfrac{1}{2}[(m^2-m'^2)^2+s(m^2+m'^2)-2s^2], \tag{12}$$

$$Q'(s)=\tfrac{1}{2}[(m^2-m'^2)^2-s(m^2+m'^2)], \tag{13}$$

$$Q''(s)=\tfrac{1}{2}[2(m^2-m'^2)^2-s(m^2+m'^2)-s^2]. \tag{14}$$

The form factors defined in Eq. (4) are calculated in the quark model at $s=-q^2=m_X^2$ where X is a vector or pseudoscalar particle in the decay $B\rightarrow B'X$, thereby taking into account recoil correction. This is in contrast to the use of the nonrelativistic quark model for the evaluation of the form factors at zero recoil $\mathbf{q}=0$ [11]. This latter approach also necessitates the extrapolation of the form factors from maximum $q^2[-q_m^2=t_m=(m_B-m_{B'})^2]$ to the desired $s=-q^2=m_X^2$. We may point out that since $|\mathbf{q}|\approx 1.75$ GeV in $\Lambda_b\rightarrow\Lambda+J/\psi$, for example, the no recoil approximation does not seem to be justified; in fact $|\mathbf{q}|\gg m_s$ in Λ, making the s quark in Λ relativistic. In our approach no recoil approximation, nor any extrapolation of the form factors at the physical point are needed. Our quark model results do satisfy the constraints imposed by the heavy quark spin symmetry.

The plan of the paper is as follows: Sec. II summarizes the calculation of the baryonic form factors within the framework of quark model at the desired value of $s=-q^2$, rather than at the zero recoil point, relegating the details in the

Appendix. In Sec. III we apply the results to some specific nonleptonic decay modes of Λ_b. Section IV summarizes our conclusions.

II. BARYONIC FORM FACTORS IN THE QUARK MODEL

In order to calculate the form factors we first reduce the matrix elements in Eq. (4) from four component Dirac spinors to Pauli spinors without making any approximation and do the same for the quark level current

$$j_\mu = i\bar{q}\gamma_\mu(1+\gamma_5)b. \tag{15}$$

We treat the b quark in B_b extreme nonrelativistically ($\mathbf{p}_b/m_b \approx 0$) and set $\mathbf{p}_b - \mathbf{p}_q = \mathbf{q} = -\mathbf{p}'$, $E_q = \sqrt{\mathbf{q}^2 + m_q^2}$, $E_b = m_b = m_3$. Then, as shown in the Appendix,

$$h_A = -f_A, \quad f_V = h_V, \tag{16}$$

$$g_V(s) = \xi_V I a(E', E_3'),$$

$$f_V(s) = \frac{1}{m}\xi_V I b(E', E_3'),$$

$$g_A(s) = \xi_A I a(E', E_3'),$$

$$f_A(s) = -\frac{1}{m}\xi_A I b(E', E_3'), \tag{17}$$

where

$$a(E', E_3') = \frac{1}{2}\sqrt{\frac{E'}{E_3'}}\frac{(E'+m')(1-m'/m)+(E_3'+m_3')(1+m'/m)}{\sqrt{(E'+m')(E_3'+m_3')}},$$

$$b(E', E_3') = \frac{1}{2}\sqrt{\frac{E'}{E_3'}}\frac{(E'+m')-(E_3'+m_3')}{\sqrt{(E'+m')(E_3'+m_3')}}, \tag{18}$$

and $E' = p_o'$, $E_3' = E_q = \sqrt{\mathbf{p}'^2 + m_3'^2}$, and $m_3' = m_q$. Note the explicit appearance of $1/m$ corrections in the above formulas. Here ξ_V and ξ_A are respectively the spin-unitary spin part of the matrix elements of the current operator (15); for example, for B_b belonging to the triplet representation of SU(3), $\xi_V = \xi_A$ and I is the overlap integral

$$I = N_f N_i \int \psi_f^*\left(\mathbf{p}_{12}, \mathbf{k} - \frac{m_1+m_2}{\tilde{m}'}\mathbf{p}'\right)\psi_i(\mathbf{p}_{12}, \mathbf{k})\, d^3p_{12} d^3k. \tag{19}$$

The recoil correction is represented by momentum mismatch $[(m_1+m_2)/\tilde{m}']\mathbf{p}'$, which arises since the rest frame of B_b is not that of B_q baryon. Here $\tilde{m}' = m_1 + m_2 + m_3'$ where m_1 and m_2 are masses of the spectator quarks and m_3' is that of q quark resulting from the decay of b. Note that the form factors in Eq. (17) are determined at the desired value of $s = -q^2$.

As already noted for B_b belonging to the triplet representation $\xi_V = \xi_A$ and then the relations (16) and (17) are consistent with those given in Eqs. (5) and (6) obtained in the heavy quark spin symmetry limit. To proceed further we use harmonic oscillator or Gaussian wave functions in Eq. (19) to obtain

$$I = \left(\frac{2\beta\beta'}{\beta^2+\beta'^2}\right)^3 \exp\left[-\frac{3}{4}\frac{(m_1+m_2)^2}{\tilde{m}'^2}\frac{p'^2}{2(\beta^2+\beta'^2)}\right]. \tag{20}$$

We take β or β' as [12]

$$\beta^2 = \sqrt{\mu_Q \kappa}, \tag{21}$$

where $\mu_Q = [M_N M_H/(M_N + M_H)]$ is the reduced mass of the bound system, M_N being the nucleon mass and M_H that of B, D, K^*, or ρ meson for Λ_b, Λ_c, Λ, and p, respectively. κ is the spring constant and its value is taken to be $(440\ \text{MeV})^3$ [13].

We summarize in Table I the form factors $g_V(s) = g_A(s) = f_1$, $f_V(s) = h_V(s) = h_A(s) = -f_A(s) = f_2/m$ for the transitions $\Lambda_b \to pD_s$, $\Lambda_b \to \Lambda\bar{D}$, $\Lambda_b \to \Lambda J/\psi$, $\Lambda_b \to \Lambda_c D_s^*(D_s)$, for $s = m_{D_s}^2$, m_D^2, $m_{J/\psi}^2$, and $m_{D_s^*}^2$ ($m_{D_s}^2$), $m_3' = m_u$, m_s, and m_c, respectively. For the numerical work we have taken the relevant masses (in GeV) as $m = m_{\Lambda_b} = 5.641$, $m_\Lambda = 1.1157$, $m_{\Lambda_c} = 2.285$, $m_p = 0.938$, $m_{J/\psi} = 3.097$, $m_{D_s^*} = 2.112$, $m_D = 1.864$, $m_{D_s} = 1.968$, $m_s = 0.510$, $m_c = 1.6$, and $m_u = 0.340$.

III. APPLICATIONS

We consider those decays of Λ_b for which baryon poles either do not contribute or their contribution is highly suppressed due to Okubo-Zweig-Iizuka (OZI) rule and that it scales as inverse of m_{Λ_b}.

For decays of the type $\Lambda_b(p) \to B_q(p')V(q)$, where V is a vector meson,

$$\rho_A(s) = 0 = \sigma_A(s), \tag{22}$$

TABLE I. The quark model predictions for baryonic form factors for Λ_b transitions. $\beta=0.51$ GeV, $\beta'=0.44$ GeV for p and Λ and $=0.48$ GeV for Λ_c. $f_1=g_V=g_A$, $f_2/f_1=(f_V/g_V)m=(h_V/g_V)m=(h_A/g_A)m=-(f_A/g_A)m$. Note that only the last column depends on the overlap integral I.

Transition	$\|p'\|$	$\xi_A=\xi_V$	$f_1(s)/I$	f_2/f_1	$\mathcal{F}_1$	$\mathcal{F}_2$	I	$f_1(s)$
pD_s	2.376	$1/\sqrt{2}$	0.720	0.123	≈ 1	≈ 1	0.119	0.086
$\Lambda\underline{D}$	2.374	$-1/\sqrt{3}$	-0.558	0.129	≈ 1	≈ 1	0.215	-0.120
$\Lambda J/\psi$	1.756	$-1/\sqrt{3}$	-0.604	0.158	0.943	0.826	0.426	-0.257
$\Lambda_c D_s$	1.766	1	1.052	0.134	0.978	0.983	0.791	0.829
$\Lambda_c D_s^*$	1.850	1	1.048	0.137	0.949	0.908	0.810	0.852

$$\rho_V(s)=F_V^2\delta(s-m_V^2), \tag{23}$$

where

$$\langle 0|J'_\mu|V\rangle=F_V\epsilon_\mu. \tag{24}$$

Then Eqs. (9), (17), and (18), on using the relations (5) and (6), give the decay rate

$$\Gamma=\frac{G'^2}{2}F_V^2\frac{|\mathbf{p}'|}{4\pi m^2}Q(m_V^2)[2f_1^2(m_V^2)]\mathcal{F}_1^V(m_V^2) \tag{25}$$

while the asymmetry

$$\alpha=\frac{-2m|\mathbf{p}'|[(m^2-m'^2)-2m_V^2]}{2Q(m_V^2)}\frac{\mathcal{F}_2^V(m_V^2)}{\mathcal{F}_1^V(m_V^2)} \tag{26}$$

where

$$\mathcal{F}_1^V(m_V^2)=\left\{1-3\frac{m'}{m}\frac{m_V^2(m^2-m'^2+m_V^2)}{Q(m_V^2)}\frac{f_2}{f_1}\right.$$
$$\left.+\frac{m_V^2}{m^2}\frac{Q''(m_V^2)}{Q(m_V^2)}\frac{f_2^2}{f_1^2}\right\}, \tag{27}$$

$$\mathcal{F}_2^V(m_V^2)=\left\{1-6\frac{m'}{m}\frac{m_V^2}{m^2-m'^2-2m_V^2}\frac{f_2}{f_1}\right.$$
$$\left.-\frac{m_V^2}{m^2}\frac{m^2+5m'^2-m_V^2}{m^2-m'^2-2m_V^2}\frac{f_2^2}{f_1^2}\right\}. \tag{28}$$

The prediction for α is independent of the value of the overlap integral and provides a test of the predictions (16) and (17) with $\xi_V=\xi_A$ through the presence of f_2/f_1. The corrections due to the form factors which scale as $1/m$ are dumped into $\mathcal{F}$ functions.

If the vector meson V is replaced by a pseudoscalar meson P, then

$$\rho_V(s)=0=\rho_A(s), \tag{29}$$

$$\sigma_A(s)=F_P^2\delta(s-m_P^2), \tag{30}$$

where

$$\langle 0|J'_\mu|p\rangle=F_Pq_\mu. \tag{31}$$

Then Eqs. (8) and (10), on using the relations (5) and (6), give

$$\Gamma_P=\frac{G'^2}{2}F_P^2\frac{|\mathbf{p}'|}{4\pi m^2}Q'(m_P^2)[2f_1^2(m_P^2)]\mathcal{F}_1^P(m_P^2), \tag{32}$$

$$\alpha_P=\frac{-2m|\mathbf{p}'|[(m^2-m'^2)]}{2Q'(m_P^2)}\frac{\mathcal{F}_2^P(m_P^2)}{\mathcal{F}_1^P(m_P^2)}, \tag{33}$$

where

$$\mathcal{F}_1^P(m_P^2)=\left\{1-\frac{m'}{m}\frac{m_P^2(m^2-m'^2+m_P^2)}{Q'(m_P^2)}\frac{f_2}{f_1}\right.$$
$$\left.+\frac{m_P^2}{m^2}\frac{m_P^2(m^2+m'^2-m_P^2)}{2Q'(m_P^2)}\frac{f_2^2}{f_1^2}\right\}, \tag{34}$$

$$\mathcal{F}_2^P(m_P^2)=\left\{1-\frac{2m'}{m}\frac{m_P^2}{(m^2-m'^2)}\frac{f_2}{f_1}\right.$$
$$\left.+\frac{m_P^4}{m^2(m^2-m'^2)}\frac{f_2^2}{f_1^2}\right\}. \tag{35}$$

We are now ready to consider the specific decays. We first consider $\Lambda_b\to\Lambda\ J/\psi$, where the first part of the Hamiltonian (1) with $q=c$ and the Fierz rearrangement give

$$G'=G_FV_{cb}V_{cs}^*(C_2+\xi C_1), \tag{36}$$

$$J'_\mu=\bar{c}\gamma_\mu(1+\gamma_5)c. \tag{37}$$

The constant $F_{J/\psi}^2$ is determined from $\Gamma(J/\psi\to e^+e^-)=(5.26\pm0.37)$ keV [14]:

$$F_{J/\psi}^2=\frac{9}{4}\left(\frac{3}{4\pi\alpha^2}\right)\Gamma(J/\psi\to e^+e^-)(m_{J/\psi})$$
$$=1.637\times10^{-1}\ \text{GeV}^2. \tag{38}$$

Using $G_F=1.16639\times10^{-5}$ GeV^{-2} and [14] $|V_{cb}|=0.0393\pm0.0028$, $|V_{cs}|=1.01\pm0.18$, we obtain, from Eqs. (23) and (24),

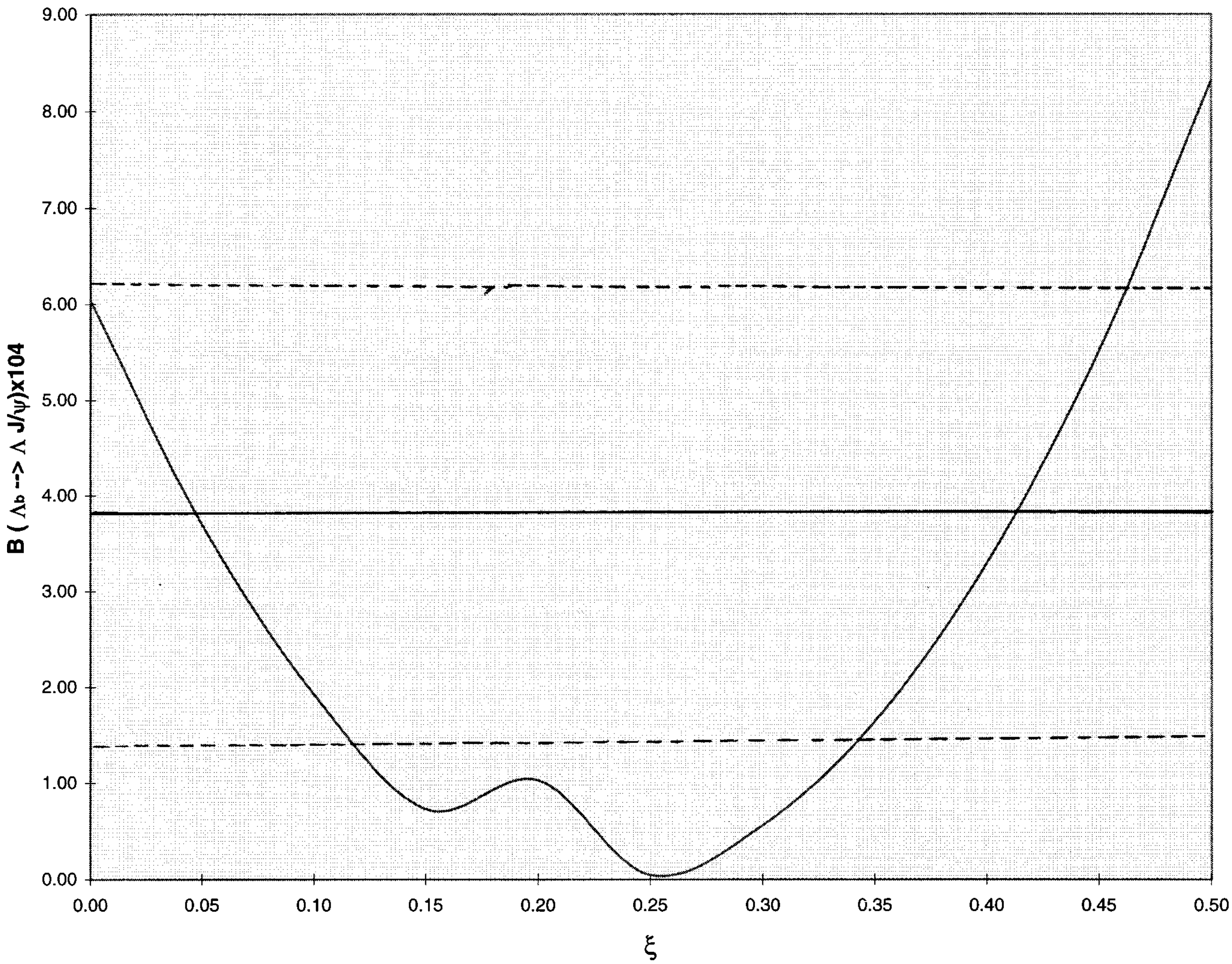

FIG. 1. Branching ratio for $\Lambda_b \to \Lambda J/\psi$ as a function of ξ. The solid line shows the central value of the CDF measurement; the dotted lines show the one sigma limits.

$$\Gamma = 8.21\times 10^{-14}(C_2+\xi C_1)^2 f_1^2(m^2_{J/\psi})\mathcal{F}_1^V(m^2_{J/\psi}), \quad (39)$$

$$\alpha = -0.21\frac{\mathcal{F}_2^V(m^2_{J/\psi})}{\mathcal{F}_1^V(m^2_{J/\psi})}. \quad (40)$$

This gives the branching ratio

$$B(\Lambda_b \to \Lambda J/\psi) = 1.47\times 10^{-1}(C_2+\xi C_1)^2 \times f_1^2(m^2_{J/\psi})\mathcal{F}_1^V(m^2_{J/\psi}), \quad (41)$$

where we have used [14] $\Gamma_{\Lambda_b} = 0.847\times 10^{10}\ \mathrm{s}^{-1} = 5.59\times 10^{-13}$ GeV. Using Table I we finally obtain

$$B(\Lambda_b \to \Lambda J/\psi) = 9.14\times 10^{-3}(C_2+\xi C_1)^2, \quad (42)$$

$$\alpha(\Lambda_b \to \Lambda J/\psi) = -0.18. \quad (43)$$

In Fig. 1, we show the branching ratio $B(\Lambda_b \to \Lambda J/\psi)$ as a function of ξ. This decay mode is sensitive to ξ and comparison with the experimental value [15] $(3.7\pm 2.4)\times 10^{-4}$ shows that ξ is restricted to $0 \leqslant \xi \leqslant 0.125$ or $0.35 \leqslant \xi \leqslant 0.45$, which lie within the range $0 \leqslant \xi \leqslant 0.5$ suggested by the combined analysis of the present CLEO data on $B \to h_1 h_2$ decay [2]. We may remark that f_2/f_1 correction to the decay rate is about 6% while that to the asymmetry parameter α is about 14%.

Other decays of interest for which the first part of the Hamiltonian (1) with $q=c$ is responsible are $\Lambda_b \to \Lambda_c^+ D_s^-$ and $\Lambda_b \to \Lambda_c^+ D_s^{*-}$. For these decays

$$G' = G_F V_{cb} V_{cs}^*(C_1+\xi C_2) \quad (44)$$

and

$$J'_\mu = \overline{s}\gamma_\mu(1+\gamma_5)c. \quad (45)$$

Then Eqs. (23), (24), (29), and (30) [on using the relations (5) and (6)] give respectively

$$\Gamma(\Lambda_b \to \Lambda_c^+ D_s^{*-}) = 2.12\times 10^{-14}(C_1+\xi C_2)^2 \times f_1^2(m^2_{D_s^*})\mathcal{F}_1^V(m^2_{D_s^*}), \quad (46)$$

$$\alpha(\Lambda_b \to \Lambda_c^+ D_s^{*-}) = -0.42 \frac{\mathcal{F}_2^V(m_{D_s^*}^2)}{\mathcal{F}_1^V(m_{D_s^*}^2)}, \tag{47}$$

$$\Gamma(\Lambda_b \to \Lambda_c^+ D_s^-) = 1.50\times 10^{-14}(C_1+\xi C_2)^2 \times f_1^2(m_{D_s}^2)\mathcal{F}_1^P(m_{D_s}^2), \tag{48}$$

$$\alpha(\Lambda_b \to \Lambda_c D_s) = -0.98 \frac{\mathcal{F}_2^P(m_{D_s}^2)}{\mathcal{F}_1^P(m_{D_s}^2)}. \tag{49}$$

Here we have used $F_{D_s}=F_{D_s^*}=232$ MeV [14] (in the normalization $F_\pi=131$ MeV). Using Table I, the above equations give

$$B(\Lambda_b \to \Lambda_c D_s^*) = 2.61(C_1+\xi C_2)^2\times 10^{-2}, \tag{50}$$

$$\alpha(\Lambda_b \to \Lambda_c D_s^*) = -0.40, \tag{51}$$

$$B(\Lambda_b \to \Lambda_c D_s) = 1.79(C_1+\xi C_2)^2\times 10^{-2}, \tag{52}$$

$$\alpha(\Lambda_b \to \Lambda_c D_s) = -0.98. \tag{53}$$

The above branching ratios are not sensitive to ξ: $2.55\times 10^{-2}\leqslant B(D_s^*)\leqslant 3.26\times 10^{-2}$ and $1.75\times 10^{-2}\leqslant B(D_s)\leqslant 2.23\times 10^{-2}$ for $0.5\geqslant\xi\geqslant 0$. The f_2/f_1 corrections are negligible when the meson in the final state is O^- while for 1^- they are about 5% for the decay rate and for the asymmetry parameter α. Previously the above decays have been analyzed in the HQET with the factorization approximation in the large N_c limit either by parametrizing the Isgure-Wise form factor $G_1(v\cdot v')$ [c.f. Eq. (5) with $f_1=G_1+(m_{\Lambda_c}/m_{\Lambda_b})G_2$, $f_2=-G_2/m_{\Lambda_b}$, where since Λ_c, Λ_b form a multiplet, the absence of the second class currents implies $G_2=0$] [5] or by evaluating it in the large N_c limit [6]. In contrast we have used quark model to fix the baryonic form factors as given in Eqs. (16) and (17). The comparison of our predicted results with the previous results mentioned above is presented in Table II.

TABLE II. Predictions for the branching ratios (BR) in % for $\Lambda_b\to\Lambda_c^+ D_s^{*-}$ and $\Lambda_b\to\Lambda_c^+ D_s$ in the large N_c limit ($\xi=0$).

Decay processes	Present BR calculation ($\xi=0$)	BR Ref. [5]	BR Ref. [6]
$\Lambda_b\to\Lambda_c^+ D_s^{*-}$	3.26	$1.73^{+0.20}_{-0.30}$	1.77
$\Lambda_b\to\Lambda_c^+ D_s^-$	2.23	$2.30^{+0.30}_{-0.40}$	1.156

Finally we consider the decays $\Lambda_b\to\Lambda\bar{D}^o$ and $\Lambda_b\to pD_s$; the interest here is that the ratio of their decay widths with $\Lambda_b\to\Lambda\ J/\psi$ and $\Lambda_b\to\Lambda_c D_s$, respectively, can fix the CKM-Wolfenstein parameter $(\rho^2+\eta^2)$ or $|V_{ub}/V_{cb}|$, independent of ξ, where η indirectly determines CP violation. For these decays the second part of the Hamiltonian (1) with $q=c$ (and the Fierz rearrangement for the former) give

$$\Gamma(\Lambda_b\to\Lambda\bar{D}^o) = \left[\frac{G_F}{\sqrt{2}}V_{ub}V_{cs}^*(C_2+\xi C_1)\right]^2 \frac{2|\mathbf{p}'|}{4\pi m_{\Lambda_b}^2}F_D^2[f_1^{\Lambda D}(m_D^2)]^2\mathcal{F}_1^P(m_D^2)Q'(m_D^2), \tag{54}$$

$$\Gamma(\Lambda_b\to pD_s) = \left[\frac{G_F}{\sqrt{2}}V_{ub}V_{cs}^*(C_1+\xi C_2)\right]^2 \frac{2|\mathbf{p}'|}{4\pi m_{\Lambda_b}^2}F_{D_s}^2[f_1^{pD_s}(m_{D_s}^2)]^2\mathcal{F}_1^P(m_{D_s}^2)Q'(m_{D_s}^2). \tag{55}$$

Using Table I, $F_D=200$ MeV and taking into consideration differences in phase space factors p', Q, and Q' we obtain

$$\frac{\Gamma(\Lambda_b\to\Lambda\bar{D}^o)}{\Gamma(\Lambda_b\to\Lambda J/\psi)} = 5.88\times 10^{-2}\left|\frac{V_{ub}}{V_{cb}}\right|^2 = 2.8\times 10^{-3}(\rho^2+\eta^2), \tag{56}$$

$$\frac{\Gamma(\Lambda_b\to p\ D_s)}{\Gamma(\Lambda_b\to\Lambda_c\ D_s)} = 2\times 10^{-2}\left|\frac{V_{ub}}{V_{cb}}\right|^2 = 9.7\times 10^{-4}(\rho^2+\eta^2). \tag{57}$$

IV. CONCLUSIONS

We have analyzed some two body nonleptonic Λ_b decays in the factorization approximation, treating $\xi=1/N_c$ (which is supposed to compensate for the neglect of color octet-octet contribution in evaluating the hadronic matrix elements) as a free parameter. In addition we have used the quark model to fix the baryonic form factors at the desired value of $s=-q^2$ without making any recoil approximation. The form factors obtained are consistent with the predictions of the heavy quark symmetry and explicitly display $1/m_b$ corrections. The experimental branching ratio for $\Lambda_b\to\Lambda J/\psi$ restricts ξ and can be understood for either $0<\xi<0.125$ or $0.3<\xi<0.45$. Our predictions for the branching ratios $\Lambda_b\to\Lambda_c D_s(D_s^*)$ are larger than the previous estimates. Future experimental data from colliders are expected to verify and distinguish the various results. Finally the parameter $|V_{ub}/V_{cb}|$ or $(\rho^2+\eta^2)$ can be determined independently of the parameter ξ from the ratio of decay widths of $\Lambda_b\to\Lambda\bar{D}$ and $\Lambda_b\to\Lambda\ J/\psi$ or that of $\Lambda_b\to p\ D_s$ and $\Lambda_b\to\Lambda_c\ D_s$, although the branching ratios expected for these decays may be hard to measure.

We want to emphasize that our derivation of Eqs. (16) and (17) does not depend on the details of the quark model. The basic assumption is that in the heavy quark limit, the velocity of heavy quark can be neglected. The details of the quark model enter in the derivation of the overlap integral I.

It may be noted from the structure of Eqs. (9) and (10), that the contribution of the form factors f_V, h_V, h_A, and f_A are proportional to s/m^2 [the same is true for the term containing $(g_V^2-g_A^2)$]. Hence when $s/m^2 \ll 1$, their contribution can be neglected and in this case asymmetry parameter α is given by

$$\alpha \simeq -\frac{2g_V g_A}{g_V^2+g_A^2}.$$

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APPENDIX

We outline the derivation of relations (16) and (17). We first reduce the matrix elements in Eq. (4) from four component Dirac-spinors to Pauli spinors. Thus in the rest frame of B_b,

$$\langle B(p')|J_o|B_b(p)\rangle = \sqrt{\frac{E'+m'}{2E'}}\left\{\left[g_V(s)-q_o h_V(s)-\frac{\mathbf{q}^2}{E'+m'}f_V(s)\right]+\left[h_A(s)+\frac{1}{E'+m'}[g_A(s)-q_o f_A(s)]\right]\sigma\cdot\mathbf{q}\right\}, \tag{A1}$$

$$\langle B(p')|\mathbf{J}|B_b(p)\rangle = \sqrt{\frac{E'+m'}{2E'}}\left\{\left[-g_A(s)+\left(q_o+\frac{\mathbf{q}^2}{E'+m'}\right)h_A(s)\right]\sigma-\left[\left(1+\frac{q_o}{E'+m'}\right)f_V(s)+\frac{1}{E'+m'}g_V(s)\right]i\sigma\times\mathbf{q} \right.$$
$$\left. -\left[h_V(s)+\frac{1}{E'+m'}[g_V(s)+q_o f_V(s)]\right]\mathbf{q}-\frac{1}{E'+m'}[h_A(s)+f_A(s)]\mathbf{q}\sigma\cdot\mathbf{q}\right\}, \tag{A2}$$

where $E'(s)=p_o'(s)$, $\mathbf{q}=-\mathbf{p}'$, $q_o=\sqrt{|\mathbf{q}|^2+s}$. It may be noted that no approximation has been made so far. On the other hand, the Pauli reduction of the quark level current,

$$j_\mu = i\overline{q}\gamma_\mu(1+\gamma_5)Q, \tag{A3}$$

is given by [with $p_Q=p_3$, $p_q=p_3'$]

$$j_o=\frac{1}{2[E_3E_3'(E_3+m_3)(E_3'+m_3')]^{1/2}}\{(E_3'+m_3')(E_3+m_3)+\mathbf{p_3'}\cdot\mathbf{p_3}+i\sigma\cdot(\mathbf{p_3'}\times\mathbf{p_3})-(E_3'+m_3')\sigma\cdot\mathbf{p_3},-(E_3+m_3)\sigma\cdot\mathbf{p_3'}\}, \tag{A4}$$

$$\mathbf{j}=\frac{1}{2[E_3E_3'(E_3+m_3)(E_3'+m_3')]^{1/2}}\{[-(E_3'+m_3')(E_3+m_3)+\mathbf{p_3'}\cdot\mathbf{p_3}]\sigma+i(\mathbf{p_3'}\times\mathbf{p_3})-(\sigma\cdot\mathbf{p_3'})\mathbf{p_3}$$
$$-\sigma\cdot\mathbf{p_3}\mathbf{p_3'}+(E_3'+m_3')(\mathbf{p_3}-i\sigma\times\mathbf{p_3})+(E_3+m_3)(\mathbf{p_3'}+i\sigma\times\mathbf{p_3'})\}. \tag{A5}$$

We now treat the quark Q extreme nonrelativistically and thus put $|\mathbf{p}_3|\simeq 0$. Then

$$j_o=\frac{1}{\sqrt{2E_3'(E_3'+m_3')}}\{(E_3'+m_3')-\sigma\cdot\mathbf{p_3'}\}, \tag{A6}$$

$$\mathbf{j}=\frac{1}{\sqrt{2E_3'(E_3'+m_3')}}\{-(E_3'+m_3')\sigma+\mathbf{p_3'}+i\sigma\cdot\mathbf{p_3'}\}, \tag{A7}$$

where

$$E_3'=\sqrt{\mathbf{p}'^2_3+m_3'^2}=\sqrt{(\mathbf{p}_3-\mathbf{q})^2+m_3'^2}\simeq\sqrt{\mathbf{q}^2+m_3'^2}.$$

Suppose that the initial baryon B contains a heavy quark Q (b in our case) and two light quarks q_1 and q_2 which behave as spectators. The final baryon B' is composed of the quark q [s, c, or u quark] and the same spectators as in B. For the initial baryon composed of quarks $Q(\equiv q_3)$, q_1, q_2, we introduce relative coordinates and momenta as

$$\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2,\quad \frac{\mathbf{p}_{12}}{m_{12}}=\frac{\mathbf{p}_1}{m_1}-\frac{\mathbf{p}_2}{m_2},\quad m_{12}=\frac{m_1m_2}{m_1+m_2},$$

$$\mathbf{R}_{12}=\frac{m_1\mathbf{r}_1+m_2\mathbf{r}_2}{m_{12}},\quad \mathbf{r}_{12,3}=\mathbf{r}_{12}-\mathbf{r}_3,$$

$$\mathbf{P}_{12}=\mathbf{p}_1+\mathbf{p}_2,\quad \frac{\mathbf{k}}{\mu}=\frac{\mathbf{P}_{12}}{m_1+m_2}-\frac{\mathbf{p}_3}{m_3},$$

$$\mu=\frac{m_3(m_1+m_2)}{\tilde{m}},$$

$$\tilde{m}=m_1+m_2+m_3,\quad \mathbf{k}=\frac{m_3}{\tilde{m}}\mathbf{P}_{12}-\frac{m_1+m_2}{\tilde{m}}\mathbf{p}_3. \tag{A8}$$

For the initial baryon, its rest frame is its center of mass frame so that $\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3=0$ which implies $\mathbf{P}_{12}=-\mathbf{p}_3=\mathbf{k}$ and then

$$\mathbf{p}_1=\mathbf{p}_{12}+\frac{m_1}{m_1+m_2}\mathbf{k},$$

$$\mathbf{p}_2=-\mathbf{p}_{12}+\frac{m_1}{m_1+m_2}\mathbf{k}. \tag{A9}$$

Denoting the relative momenta of quarks in the baryon B' by primes and noting that $p_1'=p_1$, $p_2'=p_2$ so that $\mathbf{p}_{12}'=\mathbf{p}_{12}$, $\mathbf{P}_{12}'=\mathbf{P}_{12}$, giving $\mathbf{p'}_3=-\mathbf{P}_{12}+\mathbf{p}'=-\mathbf{k}-\mathbf{q}$ and

$$\mathbf{k}'=\mathbf{k}-\frac{m_1+m_2}{\tilde{m}'}\mathbf{p}'. \tag{A10}$$

Calling ψ_s the spatial wave function in momentum space and noting that when $\mathbf{p'}_3$ in Eqs. (A6) and (A7) is replaced by $-\mathbf{k}-\mathbf{q}$, the linear terms in $\mathbf{k}$ do not contribute in the spatial integral and as such the right sides of Eqs. (A6) and (A7) are independent of integration variables $\mathbf{k}$, $\mathbf{p}_{12}$, and $\mathbf{k}'$. The comparison of hadronic matrix elements in Eqs. (A1) and (A2) with those of Eqs. (A6) and (A7) give the relations (16) and (17). The use of delta functions $\delta(\mathbf{p}_1-\mathbf{p'}_1)$, $\delta(\mathbf{p}_2-\mathbf{p'}_2)$, $\delta(\mathbf{p}_1+\mathbf{p}_2+\mathbf{p}_3)$, and $(\mathbf{p'}_1+\mathbf{p'}_2+\mathbf{p'}_3-\mathbf{p}')$ reduce the spatial integral to the form given in Eq. (20).

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