Nonspectator contribution: A mechanism for inclusive $B \to X_s \eta'$ and exclusive $B \to K^{(*)} \eta'$ decays

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We propose a mechanism that can explain the large $B(B \to X_s \eta') \approx 10^{-3}$ observed by CLEO. In this mechanism, η' is produced by the fusion of two gluons: one from the QCD penguin diagram $b \to sg$ and the other one emitted by the light quark inside the B meson. The inclusive decay rate which is calculated via the factorization assumption can easily account for the observed branching ratio. We also estimate the exclusive branching ratio $B(B \to K \eta') = 7.0 \times 10^{-5}$ which is in good agreement with the experimental data and present our prediction for the K^* mode $B(B \to K^* \eta') = 3.4 \times 10^{-5}$. [S0556-2821(98)05213-8]

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I. INTRODUCTION

The CLEO Collaboration has recently discovered an unexpectedly large branching ratio for the semi-inclusive hadronic $B \to X_s \eta'$ decay [1]:

$$B(B \to X_s \eta') = (7.5 \pm 1.5 \pm 1.1) \times 10^{-4},$$

$$2.0 \leq p_{\eta'} \leq 2.7 \ \text{GeV}. \quad (1)$$

The corresponding exclusive decay rate has also been measured:

$$B(B \to K \eta') = (7.8^{+2.7}_{-2.2} \pm 1.0) \times 10^{-5}. \quad (2)$$

Possible mechanisms behind this large production of fast η' mesons have been discussed in recent papers [2–6]. For example, Atwood and Soni (AS) [2] have suggested the subprocess $b \to sg^* \to s \eta' g$ (g^* and g are virtual and real gluons, respectively) as the main underlying mechanism. For this purpose, the standard model QCD penguin diagram is used in conjunction with a gluon anomaly driven $g^* g \eta'$ vertex. The form factor for this vertex $H(q^2, 0, m_{\eta'}^2)$, where q is the four momentum of the off-shell gluon g^*, is approximated by the constant $H(0,0,m_{\eta'}^2)$ which in turn is extracted from $J/\psi \to \eta' \gamma$ decay. However, the approximation $H(q^2,0,m_{\eta'}^2) \approx H(0,0,m_{\eta'}^2)$ turns out to be problematic for two reasons: (1) As pointed out by Hou and Tseng [5], there is an implicit factor of α_s in H which should be running with q^2 and this would suppress AS's estimate by a factor of 1/3; (2) on the other hand, Kagan and Petrov [6] have indicated that the momentum dependence of the form factor could be quite significant with the leading behavior of the form $m_{\eta'}^2/(q^2 - m_{\eta'}^2)$. As a result, including this effect further reduces AS's result to about an order of magnitude below the observed branching ratio. Consequently, it has been suggested that the remedy could be invoking new physics to increase $B(b \to sg)$ to 10 %–15 % from its standard model value of nearly 0.2 %.

In this paper, we investigate the possibility that a somewhat different process might be the underlying mechanism for $B \to X_s \eta'$. We propose a nonspectator process in which η' is produced via fusion of the gluon from the QCD penguin diagram $b \to sg^*$ and another one emitted by the light quark inside the B meson (Fig. 1). It is shown that a conservative estimate of the contribution of this mechanism can naturally account for the observed value. We also calculate the branching ratios $B(B \to K \eta')$ and $B(B \to K^* \eta')$ in the context of factorization. The latter turns out to be smaller than the former by a factor 2.

II. EFFECTIVE HAMILTONIAN

The expression for Fig. 1 is the product of three terms.

(1) The effective neutral current flavor changing vertex $b \to sg$ is as follows [7]:

$$A(b \to sg) = -i\lambda_t \frac{G_F}{\sqrt{2}} \frac{g_s}{8\pi^2} [E_0 \bar{s}(q^2 g_{\mu\nu} - q_\mu q_\nu) \times \gamma^\nu (1-\gamma_5) T^a b - E_0' \bar{s} i m_b \sigma_{\mu\nu} q^\nu (1+\gamma_5) T^a b], \quad (3)$$

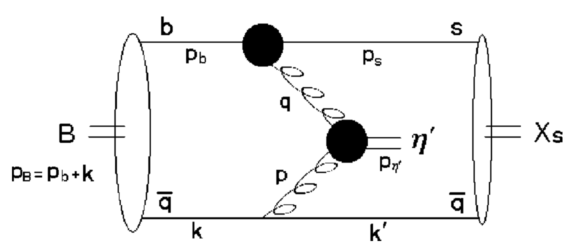

FIG. 1. Nonspectator contributions to $B \to \eta' X_s$.

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where $\lambda_t = V_{tb}V_{ts}^*$ and q is the gluon four momentum. In the process considered here, both chromoelectric and chromomagnetic operators in Eq. (3) contribute. However, since $|E_0| = |-4.92| \gg E_0' = 0.16$, the former operator is expected to have the dominant contribution. It is argued that the inclusion of the chromomagnetic operator would increase the inclusive decay rate by 20 % to 50 % [2,5]. The purpose of this paper is to show that even a conservative estimate of our proposed mechanism can account for the observed branching ratio in an order of magnitude sense. For this reason, in the rest of this work, we take into account only the dominant chromoelectric operator. The QCD correction of E_0 has been extracted from the Wilson coefficients of the four-quark operators in the leading log approximation [2]. The result points to about a 17 % reduction in $|E_0|$.

(2) The gluon-gluon-psuedoscalar meson (η' in this case) vertex can be written as

$$A^{\mu\sigma}(gg \to \eta') = H(q^2, p^2, m_{\eta'}^2)\,\delta^{ab}\epsilon^{\mu\sigma\alpha\beta}q_\alpha p_\beta, \qquad (4)$$

q and p are four momenta of the two gluons and H is the relevant form factor which contains a factor of α_s implicitly. AS made an estimate of $H(0,0,m_{\eta'}^2) \approx 1.8\ \text{GeV}^{-1}$ using the decay mode $\psi \to \eta'\gamma$ which is expected to proceed mainly via on-shell gluons. However, contrary to the $H(q^2, p^2, m_{\eta'}^2) \approx H(0,0,m_{\eta'}^2)$ assumption utilized by AS, it is claimed that the momentum dependence of H could be quite significant [5,6], resulting in a suppression by an order of magnitude. We show that the nonspectator mechanism suggested in this work can produce a large enough branching ratio which could match the observed value when such a suppression factor is taken into account.

(3) The emission of gluons by the light quark is described below.

By combining the above three terms one arrives at the effective Hamiltonian corresponding to Fig. 1:

$$H_{\text{eff}} = CH[\bar{s}\gamma_\mu(1-\gamma_5)T^a b](\bar{q}\gamma_\sigma T^a q)\frac{1}{p^2 - M_g^2}\epsilon^{\mu\sigma\alpha\beta}q_\alpha p_\beta, \qquad (5)$$

where

$$C = \lambda_t \frac{G_F}{\sqrt{2}}\frac{\alpha_s}{2\pi}E_0, \qquad (6)$$

and the effective gluon mass M_g is due to bound state effects. Alternatively, one may use the usual gluon propagator (no effective mass) along with a model that incorporates the binding effects and off shellness of the light quark inside the B meson. A rearrangement of Eq. (5) via Fierz's transformation,

$$\begin{aligned}
&[\bar{s}\gamma_\mu(1-\gamma_5)T^a b](\bar{q}\gamma_\sigma T^a q) \\
&\quad = \frac{1}{9}\Big\{[\bar{s}\gamma_\sigma\gamma_\rho\gamma_\mu(1-\gamma_5)q] \\
&\qquad \times[\bar{q}\gamma^\rho(1-\gamma_5)b] + [\bar{s}\gamma_\sigma\gamma_\mu(1+\gamma_5)q][\bar{q}(1-\gamma_5)b] \\
&\qquad - \frac{1}{2}(\bar{s}\gamma_\sigma\sigma_{\rho\eta}\gamma_\mu q)[\bar{q}\sigma^{\rho\eta}(1-\gamma_5)b] + \text{color octect}\Big\},
\end{aligned} \qquad (7)$$

simplifies the calculation of the hadronic matrix elements. In fact, using the factorization assumption, only the first two terms in Eq. (7) contribute to $\langle \eta' X_s | H_{\text{eff}} | B\rangle$. In general, the nonfactorizable contributions to two body hadronic B decays which involve large energy releases are not expected to be significant. For our nonspectator mechanism, it seems reasonable to expect the deviation from the factorization to be around $1 - B_B$ where B_B is the parameter associated with the hadronic matrix element of the neutral B meson mixing. Consequently, the nonfactorizable effects are estimated to be around 5 % [8]. Utilizing the definition of the B meson decay constant,

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 b|B(p_B)\rangle = f_B p_B^\mu, \qquad (8)$$

and its associated relation

$$\langle 0|\bar{q}\gamma_5 b|B(p_B)\rangle = -f_B\frac{M_B^2}{m_q + m_b}, \qquad (9)$$

we obtain the relevant matrix element as follows:

$$\begin{aligned}
\langle \eta' X_s|H_{\text{eff}}|B\rangle = \frac{C f_B H}{9(p^2 - M_g^2)}\Big\{&-[\bar{s}\gamma_\sigma\gamma_\rho\gamma_\mu(1-\gamma_5)q]p_B^\rho \\
&+\left(\frac{M_B^2}{m_q + m_b}\right)[\bar{s}\gamma_\sigma\gamma_\mu(1+\gamma_5)q]\Big\} \\
&\times\epsilon^{\mu\sigma\alpha\beta}q_\alpha p_\beta.
\end{aligned} \qquad (10)$$

Hereafter, we take the light quark mass $m_q = 0$.

III. RECOIL MASS DISTRIBUTION AND BRANCHING RATIO

The calculation of the differential decay rate is now straightforward, starting from the matrix element in Eq. (10). We use the usual convention for the invariant variables $s = (p_{\eta'} + k')^2$, $t = (p_s + k')^2$, and $u = (p_s + p_{\eta'})^2$:

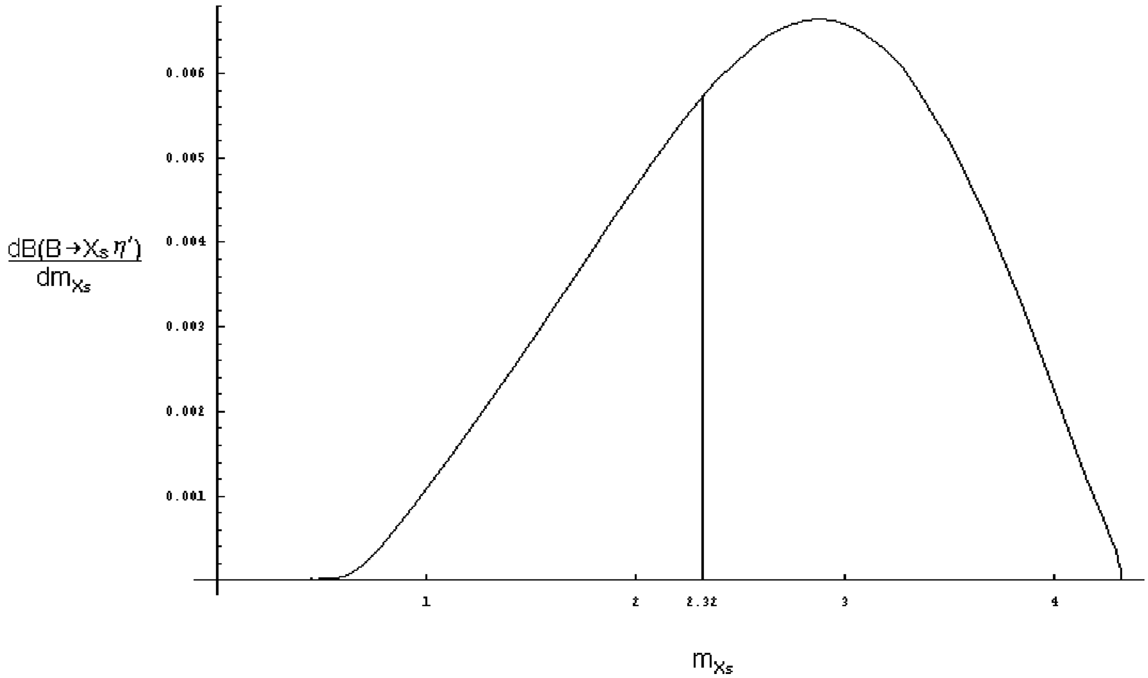

FIG. 2. The recoil mass distribution of the branching ratio for the inclusive decay $B\to\eta' X_s$.

$$\frac{d\Gamma(B\to X_s\eta')}{dtdu}=\frac{C^2f_B^2H^2}{648\pi^3M_B^3(p^2-M_g^2)^2}\Bigg[p^2X\Bigg\{\left(W-Y-\frac{p^2}{2}\right)\times(W-X)-XZ+\left(X-\frac{s+p^2}{2}\right)W\Bigg\}-q^2ZW^2+XYZW+(s-2Y-q^2)\times\left(X-\frac{s+p^2}{2}\right)W^2-\left(\frac{M_B^2}{m_b}\right)^2\times\Bigg\{p^2\left(X-\frac{s+p^2}{2}\right)(s-Y-q^2)-p^2q^2\times\left(W+Z-Y-\frac{p^2}{2}\right)+Y^2Z-(s-2Y-q^2)\times\left(W-Y-\frac{p^2}{2}\right)Y\Bigg\}\Bigg], \tag{11}$$

where $W=(u-m_b^2)/2$, $X=(m_b^2-m_s^2+s)/2$, $Y=(m_{\eta'}^2-p^2-q^2)/2$, and $Z=(t-m_s^2)/2$. $m_{X_s}^2=t$ is the invariant mass of the final state strange hadron. The differential decay rate (11) depends on the virtualities of the internal gluons both explicitly and implicitly through the form factor H. As we discussed in the previous section, $H(p^2,q^2,m_{\eta'}^2)$ is suppressed for large values of p^2 and q^2. Therefore, the dominant contribution to the decay rate is expected to arise from the small virtuality region. On the other hand, in the nonspectator mechanism of Fig. 1, because of the kinematical freedom, one can impose a constraint such as $p^2=0$. Consequently, q^2 can be expressed as

$$q^2=m_b^2+m_{\eta'}^2-u+(t-m_s^2)\frac{u-m_b^2}{m_B^2-u}. \tag{12}$$

Integrating Eq. (11) over u, we obtain the invariant mass distribution for the branching ratio, $dB(B\to X_s\eta')/dm_{X_s}$, depicted in Fig. 2. For our numerical evaluations, we have taken $m_b=4.5$ GeV, $m_s=0.15$,[1] $M_g\approx\Lambda_{\rm QCD}\approx0.3$ GeV [see our explanation following Eq. (5)], $\alpha_s=0.2$, $f_B=0.2$ GeV, and $|V_t|=|V_{tb}V_{ts}^*|\approx|V_{cb}|\approx0.04$. In order to obtain the total branching ratio with the experimental cut $2.0\leq p_{\eta'}\leq2.7$ GeV, the differential decay rate (Fig. 2) is integrated over the range $m_s^{\rm constituent}=0.45\leq m_{X_s}\leq2.32$ GeV resulting in

$$B(B\to X_s\eta')=4.7\times10^{-3},\quad 2.0\leq p_{\eta'}\leq2.7\ \ \text{GeV}. \tag{13}$$

The momentum dependence of $H(q^2,0,m_{\eta'}^2)$ has not been taken into account in the above estimate. Inserting $H(q^2,0,m_{\eta'}^2)=-H(0,0,m_{\eta'}^2)/(q^2/m_{\eta'}^2-1)$ in Eq. (11) results in the reduction of Eq. (13) by more than an order of magnitude to $B(B\to X_s\eta')\approx1.3\times10^{-4}$. We note that our proposed nonspectator mechanism can indeed explain the order of magnitude of the experimental data even though our estimate (which because of the sensitivity to the uncertain input parameters should be taken with caution) is somewhat smaller than Eq. (1).

[1]For phase space calculations, m_s is taken to be the constituent quark mass $m_s^{\rm constituent}\approx0.45$ GeV.

IV. EXCLUSIVE DECAYS $B\to K\eta'$ AND $B\to K^*\eta'$

Using Eq. (10) in conjunction with the factorization assumption, one can write the hadronic matrix element for $B\to K\eta'$ as follows:

$$\langle \eta' K|H_{\rm eff}|B\rangle = \frac{Cf_B H}{9(p^2-M_g^2)}\bigg[-\langle K|\overline{s}\gamma_\sigma\gamma_\rho\gamma_\mu(1-\gamma_5)q|0\rangle p_B^\rho + \left(\frac{M_B^2}{m_b}\right)\langle K|\overline{s}\gamma_\sigma\gamma_\mu(1+\gamma_5)q|0\rangle\bigg] \times \epsilon^{\mu\sigma\alpha\beta}q_\alpha p_\beta. \quad (14)$$

In fact, the second term in Eq. (14) does not contribute and the first term can be related to a K-meson decay constant via the definition

$$\langle K(p_K)|\overline{s}\gamma^\mu\gamma_5 q|0\rangle = f_K p_K^\mu, \quad (15)$$

along with the identity

$$\gamma_\sigma\gamma_\rho\gamma_\mu = i\epsilon_{\sigma\rho\mu\nu}\gamma_5\gamma^\nu + g_{\sigma\rho}\gamma_\mu - g_{\sigma\mu}\gamma_\rho + g_{\rho\mu}\gamma_\sigma. \quad (16)$$

As a result, the matrix element (14) can be simplified to

$$\langle \eta' K|H_{\rm eff}|B\rangle = -i\frac{CHf_Bf_K}{9(p^2-M_g^2)}(p_B\cdot q p_K\cdot p - p_B\cdot p p_K\cdot q), \quad (17)$$

leading to the exclusive decay rate

$$\Gamma(B\to K\eta') = \frac{C^2H^2f_B^2f_K^2}{1944\pi M_g^4}|\vec{p}_K|^3(m_{\eta'}^2+4|\vec{p}_K|^2)p_0^2, \quad (18)$$

which is derived by imposing the $p^2=0$ constraint. $|\vec{p}_K|$ and p_0 are the three momentum of the K meson and the energy change of the light quark in the B meson rest frame, respectively:

$$|\vec{p}_K| = \left[\frac{(m_B^2+m_K^2-m_{\eta'}^2)^2}{4m_B^2} - m_K^2\right]^{1/2}, \quad (19)$$

$$p_0 = \frac{m_B^2-m_b^2}{2m_B} - E_q, \quad (20)$$

where E_q is the energy of the light quark in the K meson. To proceed with the numerical evaluation of $B(B\to K\eta')$, one can assume an appropriate model to estimate E_q. However, roughly speaking, one would expect $E_q = m_K - m_s^{\rm constituent} \approx 0.05$ GeV. The exclusive branching ratio is then estimated to be

$$B(B\to K\eta') = 7.0\times 10^{-5}, \quad (21)$$

which is in good agreement with experimental data (2).

In the same manner, the matrix element relevant for the $B\to K^*\eta'$ decay can be obtained from Eq. (10):

$$\langle \eta' K^*|H_{\rm eff}|B\rangle = -i\frac{CHf_Bf_{K^*}}{9(p^2-M_g^2)}\bigg[p_B\cdot q\epsilon\cdot p - p_B\cdot p\epsilon\cdot q + \frac{2m_B^2m_s}{m_bm_{K^*}^2}(-i\epsilon^{\mu\sigma\alpha\beta}p_{K^*\mu}\epsilon_\sigma q_\alpha p_\beta + p_{K^*}\cdot q\epsilon\cdot p - p_{K^*}\cdot p\epsilon\cdot q)\bigg], \quad (22)$$

where the polarization vector ϵ and the decay constant f_{K^*} of K^* appear in Eq. (21) via the definition

$$\langle K^*(p_{K^*},\epsilon)|\overline{s}\gamma^\mu q|0\rangle = f_{K^*}\epsilon^\mu, \quad (23)$$

and its follow up

$$\langle K^*(p_{K^*},\epsilon)|\overline{s}\sigma^{\mu\nu}q|0\rangle = i\frac{m_s}{m_{K^*}^2}f_{K^*}(p_{K^*}^\mu\epsilon^\nu - p_{K^*}^\nu\epsilon^\mu). \quad (24)$$

Equation (22) in conjunction with the constraint $p^2=0$ leads to the following expression for the exclusive decay rate:

$$\Gamma(B\to K^*\eta') = \frac{C^2H^2f_B^2f_{K^*}^2}{1296\pi M_g^4m_B^2}|\vec{p}_{K^*}|\int_{-1}^{1}F(x)dx,$$

$$F(x) = (C_1+C_3C_5)^2\frac{C_4^2}{m_{K^*}^2} + (C_2+C_4C_5)^2\left(-q^2+\frac{C_3^2}{m_{K^*}^2}\right) - 2(C_1+C_3C_5)(C_2+C_4C_5)\left(-C_6+\frac{C_4C_3}{m_{K^*}^2}\right) + C_5^2[m_{K^*}^2C_6^2 - C_3C_6C_4 - C_4(C_3C_6-q^2C_4)], \quad (25)$$

where

$$C_1 = \frac{m_B^2+m_{\eta'}^2-m_{K^*}^2}{2} - C_2,$$

$$C_2 = m_Bp_0,$$

$$C_3 = \frac{m_B^2-m_{\eta'}^2-m_{K^*}^2}{2} - C_4,$$

$$C_4 = [(m_{K^*}^2+|\vec{p}_{K^*}|^2)^{1/2} - |\vec{p}_{K^*}|x]p_0,$$

$$C_5 = \frac{2m_sm_B^2}{m_bm_{K^*}^2},$$

$$C_6 = \frac{m_{\eta'}^2-q^2}{2},$$

$$|\vec{p}_{K*}|=\frac{\{[m_B^2-(m_{\eta'}+m_{K*})^2][m_B^2-(m_{\eta'}-m_{K*})^2]\}^{1/2}}{2m_B}.$$

p_0 is the same as in Eq. (20) and q^2 is obtained from Eq. (12) by substituting m_{K*}^2 and $m_B^2-2m_BE_q$ for t and u, respectively. In order to estimate the branching ratio for $B\to K^*\eta'$, analogous with the $B\to K\eta'$ exclusive decay, we take the energy of the light quark in K^* to be

$$E_q\approx m_{K*}-m_s^{\text{constituent}}\approx 0.44\ \text{GeV},$$

which results in[2]

$$B(B\to K^*\eta')=3.4\times 10^{-5}. \tag{26}$$

We note that the results for exclusive decays should not be altered significantly because of the momentum dependence of H. This is because of the fact that, unlike the inclusive process, q^2 for these decays is fixed at around 1–3 GeV^2. The measurement of the K^* mode will be a crucial testing ground for various mechanisms suggested for the η' production in hadronic B decays. For example, our prediction is in contrast to $\Gamma(B\to K^*\eta')\approx 2\Gamma(B\to K\eta')$ obtained from the proposed $b\to c\bar{c}s\to\eta' s$ process [3]. We should also emphasize that the ratios of decays $\Gamma(B\to K\eta')/\Gamma(B\to X_s\eta')$ and $\Gamma(B\to K^*\eta')/\Gamma(B\to K\eta')$, calculated in our mechanism, are independent of the input parameters like α_s, f_B, and M_g, and therefore, free of the uncertainties associated with these parameters.

[2]We estimate $f_{K*}\approx 205$ GeV^2 by using the experimental value of the ratios $\Gamma(\tau\to K^{*-}\nu_\tau)/\Gamma(\tau\to K^-\nu_\tau)$ and $f_K=0.167$ GeV [9].

V. SUMMARY

We calculated a nonspectator contribution to the inclusive B meson decay into η' and hadrons containing a strange quark. The result indicates that this mechanism could explain the large experimental branching ratio $B(B\to X_s\eta')$ obtained by CLEO. Our estimated exclusive branching ratio $B(B\to K\eta')$ agrees with the experiment as well. The experimental confirmation of the predicted branching ratio for $B\to K^*\eta'$ will give strong support to the suggestion that the nonspectator mechanism is indeed the underlying process for the above decay modes.

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