

# Decay constants, semileptonic and non-leptonic $B$ decays in a Bethe-Salpeter model

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We evaluate the decay constants for the  $B$  and  $D$  mesons and the form factors for the semileptonic decays of the  $B$  meson to  $D$  and  $D^*$  mesons in a Bethe-Salpeter model. From data we extract  $V_{cb} = 0.039 \pm 0.002$  from  $\bar{B} \rightarrow D^* l \bar{\nu}$  and  $V_{cb} = 0.037 \pm 0.004$  from  $\bar{B} \rightarrow D l \bar{\nu}$  decays. The form factors are then used to obtain non-leptonic decay partial widths for  $B \rightarrow D \pi(K)$  and  $B \rightarrow DD(D_s)$  in the factorization approximation. [S0556-2821(98)05313-2]

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## I. INTRODUCTION

In previous papers [1,2] we have developed a model for mesons based on the Bethe-Salpeter equation (BSE). Recently [3,4], we calculated the form factors in the semileptonic  $B \rightarrow D(D^*) l \nu$  decays and extracted the Cabibbo-Kobayashi-Maskawa (CKM) matrix element  $V_{cb}$  from data with and without taking the QCD correction into account. The key ingredient in the computation of the form factors was the construction of the physical states for the  $B$  and  $D$  mesons in terms of the wave function obtained by solving a reduced BSE. In this paper we improve upon the work of Ref. [4] in two ways. First we establish a theoretical connection between the matrix element of the bare current operator calculated using the model states constructed in [4] and the matrix elements of an effective current operator based upon arguments for contributions from neglected configurations. Second, we make an ansatz for the correspondence between the matrix elements of the bare and the effective current operator. The effective current operators are then used to calculate not only the decay constants and semileptonic form factors similar to our previous work but also the branching fractions of non-leptonic decays.

The discovery of heavy quark symmetry (HQS) in recent years [5,6,7,8,9] has generated considerable interest in the study of systems containing heavy quark(s). It has been shown that, in the heavy quark limit, the properties of systems containing a heavy quark are greatly simplified. HQS results in relations between non-perturbative quantities, such as form factors, for different processes involving transitions of a heavy quark to another quark. The development of heavy quark effective theory (HQET) [7] allows one to systematically calculate corrections to the results of the HQS limit in inverse powers of the heavy quark mass  $m_Q$ . In spite of impressive results obtained in HQET, it has not solved the problem of calculating the transition form factors in QCD. In particular, HQS reveals relations between form factors but does not provide a determination of the form factors themselves. Furthermore, the systematic expansion of the form

factors in  $1/m_Q$  in HQET involves additional non-perturbative matrix elements which are not calculable from first principles.

We are thus forced to rely on models for the non-perturbative quantities. However, the constraints of HQET, which are based on QCD, allow one to construct models which are consistent with HQET and hence QCD. We have already demonstrated the consistency of our model with the requirements of HQET [4].

The parameters in the BSE are fixed by fitting the meson spectrum. Hadronic states necessary for the calculation of form factors are constructed with the BS wave functions. In our formalism the mesons have been considered as composed of  $q\bar{q}$  constituent quarks which defines the limits of our model space. Our dual thrust in this effort is to correct for the limited model space and to carry out new applications. Higher Fock state effects are introduced through an ansatz, involving an additional parameter, connecting the bare current operator to an effective operator.

The additional parameter introduced in this process is chosen by a fit to the lattice evaluation of the decay constants  $f_B$ ,  $f_D$ , and  $f_{D_s}$ . An evaluation of the other decay constants of the  $B$  and  $D$  system along with the semi-leptonic form factors and the non-leptonic decays is performed without any additional free parameter and are, therefore, viewed as predictions of this model. Based on this new approach we again extract  $V_{cb}$  from the measured differential decay rate of  $\bar{B} \rightarrow D^* l \bar{\nu}$  and find a 20% increase over our previous results [4]. We also extract  $V_{cb}$  using recent measurement of  $\bar{B} \rightarrow D l \bar{\nu}$  by CLEO [10]. The two resulting values for  $V_{cb}$  presented in this paper are consistent with each other.

The paper is organized as follows: In Sec. II we give a brief review of Bethe-Salpeter model for mesons. In Sec. III, we discuss the formalism for the calculation of the decay constants and the form factors after establishing the connection between the bare current operator and the effective current operator. In Sec. IV, we discuss non-leptonic decays and in Sec. V we present and discuss the results of our work.

## II. BETHE-SALPETER MODEL FOR MESONS

In Ref. [1] we have developed a model for mesons based on the Bethe-Salpeter equation. The wave functions for the mesons were solved from three dimensional reductions of BSE, called the quasi-potential equations (QPE). It was found that two reductions give a good description of the meson spectrum, including open flavor mesons, over a wide range of states. Masses for 47 states were predicted using seven parameters given below with mass root mean square deviation of about 50 MeV [2].

The interaction kernel in the BSE is written as a sum of a one-gluon exchange interaction in the ladder approximation,  $V_{OGE}$ , and a phenomenological, long-range linear confinement potential,  $V_{CON}$ . In momentum space this interaction takes the form

$$V_{OGE} + V_{CON} = \frac{4}{3} \alpha_s \frac{\gamma_\mu \otimes \gamma_\mu}{(q - q')^2} + \sigma \lim_{\mu \rightarrow 0} \frac{\partial^2}{\partial \mu^2} \frac{\mathbf{1} \otimes \mathbf{1}}{-(q - q')^2 + \mu^2}. \quad (1)$$

Here,  $\alpha_s$  is the strong coupling, which is weighted by the meson color factor of  $\frac{4}{3}$ , and the string tension  $\sigma$  is the strength of the confining part of the interaction. We adopt a scalar Lorentz structure  $V_{CON}$  as discussed in [2].

In our model the strong coupling is assumed to run as in the leading log expression for  $\alpha_s$ ,

$$\alpha_s(Q^2) = \frac{4\pi\alpha_s(\mu^2)}{4\pi + \beta_1\alpha_s(\mu^2)\ln(Q^2/\mu^2)} \quad (2)$$

where  $\beta_1 = 11 - 2n_f/3$  and  $n_f$  is the number of quark flavors, with  $\alpha_s(\mu^2 = M_Z^2) \approx 0.12$  where  $Q^2$  is related to the meson mass scale through

$$Q^2 = \gamma^2 M_{meson}^2 + \beta^2, \quad (3)$$

where  $\gamma$  and  $\beta$  are parameters determined by a fit to the meson spectrum.

In our formulation of BSE there are therefore seven parameters: four masses,  $m_u = m_d, m_c, m_s, m_b$ ; the string tension  $\sigma$ , and the parameters  $\gamma$  and  $\beta$  used to govern the running of the coupling constant. Once the parameters are fixed from the mass spectrum, the meson wave functions from the BSE can be used to predict physical observables.

Table I shows the values of the parameters used in two reductions of the Bethe-Salpeter equation referred to as A, B reductions [2].

## III. DECAY CONSTANTS AND SEMI-LEPTONIC FORM FACTORS

The weak decay constants for the heavy hadrons are defined below:

TABLE I. Values of the parameters used in reductions A,B together with root mean square deviation from experimental meson masses.

	Reduction A	Reduction B
$m_b$ (GeV)	4.65	4.68
$m_c$ (GeV)	1.37	1.39
$m_s$ (GeV)	0.397	0.405
$m_u$ (GeV)	0.339	0.346
$\sigma$ (GeV <sup>2</sup> )	0.233	0.211
$\gamma$	0.616	0.444
$\beta$ (GeV)	0.198	0.187
RMS (MeV)	43	50

$$\langle 0 | J_\mu | P(p) \rangle = i f_P p_\mu$$

$$\langle 0 | J_\mu | V(p) \rangle = m_V f_V \varepsilon_\mu$$

$$J_\mu = V_\mu - A_\mu \quad (4)$$

where  $P$  and  $V$  are pseudo-scalar and vector states and  $V_\mu$  and  $A_\mu$  are the vector and axial vector currents.

The Lagrangian for the semileptonic decays involving the  $b \rightarrow c$  transition has the standard current-current form after the  $W$  boson is integrated out in the effective theory:

$$H_W = \frac{G_F}{2\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{\nu} \gamma^\mu (1 - \gamma_5) l. \quad (5)$$

The leptonic current in the effective interaction is completely known and the matrix element of the vector ( $V_\mu$ ) and the axial vector ( $A_\mu$ ) hadronic currents between the meson states are represented in terms of form factors which are defined in the equations below [11].

$$\begin{aligned} \langle D(p_D) | J_\mu | B(p_B) \rangle = & \left[ (p_B + p_D)_\mu \right. \\ & \left. - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right] F_1(q^2) \\ & + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0(q^2) \end{aligned} \quad (6)$$

where  $q = p_B - p_D$ ,

$$\begin{aligned} \langle D^*(p') | J^\mu | B(p) \rangle = & b_0 \varepsilon^{\mu\nu\alpha\beta} \varepsilon_\nu^* p_\alpha p'_\beta + b_1 \varepsilon^{*\mu} \\ & + b_2 (p + p')^\mu + b_3 (k)^\mu \end{aligned} \quad (7)$$

with

$$b_0 = \frac{2V(k^2)}{m_B + m_{D^*}}$$

$$b_1 = i(m_B + m_{D^*}) A_1(k^2)$$

$$\begin{aligned}
 b_2 &= -i\varepsilon^* \cdot k \frac{A_2(k^2)}{m_B + m_{D^*}} \\
 b_3 &= i\varepsilon^* \cdot k \frac{2m_{D^*}(A_0(k^2) - A_3(k^2))}{k^2} \\
 A_3(k^2) &= \frac{(m_B + m_{D^*})A_1(k^2) - (m_B - m_{D^*})A_2(k^2)}{2m_{D^*}}
 \end{aligned}$$

where  $k = p_B - p_{D^*}$ .  $F_0$ ,  $F_1$ ,  $V$ ,  $A_0$ ,  $A_1$ ,  $A_2$ , and  $A_3$  are Lorentz invariant form factors which are scalar functions of the momentum transfer  $(P_B - P_{D^*})^2$ . The calculation of the decay constants and form factors proceeds in two steps. In the first step, the full current from QCD is matched to the current in the effective theory (HQET) at the heavy quark mass scale [12]. Renormalization group equations are then used to run down to a low energy scale  $\mu \sim 1$  GeV where the constraints of HQET operate and where it is reasonable to calculate matrix elements in a valence constituent quark model like the one we employ here [14]. We have already described the first step, viz. the perturbative QCD corrections to the weak currents, in our previous publication [4] and therefore we will not repeat it here.

The second step is the calculation of the matrix elements of the currents in the model to obtain the decay constants and form factors. Such a calculation requires the knowledge of the meson wave functions. In our formalism the mesons are taken as bound states of a quark and an antiquark. The wave functions for the mesons, as already mentioned, are calculated by solving the Bethe-Salpeter equation [1,2]. We construct the meson states as [13]

$$\begin{aligned}
 |M(\mathbf{P}_M, J, m_J)\rangle &= \sqrt{2M_H} \int d^3\mathbf{p} \langle L m_L S m_S | J m_J \rangle \\
 &\quad \times \langle s m_s \bar{s} m_{\bar{s}} | S m_S \rangle \\
 &\quad \times \Phi_{L m_L}(\mathbf{p}) \left| \bar{q} \left( \frac{m_{\bar{q}}}{M} \mathbf{P}_M - \mathbf{p}, m_{\bar{s}} \right) \right\rangle \\
 &\quad \times \left| q \left( \frac{m_q}{M} \mathbf{P}_M + \mathbf{p}, m_s \right) \right\rangle
 \end{aligned} \quad (8)$$

where

$$\begin{aligned}
 |q(\mathbf{p}, m_s)\rangle &= \sqrt{\frac{(E_q + m_q)}{2m_q}} \begin{pmatrix} \chi^{m_s} \\ \boldsymbol{\sigma} \cdot \mathbf{p} \\ (E_q + m_q) \chi^{m_s} \end{pmatrix} \\
 M &= m_q + m_{\bar{q}} \\
 E_q &= \sqrt{m_q^2 + \mathbf{p}^2}
 \end{aligned} \quad (9)$$

and  $M_H$  is the meson mass. The meson and the constituent quark states are normalized as

$$\begin{aligned}
 \langle M(\mathbf{P}'_M, J', m'_{J'}) | M(\mathbf{P}_M, J, m_J) \rangle \\
 = 2E \delta^3(\mathbf{P}'_M - \mathbf{P}_M) \delta_{J', J} \delta_{m'_{J'}, m_J}
 \end{aligned} \quad (10)$$

$$\begin{aligned}
 \langle q(\mathbf{p}', m'_s) | q(\mathbf{p}, m_s) \rangle \\
 = \frac{E_q}{m_q} \delta^3(\mathbf{p}' - \mathbf{p}) \delta_{m'_s, m_s}.
 \end{aligned} \quad (11)$$

In constructing the meson states we maintain a constituent quark model approach as we do not include  $q\bar{q}$  sea quark states nor the explicit gluonic degrees of freedom. We also assume the validity of the weak binding approximation [13,14]. In the weak binding limit our meson state forms a representation of the Lorentz group, as discussed in Ref. [13], if the quark momenta are small compared to their masses. Assuming that the quark fields in the current create and annihilate the constituent quark states appearing in the meson state, the calculation of the matrix element of the current operator then reduces to the calculation of a free quark matrix element. In the rest frame of the  $B$  meson with a suitable choice of the four-vector indices in Eqs. (6), (7) we can construct six independent equations which we can solve to extract the six form factors.

This model space representation may be viewed as the leading characterization in an expanded representation which more accurately represents the exact states. We assume that the effects of higher Fock states, representing gluons or sea quarks, in the calculation of the matrix element of the bare current operator are represented by the matrix element of an effective operator in the truncated model space. In other words, with the notation ‘‘e’’ labeling exact states, and ‘‘m’’ labeling model states,

$$\langle M_2^e(\mathbf{P}_2) | J_\mu | M_1^e(\mathbf{P}_1) \rangle \rightarrow \langle M_2^m(\mathbf{P}_2) | J_\mu^{\text{eff}} | M_1^m(\mathbf{P}_1) \rangle. \quad (12)$$

To define clearly what we mean by the bare current operator and the effective current operator, let us write, assuming heavy quark symmetry,

$$|M_i^e(\mathbf{P}_i)\rangle = |Q_i, L_i\rangle \quad (13)$$

where  $Q_i$  is the heavy quark in the meson and  $L_i$  is the light degree of freedom which includes all possible Fock states involving light quarks and gluon degrees of freedom and  $i$  takes the value 1,2. In the model states we set  $L_i = \bar{q}_i$  which is the lowest Fock state and all the other higher Fock state effects involving the light degrees of freedom are omitted. Note that, in the heavy quark limit,

$$\langle M_2^e(\mathbf{P}_2) | J_\mu | M_1^e(\mathbf{P}_1) \rangle \sim \langle Q_2 | J_\mu | Q_1 \rangle P(L_2 | L_1). \quad (14)$$

The ratio  $\langle L_2 | L_1 \rangle / \langle \bar{q}_2 | \bar{q}_1 \rangle$  is, therefore, roughly the factor by which the matrix element of the current operator calculated with the model states have to be corrected to include higher order Fock states involving light degrees of freedom. We expect this factor to be approximately independent of the spins of the heavy quark as, in the heavy quark limit, the interaction between the heavy quark and the light degrees of freedom is independent of the spin of the heavy quark. Any effect dependent on the heavy quark spin will be higher order effect in  $1/m_Q$  and we neglect such contributions.

The effective current operator is then the bare current operator, which comes from the weak interaction Hamiltonian

defined in Eq. (5), combined with the factor representing corrections from higher Fock states with light degrees of freedom. We will include Fock state effects involving light degrees of freedom by the following replacement in the calculation of the matrix element

$$\begin{aligned}\Phi_2^e(\mathbf{p}')J_\mu\Phi_1^e(\mathbf{p}) &\equiv \Phi_2^m(\mathbf{p}')J_\mu^{\text{eff}}\Phi_1^m(\mathbf{p}) \\ &= \Phi_2^m(\mathbf{p}')\Omega^\dagger(\mathbf{p}')J_\mu\Omega(\mathbf{p})\Phi_1^m(\mathbf{p}).\end{aligned}\quad (15)$$

In the above,  $\mathbf{p}, \mathbf{p}'$  are the internal momenta of the quarks in the initial and final mesons and  $\Phi_1(\mathbf{p}), \Phi_2(\mathbf{p}')$  are the initial and final meson wave functions. We will use the very simple ansatz

$$\Omega(\mathbf{p}) = e - \alpha^2 \mathbf{p}^2/2. \quad (16)$$

A phenomenological motivation for this ansatz comes from the fact that potential models for  $\Phi_i^m(\mathbf{p})$  with Coulomb-like interaction at short distance exaggerate the values of the decay constants [15,16], which, in the non-relativistic limit, is proportional to the value of the wave function at the origin of the configuration space. We will fit the lattice calculations of the decay constants  $f_B, f_D,$  and  $f_{D_s}$  using the parameter  $\alpha$ . Our ansatz for the higher Fock state effects suppresses the tail of the wave function in the momentum space leading to smaller values of the decay constants consistent with results obtained in lattice calculations. Note that we will use the same value of  $\alpha$  for decays involving B and D decays. This is consistent with heavy quark symmetry.

The expressions of the decay constants in terms of the wave functions are given as [15]

$$f_i = \sqrt{\frac{12}{M}} \int_0^\infty \frac{p^2 dp}{2\pi^3} \sqrt{\frac{(m_q + E_q)(m_{\bar{q}} + E_{\bar{q}})}{4E_q E_{\bar{q}}}} F_i(p) \quad (17)$$

$$F_P(p) = \left[ 1 - \frac{p^2}{(m_q + E_q)(m_{\bar{q}} + E_{\bar{q}})} \right] \psi_P(p) \quad (18)$$

$$F_V(p) = \left[ 1 - \frac{p^2}{3(m_q + E_q)(m_{\bar{q}} + E_{\bar{q}})} \right] \psi_V(p) \quad (19)$$

where  $\psi_{P(V)}$  are the wave functions of the exact states. Using Eq. (12), we can then obtain the form factor in terms of the BSE wave functions.

#### IV. NON-LEPTONIC DECAYS

Non-leptonic decays arise from W exchange diagrams at tree level. Strong interactions play an important role in these decays by modifying the weak vertices through hard gluon corrections and then the long distance QCD interactions result in the binding of the quarks in the hadrons. An effective Hamiltonian of four quark operators is constructed by integrating the W-boson and the top quark from the theory. The effects of the short distance and the long distance QCD interactions are separated using the operator product expansion

where the Wilson coefficients account for the short distance effects while the long distance effects are incorporated in the matrix element of the four quark operators. The effective Hamiltonian operator for  $b \rightarrow c$  transition can be written as

$$\begin{aligned}H_{\text{eff}} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_1(\mu) O_1 + c_2(\mu) O_2] \\ O_1 &= \bar{d}_i \gamma_\mu (1 - \gamma_5) u_i \bar{c}_j \gamma^\mu (1 - \gamma_5) b_j \\ O_2 &= \bar{d}_i \gamma_\mu (1 - \gamma_5) u_j \bar{c}_j \gamma^\mu (1 - \gamma_5) b_i\end{aligned}\quad (20)$$

where  $i$  and  $j$  are the color indices. The Wilson's coefficients  $c_1$  and  $c_2$  at the scale  $\mu = m_b$  have values 1.132 and  $-0.286$  respectively [17].

The matrix element of a two body leptonic decay of the type  $B \rightarrow XY$  requires the evaluation of the matrix element

$$M = \langle X, Y | H_{\text{eff}} | B \rangle$$

where  $H_{\text{eff}}$  has a current  $\times$  current structure. The matrix element is usually calculated using the factorization assumption where one separates out the current in  $H_{\text{eff}}$  by inserting the vacuum state and neglecting any QCD interactions between the currents. The matrix element above written as a product of two current matrix elements is

$$M \sim \langle X | J_\mu | 0 \rangle \langle Y | J'^\mu | B \rangle. \quad (21)$$

In B decays, e.g.  $B \rightarrow D^+ \pi^-$ , the energetic quark-antiquark pair in the pion is created at short distance and by the time it hadronizes it is far from the other quarks so it should be a good approximation to neglect the QCD interaction between the two currents creating the final state particles. A detailed description about the validity and the corrections to the factorization assumption can be found in Ref. [17].

In this paper we will look at decays where the particle  $Y$  is a  $D$  or a  $D^*$  meson because one can then use the semi-leptonic form factors calculated in the previous section to compute  $\langle Y | J'^\mu | B \rangle$ . The  $X$  will be either a light meson ( $\pi, K, \rho, K^*$ ) or a  $D(D^*)$  meson. For the light mesons the decay constants  $f_X = \langle X | J_\mu | 0 \rangle$  are available from experiment while, for the heavy mesons, we will use the decay constants calculated in the previous section.

The expressions for the square of the matrix element for the processes  $\bar{B}^0 \rightarrow D(D^*) \pi(\rho)$  are

$$\begin{aligned}|M|^2(\bar{B}^0 \rightarrow D^+ \pi^-) &= \left( \frac{G_F}{\sqrt{2}} \right)^2 |V_{cb} V_{ud}^*|^2 (c_1 + c_2/N_c)^2 \\ &\times f_\pi^2 F_0(m_\pi^2)^2 (m_B^2 - m_D^2)^2\end{aligned}\quad (22)$$

where  $N_c$  is the number of colors,

TABLE II. Decay constants of the  $B$  and  $D$  mesons in MeV.

Decay constants	Our results	Lattice results [20]
$f_D$	209	196(9)(14)(8)
$f_{D^*}$	237	
$f_{D_s}$	213	211(7)(25)(11)
$f_{D_s^*}$	242	
$f_B$	155	166(11)(28)(14)
$f_{B^*}$	164	

$$\begin{aligned}
 |M|^2(\bar{B}^0 \rightarrow D^+ \rho^-) &= (G_F \sqrt{2})^2 |V_{cb} V_{ud}^*|^2 (c_1 + c_2/N_c)^2 \\
 &\times m_\rho^2 f_\rho^2 F_1(m_\rho^2)^2 m_B^2 \frac{p^2}{m_\rho^2} \quad (23)
 \end{aligned}$$

where  $p$  is the momentum of the decay products in the rest frame of the  $B$ ,

$$\begin{aligned}
 |M|^2(\bar{B}^0 \rightarrow D^{*+} \pi^-) &= \left( \frac{G_F}{\sqrt{2}} \right)^2 |V_{cb} V_{ud}^*|^2 (c_1 + c_2/N_c)^2 \\
 &\times f_\pi^2 4m_{D^*}^2 A_0(m_\pi^2)^2 m_B^2 \frac{p^2}{m_{D^*}^2} \quad (24)
 \end{aligned}$$

and finally,

$$\begin{aligned}
 |M|^2(\bar{B}^0 \rightarrow D^{*+} \rho^-) &= \left( \frac{G_F}{\sqrt{2}} \right)^2 |V_{cb} V_{ud}^*|^2 (c_1 + c_2/N_c)^2 \\
 &\times m_\rho^2 f_\rho^2 [T_1 + T_2 + T_3 + T_4]
 \end{aligned}$$

$$T_1 = \frac{8V^2}{(m_B + m_{D^*})^2} p^2 m_B^2$$

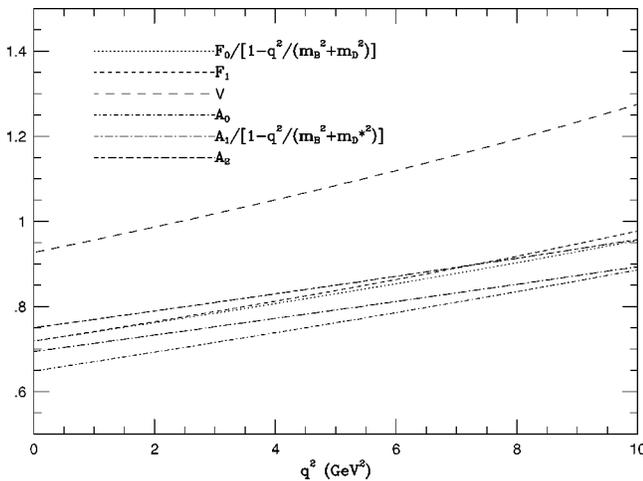


FIG. 1. The calculated form factors  $F_0$ ,  $F_1$ ,  $V$ ,  $A_0$ ,  $A_1$ , and  $A_2$  as a function of  $q^2$ .

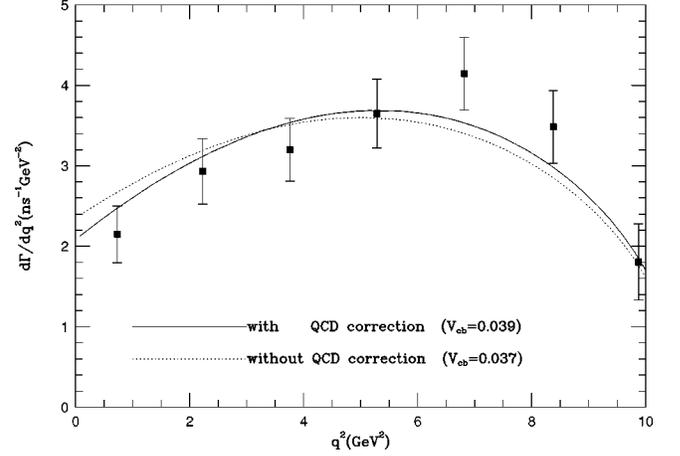


FIG. 2. The differential decay rate for  $\bar{B} \rightarrow D^* l \bar{\nu}$  with and without the QCD correction, together with the corresponding values of  $V_{cb}$ . Data from Ref. [18].

$$\begin{aligned}
 T_2 &= A_1^2 (m_B + m_{D^*})^2 \\
 &\times \left[ 2 + \frac{(m_B E_{D^*} - m_{D^*}^2)^2}{m_{D^*}^2 m_\rho^2} \right] \\
 T_3 &= \frac{4A_2^2}{(m_B + m_{D^*})^2} \frac{p^4 m_B^4}{m_{D^*}^2 m_\rho^2} \\
 T_4 &= 4A_1 A_2 \left[ \frac{E_\rho^2}{m_\rho^2} + \frac{E_{D^*}^2}{m_{D^*}^2} \right. \\
 &\quad \left. - \frac{E_{D^*} E_\rho (m_B E_{D^*} - m_{D^*}^2)}{m_{D^*}^2 m_\rho^2} - 1 \right]. \quad (25)
 \end{aligned}$$

Similar expressions can also be written down for the  $B \rightarrow DD$  decays.

As in Ref. [17] we will include the effect of corrections to the factorization assumption by the replacement

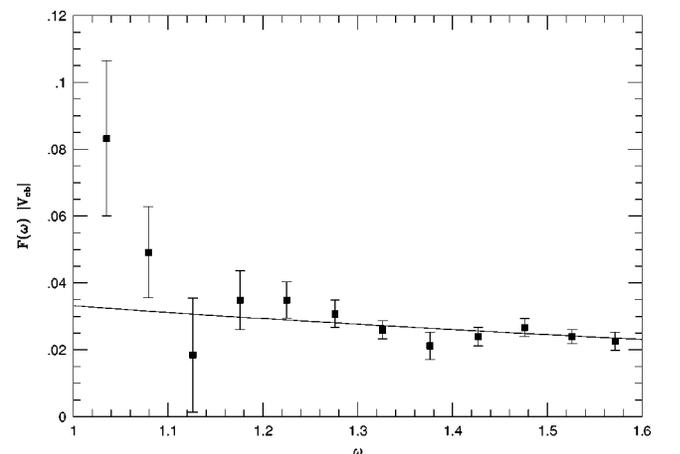


FIG. 3.  $F(\omega)|V_{cb}|$  versus  $\omega$  for  $\bar{B} \rightarrow D l \bar{\nu}$ . Data from Ref. [10].

TABLE III. Non-leptonic decay rates for the  $B$  meson.

Process	Our results	Stech-Neubert [17]	Expt. [21]
$B^0 \rightarrow D^+ \pi^-$	0.345	0.300	0.310(0.040)(0.020)
$B^0 \rightarrow D^{*+} \pi^-$	0.331	0.290	0.280(0.040)(0.010)
$B^0 \rightarrow D^+ \rho^-$	0.799	0.750	0.840(0.160)(0.070)
$B^0 \rightarrow D^{*+} \rho^-$	0.897	0.850	0.730(0.150)(0.030)
$B^0 \rightarrow D^+ K^-$	0.26	0.20	
$B^0 \rightarrow D^{*+} K^-$	0.24	0.20	
$B^0 \rightarrow D^+ K^{*-}$	0.41	0.40	
$B^0 D^{*+} K^{*-}$	0.49	0.50	
$B^0 \rightarrow D^+ D^-$	0.31	0.40	
$B^0 \rightarrow D^{*+} D^-$	0.22	0.30	
$B^0 \rightarrow D^+ D^{*-}$	0.27	0.30	
$B^0 \rightarrow D^{*+} D^{*-}$	0.65	0.80	
$B^0 \rightarrow D^+ D_s^-$	0.626	1.030	0.740(0.22)(0.18)
$B^0 \rightarrow D^{*+} D_s^-$	0.420	0.700	0.94(0.24)(0.23)
$B^0 \rightarrow D^+ D_s^{*-}$	0.514	0.950	1.140(0.42)(0.28)
$B^0 \rightarrow D^{*+} D_s^{*-}$	1.35	2.450	2.0(0.54)(0.05)
$B^- \rightarrow D^0 D^-$	0.33	0.40	
$B^- \rightarrow D^{*0} D^-$	0.210	0.30	
$B^- D^0 D^{*-}$	0.27	0.40	
$B^- D^{*0} D^{*-}$	0.64	0.90	
$B^- \rightarrow D^0 D_s^-$	0.829	1.090	1.360(0.280)(0.330)
$B^- \rightarrow D^{*0} D_s^-$	0.552	0.750	0.940(0.310)(0.23)
$B^- \rightarrow D^0 D_s^{*-}$	0.696	1.020	1.180(0.36)(0.29)
$B^- \rightarrow D^{*0} D_s^{*-}$	1.830	2.610	2.700(0.810)(0.660)

$$c_1 + c_2/N_c \rightarrow a_1$$

$$a_1 = c_1(\mu) + \frac{c_2(\mu)}{N_c} (1 + \varepsilon^1(\mu)) + c_2(\mu) \varepsilon^8(\mu).$$

The nonfactorizable corrections  $\varepsilon^1(\mu)$  and  $\varepsilon^8(\mu)$  are defined in Ref. [17] and may be process dependent. We will, however, treat  $a_1$  as a process independent free parameter that we will fit to data. RGE analysis suggests that  $a_1 \sim 1 + 0(1/N_c^2)$ .

## V. RESULTS AND DISCUSSIONS

In previous papers [3,4], a covariant reduction of the Bethe-Salpeter equation (BSE) was used to calculate the Isgur-Wise function. The BSE was solved numerically and the parameters appearing in it (the quark masses, string tension and the running coupling strength for the one gluon exchange) were determined by fitting the calculated spectrum to the observed masses of more than 40 mesons. The resulting mass spectrum of the analysis was found to agree very well with the experimental data. Once the parameters of the model were fixed, the meson wave function could be calculated from the BSE. This wave function was used to calculate the Isgur-Wise function and determine  $V_{cb}$  [3].

In our present approach we evaluate the decay constants, the form factors for the semileptonic decays  $\bar{B} \rightarrow D^* l \bar{\nu}$  and

$\bar{B} \rightarrow D l \bar{\nu}$  with the effective current operator defined in Eq. (15) treating  $\alpha$  of Eq. (16) as an adjustable parameter. The value of  $\alpha$  is fixed by fitting the leptonic decay constants  $f_B$ ,  $f_D$ , and  $f_{D_s}$ . We find  $\alpha = 0.7 \text{ GeV}^{-1}$  provides a good fit and use this value in all the calculations in this paper.

We present our results for the decay constants of the heavy mesons in Table II. For the sake of comparison we also show lattice calculations of the decay constants. The errors in the second column of Table II are, respectively, (1) the statistical errors; (2) the systematic errors of changing fitting ranges, as well as other errors within the quenched approximations; and (3) the quenching error. The results in Table II show that our calculated decay constants are similar to the lattice results.

On the other hand, our calculation for the decay constants of the light mesons  $\pi, K$  etc. are not in good agreement with the experimental numbers. In fact, the light meson decay constants are larger than experiment by a factor of 2. This is not surprising as our formalism is designed for the heavy meson system.

In Fig. 1 we show the form factors  $F_0, F_1, V, A_0, A_1$ , and  $A_2$  as a function of  $q^2$ . In Fig. 2 we show a plot of the differential decay rate for  $\bar{B} \rightarrow D^* l \bar{\nu}$ . We obtain a good agreement with the shape of the experimental data [18] and extract  $|V_{cb}| = 0.039 \pm 0.002$ . This is within the range of the presently accepted values for  $|V_{cb}|$  [19].

For the decay  $\bar{B} \rightarrow D l \bar{\nu}$ , in Fig. 3 we show a plot of  $F(\omega)|V_{cb}|$  versus  $\omega$  where the data points are taken from

measurements reported in Ref. [10]. The variable  $\omega = (M_B^2 + M_D^2 - q^2)/(2M_B M_D)$  where  $q^2$  is the invariant mass squared of the lepton neutrino system. We extract  $|V_{cb}| = 0.037 \pm 0.004$  by a  $\chi^2$  fit to the data in Fig. 3.

Note that the values of  $V_{cb}$  extracted from the two different experiments are consistent with each other. As a further test of our formalism we present our calculations of the non-leptonic decays of the B meson to  $DD$  and  $DK(\pi)$  final states. Experimental values of some of the decays are already available and new results are expected soon. We present our results in Table III. The values of the light decay constants used in our calculations are  $f_\pi = 130$  MeV,  $f_K = 159$  MeV,  $f_{K^*} = 214$  MeV and  $f_\rho = 208$  MeV.

The parameter  $a_1$  calculated on the basis of a  $\chi^2$  fit has the value 0.88 which is close to 1 as is expected from RGE analysis which gives  $a_1 \sim 1 + O(1/N_c^2)$  suggesting a value for  $a_1$  in the range 0.9–1.1

From Table III we find a good agreement of our calculation with data for the  $D^* \pi(\rho)$  final states. Our results for the  $DK$  final states are quite similar to those in Ref. [17]. This continues to be true for the  $DD$  and  $D^0 D^-$  final states. Our results for  $D^+ D_s^-$  and  $D^0 D_s^-$  final states are somewhat smaller than the central values from experiment though the measurements have large errors.

Combining the experimental errors in quadrature the difference between theory and experiment is less than  $1.5\sigma$  in all cases but the theory predictions are systematically lower for these cases. It appears that as we increase the mass of the decay products, as in the  $DD$  final states, and decrease their kinetic energy the expected deterioration of the factorization approximation may be showing up through a systematic difference between theory and experiment. This motivates fu-

ture efforts to examine corrections to the factorization approximation [17]. We have resisted the temptation to allow  $a_1$  to have a process dependence even though two values for  $a_1$  would yield an excellent description of the known non-leptonic decay rates. It is trivial to relax this restriction if the reader chooses to do so.

In conclusion, we have presented the calculation of form factors and differential decay rates in  $\bar{B} \rightarrow D(D^*) l \bar{\nu}$  transitions in a Bethe-Salpeter model for mesons. The parameters of the bound state model were fixed from the spectroscopy of the hadrons. The effects of higher Fock states in the hadron state were included in the definition of effective current operators. A simple ansatz connecting the effective current operator to the actual current operator was used involving only one parameter. After adjusting this parameter to fit certain decay constants ( $f_B$ ,  $f_D$ , and  $f_{D_s}$ ), we found good agreement with data and extracted  $|V_{cb}| = 0.039 \pm 0.002$  from  $\bar{B} \rightarrow D^* l \bar{\nu}$  and  $V_{cb} = 0.037 \pm 0.004$  from  $\bar{B} \rightarrow D l \bar{\nu}$  decays. Calculations of the decay constants of the B and D mesons were also performed with results that are similar to lattice results. Finally, the form factors were used to evaluate the non-leptonic  $B \rightarrow D \pi(K)$  and  $B \rightarrow DD(D_s)$  decays in the factorization approximation and good agreement was obtained with data.

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- [1] D. Eyre and J. P. Vary, Phys. Rev. D **34**, 3467 (1986); J. R. Spence and J. P. Vary, *ibid.* **35**, 2191 (1987); J. R. Spence and J. P. Vary, Phys. Rev. C **47**, 1282 (1993); A. J. Sommerer, J. R. Spence, and J. P. Vary, *ibid.* **49**, 513 (1994).
- [2] Alan J. Sommerer, A. Abd El-Hady, John R. Spence, and James P. Vary, Phys. Lett. B **348**, 277 (1995).
- [3] A. Abd El-Hady, K. S. Gupta, A. J. Sommerer, J. Spence, and J. P. Vary, Phys. Rev. D **51**, 5245 (1995).
- [4] A. Abd El-Hady, A. Datta, K. S. Gupta, and J. P. Vary, Phys. Rev. D **55**, 6780 (1997).
- [5] N. Isgur and M. B. Wise, Phys. Lett. B **232**, 113 (1989); **237**, 527 (1990); N. Isgur and M. B. Wise, in *B Decays*, edited by S. Stone (World Scientific, Singapore, 1991), p. 158; *Heavy Flavors*, edited by A. J. Buras and M. Lindner (World Scientific, Singapore, 1992), p. 234.
- [6] M. B. Voloshin and M. A. Shifman, Yad. Fiz. **47**, 801 (1988) [Sov. J. Nucl. Phys. **47**, 511 (1988)]; M. A. Shifman, *Lepton and Photon Interactions* in Proceedings of the 1987 International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, West Germany, 1987, edited by W. Bartel and R. Rückl [Nucl. Phys. B (Proc. Suppl.) **3**, 289 (1988)]; S. Nussinov and W. Wetzel, Phys. Rev. D **36**, 130 (1987); G. P. Lepage and B. A. Thacker, in *Field Theory on the Lattice*, Proceedings of the Conference, Seillac, France, 1987, edited by A. Billoire [Nucl. Phys. B (Proc. Suppl.) **4**, 199 (1988)]; E. Eichten, *ibid.*, p. 170; E. Shuryak, Phys. Lett. **93B**, 134 (1980); Nucl. Phys. **B198**, 83 (1982).
- [7] H. Georgi, Phys. Lett. B **240**, 447 (1990); E. Eichten and B. Hill, *ibid.* **234**, 511 (1990); M. B. Voloshin and M. A. Shifman, Sov. J. Nucl. Phys. **45**, 463 (1987); H. D. Politzer and M. B. Wise, Phys. Lett. B **206**, 681 (1988); **208**, 504 (1988); A. F. Falk, H. Georgi, B. Grinstein, and M. B. Wise, Nucl. Phys. **B343**, 1 (1990); B. Grinstein, *ibid.* **B339**, 253 (1990); M. B. Wise, in *Particles and Fields 3: Proceedings of the Banff Summer Institute (CAP)* 1988, edited by N. Kamal and F. Khanna (World Scientific, Singapore, 1989), p. 124.
- [8] C. O. Dib and F. Vera, Phys. Rev. D **47**, 3938 (1993); J. F. Amundson, *ibid.* **49**, 373 (1994); J. F. Amundson and J. L. Rosner, *ibid.* **47**, 1951 (1993); B. Holdom and M. Sutherland, *ibid.* **47**, 5067 (1993).
- [9] E. Bagan, P. Ball, V. M. Braun, and H. G. Dosch, Phys. Lett. B **278**, 457 (1992); M. Neubert, Phys. Rev. D **46**, 3914 (1993); M. Neubert, Z. Ligeti, and Y. Nir, Phys. Lett. B **301**, 101 (1993); Phys. Rev. D **47**, 5060 (1993).

- [10] M. Athanas *et al.*, Phys. Rev. Lett. **79**, 2208 (1997); We have extracted the data and errors from the published figure.
- [11] M. Wirbel, B. Stech, and M. Bauer, Z. Phys. C **29**, 637 (1985); M. Bauer, B. Stech, and M. Wirbel, *ibid.* **34**, 103 (1987).
- [12] M. Neubert, Phys. Rep. **245**, 259 (1994) and references therein; M. Neubert, Int. J. Mod. Phys. A **11**, 4173 (1996).
- [13] Nathan Isgur, Daryl Scora, Benjamin Grinstein, and Mark B. Wise, Phys. Rev. D **39**, 799 (1989).
- [14] Daryl Scora and Nathan Isgur, Phys. Rev. D **52**, 2783 (1995).
- [15] S. Veseli and I. Dunietz, Phys. Rev. D **54**, 6803 (1996).
- [16] J. Amundson, Phys. Rev. D **52**, 2926 (1995).
- [17] M. Neubert and B. Stech, in *Heavy Flavours*, 2nd ed., edited by A. J. Buras and M. Linder (World Scientific, Singapore, in press), hep-ph/9705292.
- [18] CLEO Collaboration, B. Barish *et al.*, Phys. Rev. D **51**, 1014 (1995); CLEO Collaboration, J. E. Duboscq *et al.*, Phys. Rev. Lett. **76**, 3898 (1996).
- [19] Particle Data Group, R. M. Barnett *et al.*, Phys. Rev. D **54**, 1 (1996).
- [20] C. Bernard *et al.*, in *Lattice '96*, Proceedings of the International Symposium, St. Louis, Missouri, edited by C. Bernard *et al.* [Nucl. Phys. B (Proc. Suppl.) **53**, 358 (1997)].
- [21] J. D. Rodriguez, to appear in Proceedings of the 2nd International Conference on B Physics and CP Violations, Honolulu, Hawaii, March 1997; T. E. Browder, K. Honscheid, and D. Pedrini, Annu. Rev. Nucl. Part. Sci. **46**, 395 (1996).