# Power corrections and the Gaussian form of the meson wave function

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The wave function of a light pseudoscalar meson is considered and nonperturbative corrections as signaled by perturbation theory are calculated. Two schemes are used: the massive gluon and the running coupling scheme. Both indicate the presence of leading power corrections of $\mathcal{O}(b^2)$, whose exponentiation leads to a Gaussian dependence of the wave function on the impact parameter $b$. The dependence of this correction on the light cone energy fractions of the quark and the antiquark is discussed and compared with other models for the meson. [S0556-2821(98)04113-7]

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## I. INTRODUCTION

The challenging task of explaining experimental data for exclusive processes, such as pion and nucleon electromagnetic form factors and elastic scattering, is tightly connected with our understanding of the bound states of light hadrons. In the asymptotic limit of parametrically large momentum transfer ($Q$) there is a clear theoretical picture that comes from merging the parton model with perturbative QCD (PQCD). The basic tenet of this picture is that a hadronic bound state is a superposition of virtual states with a definite number of constituent partons. In the frame where all participating hadrons are fast moving, factorization guarantees that the $Q^2$ dependence of an exclusive observable enters only through the underlying process of the elastic scattering of the partonic constituents. This has led to the, by now classic, dimensional (or quark) counting rules [1]. However, the fact that the scaling with $Q^2$ as predicted by the counting rules is actually observed by experiments at high momentum transfers generated new questions. Counting rules assume that the dominant configurations are those in which none of the constituents carries vanishingly small longitudinal momentum. But such configurations are present in the hadronic wave function and would lead to scaling with $Q^2$ that is less steeply falling as $Q^2 \to \infty$. Underestimating the contributions from these end-point regions was the main criticism against the parton model picture [2].

The next major step forward came with the work of Sterman and collaborators [3,4] who implemented the summation of leading and next to leading logarithmic radiative corrections through the introduction of Sudakov factors into the factorized expressions for exclusive processes. The generic form of such factors is $\exp\{-c \ln Q^2 \ln[\ln Q^2/\ln(1/b^2)]\}$ where $b$ is the transverse distance between constituents. It has the effect of suppressing configurations of constituents that are separated by large distance $b$, which would be the case in the end-point regions mentioned previously. We note that the Sudakov factors can be considered as the perturbative tail of the hadronic wave function, i.e. the region where the bound state properties can be reconstructed purely from PQCD. Sudakov-improved perturbation theory, implemented with a model for the hadron wave function at a low momentum scale and a reasonable prescription on how to freeze the coupling at large scales, and showed that it does give numerically stable results with the smallest possible number of phenomenological parameters [4–6]. This approach sets the benchmark but it is not the final answer. The main problem now becomes to estimate the sensitivity to soft contributions of the Sudakov-improved perturbation theory for moderate $Q^2$, that is, in the range $3\ \mathrm{GeV}^2 \leqslant Q^2 \leqslant 40\ \mathrm{GeV}^2$. To this end we need a framework for studying nonperturbative corrections to hadronic wave functions. This is the objective of this paper.

Since our approach to the problem is from the high $Q^2$ end, we consider the two-quark component of a light pseudoscalar meson wave function. This is the dominant configuration in the asymptotic limit. The specific exclusive process in which the pion participates will be of no concern here. Our goal is to calculate the structure of the nonperturbative corrections as they are signaled by perturbation theory itself. These will occur in the form of power corrections in $b$ in the Fourier-transformed wave function.

Recently, some progress has been made in understanding the power corrections by considering classes of Feynman diagrams that give rise to a factorial divergence of the perturbation series in large orders. For the case of QCD the power corrections arise from the small momentum region of the loop integrations and are associated with the so-called infrared renormalons. For a review and related references, see [7,8]. Thus, one may get an indication of the type of power corrections by looking at the infrared sensitivity of Feynman diagrams. Most investigations involving renormalons are in the context of two calculational schemes. The first is the massive gluon and the second is the running coupling scheme. In the first method, which we apply to the case of the radiative corrections to the meson wave function, a gluon mass is introduced via a dispersive parameter $\lambda$ [9] that acts as an infrared regulator. This procedure is certainly consistent at the one-loop level. Since the power corrections, in the renormalon approach, arise from the infrared-sensitive regions, our interest will be in the structure of the nonanalytic terms in $\lambda^2$ which, to the order we work in, turn out to be $\ln\lambda^2$ and $b^2\lambda^2\ln\lambda^2$. It is known [7] that such nonanalytic terms come only from the pinch singular points in the loop momentum integration with $\ln\lambda^2$ being associated with the usual logarithmic enhancements of the perturbative series and $b^2\lambda^2\ln\lambda^2$ with the leading power corrections. In the second scheme, power corrections are computed using a one-loop improved perturbation theory where infrared effects are introduced through the running coupling $\alpha_s(k_\perp^2)$. Then the Sudakov exponent is Borel transformed and its singularity structure in the Borel plane is studied. The power corrections in this case are found to be proportional to $\Lambda_{\rm QCD}^2 b^2$. This technique is the same one used for the study of power corrections to inclusive processes and has been widely discussed in recent years; see, for example, Ref. [10].

In Sec. II we calculate the leading power corrections and discuss the correspondence between the expressions derived in the two schemes. Exponentiation of leading power corrections naturally lead to the Gaussian dependence of the meson wave function on the impact parameter $b$. In Sec. III we discuss our results and compare them with previous publications where such Gaussian factors for the meson wave function have been advocated. Here, of interest will be the $x$ and $\ln Q^2$ dependence of the Gaussian factor itself, where $x$ is the light cone fraction of the momentum of the quark in the meson. Finally we summarize our conclusions in the last section.

## II. RADIATIVE CORRECTIONS TO THE MESON WAVE FUNCTION

### A. Definitions

The Bethe-Salpeter two-quark wave function of a fast moving pseudoscalar meson $M$ is defined by the following matrix element at renormalization (and factorization) scale $\mu$ [3]:

$$X(k,p,\mu)=\frac{1}{N_c}\int\frac{d^4y}{(2\pi)^4}e^{ik\cdot y}\langle 0|T[\overline{q}(0)\overline{\not v}\gamma_5 q(y)]|M(p)\rangle. \tag{2.1}$$

In the frame where the meson is moving fast we define the light cone vector along the direction of motion, $v^\mu=(1/\sqrt{2})(1,0,0,1)$, and the parity reflected direction vector $\overline{v}^\mu=(1/\sqrt{2})(1,0,0,-1)$, $\mu=0,1,2,3$, normalized as $v\cdot\overline{v}=1$. Then the light cone momenta of the meson and the quark constituent are defined as

$$p^\mu=p^+\delta_+^\mu,\quad k^+=\overline{v}\cdot k=xp^+,\quad k^-=v\cdot k,\quad k_\perp=\mathbf{k}. \tag{2.2}$$

The large scale is $p^+=Q/\sqrt{2}$, where $Q$ is the momentum transfer of the hard exclusive process in which the meson participates. Although we do not need to specify the process, we must always keep in mind that our discussion will always be in the context of a hard process. The object of interest in this work is the wave function in impact parameter space given by

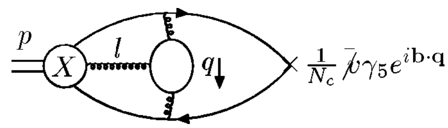


FIG. 1. Class of diagrams that contain ''nonladder'' gluons ($l$). These are suppressed in the axial gauge.

$$\begin{aligned}\mathcal{P}(x,b,p,\mu)&=\int d^2\mathbf{k}\;e^{i\mathbf{k}\cdot\mathbf{b}}\int dk^-X(k,p,\mu)\\&=\frac{1}{N_c}\int\frac{dy^-}{2\pi}e^{ixp^+y^-}\\&\quad\times\langle 0|T[\overline{q}(0)\overline{\not v}\gamma_5 q(0,y^-,\mathbf{b})]|M(p)\rangle\;.\end{aligned} \tag{2.3}$$

The quark distribution amplitude, which enters the leading order perturbative expressions, is

$$\phi(x,\mu)=\int^{|\mathbf{k}|=\mu}d^2\mathbf{k}\int dk^-X(k,p,\mu)=\mathcal{P}^{(0)}(x,b=0,p,\mu). \tag{2.4}$$

Radiative corrections to $\mathcal{P}^{(0)}$ coming from the infrared region exponentiate. Their summation leads to the Sudakov suppression factor [11]

$$\mathcal{P}(x,b,p,\mu)=\exp[-S_{wf}(x,b,Q,\mu)]\mathcal{P}^{(0)}(x,b=0,p,\mu), \tag{2.5}$$

with

$$S_{wf}=\frac{2C_F}{\beta_0}\ln\frac{Q^2}{\Lambda_{\rm QCD}^2}\ln\left(\frac{\ln Q^2/\Lambda_{\rm QCD}^2}{\ln 1/(b^2\Lambda_{\rm QCD}^2)}\right)+\mathrm{NL},$$
$$\beta_0=\frac{11}{3}N_c-\frac{2}{3}N_f, \tag{2.6}$$

where NL denotes next to leading logarithmic corrections, which are also known. Note that the above perturbative answer is really defined in the region $1/b^2\gg\Lambda_{\rm QCD}^2$; i.e., the evolution of the wave function can be reliably constructed only for small-size bound states.

The Sudakov exponent shown in Eq. (2.6) is obtained from the one-loop radiative corrections to $\mathcal{P}^{(0)}$. There exist various methods of calculating these corrections. In this paper we will mostly follow the approach of Botts and Sterman [3]. In this approach the fermions are taken to be massless and the axial gauge is used. The advantage of using the axial gauge in the derivation of an evolution equation for the wave function is that the diagrams involved in the calculation of the kernel have a simpler structure than in a covariant gauge. Specifically, diagrams in which gluons couple the kernel with the ''inside'' of the wave function, thus generating non-two-particle-irreducible (2PI) connections in the constituent quark channel (nonladder gluons), are suppressed, Fig. 1. In the covariant gauge such diagrams must be included and

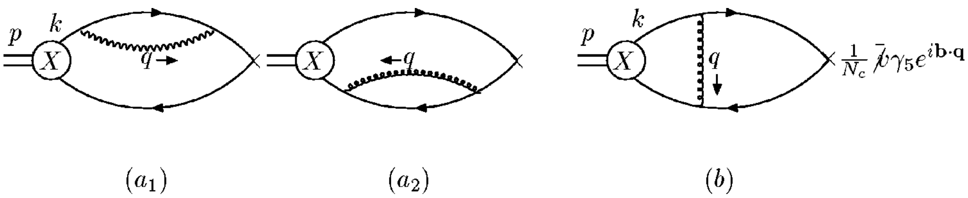


FIG. 2. One-loop diagrams for the meson wave function $\mathcal{P}$.

further reduced through the use of Ward identities, just like in the treatment of parton distribution functions in deep inelastic scattering (DIS).

In the axial gauge there are gauge ($n$) dependent contributions to $S_{wf}$ that enter in the next to leading logarithmic corrections in Eq. (2.6). Such gauge-dependent pieces cancel against gauge-dependent terms in the radiative corrections to the hard scattering. Since we do not analyze the hard subprocess here, our results will be at the level of leading logarithmic and power corrections.

### B. Meson wave function in the massive gluon scheme

As was mentioned in the Introduction, we are interested in the nonanalytic dependence of the radiative corrections to the meson wave function on the infrared cutoff. The Landau equations guarantee that infrared sensitivity arises from those momentum regions where the internal lines in a Feynman graph approach their mass shell. To regularize mass-shell divergences we introduce for the gluon a (dispersive) mass parameter $\lambda$ [9] and calculate the one-loop radiative corrections to $\mathcal{P}^{(0)}$ in this massive gluon scheme. This is the main difference with the treatment found in Ref. [3] and also the fact that we are mainly interested in the terms that are nonanalytic in $\lambda$ and vanish as powers of $\lambda$ in the exact mass-shell limit. The gluon polarization tensor $N_{\mu\nu}$ is defined via the principal value prescription for the unphysical singularities $n\cdot q$. Hence the gluon propagator reads

$$D_{\mu\nu}(q,n)=\frac{i}{q^2-\lambda^2+i\epsilon}N_{\mu\nu}(q,n),$$

$$N_{\mu\nu}(q,n)=-g_{\mu\nu}+\frac{n\cdot q}{(n\cdot q)^2+\eta^2}n_{(\mu}q_{\nu)}-\frac{n^2}{(n\cdot q)^2+\eta^2}q_\mu q_\nu, \tag{2.7}$$

with $\eta\to 0$. At the end of this subsection we comment on the possibility of using the same parameter $\lambda$ to regularize the unphysical $n\cdot q$ singularities. The graphs to be calculated are depicted in Fig. 2 and the Dirac structure of the meson-$q$-$\bar{q}$ vertex is $\mathbf{X}=-(1/4)\not{v}\gamma_5 X$, where $X$ is the scalar wave function defined in Eq. (2.1).

Contribution to $\mathcal{P}$ from vertex correction, Fig. 2 graph ($b$): The vertex correction to the meson wave function is

$$\mathcal{P}_{(b)}=i(4\pi\alpha_s)C_F\int\frac{d^4q}{(2\pi)^4}e^{i\mathbf{b}\cdot\mathbf{q}}\mathrm{tr}\left[\mathbf{X}\gamma^\nu\frac{1}{-\not{p}+\not{k}-\not{q}+i\epsilon}\times\bar{\not{v}}\gamma_5\frac{1}{\not{k}-\not{q}+i\epsilon}\gamma^\mu\right]N_{\mu\nu}\frac{1}{q^2-\lambda^2+i\epsilon}. \tag{2.8}$$

The impact parameter $b$ acts as an ultraviolet regulator so that the loop integral can be kept at $D=4$ dimensions. Standard manipulations for the $-g_{\mu\nu}$ piece of the gluon propagator yield

$$\mathcal{P}_{(b)}|_{-g_{\mu\nu}}=-\frac{\alpha_s}{4\pi}C_F\mathcal{P}^{(0)}\left[\ln(b^2\lambda^2)+\frac{1}{6}b^2\lambda^2\ln(b^2\lambda^2)\right]+\mathcal{O}(b^4\lambda^4\ln(b\lambda))+\mathcal{O}(\ln^0(b\lambda)). \tag{2.9}$$

Without the gluon mass regulator, the $\ln(b^2\lambda^2)$ term would turn into an IR divergence that should be dimensionally regularized. Here, as well as in the rest of the calculations, the interest lies in the presence of the $b^2\lambda^2\ln(b^2\lambda^2)$ nonanalytic term.

The calculation involving the $n$-dependent pieces of the gluon propagator is somewhat subtler. Evaluation of the traces in Eq. (2.8) yields

$$\mathcal{P}_{(b)}|_{n_{(\mu}q_{\nu)}}=i(4\pi\alpha_s)C_F\mathcal{P}^{(0)}(I_1+I_2), \tag{2.10}$$

with

$$I_1=\int\frac{d^4q}{(2\pi)^4}e^{i\mathbf{b}\cdot\mathbf{q}}\times\frac{[n\cdot(q-k)+v\cdot n\bar{v}\cdot(q-k)-\bar{v}\cdot nv\cdot q]n\cdot q}{[(q-k)^2+i\epsilon](q^2-\lambda^2+i\epsilon)[(n\cdot q)^2+\eta^2]} \tag{2.11}$$

and

$$I_2=I_1|_{k\to k-p}. \tag{2.12}$$

The two integrals $I_1$ and $I_2$ arise from the two pieces $n_\mu q_\nu$ and $n_\nu q_\mu$ of the gluon propagator, respectively. At this stage it is advantageous to *fix the gauge* by the choice

$$n^\mu=\frac{1}{\sqrt{2}}(v^\mu-\bar{v}^\mu). \tag{2.13}$$

This gauge choice has the advantage of simplifying the corresponding integrals by removing all dependence on $q^-$ and $\mathbf{q}$, from the numerator of Eq. (2.11), i.e., the normal coordinates in the collinear limit for the gluon momentum. In this gauge $I_1$ becomes

$$I_1=\int\frac{d^4q}{(2\pi)^4}e^{i\mathbf{b}\cdot\mathbf{q}}\times\frac{[2n\cdot q+(k\cdot q)/(n\cdot k)-2n\cdot k]n\cdot q}{[(q-k)^2+i\epsilon](q^2-\lambda^2+i\epsilon)[(n\cdot q)^2+\eta^2]}. \quad (2.14)$$

The above integral is evaluated via the introduction of Schwinger parameters. The intermediate steps can be reconstructed by using the techniques and the integral relations presented in Ref. [12] and the result is

$$I_1=\frac{i}{(4\pi)^2}\int_0^1 dx_1 x_1^{-1/2}\int_0^1 dx_2\left(K_0(\sqrt{B})+A\frac{1}{\sqrt{B}}K_1(\sqrt{B})\right), \quad (2.15)$$

with

$$A=x_2(-2+x_2+x_1x_2)b^2(n\cdot k)^2,$$

$$B=b^2\left[(1-x_1)x_2^2(n\cdot k)^2+(1-x_2)\lambda^2+\frac{1-x_1}{x_1}\eta^2\right]. \quad (2.16)$$

Since we are interested in the limit $\lambda\to 0$, we will calculate leading and nonanalytic $\lambda$ dependence of the Feynman integrals with the help of the Mellin transformation defined as

$$M[F(t)](\beta)=\int_0^\infty dt\, t^{-\beta-1}F(t). \quad (2.17)$$

It should be noted that the Mellin transform method has been used extensively in the study of the high energy behavior of Feynman diagrams. For a review of the method and for further references, see [13]. The definition (2.17) yields the following correspondence between poles in $\beta$ and (large) $t$ dependence:

$$M[F](\beta)=\frac{r}{(\beta-\beta_0)^n}\leftrightarrow F(t)=\frac{rt^{\beta_0}\ln^{n-1}t}{\Gamma(n)}. \quad (2.18)$$

Nonanalytic terms in $t$ (logarithmic) are generated by at least double poles in the Mellin image. We define the dimensionless ratios

$$t=\frac{(n\cdot k)^2}{\lambda^2},\quad s=b^2(n\cdot k)^2, \quad (2.19)$$

and we note that the parameter $\beta$ also acts as an IR regulator. Then, in Eqs. (2.16) the $\eta\to 0$ limit for the gluon propagator can be taken. The Mellin transformation of the Bessel $K$ functions can be found in Ref. [14] and the Mellin image of Eq. (2.15) turns out to be

$$M[I_1]=\frac{i}{(4\pi)^2}(J_0+J_1), \quad (2.20)$$

where

$$J_0(\beta)=2^\beta\Gamma(\beta)s^{-\beta/2}\int_0^1 dx_1 x_1^{\beta/2}(1-x_1)^{-1/2}\times\int_0^1 dx_2 x_2^\beta(1-x_2)^{-\beta}K_{-\beta}(\sqrt{sx_1}x_2), \quad (2.21)$$

$$J_1(\beta)=2^\beta\Gamma(\beta)s^{1/2-\beta/2}\int_0^1 dx_1 x_1^{-1/2+\beta/2}(1-x_1)^{-1/2}\times\int_0^1 dx_2 x_2^\beta(1-x_2)^{-\beta}(-2+2x_2-x_1x_2)\times K_{1-\beta}(\sqrt{sx_1}x_2). \quad (2.22)$$

Inspecting the above integrals we see that for $\beta>0$ there are no singularities. Singularities arise only for $\beta\leqslant 0$ and they are generated from the integration end points $x_1\to 0$ and/or $x_2\to 0$. The extraction of such singularities is straightforward because the series expansions of the $K$ functions for a small argument can be used. From Eq. (2.18) it follows that the relevant nonanalytic terms that we are interested in can arise from at least double poles at $\beta=0,-1$.[1] We examine these two points in turn.

For the poles at $\beta=0$ we set $\beta=\delta$, $\delta\to 0$ and obtain

$$J_0(\beta=\delta)=\mathcal{O}\left(\frac{1}{\delta}\right),$$

$$J_1(\beta=\delta)=-\frac{1}{\delta^3}+\frac{2}{\delta^2}(1-\ln 2)+\mathcal{O}\left(\frac{1}{\delta}\right). \quad (2.23)$$

Combining Eqs. (2.23), (2.20) and inverting the Mellin transformation we get

$$I_1|_{\beta\to 0}=\frac{-i}{(4\pi)^2}\left[\frac{1}{2}\ln^2\frac{\lambda^2}{(n\cdot k)^2}+2(1-\ln 2)\ln\frac{\lambda^2}{(n\cdot k)^2}\right]+R. \quad (2.24)$$

[1]It is straightforward to see that there are no other points in the range between $-1$ and 0 that would give double poles, thus excluding any linear in $\lambda$ corrections arising from poles at $\beta=-1/2$.

The residue $R$ generically contains all terms that are analytic in $\lambda^2$. Similarly, the poles are $\beta=-1$ are obtained by setting $\beta=-1+\delta$ and power expanding the $K$ functions. The results are

$$J_0(\beta=-1+\delta)=(s\ \ \text{independent}) \tag{2.25}$$

and

$$J_1(\beta=-1+\delta)=(s\ \ \text{independent})-\frac{s}{4\delta^3}+\frac{s}{2\delta^2}(1-\ln 2)+\mathcal{O}(1/\delta). \tag{2.26}$$

Again, combining the above two equations with Eq. (2.20) and inverting the Mellin transformation we have

$$\begin{aligned} I_1|_{\beta\to-1}&=(b\ \ \text{independent})+\frac{i}{(4\pi)^2}\left[-\frac{1}{8}b^2\lambda^2\ln^2\frac{\lambda^2}{(n\cdot k)^2}-\frac{1}{2}(1-\ln 2)b^2\lambda^2\ln\frac{\lambda^2}{(n\cdot k)^2}\right]+R\\ &=(b\ \ \text{independent})-\frac{i}{(4\pi)^2}b^2\lambda^2\left[\frac{1}{8}\ln^2\left(\frac{\lambda^2}{x^2Q^2}\right)+\frac{1}{2}(1-\ln 2)\ln\left(\frac{\lambda^2}{x^2Q^2}\right)\right]+R. \end{aligned} \tag{2.27}$$

In the last step we have used that $(n\cdot k)^2=x^2Q^2$. By ($b$ independent) we denote all terms that are nonanalytic in $\lambda^2$, $\mathcal{O}(\lambda^2\ln\lambda^2)$ in this case, and are independent of $b$. Analytic in $\lambda$ terms reside in $R$. We make this distinction because, as we will see later, the terms denoted by ($b$ independent) are of infrared origin and cancel against the self-energy contribution. The expression for $I_2$, Eq. (2.12), is obtained from the above by simply $x^2\to(1-x)^2$. The overall contribution to the vertex from the $n_{(\mu}q_{\nu)}$ piece of the gluon propagator is [Eq. (2.10)]

$$\begin{aligned} \mathcal{P}_{(b)}|_{n_{(\mu}q_{\nu)}}=\frac{\alpha_s}{4\pi}C_F\mathcal{P}^{(0)}b^2\lambda^2&\left[\frac{1}{8}\left(\ln^2\frac{\lambda^2}{x^2Q^2}+\ln^2\frac{\lambda^2}{(1-x)^2Q^2}\right)\right.\\ &\left.+(1-\ln 2)\ln\frac{\lambda^2}{x(1-x)Q^2}\right]\\ &+(b\ \ \text{independent})+R. \end{aligned} \tag{2.28}$$

Finally, the $q_\mu q_\nu$ piece of the gluon propagator gives $b$-independent leading contributions and hence it cancels against the corresponding piece of the self-energy, as we will point out below.

Contribution to $\mathcal{P}$ from self-energy correction, Fig. 2 graphs ($a_1$) and ($a_2$): Because of UV divergences, we employ dimensional regularization with $D=4-2\epsilon$ and renormalization scale $\mu$:

$$\begin{aligned} \mathcal{P}_{(a_1)}=i(4\pi\alpha_s)C_F\ \mu^{2\epsilon}\int\frac{d^Dq}{(2\pi)^D}\\ \times\text{tr}\left[\mathbf{X}\bar{\not v}\gamma_5\frac{1}{\not k+i\epsilon}\gamma^\nu\frac{1}{\not k-\not q+i\epsilon}\gamma^\mu\right]N_{\mu\nu}\frac{1}{q^2-\lambda^2+i\epsilon}. \end{aligned} \tag{2.29}$$

It is straightforward to find that, even before any gauge fixing, the self-energy contribution is such that

$$\mathcal{P}_{(a_1)}+\mathcal{P}_{(a_2)}+\mathcal{P}_{(b)}(b\to 0)=R_{\rm UV}, \tag{2.30}$$

where the residue $R_{UV}$ contains terms of purely ultraviolet origin and it is due to the mismatch in the UV regularization of the two sets of diagrams. Note that the vertex diagram is UV regularized by a finite $b$ whereas the self-energy diagrams are dimensionally regularized. Had we taken the limit $b\to 0$ before calculating the vertex integrals, then this would have required us to also dimensionally regularize the vertex and then in Eq. (2.30) $R_{\rm UV}$ would have been exactly zero. In any case, the net result of this analysis is that the self-energy diagrams will cancel all $b$-independent terms in the vertex, such as the ones shown in Eq. (2.28), as well as the vertex contributions from the $q_\mu q_\nu$ piece of the gluon propagator.

The final result for the one-loop radiative correction to the wave function in the massive gluon scheme is obtained by combining the partial results of Eqs. (2.9), (2.28), and (2.30). Retaining only the nonanalytic in $\lambda$ terms up to $\mathcal{O}(\lambda^2)$ we obtain

$$\mathcal{P}^{(1)}=\frac{\alpha_s}{4\pi}C_F\mathcal{P}^{(0)}(C_1b^2\lambda^2\ln\lambda^2+C_2b^2\lambda^2\ln^2\lambda^2)+R, \tag{2.31}$$

with

$$C_1=-\frac{1}{2}\ln[x(1-x)Q^2],\quad C_2=\frac{1}{4}. \tag{2.32}$$

The familiar Sudakov factor $S_{wf}\sim\alpha_s\ln^2Q^2$, calculated for fixed coupling $\alpha_s$, is analytic in $\lambda^2$ and it is contained in the residue $R$. In the coefficients $C_1, C_2$ we have only kept the parts that are $n$ independent to leading logarithmic in $Q$ order. Note that the $n$-independent leading contributions to $C_1$, $C_2$ come only from Eq. (2.28). The $n$ dependence enters through the combination $\ln(n\cdot k)^2+\ln[n\cdot(k-p)]^2$, and any variation of $n$ will lead to change that is subleading in $Q$; i.e., it will be down by a $\ln[x(1-x)Q^2]$. Fixing the gauge as in Eq. (2.13) organizes conveniently the leading and subleading $\ln Q^2$ contributions but the result that we quote in Eqs. (2.31), (2.32) is the gauge-independent leading piece.

We now return to the question of regularizing infrared singularities in the axial gauge and within the massive gluon scheme. In the massive gluon propagator, Eq. (2.7), we used the parameter $\lambda$ to regularize on-shell singularities whereas the polarization tensor $N_{\mu\nu}$ was defined through the principal value prescription and the regulator $\eta$. If we treat the parameter $\lambda$ as a Lagrangian mass, instead of a dispersive parameter (this is at the one-loop level only) then the propagator will be modified by the addition of a fourth term of the form $n_\mu n_\nu \lambda^2/[(n\cdot k)^2-\lambda^2 n^2]$. A similar prescription has been considered in the leading $\lambda\to 0$ limit in Ref. [15]. However, beyond the logarithmic in $\lambda$ level, this term yields $b$-dependent $\mathcal{O}(\lambda^2\ln^3\lambda^2)$ contributions that cannot be interpreted as arising from some pinch singularity in the limit $\lambda\to 0$. Such artifacts of the regularization prescription are avoided once one keeps in mind that $\lambda$ is not a Lagrangian gluon mass but just an on-shell regulator, as it appears in Eq. (2.7).

### C. Running $\alpha_s$ scheme and IR renormalons

The running $\alpha_s$ scheme for estimating nonperturbative corrections starts with the expression for the one-loop radiative corrections to the meson wave function. The leading logarithmic term is well known and can be readily obtained from the expressions in Eqs. (2.8), (2.29) after rationalizing the fermion propagators and applying the collinear approximation to the numerator factors. Equivalently it can be computed by calculating emission from two, almost parallel, Wilson lines. All infrared divergences are dimensionally regularized and no $\lambda$ regulator need be introduced. A principal value prescription is used for the axial gauge gluon propagator which now reads

$$D_{\mu\nu}(q,n)=\frac{i}{q^2+i\epsilon}\left[-g_{\mu\nu}+\frac{1}{n\cdot q}n_{(\mu}q_{\nu)}-\frac{n^2}{(n\cdot q)^2}q_\mu q_\nu\right]. \tag{2.33}$$

After performing the collinear approximation to the numerator factors and then integrating over $q^-$ by closing around its poles in the $q^-$ complex plane we obtain the result [3]

$$\mathcal{P}^{(1)}(x,b,p,\mu)=\frac{\alpha_s}{2\pi^2}C_F\mathcal{P}^{(0)}(x,b=0,p,\mu)\int^{\mathbf{q}^2=Q^2}\frac{d^2\mathbf{q}}{\mathbf{q}^2}(e^{i\mathbf{b}\cdot\mathbf{q}}-1)\int_{|\mathbf{q}|}^{xp^+}\frac{dq^+}{q^+}+(x\to 1-x). \tag{2.34}$$

The Sudakov exponent due to perturbative evolution of the wave function, Eq. (2.5), is determined to leading order in $\alpha_s$ by the above expression. At this stage the running coupling is introduced. A perturbative analysis at the logarithmic level [3] indicates that the scale of the coupling is set by the transverse momentum of the emitted gluon and it encodes the information that the strength of the interactions increases for emission at large impact parameter. The Sudakov exponent now reads

$$\begin{aligned}S_{wf}=&-\frac{C_F}{2\pi^2}\int\frac{d^2\mathbf{q}}{\mathbf{q}^2}\alpha_s(\mathbf{q}^2)(e^{i\mathbf{b}\cdot\mathbf{q}}-1)\\&\times\left(\int_{|\mathbf{q}|}^{xp^+}\frac{dq^+}{q^+}+\int_{|\mathbf{q}|}^{(1-x)p^+}\frac{dq^+}{q^+}\right)\\=&-\frac{C_F}{2\pi^2}\int\frac{d^2\mathbf{q}}{\mathbf{q}^2}\alpha_s(\mathbf{q}^2)\ln\frac{x(1-x)(p^+)^2}{\mathbf{q}^2}(e^{i\mathbf{b}\cdot\mathbf{q}}-1).\end{aligned} \tag{2.35}$$

When the lower limit of the $d\mathbf{q}^2$ integration is set to $1/b^2$ the result in Eq. (2.6) is recovered. However, apart from the case of small-size quarkonia, for the usual light mesons $1/b\sim\Lambda_{\mathrm{QCD}}$. It is then apparent that after the introduction of the running coupling the Sudakov exponent will become dominated by the lower end of the $d\mathbf{q}^2$ integration where perturbation theory itself is ill defined. Nevertheless, what we are interested in here is not the numerical stability of the perturbative results but signals of nonperturbative corrections. We therefore proceed by allowing the infrared regulator to become $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ and introduce the Borel transformation of the Sudakov exponent defined as

$$S_{wf}(x,b,Q,\mu;\alpha_s)=\int_0^\infty d\sigma\,\tilde{S}_{wf}(x,b,Q,\mu;\sigma)e^{-\sigma/\alpha_s}. \tag{2.36}$$

The Borel image $\tilde{S}_{wf}$ can be obtained by first using the following representation of the one-loop running coupling:

$$\alpha_s(\mathbf{q}^2)=\int_0^\infty d\sigma e^{-\sigma\beta_0\ln(\mathbf{q}^2/\Lambda^2)}. \tag{2.37}$$

Substituting the above expression into Eq. (2.35) and integrating over $\mathbf{q}$ we get

$$S_{wf}=\frac{C_F}{2\pi^2}\ln\frac{x(1-x)(p^+)^2}{\Lambda^2}$$

$$\times \int_0^\infty d\,\sigma \frac{\Gamma(1-\sigma\beta_0)}{\sigma\beta_0\Gamma(1+\sigma\beta_0)} e^{-\sigma/\alpha_s(4/b^2)}. \quad (2.38)$$

Comparing the result with the definition (2.36) we read off the Borel image

$$\tilde{S}_{wf} = \frac{C_F}{2\pi^2} \ln \frac{x(1-x)(p^+)^2}{\Lambda^2} \frac{\Gamma(1-\sigma\beta_0)}{\sigma\beta_0\Gamma(1+\sigma\beta_0)}. \quad (2.39)$$

The pole at $\sigma=0$ is of UV origin. Infrared sensitivity is parametrized by the presence of IR renormalon poles at $\sigma = n/\beta_0$, $n=1,2,\ldots$ . The leading IR renormalon is at $\sigma = 1/\beta_0$ which leads to $\mathcal{O}(\Lambda_{\rm QCD}^2 b^2)$ power corrections of the form

$$S_{wf} = S_{wf}^{PT} + C\Lambda_{\rm QCD}^2 b^2 \ln \frac{x(1-x)Q^2}{\Lambda_{\rm QCD}^2}. \quad (2.40)$$

Terms that are $Q$ independent have been omitted in the above equation. As in the case of the massive gluon scheme, it is this part of the coefficient that is $n$ independent to leading logarithmic in $Q$ order and it arises from soft gluon emission; hence, it is universal. The coefficient $C$ contains the principal value integral over $\sigma$, which is one possible way to define it. This ambiguity is anticipated since perturbation theory is ill defined and therefore it cannot predict the size of nonperturbative corrections, only their scaling with $Q$. We emphasize that the manipulations that led to Eq. (2.39) are formal and the transformation cannot be inverted back to $\alpha_s$ space without the use of some prescription (or convention). The value of the coefficient $C$ will depend on this prescription. In addition, as with the massive gluon case, higher orders in $\alpha_s$ are not suppressed, unless one invokes an additional assumption of, say, the freezing of the coupling at small momenta [9,16].

The connection between the running coupling and the massive gluon calculation is established by comparing Eq. (2.40) with Eq. (2.31). Note that in the massive gluon result we have not written the analogue of the $S_{wf}^{PT}$ that we see above. This is because in the Mellin transformation analysis of the massive gluon integrals we looked only for contributions that are nonanalytic in the regulator $\lambda^2$ and not in the impact parameter $b^2$. Had we chosen the latter we would have generated the perturbative answer $S_{wf}^{PT}$ in the massive gluon scheme as well. The nonperturbative pieces in the two calculations can be mapped onto each other, up to a multiplicative constant, via the identification

$$\lambda^2 \ln \lambda^2 \leftrightarrow \Lambda_{\rm QCD}^2. \quad (2.41)$$

In Eq. (2.31), the $\lambda^2 \ln^2 \lambda^2$ term indicates that the coefficient of the term $\lambda^2 \ln \lambda^2$ will depend on $\ln Q^2$, $Q^2$ being the only other scale involved; hence, we also identify $(1/2)\lambda^2\ln^2\lambda^2 \leftrightarrow \Lambda_{\rm QCD}^2 \ln \Lambda_{\rm QCD}^2$. The existence of $\lambda^2 \ln \lambda^2$ contributions is a signal for the presence of a condensatelike term. Had only this term been present, then one could expect that the power corrections could be captured from an operator product expansion (OPE) in terms of local operators. The existence of $\lambda^2 \ln^2 \lambda^2$ contributions, though, signals that such an expansion is not possible for the wave function of a light hadron. An operator expansion formalism for the wave function must necessarily involve nonlocal operators that extend along the light cone and, in analogy with the usual OPE, their expectations are to be parametrized by nonlocal condensates. The possibility of parametrizing power corrections to the meson electromagnetic form factor in terms of nonlocal condensates has been discussed in Ref. [17].

The other important piece of information in the leading power correction term is that its $b^2$ dependence leads to a Gaussian factor for the meson wave function. This is obtained from Eqs. (2.5) and (2.40):

$$\mathcal{P} = \mathcal{P}^{PT} \exp\left[ -C \ln\left( \frac{x(1-x)Q^2}{\Lambda_{\rm QCD}^2} \right) \Lambda_{\rm QCD}^2 b^2 \right], \quad (2.42)$$

with $\mathcal{P}^{PT}$ denoting the wave function perturbatively evolved with exponent given in Eq. (2.6). We turn to a discussion of this form in the next section.

## III. GAUSSIAN FORM OF THE MESON WAVE FUNCTION

The wave function in Eq. (2.31) and Eq. (2.42) can be rewritten in the following suggestive form:

$$\mathcal{P}(x,b,Q) = \phi(x,Q) \exp\{ -S_{wf}^{PT} - b^2 S_2[x(1-x)Q^2] - b^4 S_4[x(1-x)Q^2] + \cdots \}. \quad (3.1)$$

From the infrared renormalons at integer values of $\sigma\beta_0$ we obtain for the Sudakov exponent a power series expansion in the small parameter $\Lambda_{\rm QCD}^2 b^2 \ll 1$ with $Q$-dependent coefficients. Within perturbation theory this $Q$ dependence is logarithmic. Truncation of the above power series to the first two terms is accurate in the small $b$ region up to corrections of order $\mathcal{O}(\Lambda_{\rm QCD}^4 b^4)$. This form of the exponent is the same as the one seen in the Drell-Yan process at measured $Q_\perp$ and has been derived by Korchemsky and Sterman in Ref. [10]. These authors have also given an operator definition of the power corrections in terms of pairs of Wilson lines and their transverse derivatives. For the same Drell-Yan process, the Gaussian form in impact parameter space was noted years ago and outside the renormalon context by Collins and Soper [18] in their treatment of the infrared sensitivity of the Sudakov exponent.

The dimensionful functions $S_{2n}$, $n=1,2,\ldots$, scale as $\Lambda_{\rm QCD}^{2n}$ but their absolute normalization cannot be fixed within perturbation theory. However, we have obtained some information beyond the summation of logarithmic corrections. If we assume that in Eq. (2.42) the unknown coefficient $C$ is a number of $\mathcal{O}(1)$ and that our methods capture correctly the $x$ dependence of the leading nonperturbative corrections, then

$$S_2(Q) \sim \Lambda_{\rm QCD}^2 \ln[x(1-x)Q^2], \quad (3.2)$$

where only the universal piece is retained, as has been argued in the previous section. For fixed $x \neq 0$, power corrections lead to increasing suppression of emission at a large impact parameter with increasing $Q$. This suppression is *in addition* to the one generated by $S_{wf}^{PT}$. However, for fixed $Q$, it is seen that this additional suppression becomes weak in the end-point region. The end-point region is enhanced relative to the central region by the Gaussian dependence of the meson wave function. This is not surprising since it is in the end-point region that the effective hard scale $x(1-x)Q^2$ becomes small and power corrections become more important. We emphasize that all this is on top of the Sudakov suppression, $S_{wf}^{PT}$, as mentioned above. In this respect, the pattern of power corrections is very similar to that observed, for example, in event shape variables where the leading power corrections [22] also come from the end-point region (two-jet limit in this case), which is itself Sudakov suppressed.

For exclusive processes there exist in the literature various models for the wave function with which we may compare our results. In the case of the meson, the most popular is the one obtained from an oscillator model of two constituent quarks boosted in the light cone frame. This leads to a wave function of the form [19,20]

$$\mathcal{P}(x,b,Q)=\phi(x,Q)\exp[-S_{wf}^{PT}-\langle k_\perp^2\rangle x(1-x)b^2], \tag{3.3}$$

where the average $\langle k_\perp^2\rangle^{-1}$ is the oscillator parameter. Note that this Gaussian form is such that it can interpolate between the perturbative tail of the wave function given by $\exp(-S_{wf}^{PT})$ and the nonperturbative region. The $\mathcal{O}(b^2)$ term in the exponent has been found to be numerically important in order for Sudakov resummed perturbative expressions for the meson form factor to be applicable in the subasymptotic $Q$ region [21]. It is therefore interesting to compare its $x$ dependence with the expression we get from perturbation theory. We observe that like in Eq. (3.2), the exponent in Eq. (3.3) enhances the end-point regions compared to the central region. However, from the renormalon-based approach, which is rooted in perturbation theory, one can expect a dependence on $x(1-x)$ which is logarithmic but not proportional to it. The analyses of nonperturbative corrections as signaled by perturbation theory and phenomenological models for the meson wave function in the nonperturbative regime have similar qualitative behavior although the end-point enhancement in the former case is much milder than in the latter. For our approach to give results in exact correspondence with, say, the oscillator model would require a resummation of the higher order in $\alpha_s$ contributions to the $\lambda^2\ln\lambda^2$ term at the level of the *exponent*. These could turn the $\ln x(1-x)$ into a $x^n(1-x)^n$ dependence. It is not obvious to us how such a resummation would be implemented. Recall that in the region where nonperturbative corrections are generated, the coupling is $\alpha_s=\mathcal{O}(1)$ and there appears to be no small parameter around which to build and then resum a perturbative series. More importantly, it is not known how to define the massive gluon scheme for higher loop corrections, and for the running coupling scheme it is only an assumption that $\alpha_s=\alpha_s(k_\perp^2)$ all the way into the deep infrared region. We can say for sure, though, that any answer obtained from perturbation theory would necessarily have $Q$ dependence in the Gaussian form that it generates, unlike Eq. (3.3) which is $Q$ independent beyond the logarithmic corrections residing in $S_{wf}^{PT}$. It should be pointed out here that, recently, renormalon-based models have been used to predict the $x$ dependence of the higher twist structure functions in deeply inelastic scattering with some phenomenological success [23] and this was one of our motivations for pursuing the above analysis.

## IV. SUMMARY

In this paper we studied the nonperturbative corrections to the meson wave function using the methods that have already been developed for semi-inclusive cases, namely, the massive gluon and the renormalon methods. We found that the leading nonperturbative corrections are of order $\mathcal{O}(\Lambda_{\rm QCD}^2 b^2)$ for the two-quark wave function at transverse separation $b$. The exponentiation of such contributions leads to a Gaussian factor in addition to the Sudakov resummed logarithmic enhancements. Of particular interest is the $x$ dependence of this Gaussian factor. It leads to the conclusion that the power corrections arise from the end-point regions. This $x$ dependence has been compared with low energy Gaussian models for the wave function.

It must be emphasized that both methods for obtaining the leading nonperturbative corrections have their origin in perturbation theory. They predict correctly the type of the power correction, but since they are applied in a region where the coupling is normalized at low scales, they have limited predictability for the coefficient of the power corrections unless additional assumptions are introduced, such as freezing of the coupling. Thus our predictions for the $x$ dependence should be considered as another model. A similar approach for the $x$ dependence of the higher twist structure functions in DIS has met with some phenomenological success. It would be interesting to apply this model to the phenomenological study of the meson electromagnetic form factor and elastic scattering.

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