

**Measuring  $|V_{td}/V_{ub}|$  ( $= \sin \gamma/\sin \beta$  within the SM) through  $B \rightarrow M \nu \bar{\nu}$  ( $M = \pi, K, \rho, K^*$ ) decays**

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We propose a new method for the precise determination of  $|V_{td}/V_{ub}|$  from the ratios of branching ratios  $\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})/\mathcal{B}(B \rightarrow \rho l \nu)$  and  $\mathcal{B}(B \rightarrow \pi \nu \bar{\nu})/\mathcal{B}(B \rightarrow \pi l \nu)$ . These ratios depend only on the ratio of the Cabibbo-Kobayashi-Maskawa (CKM) elements  $|V_{td}/V_{ub}|$  with little theoretical uncertainty, when very small isospin breaking effects are neglected. As is well known,  $|V_{td}/V_{ub}|$  equals  $(\sin \gamma/\sin \beta)$  for the CKM version of  $CP$  violation within the standard model. We also give in detail analytical and numerical results on the differential decay width  $d\Gamma(B \rightarrow K^* \nu \bar{\nu})/dq^2$  and the ratio of the differential rates  $d\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})/dq^2/d\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})/dq^2$  as well as  $\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})/\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})$  and  $\mathcal{B}(B \rightarrow \pi \nu \bar{\nu})/\mathcal{B}(B \rightarrow K \nu \bar{\nu})$ . [S0556-2821(98)00513-X]

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**I. INTRODUCTION**

The determination of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is one of the most important issues of quark flavor physics. The precise determination of  $V_{td}$  and  $V_{ub}$  elements has a principal meaning, since they are closely related to the origin of  $CP$  violation in the CKM version of  $CP$  violation within the standard model (SM). Furthermore, the accurate knowledge of these matrix elements can be useful in relating them to the fermion masses and also in searches for hints of new physics beyond the SM. Therefore, strategies for the accurate determination of  $V_{td}$  and  $V_{ub}$  are urgently required. In the existing literature, we can find proposals of different methods for the precise determination of  $V_{ub}$  and  $V_{td}$  from inclusive and exclusive, semileptonic and nonleptonic decays of the  $B$  meson (see [1] for a recent review).

The quantity  $|V_{ub}/V_{cb}|$  has been historically measured by looking at the end point of the inclusive lepton spectrum in semileptonic  $B$  decays, or from the exclusive semileptonic decays  $B \rightarrow \rho l \nu$ . It has been suggested that the measurements of the hadronic invariant mass spectrum [2,3] as well as hadronic energy spectrum [4] in the inclusive  $B \rightarrow X_{c(u)} l \nu$  decays can be useful in extracting  $|V_{ub}|$  with better theoretical understanding. The measurement of the ratio  $|V_{ub}/V_{ts}|$  from the differential decay widths of the processes  $B \rightarrow \rho l \nu$  and  $B \rightarrow K^* l \bar{l}$  by using SU(3)-flavor symmetry and heavy quark symmetry has also been proposed [5]. There has also been recent theoretical progress on the exclusive  $b \rightarrow u$  semileptonic decay form factors using heavy quark effective theory (HQET) based scaling laws to extrapolate the form factors from semileptonic  $D$  meson decays [6]. The element  $V_{td}$  can be extracted indirectly from  $B_d - \bar{B}_d$  mixing. However, in

$B_d - \bar{B}_d$  mixing the large uncertainty of hadronic matrix elements prevents one from extracting  $V_{td}$  with good accuracy. A better extraction of  $|V_{td}/V_{ts}|$  can be made if  $B_s - \bar{B}_s$  mixing is measured as well, since the ratio  $(f_{B_d}^2 B_{B_d})/(f_{B_s}^2 B_{B_s})$  can be determined much better. Another method to determine  $|V_{td}/V_{ts}|$  comes from the analysis of the invariant dilepton mass distributions of  $B \rightarrow X_{d,s} l^+ l^-$  decays [7]. An interesting strategy for measuring  $|V_{td}/V_{us}|$  was proposed in [8], which uses isospin symmetry to relate the decay  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  to the well measured decay  $K^+ \rightarrow \pi^0 l \nu$ .

We propose a new method to determine the ratio  $|V_{td}/V_{ub}|$  from an analysis of exclusive  $B \rightarrow M \nu \bar{\nu}$  decays, where  $M$  means pseudoscalar  $\pi, K$  and vector  $\rho, K^*$  mesons. The inclusive  $B \rightarrow X_q \nu \bar{\nu}$  decay is theoretically very clean because of the absence of any long distance effects and very small QCD corrections ( $\sim 3\%$ ) [1,9], and is therefore practically free from the scale ( $\mu$ ) dependence. However, in spite of such theoretical advantages, it would be very difficult to detect this inclusive decay in experiments because the final state contains two missing neutrinos and (many) hadrons.

This paper is organized as follows. In Sec. II we give the necessary theoretical framework to describe  $B \rightarrow M \nu \bar{\nu}$  decays. In Sec. III we study the ratios of branching fractions

$$\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})/\mathcal{B}(B \rightarrow \rho l \nu) \quad \text{and} \quad \mathcal{B}(B \rightarrow \pi \nu \bar{\nu})/\mathcal{B}(B \rightarrow \pi l \nu).$$

We also study the  $q^2$  dependence of the differential decay rate of  $B \rightarrow K^* \nu \bar{\nu}$ , and the ratio of the differential decay rates

$$\frac{d\Gamma(B \rightarrow \rho \nu \bar{\nu})}{dq^2} \bigg/ \frac{d\Gamma(B \rightarrow K^* \nu \bar{\nu})}{dq^2},$$

as well as  $\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})/\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})$  and  $\mathcal{B}(B \rightarrow \pi \nu \bar{\nu})/\mathcal{B}(B \rightarrow K \nu \bar{\nu})$ . Section IV is devoted to a discussion of our results and conclusion.

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## II. THEORY OF $B \rightarrow M \nu \bar{\nu}$ ( $M = \pi, K, \rho, K^*$ ) DECAYS

In the standard model (SM), the process  $B \rightarrow M \nu \bar{\nu}$  is described at quark level by the  $b \rightarrow q \nu \bar{\nu}$  transition, and receives contributions from  $Z$ -penguin and box diagrams, where dominant contributions come from intermediate top quarks. The effective Hamiltonian responsible for  $b \rightarrow q \nu \bar{\nu}$  decays is described by only one Wilson coefficient, namely  $C_{10}^{\nu}$ , and its explicit form is

$$H_{eff} = \frac{G_F \alpha}{2\pi\sqrt{2}} C_{10}^{\nu} (V_{tb} V_{tq}^*) \bar{q} \gamma^{\mu} (1 - \gamma_5) b \bar{\nu} \gamma_{\mu} (1 - \gamma_5) \nu, \quad (1)$$

where  $G_F$  is the Fermi constant,  $\alpha$  is the fine structure constant (at the  $Z$  mass scale), and  $V_{ij}$  are elements of the CKM matrix. In Eq. (1), the Wilson coefficient  $C_{10}^{\nu}$  has the following form, including  $\mathcal{O}(\alpha_s)$  corrections:

$$C_{10}^{\nu} = X(x_t) / \sin^2 \theta_w, \quad (2)$$

where

$$X(x_t) = X_0(x_t) + (\alpha_s / 4\pi) X_1(x_t). \quad (3)$$

In Eq. (3),

$$X_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \ln(x_t) \right]$$

is the Inami-Lim function [10], and

$$\begin{aligned} X_1(x_t) = & \frac{4x_t^3 - 5x_t^2 - 23x_t}{3(x_t - 1)^2} - \frac{x_t^4 + x_t^3 - 11x_t^2 + x_t}{(x_t - 1)^3} \ln(x_t) \\ & + \frac{x_t^4 - x_t^3 - 4x_t^2 - 8x_t}{2(x_t - 1)^3} \ln^2(x_t) \\ & + \frac{x_t^3 - 4x_t}{(x_t - 1)^2} \text{Li}_2(1 - x_t) + 8x_t \frac{\partial X_0(x_t)}{\partial x_t} \ln(x_t), \end{aligned}$$

where

$$\text{Li}_2(1 - x_t) = \int_1^{x_t} dt \frac{\ln(t)}{1 - t}$$

is the Spence function, and  $x_t = m_t^2 / m_W^2$ , and  $x_{\mu} = \mu^2 / m_W^2$ . Here  $\mu$  describes the scale dependence when leading QCD corrections are taken into account. The term  $X_1(x_t)$  is calculated in Ref. [9]. The presence of only one operator in the effective Hamiltonian makes the process  $b \rightarrow q \nu \bar{\nu}$  very attractive, because the estimated theoretical uncertainty is related only to the value of the Wilson coefficient  $C_{10}^{\nu}$  (i.e., the uncertainty due to the top quark mass), which is well under control. Another favorable property of this decay is the absence of any long distance effects, which make the  $b \rightarrow q l^+ l^-$  process on the other hand considerably more

complicated. In spite of such theoretical advantages, in practice the inclusive channel  $B \rightarrow X_q \nu \bar{\nu}$  would be very difficult to detect in experiments.

At this point we consider the problem of computing the matrix elements of the effective Hamiltonian (1) between  $B$  and  $M$  states. This problem is related to the nonperturbative sector of QCD, and it can be solved only by using nonperturbative methods. The matrix element  $\langle M | H_{eff} | B \rangle$  has been investigated through different approaches, such as chiral perturbation theory [11], three point QCD sum rules [12], relativistic quark model by the light front formalism [13], effective heavy quark theory [14], light-cone QCD sum rules [15–17], etc.

The hadronic matrix elements for  $B \rightarrow P \nu \bar{\nu}$  ( $P$  is a pseudoscalar meson,  $\pi$  or  $K$ ) decays can be parametrized in terms of the form factors  $f_+^P(q^2)$  and  $f_-^P(q^2)$ :

$$\langle P(p_2) | \bar{q} \gamma_{\mu} (1 - \gamma_5) b | B(p_1) \rangle = p_{\mu} f_+^P(q^2) + q_{\mu} f_-^P(q^2), \quad (4)$$

where  $p = p_1 + p_2$  and  $q = p_1 - p_2$ . For  $B \rightarrow V \nu \bar{\nu}$  ( $V$  is the vector  $\rho$  or  $K^*$  meson) decays, the hadronic matrix element can be written in terms of five form factors:

$$\begin{aligned} & \langle V(p_2, \varepsilon) | \bar{q} \gamma_{\mu} (1 - \gamma_5) b | B(p_1) \rangle \\ & = -\varepsilon_{\mu\nu\alpha\beta} \varepsilon^{*\nu} p_2^{\alpha} q^{\beta} \frac{2V(q^2)}{m_B + m_V} - i \left[ \varepsilon_{\mu}^*(m_B + m_V) A_1(q^2) \right. \\ & \quad - (\varepsilon^* q)_{(p_1 + p_2)\mu} \frac{A_2(q^2)}{m_B + m_V} \\ & \quad \left. - q_{\mu} (\varepsilon^* q) \frac{2m_V}{q^2} (A_3(q^2) - A_0(q^2)) \right] \end{aligned} \quad (5)$$

with the condition

$$A_3(q^2 = 0) = A_0(q^2 = 0). \quad (6)$$

Note that after using the equations of motion the form factor  $A_3(q^2)$  can be written as a linear combination of the form factors  $A_1$  and  $A_2$  (for more details see the first reference in [12]):

$$A_3(q^2) = \frac{1}{2m_V} [(m_B + m_V) A_1(q^2) - (m_B - m_V) A_2(q^2)]. \quad (7)$$

In Eq. (5),  $\varepsilon_{\mu}$ ,  $p_2$ , and  $m_V$  are the polarization 4-vector, 4-momentum, and mass of the vector particle, respectively. Using Eqs. (1), (4), and (5), and after performing summation over vector meson polarization and taking into account the number of light neutrinos  $N_{\nu} = 3$ , we have

$$\begin{aligned} \frac{d\Gamma}{dq^2}(B^{\pm} \rightarrow P^{\pm} \nu \bar{\nu}) = & \frac{G_F^2 \alpha^2}{2^8 \pi^5} |V_{tq} V_{tb}^*|^2 \lambda^{3/2}(1, r_P, s) \\ & \times m_B^3 |C_{10}^{\nu}|^2 |f_p^+(q^2)|^2 \end{aligned} \quad (8)$$

and

$$\begin{aligned} \frac{d\Gamma}{dq^2}(B^\pm \rightarrow V^\pm \nu \bar{\nu}) &= \frac{G_F^2 \alpha^2}{2^{10} \pi^5} |V_{tq} V_{tb}^*|^2 \lambda^{1/2}(1, r_V, s) m_B^3 |C_{10}^\nu|^2 \\ &\times \left( 8\lambda s \frac{V^2}{(1 + \sqrt{r_V})^2} \right. \\ &+ \frac{1}{r_V} \left[ \lambda^2 \frac{A_2^2}{(1 + \sqrt{r_V})^2} \right. \\ &+ (1 + \sqrt{r_V})^2 (\lambda + 12r_V s) A_1^2 \\ &\left. \left. - 2\lambda(1 - r_V - s) \text{Re}(A_1 A_2) \right] \right). \end{aligned} \quad (9)$$

In Eqs. (8) and (9),  $\lambda(1, r_M, s)$  is the usual triangle function  $\lambda(1, r_M, s) = 1 + r_M^2 + s^2 - 2r_M - 2s - 2r_M s$  with  $r_M = m_M^2/m_B^2$ ,  $s = q^2/m_B^2$ .

Similarly, calculations for the  $B^\pm \rightarrow M^0 e^\pm \nu$  decay lead to

$$\begin{aligned} \frac{d\Gamma}{dq^2}(B^\pm \rightarrow P^0 e^\pm \nu) &= \frac{G_F^2}{192 \pi^3} |V_{qb}|^2 \lambda^{3/2}(1, r_P, s) m_B^3 |f_P^+(q^2)|^2, \end{aligned} \quad (10)$$

$$\begin{aligned} \frac{d\Gamma}{dq^2}(B^\pm \rightarrow V^0 e^\pm \nu) &= \frac{G_F^2 |V_{qb}|^2 \lambda^{1/2} m_B^3}{768 \pi^3} \left( 8\lambda s \frac{V^2}{(1 + \sqrt{r_V})^2} + \frac{1}{r_V} \right. \\ &\times \left[ \lambda^2 \frac{A_2^2}{(1 + \sqrt{r_V})^2} + (1 + \sqrt{r_V})^2 (\lambda + 12r_V s) A_1^2 \right. \\ &\left. \left. - 2\lambda(1 - r_V - s) \text{Re}(A_1 A_2) \right] \right). \end{aligned} \quad (11)$$

### III. NUMERICAL ANALYSIS

In deriving Eqs. (8)–(11), we set the masses of  $M^+$  and  $M^0$  equal and the electron mass is neglected. Using isospin symmetry the branching ratio for  $B^\pm \rightarrow \rho^\pm \bar{\nu} \nu$  can be related to that for  $B^\pm \rightarrow \rho^0 \bar{e} \nu$ . It is clear that their ratio is independent of form factors, i.e., free of hadronic long-distance uncertainties in the limit  $m_{\rho^\pm} = m_{\rho^0}$ . Corrections to the strict isospin symmetry, which come from phase space factors due to the difference of masses of  $\rho^\pm$  and  $\rho^0$ , isospin violation in the  $B \rightarrow \rho$  form factors and electromagnetic radiative corrections to the  $b \rightarrow q e \nu$  transition, are all small. In the following discussions we shall neglect these small isospin violation effects. Also note that these corrections for  $K \rightarrow \pi$  transition have been calculated in [18] and found to be small,  $\sim 5\%$ .

Now we relate the branching ratio  $\mathcal{B}(B^\pm \rightarrow \rho^\pm \bar{\nu} \nu)$  with  $\mathcal{B}(B^\pm \rightarrow \rho^0 e^\pm \nu)$ . From Eqs. (9) and (11), we have

$$\frac{\mathcal{B}(B^\pm \rightarrow \rho^\pm \bar{\nu} \nu)}{\mathcal{B}(B^\pm \rightarrow \rho^0 e^\pm \nu)} = 6 \frac{\alpha^2}{4 \pi^2} |C_{10}^\nu|^2 \left| \frac{V_{td}}{V_{ub}} \right|^2. \quad (12)$$

Here the numerical factor 6 comes from the number of light neutrinos, and isospin symmetry relation between the form factors of  $B^\pm \rightarrow \rho^\pm$  and  $B^\pm \rightarrow \rho^0$ . In Eq. (12), we also put  $|V_{tb}| = 1$ . From Eq. (12), we get

$$\left| \frac{V_{td}}{V_{ub}} \right|^2 = \frac{1}{6C} \frac{\mathcal{B}_{exp}(B^\pm \rightarrow \rho^\pm \bar{\nu} \nu)}{\mathcal{B}_{exp}(B^\pm \rightarrow \rho^0 e^\pm \nu)} = \left( \frac{\sin \gamma}{\sin \beta} \right)^2, \quad (13)$$

where  $C = (\alpha^2/4\pi^2) |C_{10}^\nu|^2$ .

The second relation in Eq. (13) holds only for the CKM version of  $CP$  violation within the SM.

From Eq. (13), we can see that measurements of the ratio of the branching fractions allow us to determine the ratio of  $\sin \gamma$  and  $\sin \beta$ . Up to now,<sup>1</sup> various methods for measuring each angle separately have been proposed, e.g., the angle  $\beta$  will be measured from  $B \rightarrow J/\psi K_s$  decay with high accuracy, and angle  $\gamma$  is from the charged  $B$  decay  $B^\pm \rightarrow DK^\pm$  with larger uncertainty. As follows from Eq. (13), one can measure the angle  $\gamma$  with small theoretical uncertainty, if  $\sin \beta$  is measured independently with high accuracy. The following relations will also be useful for extracting the phase angle  $\gamma$  precisely:

$$\frac{\mathcal{B}(B^0 \rightarrow \rho^0 \nu \bar{\nu})}{\mathcal{B}(B^0 \rightarrow \rho^\pm e^\mp \nu)} = \frac{3}{2} \left( \frac{\sin \gamma}{\sin \beta} \right)^2 C, \quad (14)$$

$$\frac{\mathcal{B}(B^\pm \rightarrow \pi^\pm \nu \bar{\nu})}{\mathcal{B}(B^\pm \rightarrow \pi^0 e^\pm \bar{\nu})} = 6 \left( \frac{\sin \gamma}{\sin \beta} \right)^2 C, \quad (15)$$

$$\frac{\mathcal{B}(B^0 \rightarrow \pi^0 \nu \bar{\nu})}{\mathcal{B}(B^0 \rightarrow \pi^\pm e^\mp \bar{\nu})} = \frac{3}{2} \left( \frac{\sin \gamma}{\sin \beta} \right)^2 C, \quad (16)$$

$$\frac{\mathcal{B}(B^\pm \rightarrow K^{*\pm} \nu \bar{\nu})}{\mathcal{B}(B^\pm \rightarrow \rho^0 e^\pm \nu)} \approx 6 \left| \frac{V_{ts}}{V_{ub}} \right|^2 C. \quad (17)$$

In derivation (14)–(17), we assumed that the mass of charged and neutral final states mesons are equal.

Now we consider the differential decay widths,  $(d\Gamma/dq^2)(B \rightarrow \rho, K^* + \nu + \bar{\nu})$ . For the hadronic form factors we have used the results of the works [15–17], i.e., the monopole type form factors based on light cone QCD sum rules. The values of the form factors at  $q^2 = 0$  are (see also Ref. [20])

$$A_1^{B \rightarrow K^*}(0) = 0.36 \pm 0.05,$$

$$A_2^{B \rightarrow K^*}(0) = 0.40 \pm 0.05,$$

<sup>1</sup>See also the recent work [19] on the simultaneous determination of  $\sin \alpha$  and  $\sin \gamma$  from  $B_{d,s}^0 \rightarrow K, \pi$  decays.

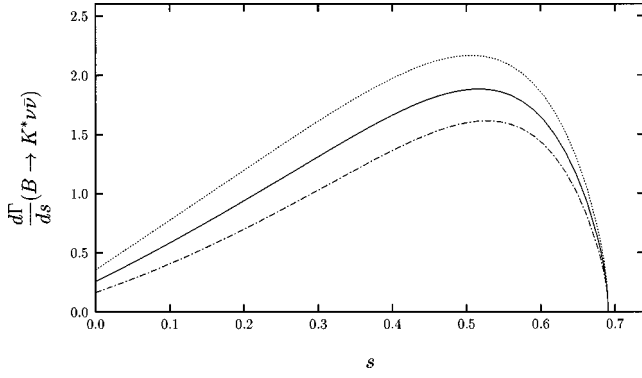


FIG. 1. Differential decay width  $(d\Gamma/ds)(B \rightarrow K^* \nu \bar{\nu})$  as a function of the normalized momentum transfer square,  $s \equiv q^2/m_B^2$ , in units of  $[8.84 \times 10^{-18} (|V_{tb} V_{ts}^*|/0.045)^2]$  GeV. Dotted and dash-dotted curves correspond to the cases when the uncertainty is added and subtracted from the central values of all form factors, respectively.

$$\begin{aligned}
 V^{B \rightarrow K^*}(0) &= 0.45 \pm 0.08, \\
 A_1^{B \rightarrow \rho}(0) &= 0.30 \pm 0.05, \\
 A_2^{B \rightarrow \rho}(0) &= 0.325 \pm 0.05, \\
 V^{B \rightarrow \rho}(0) &= 0.37 \pm 0.07, \\
 f_+^{B \rightarrow K}(0) &= 0.29 \pm 0.05, \\
 f_+^{B \rightarrow \pi}(0) &= 0.32 \pm 0.05. \quad (18)
 \end{aligned}$$

A few words about error analysis in the differential decay rates and their ratios are in order. In both cases the errors which come from different form factors are added quadratically since they are theoretically independent of each other. Note also that all errors, which come from the uncertainties of the  $b$  quark mass, the Borel parameter variation, wave functions, non-inclusion of higher twists and radiative corrections, are added in quadrature. The uncertainty in the ratio is estimated as the half distance between the maximum and minimum values of the ratio.

In Fig. 1 we present the differential decay width  $d\Gamma/ds(B \rightarrow K^* \nu \bar{\nu})$  as a function of the normalized momentum transfer square,  $s \equiv q^2/m_B^2$ . In Fig. 2 we show the  $q^2$  dependence of the ratio of the differential decay rates  $B \rightarrow \rho \nu \bar{\nu}$  and  $B \rightarrow K^* \nu \bar{\nu}$ , normalized to  $|V_{td}/V_{ts}|^2$ . In these figures, dotted and dash-dotted curves correspond to the cases when the uncertainty is added and subtracted from the central values of all form factors, respectively. For the central solid curve we use the central values of form factors. We note that the errors in the differential decay width of Fig. 1 due to the form factors uncertainties are about  $\sim \pm 20\%$ . However, the errors in the ratio of Fig. 2 are reduced to about  $\sim \pm 10\%$ . We conclude that even though the errors from uncertainties of the form factors for each channel are substantial, those in the corresponding ratio are comparatively small, and that for precise determination of the elements of the CKM matrix the investigation of the corresponding ratio

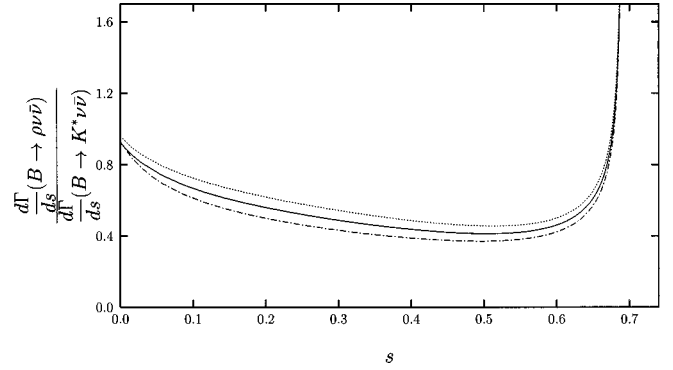


FIG. 2. Ratio of the differential decay rates  $B \rightarrow \rho \nu \bar{\nu}$  and  $B \rightarrow K^* \nu \bar{\nu}$ , in units of  $|V_{td}/V_{ts}|^2$ , as a function of the normalized momentum transfer square,  $s \equiv q^2/m_B^2$ .

is very suitable. We also note that the uncertainties for our main results, Eqs. (12)–(16), where we only assume flavor SU(2) (isospin), should be even much smaller than that shown in Fig. 2, since there we had to assume flavor SU(3) symmetry.

For completeness we present the integrated value for the branching fractions of  $B \rightarrow K^* \nu \bar{\nu}$  and  $B \rightarrow K \nu \bar{\nu}$  as well as the value of the ratio  $\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})/\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})$  and  $\mathcal{B}(B \rightarrow \pi \nu \bar{\nu})/\mathcal{B}(B \rightarrow K \nu \bar{\nu})$ :

$$\begin{aligned}
 \mathcal{B}(B \rightarrow K^* \nu \bar{\nu}) &= 1.7 \times (1 \pm 0.16) \times 10^{-5} \left| \frac{V_{ts} V_{tb}}{0.045} \right|^2, \\
 \frac{\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})}{\mathcal{B}(B \rightarrow K^* \nu \bar{\nu})} &= 0.52 \times (1 \pm 0.1) \left| \frac{V_{td}}{V_{ts}} \right|^2, \\
 \mathcal{B}(B \rightarrow K \nu \bar{\nu}) &= 7.8 \times (1 \pm 0.25) \times 10^{-6} \left| \frac{V_{ts} V_{tb}}{0.045} \right|^2, \\
 \frac{\mathcal{B}(B \rightarrow \pi \nu \bar{\nu})}{\mathcal{B}(B \rightarrow K \nu \bar{\nu})} &= 1.29 \times (1 \pm 0.2) \left| \frac{V_{td}}{V_{ts}} \right|^2. \quad (19)
 \end{aligned}$$

The values of the main input parameters, which appear in the expressions for the decay widths, are  $m_b = (4.8 \pm 0.1)$  GeV,  $m_\rho \approx 0.77$  GeV,  $m_{K^*} = 0.892$  GeV. For the  $B$  meson lifetime, we take  $\tau(B_d) = 1.56 \times 10^{-12}$  sec [21].

#### IV. DISCUSSIONS AND CONCLUSIONS

We proposed a new method for the precise determination of  $|V_{td}/V_{ub}|$  from the ratios of the branching fractions

$$\mathcal{R}_\rho = \frac{\mathcal{B}(B \rightarrow \rho \nu \bar{\nu})}{\mathcal{B}(B \rightarrow \rho \nu e)} \quad \text{and} \quad \mathcal{R}_\pi = \frac{\mathcal{B}(B \rightarrow \pi \nu \bar{\nu})}{\mathcal{B}(B \rightarrow \pi \nu e)}.$$

As is well known, each partial decay width depends very strongly on hadronic form factors. However, as also shown in Eqs. (9)–(13), these ratios,  $\mathcal{R}_\rho, \mathcal{R}_\pi$ , are free of any hadronic uncertainties, if small isospin breaking effects are ne-

glected. Measurements of  $\mathcal{R}_{\rho,\pi}$  allow us to determine  $|V_{td}/V_{ub}|$  with little theoretical error, which equals  $(\sin \gamma/\sin \beta)$  for the CKM version of  $CP$  violation within the standard model. Therefore,  $\mathcal{R}_{\rho,\pi}$  measures a relation between two different phases angles, which can be measured separately by experiments. We also found that each exclusive channel  $B \rightarrow (K, K^*, \rho, \pi) \nu \bar{\nu}$  has rather large theoretical uncertainties due to the unknown hadronic form factors, as shown in Fig. 1. In order to reduce these uncertainties we have considered the ratio of the corresponding exclusive channels, e.g.  $(B \rightarrow \rho \nu \bar{\nu})/(B \rightarrow K^* \nu \bar{\nu})$ , as shown in Fig. 2.

A few words about experimental statistics for detecting the  $B \rightarrow \rho \nu \bar{\nu}$  decay follow: Future symmetric and asymmetric  $B$  factories should produce much more than  $\sim 10^9$   $B\text{-}\bar{B}$  mesons by the year 2010. With  $10^9$   $B$  mesons effectively reconstructed, the number of expected events for  $B^\pm \rightarrow \rho^\pm \nu \bar{\nu}$  channel is  $N \equiv \mathcal{B}(B \rightarrow \rho \nu \bar{\nu}) \times 10^9 \sim 100$  [and  $N(B \rightarrow K^* \nu \bar{\nu}) \sim 2 \times 10^4$ ]. And the statistically estimated error for  $B \rightarrow \rho \nu \bar{\nu}$  decay is approximately  $1/\sqrt{N}$

$\approx 1/10 = 10\%$ . We argue that within the next decade the decay channel  $B^\pm \rightarrow \rho^\pm \nu \bar{\nu}$  has a good chance for being detected in future  $B$  factories. Note that the inclusive channels  $B \rightarrow X_{d,s} \nu \bar{\nu}$  are also free of any theoretical uncertainties. However, measuring inclusive channels in experiments would be very difficult because of the two missing neutrinos and (many) hadrons.

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