Fermion masses in $SO(10)$ with a single adjoint Higgs field

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It has recently been shown how to break $SO(10)$ down to the standard model in a realistic way with only one adjoint Higgs field. The expectation value of this adjoint must point in the $B-L$ direction. This has consequences for the possible form of the quark and lepton mass matrices. These consequences are explored in this paper, and it is found that one is naturally led to consider a particular form for the masses of the heavier generations. This form implies typically that there should be large (nearly maximal) mixing of the μ and τ neutrinos. An explanation that does not involve large $\tan\beta$ also emerges for the fact that b and τ are light compared to the top quark. [S0556-2821(98)01413-1]

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I. INTRODUCTION

For a number of reasons, $SO(10)$ is widely considered to be the most attractive grand unified group. It achieves complete quark-lepton unification for each family, explains the existence of right-handed neutrinos and of ''seesaw'' neutrino masses, has certain advantages for baryogenesis, in particular, since $B-L$ is broken [1], and has the greatest promise for explaining the pattern of quark and lepton masses [2–6]. Some progress has been made in constructing $SO(10)$ models in superstring theory, it now being known that there are perturbative ground states of the heterotic string with three generations of quarks and leptons [7].

It has been shown that there are limitations in the context of perturbative superstring theory on supersymmetric grand unified models which have more than a single adjoint Higgs field. In particular, it had been argued that if there are multiple adjoints in realistic models, they must have the same charges under local symmetries. (They may have different discrete gauge charges, however.) This makes it significantly harder to construct realistic models in which there are several adjoints which couple in different ways [8]. On the other hand, until recently, it was not known how to break $SO(10)$ without either using three adjoint Higgs fields [9] or having colored pseudo-goldstone fields that largely vitiated the unification of gauge couplings [10,11]. However, in a recent paper [12], a satisfactory mechanism was proposed for achieving natural breaking of $SO(10)$ without more than one adjoint Higgs field. But in that paper, only the Higgs sector was considered. This raises the question of whether quarks and leptons can be incorporated in a satisfactory way into models which employ that mechanism of symmetry breaking.

There are two aspects to this question. First, it is not obvious whether a single adjoint Higgs field is sufficient to give a realistic pattern of quark and lepton masses. If there is only one adjoint Higgs field in $SO(10)$, its vacuum expectation value must point in the $B-L$ direction in order to produce the doublet-triplet splitting [13]. This greatly constrains the possibilities for the quark and lepton masses, as this adjoint vacuum expectation value (VEV) is the only one that breaks the $SU(5)$ subgroup of $SO(10)$ at the unification scale, and therefore the only one that can break the ''bad'' $SU(5)$ relations such as $m^0_\mu = m^0_s$. (The superscript ''0'' refers throughout to parameters at the unification scale.) All models in the literature which attempt to explain the pattern of fermion masses in the context of $SO(10)$ make use of adjoint VEVs that point in directions other than $B-L$ [2–6].

The second issue has to do with the stability of the gauge hierarchy. In $SO(10)$, as in any unified model, there are higher-dimension operators that would destabilize the hierarchy, and which must therefore be forbidden by some local symmetry or other principle. These local symmetries constrain the possible couplings of the Higgs fields and therefore the possible Yukawa couplings of the quarks and leptons. Conversely, the existence of realistic quark and lepton Yukawa interactions may be incompatible with any symmetry that could stabilize the hierarchy, and may therefore imply the presence (because of Planck-scale effects) of operators that destroy the hierarchy.

In this paper we show that a realistic pattern of quark and lepton masses can be achieved in a natural way using only one adjoint Higgs field and the mechanism for symmetry-breaking proposed in [12]. We find, indeed, that the possibilities are tightly constrained, and under certain reasonable requirements the basic structure that we find may be unique. This structure is fairly simple: it does not require that there be any Higgs fields or any symmetries beyond those introduced in [12] to achieve $SO(10)$ breaking to $SU(3)\times SU(2)\times U(1)$. It also provides an explanation of many of the qualitative and quantitative features of the quark and lepton masses and mixings.

There are two interesting features of the structure to

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which we are led. First, it typically gives large, and indeed nearly maximal, mixing of ν_μ with ν_τ. This is possibly of great significance in light of the evidence of such mixing coming from atmospheric neutrino observations. Second, an interesting explanation emerges of the smallness of m_b and m_τ compared to m_t that does not involve large $\tan\beta$.

II. REVIEW OF THE BREAKING OF $SO(10)$

Before turning to the problem of quark and lepton masses, let us briefly review the mechanism proposed in [12] for breaking $SO(10)$ with only a single adjoint. The Higgs superpotential has the form

$$W = T_1 A T_2 + M_T T_2^2 + W_A + W_C + W_{CA} + W_{TC}, \qquad (1)$$

where T_1 and T_2 are **10**'s and A is a **45**. W_A is a set of terms that produces the ''Dimopoulos-Wilczek'' form for the expectation value of A: $\langle A\rangle = \mathrm{diag}(0,0,a,a,a)\times i\tau_2$, where $a \sim M_G$. This is equivalent to saying that the VEV of A is proportional to the generator $B-L$. This form for $\langle A\rangle$ couples the color-triplets in T_1 and T_2, but not the weak-doublets. The effect of the first two terms in Eq. (1) is to give superheavy masses to all the color triplets in T_i, but leave the pair of weak-doublets in T_1 light. The simplest form for W_A that works is

$$W_A = \mathrm{tr}A^4/M + M_A \mathrm{tr}A^2. \qquad (2)$$

Here and in the following, all explicit denominator masses are regarded as Plank scale masses, i.e., M_P.

To break $SO(10)$ completely to the standard model requires also Higgs fields in the spinor representation which must get vacuum expectation values in the $SU(5)$-singlet direction. If C and $\overline{C}$ are, respectively, a **16** and $\overline{\mathbf{16}}$, then a simple form for W_C is

$$W_C = X(\overline{C}C)^2/M_C^2 + f(X), \qquad (3)$$

where X is a singlet field, and $f(X)$ is a polynomial in X that has at least a linear term. Then the f-flat condition $F_X=0$ forces C and $\overline{C}$ to get VEVs.

The terms W_{CA} couple the spinor sector (C, $\overline{C}$) to the adjoint sector (A). This is necessary [12] to prevent light, color-singlet pseudo-goldstone fields from being produced by breaking of the unified symmetry. The only mechanism known to do this without involving several adjoint fields was proposed in [12]. The form of W_{CA} given there is

$$W_{CA} = \overline{C}'(PA/M_1 + Z_1)C + \overline{C}(PA/M_2 + Z_2)C'. \qquad (4)$$

Here C' and $\overline{C}'$ are an additional $\mathbf{16}+\overline{\mathbf{16}}$ pair, and P, Z_1 and Z_2 are singlets. C' and $\overline{C}'$ have vanishing VEVs, which ensures that W_{CA} does not destabilize the hierarchy (*i.e.* the Dimopoulos-Wilczek form of $\langle A\rangle$) by contributing to F_A. The $F_{C'}=0$ and $F_{\overline{C}'}=0$ equations lead to the conditions $(PA/M_1+Z_1)C = \overline{C}(PA/M_2+Z_2)=0$ having a discrete number of solutions, for one of which $\langle C\rangle$ and $\langle\overline{C}\rangle$ point in the $SU(5)$-singlet direction. These two equations then fix the relative magnitudes of the VEVs of the singlets P and Z_i. There is one linear combination of these singlets that is not fixed by the terms in Eq. (1), but this can be fixed by radiative effects after supersymmetry breaks [12].

Finally, the W_{TC} term which was not included in [12] is added here in order to induce an electroweak-breaking VEV in the spinor C'. This VEV will help to generate the desired texture in the fermion mass matrices. For this purpose we set

$$W_{TC} = \lambda T_1 \overline{CC} \qquad (5)$$

where λ is a dimensionless coefficient which, as we shall see later, must be somewhat smaller than one—about 1/20. From the $F^*_C=0$ equation,

$$0 = 2\lambda T_1 \overline{C} + (PA/M_2 + Z_2)C'. \qquad (6)$$

It then follows that since $\overline{C}$, P, A, and Z_i all have superlarge VEVs in the $SU(5)$ **1** direction, while the Higgs doublets of T_1 are assumed to develop weak-scale VEV's in the $SU(5)$ **5** and $\overline{\mathbf{5}}$ directions, the $SU(2)_L$-doublet in C' must also develop a weak-scale VEV in the $SU(5)$ $\overline{\mathbf{5}}$ direction.

This set of terms gives a complete breaking of $SO(10)$ down to the standard model group without fine-tuning of parameters and without pseudo-goldstone fields. The mass M_T appearing in Eq. (1) must arise from the expectation value of some field or product of fields. Two viable possibilities are P^2 and Z_i.

The stability of the hierarchy requires that certain types of higher-dimension terms not arise, in particular, terms that give effectively T_1^2, $\overline{C}AC$, $\overline{C}CA^2/M$, or Z_i^n. The first of these, T_1^2, would directly give superheavy mass to the doublet Higgs fields. Both $\overline{C}AC$ and $\overline{C}CA^2/M$ would destabilize the Dimopoulos-Wilczek form of $\langle A\rangle$; hence the choice of a higher order term in the W_C superpotential of Eq. (3). The appearance of Z_i^n would cause a conflict between the $F_{Z_i}=0$ equations and the $F_{C'}=0$ and $F_{\overline{C}'}=0$ equations. In [12] it was shown that a simple $U(1)\times Z_2\times Z_2$ symmetry is sufficient to rule out all dangerous operators. In order to obtain the desired appearance of the $\lambda T_1\overline{CC}$ term in W_{TC} along with the rest of the Higgs superpotential, the $U(1)\times Z_2\times Z_2$ charges are reassigned as follows:

$$A(0^{+-}), \quad T_1(1^{++}), \quad T_2(-1^{+-})$$
$$C(\tfrac{1}{2}^{-+}), \quad \overline{C}(-\tfrac{1}{2}^{++}), \quad C'([\tfrac{1}{2}-p]^{++}),$$
$$\overline{C}'([-\tfrac{1}{2}-p]^{-+}) \qquad (7)$$
$$X(0^{++}), \quad P(p^{+-}), \quad Z_1(p^{++}), \quad Z_2(p^{++}).$$

III. *B-L* GENERATOR AND FERMION MASS MATRIX TEXTURES

We have succeeded in constructing a simple superpotential for the quark and lepton fields that gives the fermions realistic masses and makes use of no Higgs superfields beyond the set found necessary to achieve a satisfactory breaking of $SO(10)$ in [12], namely T_i, A, C, $\overline{C}$, C', $\overline{C}'$, and the singlets X, P, Z_1 and Z_2. To help understand this superpotential before writing it down, we explain the kind of textures that are needed if only one adjoint is available with its VEV in the $B-L$ direction. The desired textures for the mass matrices U, D, and L are of the form

$$U \cong \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & F/3 \\ 0 & -F/3 & E \end{bmatrix} v_u, \tag{8}$$

$$D \cong \begin{bmatrix} 0 & 0 & G' \\ 0 & 0 & F/3+G \\ 0 & -F/3 & E \end{bmatrix} v_d, \tag{9}$$

and

$$L \cong \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -F \\ G' & F+G & E \end{bmatrix} v_d. \tag{10}$$

These matrices are written so that the left-handed antifermions multiply them from the left and the left-handed fermions from the right. We imagine that some of the zero entries in the first row and column actually get small contributions from higher order terms so that the first generation will not remain exactly massless. This will be discussed later. Note that the parameter F is multiplied by a factor of $B-L$ everywhere. Suppose that we assume that $G \sim E \gg F$. Denote the small parameter F/E by the symbol ϵ, and the $O(1)$ parameter $\sqrt{G^2+G'^2}/E$ by ρ. Then it is easy to see that the following relations hold:

$$\begin{aligned} m_c^0/m_t^0 &\cong \epsilon^2/9, \\ m_s^0/m_b^0 &\cong \epsilon\rho/3(1+\rho^2) \sim \epsilon/3, \\ m_\mu^0/m_\tau^0 &\cong \epsilon\rho/(1+\rho^2) \sim \epsilon, \\ m_\tau^0 &\cong m_b^0, \\ m_\mu^0/m_s^0 &\cong 3, \\ V_{cb} &\cong \epsilon\rho^2/3(1+\rho^2) \sim \epsilon/3. \end{aligned} \tag{11}$$

Thus the following facts would be explained: the equality at the grand unified theory (GUT) scale of the b and τ masses, the Georgi-Jarlskog factor of 3 between the μ and s masses at the GUT scale [14], why V_{cb} is of order m_s/m_b, why m_c/m_t is much smaller than both m_s/m_b and m_μ/m_τ, and why the second generation masses are small compared to the third, and the first generation masses are very small compared to the second. This list contains most of the salient features of the quark and lepton spectrum. It is important to note how some of these relations are achieved, and therefore the rational for the form of the textures.

In our model the only generator of $SO(10)$ available for constructing the textures is $B-L$. As we shall see, it is a simpler matter for this generator to appear in the off-diagonal entries than in the diagonal ones. However, if the 23 and 32 entries are just proportional to $B-L$, while the 33 entries are proportional to the identity, then the ratio $(m_\mu/m_\tau)/(m_s/m_b)$ is 9 instead of the Georgi-Jarlskog value of 3. It is therefore essential to have asymmetrical entries like those denoted by G and G'. With G or G' being much larger than F and *not* depending on $B-L$, the desired ratio of 3 for m_μ^0/m_s^0 is obtained. As we will see, such asymmetrical entries can be achieved simply by integrating out $SO(10)$ **10**'s of fermions, since these contain $SU(5)$ $\overline{\mathbf{5}}+\mathbf{5}$ (which contain d_L^c and l_L), but not $SU(5)$ **10** (which contain d_L and l_L^c). Moreover, entries produced in this way will appear only in the down quark and charged lepton mass matrices, D and L; but not in the up quark and Dirac neutrino mass matrices, U or N. [This follows from the fact that they come from effective operators of the form $\mathbf{16}\mathbf{16}\mathbf{16}_H\mathbf{16}_H$, where $\mathbf{16}_H$ contains the $\overline{\mathbf{5}}$, but not the **5** of $SU(5)$.] This then automatically explains why the ratio m_c/m_t is much smaller than the m_s/m_b and m_μ/m_τ ratios. The fact that the entries G and G' appear in D, but not in U also explains why V_{cb} does not vanish. [Of course, $V_{cb}=0$ is a minimal $SO(10)$ relation.]

IV. IMPORTANT CONCLUSION ABOUT NEUTRINO MIXING

Careful consideration of those possibilities available that use only the generator $B-L$ leads to the conclusion that the textures given above are likely to be the only ones that satisfy the requirements of simplicity and realism. Other structures tend to be more complicated, or require artificial numerical relationships among parameters to reproduce the qualitative and quantitative features of the spectrum of quarks and leptons.

These textures already have an interesting phenomenological consequence, namely, that they predict large mixing of ν_μ and ν_τ. The neutrino mixing angles arise from the mismatch between the unitary transformations required to diagonalize the charged leptom mass matrix, L, and the neutrino mass matrix, M_ν. The neutrino mass matrix can be written in the familiar seesaw form: $M_\nu = -N^T M_R^{-1} N$, where M_R is the superheavy Majorana mass matrix of the right-handed neutrinos, and N is the Dirac mass matrix for the neutrinos. Little can be said at present about the form of M_R as there are many possible ways that the right-handed neutrinos can get mass. However, the form of N is closely connected to the forms of U, D, and L. In fact, given the forms shown in Eqs. (8)–(10), one expects N to have the form

$$N = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -F \\ 0 & F & E \end{pmatrix}. \tag{12}$$

Precisely this form will indeed arise from the superpotential that we shall discuss in the next section. The similarity of structure of N and U is a typical feature of $SU(5)$ and $SO(10)$ models. The difference in the coefficient of the F term is, of course, just due to the generator $B-L$. The G and G' terms are absent from N just as they are from U for the reasons explained above.

One sees immediately that the 13 and 23 angles required to diagonalize M_ν vanish in the limit that the second generation masses go to zero (*i.e.* $F/E \equiv \epsilon \to 0$) and the first generation masses go to zero, no matter what the form of M_R. Nevertheless, it is possible that the texture of M_R is such that these angles are numerically large in spite of being formally of order ϵ. However, we will assume that M_R does not have such a special form, and therefore that one can neglect these angles. With this plausible assumption, the mixing angle between ν_μ and ν_τ can be read off directly from the matrix L. It is given by $\tan\theta_{\mu\tau} \cong \sqrt{G^2+G'^2}/E = \rho$. One then finds that

$$\tan\theta_{\mu\tau} \cong \rho \equiv 3V^0_{cb}/(m^0_\mu/m^0_\tau) \cong 1.8. \tag{13}$$

It is quite striking that the constraint of having $SU(5)$ broken only by an adjoint pointing in the $B-L$ direction, which is in essence a minimality condition on the Higgs sector, leads in a natural way to textures for the quark and lepton mass matrices that predict large mixing of the μ and τ neutrinos. The consequences of this implication for neutrino mixing will be explored more fully elsewhere [15].

V. YUKAWA SUPERPOTENTIAL YIELDING THE DESIRED TEXTURES

We will now show how these textures arise in a straightforward way from a few terms in the superpotential. We distinguish the third generation quarks and leptons, which we denote $\mathbf{16}_3$, from the other two generations, which we denote $\mathbf{16}_i$, $i=1,2$. In addition, we posit the existence of some ''vectorlike'' sets of quarks and leptons to be ''integrated out,'' namely $\mathbf{16}+\overline{\mathbf{16}}$, $\mathbf{10}$ and $\mathbf{10}'$. The proposed Yukawa superpotential has the following form:

$$W_{Yukawa} = \mathbf{16}_3\mathbf{16}_3T_1 + \mathbf{16}\overline{\mathbf{16}}P + \mathbf{16}_3\overline{\mathbf{16}}A + a_i\mathbf{16}_i\mathbf{16}T_1 + \mathbf{10}\mathbf{10}'\overline{C}C/M_P + c_i\mathbf{16}_i\mathbf{10}C + \mathbf{16}_3\mathbf{10}'C'. \tag{14}$$

As in the Higgs superpotential, we have suppressed most of the dimensionless coefficients, which are assumed to be of order unity. However, we have explicitly written the two Yukawa coefficients that carry the family index i, which, of course, is summed over. Recall that the Higgs fields T_1 and C' each develop weak-scale VEV's, while A, C, $\overline{C}$, P, Z_1 and Z_2 all acquire superlarge VEVs. No VEV's appear for $\overline{C'}$ or X.

The 33 elements denoted by E in the U, D and L matrices of Eqs. (8)–(10) obviously arise directly from the first term in Eq. (14) as illustrated in Fig. 1(a). The F contributions to the matrix elements arise from the next three terms in Eq. (14), which contain the spinors $\mathbf{16}$ and $\overline{\mathbf{16}}$. This is easiest to see diagrammatically by considering Fig. 1(b). By

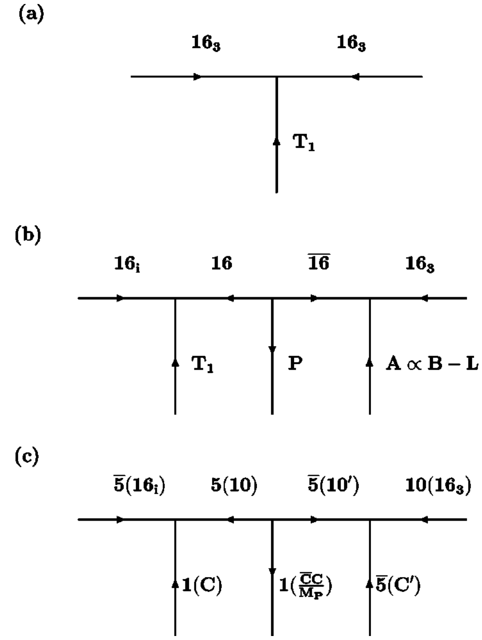

FIG. 1. Diagrams that generate the entries in the quark and lepton mass matrices shown in Eqs. (8)–(10). (a) The 33 elements denoted ''E.'' (b) The 23 and 32 elements denoted ''G.'' Note that because of the VEV of A, they are proportional to the $SO(10)$ generator $B-L$. (c) The asymmetric entries denoted ''G'' and ''G''' arise from these diagrams. That they do not contribute to the up quark masses, and contribute asymmetrically to the down quark and lepton mass matrices, are consequences of the fact that the $SO(10)$ $\mathbf{10}$'s contain $\overline{\mathbf{5}}$, but not $\mathbf{10}$ of $SU(5)$.

integrating out those spinors, one effectively obtains a term of the form $a_i\mathbf{16}_i\mathbf{16}_3\langle A\rangle\langle T_1\rangle/M_G$. Because the vacuum expectation value of A is proportional to $B-L$, this term will have a factor of $B-L$ of the field contained in $\mathbf{16}_3$ [or, equivalently, $-(B-L)$ of the field contained in $\mathbf{16}_i$]. Without loss of generality, one can take the Yukawa coefficient a_i to point in the 2 direction. Thus one has $F[(B-L)_f f^c_2 f_3 + (B-L)_{f^c} f^c_3 f_2]\langle T_1\rangle$, where F is a dimensionless combination of VEVs and Yukawa couplings. This form also explains why it is hard for the generator $B-L$ to appear in a diagonal element of the mass matrices, for the combination $[(B-L)_f + (B-L)_{f^c}]f^c_i f_i$ vanishes for the diagonal ii matrix element.

The G and G' contributions to the mass matrices in Eqs. (8)–(10) arise from the last three terms in Eq. (14), which contain the vector fields $\mathbf{10}$ and $\mathbf{10}'$ as can be seen diagrammatically from Fig. 1(c). Having defined the 2 direction to be that of a_i, there is no freedom left, and c_i will have components in both the 1 and 2 directions. Since as noted earlier, the VEV's of C and C' point, respectively, in the $\mathbf{1}$ and $\overline{\mathbf{5}}$ $SU(5)$ directions, it is clear that only the $\overline{\mathbf{5}}(\mathbf{16}_i)\mathbf{5}(\mathbf{10})\langle\mathbf{1}(C)\rangle$ and $\overline{\mathbf{10}}(\mathbf{16}_3)\overline{\mathbf{5}}(\mathbf{10}')\langle\overline{\mathbf{5}}(C')\rangle$ components of the last two terms in the superpotential of (14) can contribute to the mass diagram in Fig. 1(c). [Here and throughout $\mathbf{p}(\mathbf{q})$ denotes an $SU(5)$ $\mathbf{p}$ contained in an $SO(10)$ $\mathbf{q}$.] Hence with the convention that the mass matrices are to be multiplied from the left by left-handed antifermions and from the right by left-handed fermions, the diagram depicted in Fig. 1(c) can only contribute to the 13 and 23 elements of the down quark mass matrix D and the 31 and 32 elements of the charged lepton mass matrix L. The up quark mass matrix U and the Dirac neutrino mass matrix N receive no such contributions.

One can also easily see the origin of the G and G' terms directly from the superpotential terms in Eq. (14). The $\mathbf{5}(\mathbf{10})$ has a mass term with the linear combination of superfields $\langle \overline{C}C/M_P\rangle \overline{\mathbf{5}}(\mathbf{10}') + c_i\langle C\rangle \overline{\mathbf{5}}(\mathbf{16}_i)$. But this linear combination lies nearly exactly in the $c_i\mathbf{16}_i$ direction, because of the M_P^{-1} Planck scale suppression factor. Thus $\overline{\mathbf{5}}(\mathbf{10}')$ is almost purely one of the light (*i.e.* weak-scale) multiplets, and in generation space points partly in the 1 and partly in the 2 directions. It then follows directly that the term $\mathbf{16}_3\mathbf{10}'C'$ gives the G and G' entries. Note that direct calculation of the mass matrix elements shows these entries are not suppressed by powers of M_P as one might naively think from Fig. 1(c).

Before turning to the question of how the small first generation masses arise, we note that the terms in the Yukawa superpotential of 13 do not destabilize the gauge hierarchy. With the assignments given in Eq. (7) for the Higgs multiplets, the charges of the chiral multiplets are completely determined by the terms appearing in Eq. (14):

$$\mathbf{16}_3(-\tfrac{1}{2}^{++}), \quad \mathbf{16}_i([-\tfrac{1}{2}+p]^{++}), \quad i=1,2$$

$$\mathbf{16}(-\tfrac{1}{2}^{++}), \quad \overline{\mathbf{16}(\tfrac{1}{2}^{++})} \tag{15}$$

$$\mathbf{10}(-p^{-+}), \quad \mathbf{10}'(p^{++}).$$

The value of the charge p depends on which field or fields couple to T_2^2. Two viable choices are $p=1$ or $p=2$, giving, respectively, that the mass term for T_2 is of the form $T_2^2P^2/M_P$ or $T_2^2Z_i$. It is easily checked that the $U(1)\times Z_2\times Z_2$ forbids any destabilizing terms, such as those containing factors of T_1^2, $\overline{C}AC$, and Z_i^n as discussed in the pure Higgs field case. There are some higher-dimension terms not included in Eq. (14) that are allowed by the symmetry, such as $\mathbf{10}^2P^2/M_P$, but these prove to be harmless.

The requirement of stability of the gauge hierarchy does dictate an important feature of the structure of the Yukawa superpotential in Eq. (14), namely that C' acquires a weak-scale $\overline{\mathbf{5}}(\mathbf{16})$ $SU(2)_L\times U(1)_Y$-breaking VEV, and that C' and T_1 therefore mix. One might imagine that the G and G' terms in the matrices of Eqs. (8)–(10) could be generated without a spinor Higgs field acquiring an $SU(2)_L\times U(1)_Y$-breaking VEV. This could happen via the diagram in Fig. 2, if instead of the terms in Eq. (14), there were the following terms: $\mathbf{16}_3\mathbf{16}_3T_1+\mathbf{16}\overline{\mathbf{16}}P+a_i\mathbf{16}_i\overline{\mathbf{16}}A+\mathbf{16}_3\mathbf{16}T_1+\mathbf{10}\mathbf{10}S+c_i\mathbf{16}_i\mathbf{10}C+\overline{\mathbf{16}}\mathbf{10}\overline{C}$. However, it is easy to see that the existence of the terms $\mathbf{10}\mathbf{10}S$, $\mathbf{16}_i\mathbf{10}C$, $\overline{\mathbf{16}}\mathbf{10}\overline{C}$, $\mathbf{16}_i\overline{\mathbf{16}}A$, and $X(\overline{C}C)^2/M_P^2$ would imply that the term $\overline{C}ACS/M_P$ is allowed by the symmetry; this term would

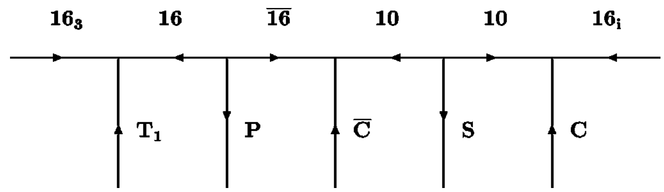

FIG. 2. A diagram that could generate the "E" and "E'" entries of the mass matrices in an alternative version of the model. However, this version has an unstable gauge hierarchy. Thus the diagram in Fig. 1(c) is necessary, implying that C' must break the weak interactions.

destroy the gauge hierarchy and such a form for the Yukawa superpotential is unacceptable for the doublet-triplet splitting solution.

Thus it seems that generating simple and realistic textures for the quark and lepton mass matrices requires that C' break the electroweak symmetry and mix with T_1. This is an important fact, for it may also hold the key to explaining why t is much heavier than b and τ, which is otherwise somewhat mysterious in the context of $SO(10)$. This point can be seen from Eq. (6), which says that the linear combination of $\overline{\mathbf{5}}(T_1)\cos\theta+\overline{\mathbf{5}}(C')\sin\theta$, where $\tan\theta=\langle(PA/M_2+Z_2)\rangle/(2\lambda)\langle\overline{C}\rangle$, has a vanishing VEV. In fact, from the term $|F_{\overline{C}}|^2$ in the scalar potential, it is clear that this linear combination is superheavy. The orthogonal linear combination is the field H' of the minimal supersymmetric standard model (MSSM), while H has the usual definition:

$$H'=\overline{\mathbf{5}}(C')\cos\theta-\overline{\mathbf{5}}(T_1)\sin\theta$$

$$H=\mathbf{5}(T_1). \tag{16}$$

Therefore the ratio of the b to t masses is determined by the angle θ; in particular,

$$m_b^0/m_t^0=m_\tau^0/m_t^0=\sin\theta(\langle H'\rangle/\langle H\rangle)=\sin\theta/\tan\beta. \tag{17}$$

But from the fact that $\langle P\rangle\sim\langle A\rangle\sim\langle\overline{C}\rangle\sim M_G$, while $\langle Z_i\rangle\sim M_G^2/M_P$, one finds $\tan\theta\sim\lambda^{-1}M_G/M_P$. Therefore, the smallness of the mass ratios in Eq. (17) may be due to small $\sin\theta$ rather than large $\tan\beta$. The authors of [16] pursued a similar attempt to lower $\tan\beta$ by reducing the ratio of the bottom to top Yukawa couplings in $SO(10)$ models. Here with $\lambda\sim1/20$ the correct mass ratios are obtained with $\tan\beta\sim1$. This would alleviate the problem of Higgsino-mediated proton-decay, the amplitude for which is proportional to $\tan\beta$ for the large $\tan\beta$ case. To suppress Higgsino-mediated proton decay then requires that M_T [see Eq. (1)] be made small compared to M_G. This, however, tends to increase α_s. Thus, the problems of $SO(10)$ are alleviated if $\tan\beta$ is small.

So far we have not specified how the quarks and leptons of the first generation get masses. There are a number of possibilities, all of which require integrating out additional vectorlike quark/lepton representations to get effective higher-dimensional Yukawa operators. One such effective operator is

$$W'=\mathbf{16}_i\mathbf{16}_j\overline{C}^\dagger C'Z_k^\dagger. \tag{18}$$

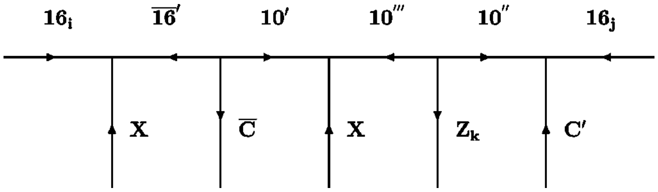

FIG. 3. A diagram that can generate small masses for the first generation quarks and leptons.

This operator can be obtained by integrating out the vectorlike representations $\mathbf{16}'$, $\overline{\mathbf{16}'}$, $\mathbf{10}''$ and $\mathbf{10}'''$, as shown in Fig. 3. This operator contributes only to D and L, and thus explains why $m_u/m_t \ll m_d/m_b, m_e/m_\tau$. The $U(1)\times Z_2\times Z_2$ charges of these additional vectorlike representations can be read off from Fig. 3, using the charges that have already been given. It is straightforward to show that these additional representations do not lead to any destabilization of the gauge hierarchy.

An alternative possibility is the operator $\mathbf{16}_i\mathbf{16}_j T_1 P^{\dagger 2}$, which can be obtained by introducing the fields $\mathbf{16}'$ $(-\frac{1}{2}{}^{+-})$ and $\overline{16}'(\frac{1}{2}{}^{++})$. Again, the addition of these fermions does not destabilize the gauge hierarchy. The subject of suitable higher-order diagrams for the vanishing first and second generation elements of the mass matrices in Eqs. (8)–(10) and Eq. (12) is under investigation, and the results will be reported elsewhere.

We have calculated the effect of the superheavy quarks and leptons on the running of the gauge couplings. Defining $\epsilon_3 \equiv [\alpha_3(M_G)-\tilde{\alpha}_G]/\tilde{\alpha}_G$, as in [17], we find that the quarks and leptons contribute -0.004. Though this is in the right direction to improve the fit to the data, it is too small to be significant as the discrepancy is on the order of 2 or 3% in supersymmetric (SUSY) GUTs [17].

VI. SUMMARY

We have thus been able to show that it is possible to construct a realistic set of mass matrices for the quarks and leptons which makes use of precisely the Higgs fields necessary to solve the doublet-triplet splitting problem in the $SO(10)$ framework: one $\mathbf{45}$ adjoint Higgs field with its VEV pointing in the $B-L$ direction; two pairs of $\mathbf{16}+\overline{\mathbf{16}}$ spinor Higgs fields, one of which gets VEV's at the GUT scale, while the $\mathbf{16}$ of the other develops an electroweak-breaking VEV in the $SU(5)$ $\overline{\mathbf{5}}$ direction; and a pair of $\mathbf{10}$ vector Higgs fields, one of which develops a pair of electroweak-breaking doublets. The $\overline{\mathbf{5}}(\mathbf{16})$ and $\overline{\mathbf{5}}(\mathbf{10})$ mix with the mixing angle possibly serving to achieve a small m_b^0/m_t^0 ratio without necessitating a large $\tan\beta$. Just one pair of vectorlike superheavy fermions in the $\mathbf{16}+\overline{\mathbf{16}}$ spinor and $\mathbf{10}+\mathbf{10}'$ vector representations are required to generate masses for the second and third generations of quarks and leptons. Higher-order radiative corrections will give masses to the first generation fermions and are under study.

An interesting consequence of the incorporation of the Georgi-Jarlskog factor of three in the quark and charged lepton mass matrices is the prediction of sizable $\nu_\mu-\nu_\tau$ mixing in the neutrino sector without the imposition of a special texture for the right-handed Majorana matrix. This has a direct bearing on the large $\mu-\tau$ neutrino mixing observed with atmospheric neutrinos and in future long-baseline experiments.

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