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Soft modes associated with chiral transition at finite temperature

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Using a novel resummation procedure of thermal loops, real-time correlations in the scalar and pseudoscalar channels are studied in the O(4) linear σ model at finite temperature. A threshold enhancement of the spectral function in the scalar channel is shown to be a noticeable precritical phenomenon of the chiral phase transition. [S0556-2821(98)50101-4]

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Observing the restoration of chiral symmetry at finite temperature (T) is one of the central aims in the future relativistic heavy-ion experiments at the BNL Relativistic Heavy Ion Collider (RHIC) and CERN Large-Hadron Collider (LHC) [1]. Numerical studies of quantum chromodynamics (QCD) on the lattice for two and three flavors are actively pursued to reveal the nature of the chiral transition [2].

From the theoretical point of view, one of the ideal signals of the second-order (or weakly first-order) chiral transition is the existence of the soft modes associated with chiral order parameters $\phi(\mathbf{x},t)=(\overline{q}q(\mathbf{x},t),\overline{q}i\gamma_5\vec{\tau}q(\mathbf{x},t))$ [3]. The softening is a *dynamical* phenomenon characterized by the anomalous enhancement of the dynamic susceptibility defined by

$$D_\phi^R(\omega,\mathbf{k})=-i\int d^4x e^{ikx}\theta(t)\langle[\phi(\mathbf{x},t)\phi(0,0)]\rangle_T. \quad (1)$$

This phenomenon at finite T was first discussed in Ref. [4] using an effective theory of QCD. Also there are subsequent theoretical and phenomenological studies on this problem [5]. In condensed matter and solid state physics, the soft modes have been studied extensively in neutron and light scattering experiments [6].

The purposes of this paper are twofold. The first one is to study the spectrums of the soft modes at finite T by taking into account the strong coupling between the scalar and pseudoscalar channels. Despite the fact that this has a considerable effect on the qualitative behavior of D_ϕ^R, it has not been studied seriously so far. The second purpose is to present a systematic resummation procedure of thermal loops which is applicable even when the dynamical symmetry breaking takes place. We will show that this procedure is inevitable for studing the first problem.

To demonstrate the above points, we adopt the O(4) linear σ model with explicit symmetry breaking. The model has been used to study critical exponents of the chiral transition [7] on the basis of the static and dynamical universality. The Lagrangian density reads

$$\mathcal{L}=\frac{1}{2}[(\partial_\mu\phi_i)^2+\mu^2\phi_i^2]-\frac{\lambda}{4!}(\phi_i^2)^2+h\phi_0+\text{counterterms}, \quad (2)$$

with $\phi_i=(\sigma,\vec{\pi})$, and $h\phi_0$ being the explicit symmetry-breaking term. All the divergences are removed by counterterms in the symmetric phase ($\mu^2<0$) in the modified minimal subtraction ($\overline{\text{MS}}$) scheme. In this symmetric and mass independent scheme, the renormalization constants are μ^2 independent [8]. The Lagrangian after dynamical symmetry breaking ($\mu^2>0$) is obtained from Eq. (2) by the replacement $\sigma\to\sigma+\xi$, where ξ is determined by the stationary condition for the effective potential $\partial V(\xi)/\partial\xi=0$.

Causal meson propagators at $T=0$ have a general form $D_\phi(q)=[q^2-m_{0\phi}^2-\Sigma_\phi(q^2)+i\epsilon]^{-1}$, where $m_{0\phi}$ is the tree-level mass, namely $m_{0\sigma}^2=-\mu^2+\lambda\xi^2/2$ and $m_{0\pi}^2=-\mu^2+\lambda\xi^2/6$. The one-loop calculation of the self-energy Σ_ϕ is standard [8] and we do not recapitulate it here.

The renormalized couplings μ^2, λ, and h are determined by the following renormalization conditions: (i) on-shell condition for the pion, $D_\pi^{-1}(q^2=m_\pi^2)=0$ with $m_\pi=140$ MeV, (ii) partial conservation of axial-vector current (PCAC) in one-loop order, $f_\pi m_\pi^2=h\sqrt{Z_\pi}$ with Z_π being the wave function renormalization constant for pion and $f_\pi=93$ MeV, (iii) the peak position of the spectral function $\rho_\sigma=-(1/\pi)\text{Im}\,D_\sigma$ is chosen to be 550 MeV; this number corresponds to the value obtained in the recent reanalyses of the π-π scattering phase shift [9]. We have also taken 750 MeV and 1 GeV [4], and checked that our main conclusions do not suffer qualitative change. The arbitrary renormalization point κ is chosen so that $\rho_\sigma(\omega)$ starts from the correct continuum threshold at $\omega=2m_\pi$: This is achieved by demanding $m_{0\pi}=m_\pi$. The resultant values read $\mu^2=(283\text{ MeV})^2$, $\lambda=73.0$, $h=(123\text{ MeV})^3$, and $\kappa=255$ MeV.

As has been known since the works of Weinberg, and Kirzhnits and Linde [10], naive perturbation theory breaks down for sufficiently high T. In the linear σ model, this is easily seen from the behavior of $m_{0\phi}$ defined above which becomes tachyonic as one approaches the symmetric phase ($\xi\to0$). This not only destroys the credibility of the loop expansion at high T, but also leads to an unphysical threshold in $\rho_\sigma(\omega)$ even at low T. The latter aspect is particularly harmful for the purpose of this paper.

To cure these problems, we developed a method based on a resummation procedure proposed by Banerjee and Mallik [11]. It is a generalized mean-field theory and allows one to carry out systematic perturbation theory and renormalization at finite T [12]. In high T limit, the method reduces to the resummation of hard thermal loops [13]. We will report the technical details elsewhere [14], and recapitulate here only the essential parts needed for subsequent discussions.

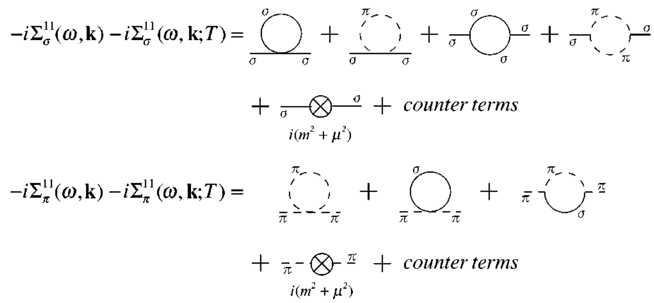

FIG. 1. One-loop self-energy Σ^{11} for σ and π in the modified loop-expansion at finite T.

Let us rewrite Eq. (2) by adding and subtracting a chiral invariant mass term with an arbitrary mass m;

$$\mathcal{L}=\frac{1}{2}[(\partial_\mu\phi_i)^2-m^2\phi_i^2]-\frac{\lambda}{4!}(\phi_i^2)^2+h\phi_0$$
$$+\frac{1}{2}(A-1)(\partial_\mu\phi_i)^2-\frac{1}{2}Bm^2\phi_i^2-\frac{\lambda}{4!}C(\phi_i^2)^2$$
$$+\frac{1}{2}(m^2+\mu^2)(1+B)\phi_i^2+h(\sqrt{A}-1)\phi_0. \qquad (3)$$

A, B, and C are renormalization constants: In one-loop in the $\overline{\rm MS}$ scheme, $A=1$, $B=\lambda/16\pi^2\bar{\epsilon}$, and $C=\lambda/8\pi^2\bar{\epsilon}$. The loop expansion at finite T can be done in the same way as that at $T=0$ except that (i) m^2 should be used instead of $-\mu^2$, (ii) a new vertex proportional to $m^2+\mu^2$ should be considered as one-loop order, and (iii) the term proportional to $(m^2+\mu^2)B$ should be considered from the two-loop order. m^2 is a T dependent mass parameter to be determined later by the dynamics.

At finite T, retarded meson propagators read

$$D^R_\phi(\omega,\mathbf{k})=[k^2-m^2_{0\phi}(T)-\Sigma^R_\phi(\omega,\mathbf{k};T)]^{-1}, \qquad (4)$$

where $k^2=\omega^2-\mathbf{k}^2$ and

$$m^2_{0\pi}(T)=m^2+\frac{\lambda\xi^2}{2},\quad m^2_{0\sigma}(T)=m^2+\frac{\lambda\xi^2}{6}. \qquad (5)$$

The self-energy Σ^R_ϕ is obtained either from the imaginary-time or the real-time formalism. We adopt the latter in which $\mathrm{Re}\,\Sigma^R_\phi(\omega,\mathbf{k};T)=\mathrm{Re}\{\Sigma^{11}_\phi(\omega,\mathbf{k})+\Sigma^{11}_\phi(\omega,\mathbf{k};T)\}$ and

$$\mathrm{Im}\,\Sigma^R_\phi(\omega,\mathbf{k};T)=\tanh(\omega/2T)\mathrm{Im}\{\Sigma^{11}_\phi(\omega,\mathbf{k})+\Sigma^{11}_\phi(\omega,\mathbf{k};T)\}.$$

Here $\Sigma^{11}_\phi(\omega,\mathbf{k})(\Sigma^{11}_\phi(\omega,\mathbf{k};T))$ is a T-independent (T-dependent) part of the 11-component of the 2×2 self-energy in the real-time formalism [15]. Associated diagrams in one-loop with our modified loop-expansion are shown in Fig. 1.

m^2 is a fictitious parameter and physical quantities should not depend on it. This leads us to several procedures to choose optimal m^2 in perturbation theory, such as the principle of minimal sensitivity (PMS), the fastest apparent convergence (FAC) and so on [16]. We find that a variance of FAC is most suited for the purpose of this paper: A condition for making the loop-correction to the real part of the pion propagator as small as possible reads

$$[\omega^2-m^2_{0\pi}-\mathrm{Re}\{\Sigma^{11}_\pi(\omega,0)+\Sigma^{11}_\pi(0,0;T)\}]_{\omega=m_{0\pi}}=0. \quad (6)$$

ω in $\Sigma^{11}_\pi(\omega,0;T)$ is chosen to be zero by a technical reason for getting a continuous solution of m^2 as a function of T [14]. Our procedure naturally gives $m^2\to-\mu^2$ as $T\to0$ and $m^2\to\lambda T^2/12$ as $T\to\infty$. The former guarantees that the loop-expansion at $T=0$ is not spoiled, and the latter guarantees that thermal tadpole diagrams are resummed correctly.

In Fig. 2(A), the chiral condensate $\xi(T)$ obtained by minimizing the effective potential is shown for $m_\pi(T=0)=140$ MeV as well as for $m_\pi(T=0)=50$ MeV. The latter corresponds to a fairly small quark mass compared to the physical value $m_q^{\rm phys}$: $m_q/m_q^{\rm phys}\simeq(50\ \mathrm{MeV}/140\ \mathrm{MeV})^2=0.13$ [17].

In Fig. 2(B), $m_{0\pi}(T)$, $m_{0\sigma}(T)$, and $m^2(T)$ are shown. Because of our resummation procedure, *tree-level masses* $m_{0\pi}(T)$ and $m_{0\sigma}(T)$ do not show tachyonic behavior and both approach to the classical plasma limit $\lambda T^2/12$ at high T. This behavior is important to have physical continuum

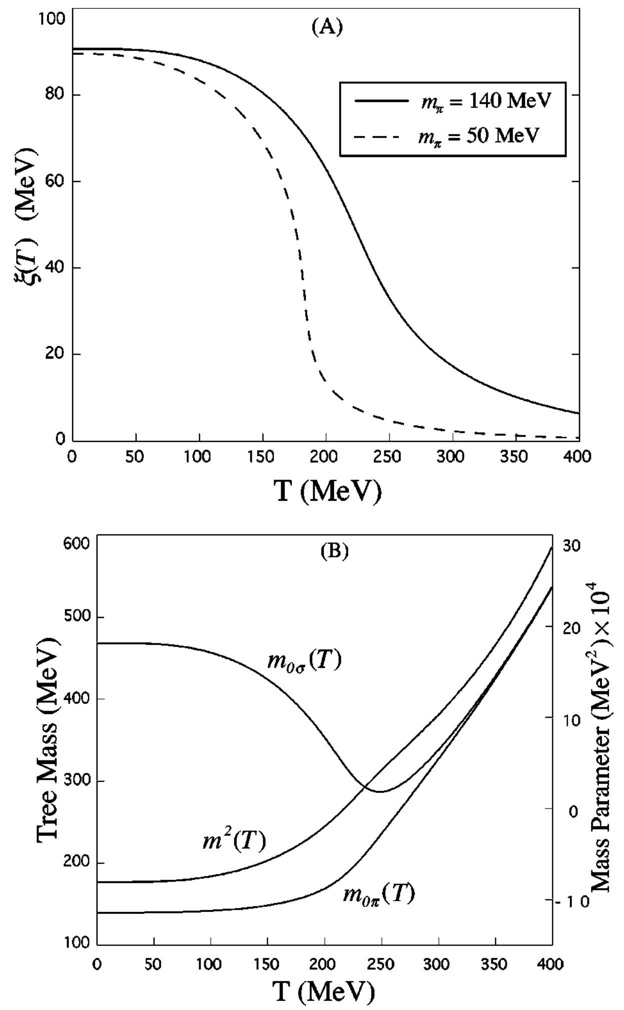

FIG. 2. (A) $\xi(T)$ for $m_\pi=140$ MeV and 50 MeV. (B) Masses in the tree-level $m_{0\pi}(T)$ and $m_{0\sigma}(T)$ shown with left vertical scale, and the mass parameter $m^2(T)$ with right vertical scale.

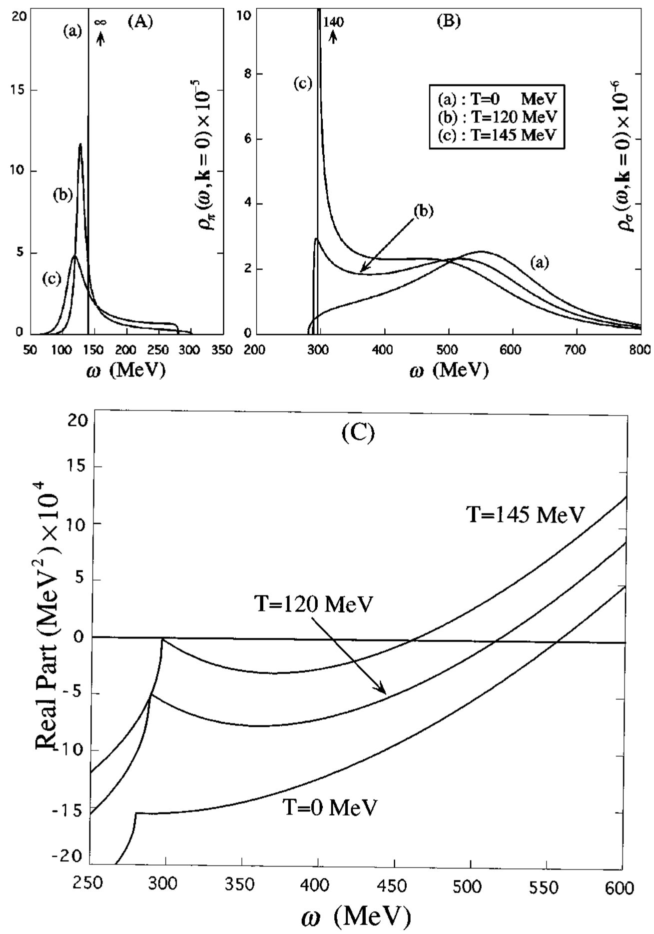

FIG. 3. Spectral function in π channel (A) and in σ channel (B) for $T=0$, 120, 145 MeV. The real part of $(D_\sigma^R(\omega,0;T))^{-1}$ as a function of ω is shown in (C).

threshold in the spectral functions, because the threshold is dictated by the particle masses running inside the loops (see Fig. 1).

Let us now turn to the discussion on the spectral functions (imaginary part of the dynamical susceptibility) defined by $\rho_\phi = -(1/\pi)\mathrm{Im}\, D_\phi^R$;

$$\rho_\phi(\omega,\mathbf{k}) = -\frac{1}{\pi}\frac{\mathrm{Im}\,\Sigma_\phi^R}{(k^2-m_{0\phi}^2-\mathrm{Re}\,\Sigma_\phi^R)^2+(\mathrm{Im}\,\Sigma_\phi^R)^2}. \quad (7)$$

In Fig. 3(A) and (B), $\rho_\phi(\omega,\mathbf{k}=0)$ is shown in π and σ channels for $T=0$, 120, 145 MeV. In the π channel at $T=0$, there is a distinct pion pole as well as a continuum starting from $m_{0\pi}+m_{0\sigma}\simeq 690$ MeV. In the σ channel at $T=0$, continuum starts at $2m_{0\pi}=280$ MeV. Also, there is a broad peak centered at $\omega=550$ MeV with a total width of 260 MeV. This corresponds to a σ-pole located far from the real axis in the complex ω plane. The large width is due to a strong coupling of σ with 2π in the linear σ model. If we choose parameters so that the peak position of σ are 750 MeV (1 GeV), corresponding width reads 657 MeV (995 MeV).

In the π-channel for $T\neq 0$, a continuum arises in $0<\omega<m_{0\sigma}-m_{0\pi}$. This is caused by the induced ''decay'' through the process $\pi+\pi\rightarrow\sigma$. Besides this, the pion still has its quasiparticle feature with no appreciable modification of the mass. This is in accordance with the Nambu-Goldstone nature of the pion.

In the σ-channel for $0<T<145$ MeV, there are two noticeable modifications of the spectral function. One is the shift of the σ-peak toward the low mass region. The other is the sharpening of the spectral function just above the continuum threshold starting from $\omega=2m_{0\pi}(T)$.

These features are actually related with each other and can be understood in the following way. Because of the partial restoration of chiral symmetry at finite T together with the strong σ-2π coupling, the real part of $(D_\sigma^R)^{-1}$ [which appears in the first term of the denominator of Eq. (7)] approaches zero at $\omega=2m_{0\pi}$ for $T\sim 145$ MeV as shown in Fig. 3(C). (The cusp structure in this figure originates from the pion-loop contribution to $\mathrm{Re}\,\Sigma_\sigma^R$.) In this situation, the spectral function (7) is almost *inversely* proportional to $\mathrm{Im}\,\Sigma_\sigma^R$; namely $\rho_\sigma(\omega\sim 2m_{0\pi})\propto 1/\{\epsilon^2/\sqrt{1-(2m_{0\pi}/\omega)^2}+\sqrt{1-(2m_{0\pi}/\omega)^2}\}$. Here ϵ is an infinitesimal quantity proportional to $\mathrm{Re}(D_\sigma^R)^{-1}$ and $\sqrt{1-(2m_{0\pi}/\omega)^2}$ originates from the phase space factor in $\mathrm{Im}\,\Sigma_\sigma^R$. This shows that a large enhancement of ρ_σ occurs just above the threshold.

In other words, the threshold enhancement in Fig. 3(B), although it occurs at relatively low T, is caused by a combined effect of the partial restoration of chiral symmetry and the strong σ-2π coupling. Note also that, near the chiral limit, the continuum threshold $2m_{0\pi}$ approaches zero and the threshold enhancement occurs at the critical temperature where the chiral transition takes place.

The spectral functions of π (σ) for $T>165(145)$ MeV behave in a standard way as expected from the previous analyses [4,5]: Simple σ and π poles without width appear, because the decay ($\sigma\rightarrow 2\pi$) and induced decay ($\pi+\pi\rightarrow\sigma$) are forbidden kinematically. Also, the continuum part does not have strong threshold enhancement anymore. As T increases, these simple poles gradually merge into a degenerate (chiral symmetric) state. For sufficiently high T, the system is supposed to be in the deconfined phase and the decay $(\sigma,\pi)\rightarrow q\bar{q}$ starts to occur. This is not taken into account in the present linear σ model. A calculation based on the Nambu-Jona-Lasinio model shows, however, that there is still a chance for collective modes to survive as far as T/T_c is not so far from unity [4].

In summary, we have studied the spectral function of the soft modes associated with chiral transition on the basis of a special resummation method at finite T. An enhancement of the continuum threshold in the scalar channel are shown as a typical signal of the partial restoration of chiral symmetry. Detectability of this phenomenon in experiments through the decays such as $\sigma\rightarrow 2\pi, 2\gamma, e^+e^-$ remains as an interesting future problem [18].

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[17] Within the one-loop approximation of our modified expansion scheme, $\xi(T)$ shows a weak first-order transition below $m_q/m_q^{\rm phys}<0.08$. It is not clear whether this still persists in higher orders.
[18] We have analyzed a diphoton spectrum from the hot plasma of π and σ. We found that threshold enhancement from the process $\sigma\rightarrow 2\gamma$ over the main background ($\pi^+\pi^-\rightarrow 2\gamma$) can be seen only in a very narrow region of T and $M_{\gamma\gamma}$ (diphoton invariant mass).