

Light-front partons and dimensional reduction in relativistic field theory

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We present a simple discussion of the appearance of light-front partons in local field theory. The description in terms of partons provides a dimensional reduction which relates a (2+1)- with a (3+1)-dimensional theory, for example. The possibility for the existence of Lorentz symmetry and a connection to the relativistic membrane is described. It is shown how the reconstruction of the full relativistic field theory is possible with a proper treatment of the parton configuration space. Compared with the case of identical particles, this involves keeping configurations, where the partons are at the same points. [S0556-2821(98)50110-5]

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It was originally suggested by 't Hooft [1] that in order to reconcile gravity with quantum mechanics, there should be a reduction of degrees of freedom from (3+1) to (2+1) dimensions. This reduction, operating as a holographic mapping should allow for an inverse namely a recovery of the 3D data from 2D. 't Hooft presented a model based on cellular automata as an implementation of this idea. One of the main challenges is the demonstration of Lorentz invariance in 3+1D. In several papers Susskind emphasized [2] the use of light-front quantization and partons for obtaining the holographic projection. Indeed a string theory in the light cone frame can be formulated [3,4] without the explicit use of a longitudinal dimension. In general one has the basic idea of light-front partons which would serve as most fundamental degrees of freedom. This would, among other things, significantly improve the ultraviolet properties of the theory and allow a consistent microscopic understanding of the physics of black holes. The recently introduced M(atr)ix models belong to the above category of theories [5,6].

In this note we point out that a similar mechanism could be operational already in local field theory. We present a simple discussion of the appearance of light-front partons and the corresponding dimensional reduction in the example of a relativistic scalar field. Our discussion will be at classical level with the interactions ignored. This will be sufficient for our purpose of presenting the basic ideas. We are then able to describe a mechanism for recovering the full theory from the lower dimensional (parton) picture.

The initial idea is to consider a relativistic field in the light-front frame and after compactifying the longitudinal direction x^- truncate the fields by keeping only oscillators with the lowest value of p^+ . These would represent the basic partonic degrees of freedom and the question is if it is possible to build the full theory in terms of them. We first show that even though this represents a seemingly huge truncation, one still has the possibility of a (3+1). Lorentz symmetry in the emerging (2+1)-dimensional theory. We give a set of generators closing a 4D Poincare algebra. These are closely related to generators of the 4D relativistic membrane and we explain in some detail this connection. We then proceed to our main topics and study the N -body problem of free partons. We show that using appropriately defined (collective) fields, the full 4D relativistic scalar field theory is reconstructed. This reconstruction is based on a special treatment

of the N -parton configuration space. As opposed to the standard case of identical particles, we include in this space configurations of coinciding partons. This comes in analogy with a similar treatment of light cone strings. The fact that a relativistic field can be represented in terms of nonrelativistic partons and described in lower dimensional terms is likely to give us new insight into the properties of local field theory.

Consider a relativistic scalar field theory $\varphi(x_\mu)$ in (3+1) dimensions:

$$\mathcal{L} = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi).$$

On the light-front plane $x^+ = \text{const}$, [$x^\pm = (1/\sqrt{2})(x^0 \pm x^3)$], one has the Dirac brackets

$$\{\varphi(x^-, x), \varphi(y^-, y)\} = (\partial_-)^{-1} \delta(x^- - y^-) \delta(x - y).$$

Here and in what follows $x = (x^1, x^2)$ stands for all the transverse directions. The Hamiltonian and the longitudinal momentum (generating translation in x^+ and x^- , respectively) read

$$H \equiv P^- = \int dx^- d^2x \left\{ \frac{1}{2} \nabla \varphi \nabla \varphi + V(\varphi) \right\}, \tag{1}$$

$$P^+ = \int dx^- d^2x \partial_- \varphi \partial_- \varphi.$$

When we compactify the x^- direction for convenience with a radius $R = 2\pi$, the conjugate momenta $p^+ = 2\pi n/R$ are given by positive integers. Expanding in the p^+ modes,

$$\begin{aligned} \varphi(x^-, x) = & \sum_{n>0} \frac{1}{\sqrt{4\pi p_n^+}} [A_n(x) e^{-ip_n^+ x^-} + A_n^+(x) e^{ip_n^+ x^-}] \\ & + \frac{1}{\sqrt{2\pi}} \tilde{\phi}(0, x). \end{aligned} \tag{2}$$

In order to obtain something like the parton picture with fundamental degrees of freedom, we now truncate the system to just the $p^+ = 1$ mode (we ignore all the other modes including the zero mode):

$$\varphi(x^-,x) \approx \frac{1}{\sqrt{4\pi}} [A_1(x)e^{-ix^-} + A_1^+(x)e^{ix^-}].$$

The Hamiltonian and the longitudinal momentum then reduce to

$$\begin{aligned} H &= \int d^2x [\frac{1}{2} \nabla A^+(x) \nabla A + V], \\ P^+ &= \int d^2x A^+(x) A(x), \end{aligned} \quad (3)$$

where we have dropped the index ($p^+=1$) from our creation-annihilation operators. In general, it is expected that if one interprets the reduction as integrating out degrees of freedom that one would have changes to the interaction term. The creation-annihilation operators obey the Poisson brackets of a nonrelativistic second quantized Schrodinger field,

$$\{A(x), A^+(x')\} = \delta^{(2)}(x-x'), \quad (4)$$

and give the density function $\rho(x) \equiv A^+A$. We see that the longitudinal momentum is the integral of the density and equal to the number operator:

$$P^+ = \int d^2x \rho(x) = \hat{N}.$$

Changing to the density and phase variables,

$$\begin{aligned} A(x) &= \sqrt{\rho} e^{i\pi}, \\ A^+(x) &= \sqrt{\rho} e^{-i\pi}, \end{aligned}$$

we write the Hamiltonian as

$$H = \int d^2x [\frac{1}{2} \rho \nabla \pi \nabla \pi + V'(\rho)],$$

where an additional ρ -dependent term was added to the potential energy. As we have mentioned, we will ignore interactions and \hbar -quantum effects in what follows.

To conclude, by truncating the relativistic field $\varphi(x^-,x)$ to just the $p^+=1$ mode, we have a nonrelativistic type theory in (2+1) dimensions with $\rho(x)$ and $\pi(x)$ as conjugate fields. These are the density fields of partons.

We will now point out that there can exist a 4D Lorentz symmetry still operating in this reduced theory. It is given by the following set of generators:

$$\begin{aligned} J^{ab} &= \int d^2x \pi (x^a \partial_b - x^b \partial_a) \rho \quad a,b=1,2, \\ J^{a-} &= \int d^2x \frac{1}{2} \{x^a \rho (\nabla \pi)^2 - \rho \partial_a (\pi^2)\}, \\ J^{a+} &= \int d^2x x^a \rho, \\ J^{+-} &= - \int d^2x \rho \pi. \end{aligned}$$

These together with

$$\begin{aligned} P^- &= \int d^2x \frac{1}{2} \rho (\nabla \pi)^2, \\ P^+ &= \int d^2x \rho, \\ P^a &= \int d^2x \pi \partial_a \rho, \end{aligned}$$

can be seen to close the (3+1)-dimensional Poincare algebra.

This algebra is directly related to the Poincare algebra of the 4D membrane ($X^\mu(\tau, \sigma_1, \sigma_2)$; $\mu=0,1,2,3$) given by Bordemann and Hoppe in [7]. In the light cone gauge after fixing the area preserving diffeomorphisms, there remains a scalar field $(q(x), p(x))$ and also the zero modes (x_0^-, p_0^+) corresponding to the $X^-(\sigma)$ and $P^+(\sigma)$ coordinates:

$$\begin{aligned} \{q(x), p(x')\} &= \delta^{(2)}(x-x'), \\ \{x_0^-, p_0^+\} &= -1. \end{aligned}$$

The full set of Poincaré generators constructed in [7] is in terms of these sets of variables. While our fields $(\rho(x), \pi(x))$ are analogous to the continuous variables $(q(x), p(x))$ of the membrane, we do not have any discrete zero modes (x_0^-, p_0^+) . It is somewhat puzzling that a Lorentz symmetry still exists even without the longitudinal zero modes. This can be explained as follows.

The Hamiltonian in the case of the membrane reads

$$P^- = \frac{1}{2p_0^+} \int d^2x q \left((\nabla \pi)^2 + \frac{1}{q^2} \right).$$

The other generators and especially the most relevant longitudinal boost operator J^{a-} involve in a nontrivial way both the discrete (x_0^-, p_0^+) and the continuum (q, p) degrees of freedom. One notices, however, a scaling a symmetry in the membrane Hamiltonian,

$$\begin{aligned} q &\rightarrow \lambda q, \quad p \rightarrow \lambda^{-1} p, \\ x_0^- &\rightarrow \lambda x_0^-, \quad p_0^+ \rightarrow \lambda^{-1} p_0^+, \end{aligned}$$

generated by

$$Q_s = p_0^+ x_0^- - \int q p d^2x.$$

Using this symmetry, one can perform a Hamiltonian reduction¹ setting

$$Q_s = 0 \quad \text{and} \quad p_0^+ = 1.$$

This results in elimination of the zero mode degrees of freedom leaving just the 2D field $(q(x), p(x))$ which is identified with our $(\rho(x), \pi(x))$. Incidentally, this reduction also explains the sometimes mysterious identification

¹This observation is due to Jean Avan.

$$P^+ = \int d^2x \rho = N$$

made in M(atr)ix theory.

We would now like to demonstrate that it is possible to recover the full (relativistic) field theory from the N -body system of partons. Consider the kinetic energy

$$H = \sum_{i=1}^N \frac{1}{2} p_i^2,$$

where the coordinates are in $d = D - 2$ dimensions

$$x_i = (x_i^1; x_i^2, \dots, x_i^d).$$

The time conjugate to the Hamiltonian is the null-front time $x^+ = x^0 + x^{d+1}$ and consequently $H = P^-$. The total longitudinal momentum is

$$P^+ = \sum_{i=1}^N 1 = N.$$

We will now define a sequence of collective fields $\rho_n(x)$ on the coordinate space

$$X^d \times X^d \times \dots \times X^d / S_N$$

by treating this space as an orbifold² in parallel with the case of strings and the string light cone N -body dynamics as given in [8,9].

The idea is to treat in a special way the configurations, when locations of partons coincide. It is well known that in the standard treatment of identical particles [11] one excludes from the configuration space the points, when the particles are at the same location. In the present orbifold logic, these points are included and reinterpreted as different observables. This then allows us to construct a sequence of (collective) density fields as follows. First

$$\rho_1(x) = \sum_i' \delta(x - x_i)$$

with the prime denoting the fact that in the sum we only have *noncoincidental* coordinates. To define this we can simply introduce an infinitesimal distance ϵ and define a minimal separation by $|x_{i'} - x_{i''}| > \epsilon$. In turn, if two coordinates are within ϵ , we will treat this as a composite object with $p^+ = 2$. Consequently, all pairs of coordinates such that

$$|x_{i'} - x_{i''}| < \epsilon$$

will be taken to contribute to

$$\rho_2(x) = \sum_{(i', i'')} \frac{1}{2} [\delta(x - x_{i'}) + \delta(x - x_{i''})].$$

Generally,

$$\rho_n(x) = \sum_{(i_1, i_2, \dots, i_n)} \frac{1}{n} \left(\sum_{k=1}^n \delta(x - x_{i_k}) \right)$$

which receives a contribution only if some n - of the particles are at the same point. This defines the $p^+ = n$ density field. The sequence of densities so constructed is to be identified directly with the creation-annihilation operators in the expansion of the relativistic field $\varphi(x^-, x)$. More precisely,

$$\rho_n(x) = A_n^+(x) A_n(x)$$

and we have correctly reconstructed the total longitudinal momentum operator

$$P^+ = \sum_{n>0} n \int \rho_n(x) dx = \sum_{n>0} n \int A_n^+ A_n dx^d.$$

What is less trivial is the Hamiltonian. Here we use the methods of [12] and write

$$\begin{aligned} \sum_{i=1}^N \frac{\partial^2}{\partial x_i^2} &= \sum_n \sum_i \int dx dy \frac{\partial \rho_n(x)}{\partial x_i} \frac{\partial \rho_n(y)}{\partial x_i} \frac{\partial}{\partial \rho_n(x)} \frac{\partial}{\partial \rho_n(y)} \\ &+ \frac{\partial^2 \rho_n}{\partial x_i^2} \frac{\partial}{\partial \rho_n}. \end{aligned}$$

The second term is known to lead to potential type contributions so let us just follow the first kinetic term (for details see [12]). A simple calculation gives

$$\sum_i p_i^2 \rightarrow \int d^2x \sum_n \frac{1}{n} \rho_n(x) \nabla \pi_n(x) \nabla \pi_n(x) + \dots$$

so that with

$$A_n = \sqrt{\rho_n} e^{i\pi_n}, \quad A_n^+ = \sqrt{\rho_n} e^{-i\pi_n},$$

we have

$$\begin{aligned} P^- &= \sum \frac{1}{2} p_i^2 \rightarrow \int d^d x \sum_n \frac{1}{2n} \nabla A_n^+ \nabla A_n \\ &= \int dx^- dx^d \nabla \varphi \nabla \varphi. \end{aligned}$$

We have shown that the relativistic field theory expression is recovered from the nonrelativistic parton picture.

To summarize the basic point of this treatment rests on the question of statistics obeyed by the so called parton coordinates. For a usual system of identical particles, we exclude from the coordinate space configurations, where the particles coincide, indeed this assumption is crucial for the existence of anyon statistics in 2D for example. If for the light front partons, we decide to keep the configurations with coinciding points, these configurations then carry additional information: that is the origin for a sequence of density fields of growing p^+ which we have built to reconstruct the single relativistic field. If on the other hand, partons obey some standard statistics, this construction would not be applicable.

²After completion of this work, we have learned of a similar orbifold treatment of the $N=1$ instanton moduli space by J. Brodie and S. Ramgoolam. We have also learned of the work of Aharony, Berkooz, and Seiberg [10].

The present picture of local fields is now in line with that of (light-cone) strings. One hopes that this can play a role in reconciling the loop properties of point particles and strings. One of the basic properties of closed string loops is modular invariance a symmetry which has no counterpart in field theory. We have presented our construction with little attention to interactions and one would clearly like to have a full interacting theory at hand. The Lorentz generators of the

membrane allow an additional singular interaction, this would represent a fine tuning or possibly a critical point in the relativistic scalar case. A much more interesting example which we would like to bring up is the theory of self-dual gravity which is known to possess certain large N composite structure [13].

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