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Nonuniqueness of the third post-Newtonian binary point-mass dynamics

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It is shown that the recently found nonuniqueness of the third post-Newtonian binary point-mass Arnowitt-Deser-Misner (ADM) Hamiltonian is related to the nonuniqueness at the third post-Newtonian approximation of the applied ADM-coordinate conditions. [S0556-2821(98)50310-4]

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In a recent paper [1] the authors reported on the nonuniqueness of the 3rd post-Newtonian Arnowitt-Deser-Misner (ADM) Hamiltonian for binary point-mass systems. The term in the Hamiltonian which came out to be ambiguous, in the center-of-mass reference frame, is given by [see Eqs. (71) and (75) in [1]]

$$\omega\frac{G^3m_1m_2}{c^6}p_{1i}p_{1j}\partial_{1i}\partial_{1j}\left(-\frac{1}{r_{12}}\right). \tag{1}$$

In Eq. (1) m_1 and m_2 denote the masses of the bodies 1 and 2, respectively, $r_{12}=|\mathbf{x}_1-\mathbf{x}_2|$ is their relative coordinate distance, where $\mathbf{x}_a$ $(a=1,2)$ denotes the position of the ath body. For the momenta of the bodies $p_{1i}=-p_{2i}$ holds ($i=1,2,3$); ∂_{ai} denotes the partial derivative with respect to x_a^i. G and c are the Newtonian gravitational constant and the speed of light, respectively. The ambiguity in the Hamiltonian is expressed by an unspecified finite number ω.

In our treatment we applied the following generalized isotropic ADM-coordinate conditions [see Eqs. (7-4.22) and (7-4.23) in [2], respectively, Eqs. (3) and (4) in [1]]:

$$g_{ij}=\left(1+\frac{1}{8}\phi\right)^4\delta_{ij}+h_{ij}^{TT}, \tag{2}$$

$$\pi^{ii}=0, \tag{3}$$

where g_{ij} denotes the 3-metric and π^{ij} the field momentum (canonical conjugate to g_{ij}), h_{ij}^{TT} is the transverse-traceless (with respect to the flat-space metric) part of $g_{ij}-\delta_{ij}$. The form of the isotropic part of Eq. (2) stems from the Schwarzschild metric in isotropic coordinates.

To leading order in powers of $1/c$ (this is enough for the following) the coordinate condition (3) reads ($x^0=ct$)

$$2\partial_i g_{0i}-\partial_0 g_{ii}=0. \tag{4}$$

Note that the coordinate conditions (2) can exactly be written as

$$3\partial_j g_{ij}-\partial_i g_{jj}=0. \tag{5}$$

Let us define the following infinitesimal coordinate transformation

$$x'^{\mu}=x^{\mu}+\epsilon^{\mu},\quad \epsilon^0=0,\quad \epsilon^i=\alpha\frac{G^3m_1^2m_2^2}{2c^6M}\partial_i(r_1^{-1}+r_2^{-1}), \tag{6}$$

where $M=m_1+m_2$ denotes the total mass of the system, $r_a=|\mathbf{x}-\mathbf{x}_a|$, and α is a pure number. This transformation induces the following leading order in powers of G transformation in the metric coefficients, keeping the independent variables fixed [see, e.g., Eq. (16) in [3]]:

$$g'_{\mu\nu}(t,x^i,x_a^i(t),p_{ai}(t))=g_{\mu\nu}(t,x^i,x_a^i(t),p_{ai}(t))-g_{\mu\lambda}\partial_\nu\epsilon^\lambda-g_{\nu\lambda}\partial_\mu\epsilon^\lambda. \tag{7}$$

Hereof the transformation of the Hamiltonian results, plugging the expression (7) into the test-mass Hamiltonian [see, e.g., Eq. (5.2) in [4]] and identifying in turn the test mass with the source masses. This gives

$$H'=H+\alpha\frac{G^3m_1m_2}{c^6}p_{1i}p_{1j}\partial_{1i}\partial_{1j}\left(\frac{1}{r_{12}}\right). \tag{8}$$

Outside the mass points, the transformation (6) keeps invariant the Eqs. (2) and (3), respectively, Eqs. (5) and (4). Towards spacelike infinity, the perturbation (6) dies out very fast, like $1/r^2$, implying an $1/r^3$ decay for the metric perturbation (7).

The shift in Eq. (8) is identical with Eq. (1). This shows that the dynamical ambiguity found in [1] is related to the ambiguity in the coordinate system (in quantum field theory

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those gauge ambiguities are well known and result in the DeWitt-Faddeev-Popov ghost fields). A corresponding ambiguity is likely to exist also in harmonic coordinates as one can infer from [5], p. 120. For extended bodies, neither the coordinate-system ambiguity arises nor is the dynamical ambiguity present.

Note added in proof. The transformation (8) of the Hamiltonian is not a gauge transformation because of the uninvolved and dropped terms in Eq. (7) of the order G^4/c^6.

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