

Soft dynamics and gauge theories

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Infrared divergences obscure the underlying soft dynamics in gauge theories. They remove the pole structures associated with particle propagation in the various Green's functions of gauge theories. Here we present a solution to this problem. We give two equations which describe how charged particles must be dressed by gauge degrees of freedom. One follows from gauge invariance, the other, which is new, from velocity superselection rules familiar from the heavy quark effective theory. The solution to these equations in the Abelian theory is proven to lead to on-shell Green's functions that are free of soft divergences at all orders in perturbation theory. [S0556-2821(98)50108-7]

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A widespread belief in particle physics is that the relativistic concept of a *charged* particle does not exist due to the infrared (IR) structure of gauge theories (see, for example, the conclusions to Ref. [1]). This point of view is based on the observation [2] that the Green's functions for the charged matter fields do not have a simple polelike structure, but rather a branch cut, thus signaling the need for a form of asymptotic dynamics more complicated than that of the free theory [1]. The interpretation of this result is that in QED a soft photon cloud always surrounds each physical charge (see the discussion on page 524 of Ref. [3]). Although there is now a well-developed industry which, at the level of cross sections, avoids most of the problems associated with our lack of knowledge of the form of this asymptotic dressing [4], even in QED it is still not completely understood how to circumvent these difficulties [5–7]. The situation in non-Abelian theories is much more complicated as the form of the asymptotic dynamics is poorly understood [8].

Here we will show that a relativistic particle description of charged matter is, in fact, possible. Our starting point is the observation that one cannot talk about a physical charge without including some form of gauge dressing, and it is this *combined system* that must be identified with a charged particle. To this end, we present two equations which determine how gauge degrees of freedom, such as glue, surround charged matter, such as valence quarks. We solve these equations for the Abelian theory and demonstrate that the on-shell Green's functions of the solutions are free of soft divergences and have a pole structure at all orders in perturbation theory. This is the first formulation of QED with this important property. We conclude with a discussion of the solutions in QCD.

In a gauge theory physical fields must be gauge invariant and, in particular, charged matter fields [9] are identified

with products of the form $h^{-1}(x)\phi(x)$ where the dressing, $h^{-1}(x)$, which surrounds the charged matter, $\phi(x)$, transforms as

$$h^{-1}(x) \rightarrow h^{-1}(x)U(x) \quad (1)$$

under the gauge transformation described by the group element $U(x)$. Since ϕ transforms as $\phi(x) \rightarrow U^{-1}(x)\phi(x)$, the product is gauge invariant. The explicit x dependence in the charged matter fields does not imply that this is an operator creating a charge at the spacetime point x . Rather, as discussed in Ref. [9] such fields must necessarily be nonlocal and the x dependence is simply labeling the matter field core to the charged field and thus the source of the dressing. For a particle description of the charged field, we need to investigate whether there is a choice of dressing for which the field has a well-defined momentum.

We will say that the charged field $h^{-1}(x)\phi(x)$ has a sharp momentum p^μ if the state created by it is a (generalized) eigenstate of the momentum operator $\hat{P}^\mu = i\partial^\mu$, with eigenvalue p^μ . Thus the sharpness condition is $\partial^\mu(h^{-1}(x)\phi(x)) = -ip^\mu h^{-1}(x)\phi(x)$. In terms of the four velocity, u^μ , we can write this sharpness condition as

$$u \cdot \partial(h^{-1}(x)\tilde{\phi}(x)) = 0, \quad (2)$$

where $\tilde{\phi}(x) = e^{imu \cdot x}\phi(x)$, which is familiar from heavy quark effective theory.

We now need to find such sharp charged states which we can then identify, in an appropriate asymptotic regime, with the in and out charged particle states of the theory. For massive charged matter (which is the only case we will consider in this paper), the heavy matter sector (or equivalently the soft gauge sector) is the region where the velocity becomes a superselection rule [10] and a particle description can emerge. In this sector the matter fields satisfy the equation

$$u \cdot D(\tilde{\phi}(x)) = 0. \quad (3)$$

[In this expression the covariant derivative is taken to be $D_\mu = \partial_\mu + gA_\mu$, where A_μ is the Lie algebra valued (anti-

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Hermitian) gauge potential.] It then follows that the dressed charge has a sharp momentum in this heavy sector if the dressing satisfies the equation

$$u \cdot \partial(h^{-1}(x)) = gh^{-1}(x)u \cdot A(x). \quad (4)$$

Equations (1) and (4) are the fundamental equations for determining the dressing and hence the soft dynamics of charged particles in gauge theories. The first of these is a minimal demand: physical charges are gauge invariant. Previous work on these various solutions, in the framework of the standard model, was summarized and extended in Ref. [9] (see also Refs. [11–15]). The second equation is a new, kinematical requirement which removes the ambiguity associated with the plethora of solutions to Eq. (1). We now note that in QED we can explicitly solve *both* of these equations. The resulting dressing can be shown to factor into the product of two terms:

$$h^{-1}(x) = e^{-ieK(x)}e^{-ie\chi(x)}. \quad (5)$$

The K -dependent factor is gauge invariant and describes the response of the charge to the other charges in the system. The second term in the dressing represents the soft core of the dressing and its form was essentially guessed in Ref. [9] through an analysis of the resulting electric and magnetic fields of the dressed particle. The explicit forms of these terms are

$$K(x) = - \int_{\Gamma} d\Gamma(\eta+v)^{\mu} \frac{\partial^{\nu} F_{\nu\mu}}{\mathcal{G}^{\nu} \cdot \partial}$$

$$\chi(x) = \frac{\mathcal{G}_{\mu}^{\nu} A^{\mu}}{\mathcal{G}^{\nu} \cdot \partial}(x) := - \frac{\gamma}{4\pi} \int d^3z \frac{\mathcal{G}_{\mu}^{\nu} A^{\mu}(x^0, \underline{z})}{\|\underline{z} - \underline{x}\|_v}. \quad (6)$$

In these expressions $\mathcal{G}_{\mu}^{\nu} = (\eta+v)_{\mu}(\eta-v) \cdot \partial - \partial_{\mu}$ with $\eta = (1, 0)$, a unit timelike vector; v the orthogonal velocity vector $v = (0, \underline{v})$, where \underline{v} is the velocity three vector of the particle; $\gamma = (1 - \underline{v}^2)^{-1/2}$ and the metric convention is $(+, -, -, -)$. The contour, Γ , in the definition of K is the past world line of a particle moving with four velocity u^{μ} . The explicit form of the inverse to $\mathcal{G}^{\nu} \cdot \partial$ is given by

$$\frac{1}{\|\underline{z}\|_v} := \frac{1}{2\pi^2\gamma} \int d^3k \frac{e^{ik \cdot \underline{z}}}{V^{\nu} \cdot k}, \quad (7)$$

where $V_{\mu}^{\nu} = (\eta+v)_{\mu}(\eta-v) \cdot k - k_{\mu}$. This is well defined for massive matter ($|\underline{v}| < 1$). As $v \rightarrow 0$ we obtain the usual inverse to the three-dimensional Laplacian. Equation (6) makes manifest the spatial nonlocality of the physical charges.

On physical states the K term of the dressing may be reexpressed in terms of the matter current. As such it is the analogue of the Coulombic term in infrared dynamics [3] and, as can be verified explicitly, does not take part in the soft dynamics of the Abelian theory.

In the remainder of this paper we will show that, using the non-Coulombic, χ -dependent part of the dressing, we remove the soft divergences due to virtual photons in the Green's functions of these dressed charges, $e^{-ie\chi(x)}\phi(x)$, at all orders in scalar QED. Note that there are no collinear

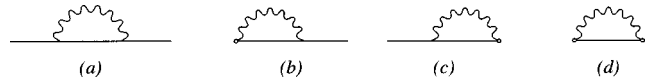


FIG. 1. The one-loop propagator for the dressed field, $e^{-ie\chi}\phi$.

divergences as we assume massive electrons and that we use the scalar theory for simplicity: the electron's spin does not affect the infrared structure. We will consider the terms in the dressed Green's functions that have a simple pole for each external leg. Generally an infrared divergence appears in the residues of these pole structures when we go on-shell. We will show that these soft divergences cancel when we include our dressings. We start with the propagator of the above (dressed) scalar electron which at one loop is given by the four diagrams of Fig. 1 [with (b) and (c) identical in the scalar theory]. The new Feynman rule for n photons at a dressing vertex is shown in Fig. 2. We will work in Feynman gauge, but gauge invariance will be always apparent in our final results. Note that the dressed fermion propagator is treated in detail in Ref. [16] and the scalar case in Ref. [17]: IR-finite results were obtained in both cases and the renormalization constants were calculated. We therefore extract the IR-divergent terms in the residue of the pole in the propagator. After some algebra they are found to be

$$\text{Res}[iS_{\text{pole}}^{\text{IR}}] = \frac{ie^2}{(2\pi)^4} \int d^4k \left(\frac{p_{\mu}}{p \cdot k} - \frac{V_{\mu}^{\nu}}{V^{\nu} \cdot k} \right) \frac{g^{\mu\nu}}{k^2}$$

$$\times \left(\frac{p_{\nu}}{p \cdot k} - \frac{V_{\nu}^{\nu}}{V^{\nu} \cdot k} \right), \quad (8)$$

where we are only integrating over soft momenta less than some cutoff. Note that the divergences in Eq. (8) are of logarithmic type. Replacing $g^{\mu\nu}/k^2$ by a more general propagator shows the gauge invariance of this result. We now need to show that it in fact vanishes: this may be done either by direct calculation or by realizing that effectively we may write [with $p^* = (p^0, -p)$]

$$\frac{V_{\mu}^{\nu}}{V^{\nu} \cdot k} \rightarrow \frac{p_{\mu}}{p \cdot k} - \frac{m^2 \gamma^2 k_{\mu}}{p \cdot k p^* \cdot k}, \quad (9)$$

which holds when we evaluate at the residue of the k^2 poles (which are well known to be the only poles that yield soft divergences) and go on shell in the correct fashion: i.e., at $p = m\gamma(1, \underline{v})$. The second term clearly will not contribute to Eq. (8) and we see that *the soft divergences all cancel*.

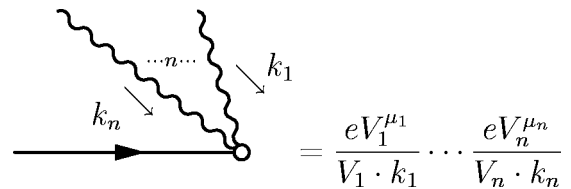


FIG. 2. The Feynman rule for the dressing vertex.

$$Z_2 \text{--- pole ---} = \text{---} + \text{---} + 2 \text{---} + \text{---}$$

 FIG. 3. Diagrammatic expression for Z_2 .

The pole structure that we obtain has, as a special case, the limit $p \rightarrow 0$, where the dressed charge and the undressed one coincide in Coulomb gauge. Thus we see that our construction explains and extends a result due to Kibble [2], who noted that the propagator in the Coulomb gauge has a pole if we are at the static point on the mass shell, $p = (m, 0)$. We also remark that we may use Sect. 3D of Ref. [18] to see that the dressed propagator is indeed IR-finite at all orders if we choose the correct point on the mass shell.

We now consider an interaction with a source. We will assume that the vertex has the form $m^2 \phi^* \phi$, which is renormalization group invariant. For simplicity we take a charge initially at rest which emerges from the interaction with velocity v . We now multiply in a factor of $1/\sqrt{Z_2^0 Z_2^v}$, where Z_2^v is the infrared finite [16,17], v -dependent wave-function renormalization constant associated with a particle moving with three velocity v . This infrared finite factor will lead to various useful cancellations of diagrams. As far as the residues of the poles are concerned, their diagrammatic form is shown in Fig. 3. Using this, we obtain the covariant contribution from Fig. 4, while the explicitly noncovariant terms are expressed by Fig. 5. Note the amputated legs on the last two terms of this figure (which are each to be understood as the product of their two parts). Calculating these latter diagrams we find for the IR divergences in the residue the gauge-invariant result

$$-\frac{m^2 e^2}{2} \frac{1}{p_1^2 - m^2} \frac{1}{p_2^2 - m^2} \int \frac{d^4 k}{(2\pi)^4} \left(\frac{V_\mu^v}{V^v \cdot k} - \frac{V_\mu^0}{V^0 \cdot k} \right) \times \frac{-i g^{\mu\nu}}{k^2} \left(\frac{V_\nu^v}{V^v \cdot k} - \frac{V_\nu^0}{V^0 \cdot k} \right). \quad (10)$$

Using Eq. (9) we may easily see that this cancels the standard covariant integral which is just Eq. (10) with an opposite sign and the V 's replaced by the appropriate momentum p_i . The extension of this argument to other processes is direct.

To show that this is true at all orders, we consider the structure of the propagator. As well as the usual dressing-independent diagrams which yield the form, $i/(p^2 - m^2 - \Sigma)$, we can have (at either end) a ‘blob’ with lines emanating from a dressing vertex, this we denote by $\tilde{\Sigma}$. This then adds to the propagator the term, $2i\tilde{\Sigma}/(p^2 - m^2 - \Sigma)$. A blob at both ends contributes, $i\tilde{\Sigma}^2/(p^2 - m^2 - \Sigma)$. If these end blobs overlap, there is no pole contribution to the residue, except in one important case: when this overlap is solely due

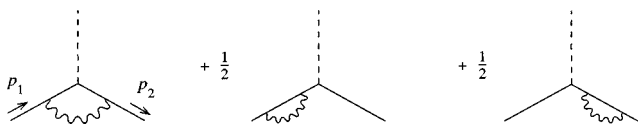


FIG. 4. Covariant part of the vertex.

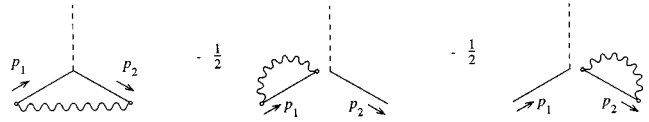


FIG. 5. Noncovariant part of the vertex.

to lines directly linking one dressing vertex to the other. This is a consequence of the Feynman rules for the dressing vertex (see Refs. [16] and [17] for the one loop example). It may be seen that the IR-divergent contribution of such ‘rainbow’ lines to the residue factorize, yielding a factor of $-C_{vv}$ for each line where we define

$$C_{vv'} = \int \frac{d^4 k}{(2\pi)^4} \frac{V^v \cdot V^{v'}}{(V^v \cdot k)(V^{v'} \cdot k)k^2}. \quad (11)$$

Summing up terms, and noting the $1/n!$ symmetry factor for n such lines, yields an exponential. The general structure of the IR-divergent part of the propagator pole is thus

$$\frac{i(1 + \tilde{\Sigma})^2}{p^2 - m^2 - \Sigma} \exp(-C_{vv}). \quad (12)$$

We may conclude that dressings do not change the mass shift (as seen [16,17] in explicit one-loop calculations) and that the wave-function renormalization constant factors into a product of a covariant and a noncovariant part, $Z_2^v = Z_2^{\text{cov}}(1 + \tilde{\Sigma})^2 \exp(-C_{vv})$.

Armed with this we consider the vertex at all orders. The general class of diagrams with two simple poles and possible IR divergences is shown in Fig. 6, i.e., the diagrams where there are possible covariant vertex corrections from one leg to another (in the left-most blob), covariant corrections on the external legs, ‘end blob’ corrections on the legs and possible rainbow corrections from one dressing to the other. The use of a black dot, \bullet , here denotes that there may or may not be an ‘end blob’ ($\tilde{\Sigma}$) at the ends of one or both of the lines. Other diagrams are either IR finite or will not yield both poles. For example, if a line from, say, the upper dressing vertex is attached to a covariant vertex on the lower scalar matter leg, there will not be a pole associated with the upper leg.

Factorizing the rainbow dressings yields the diagrams of Fig. 7. Since there may or may not be an end blob in these lines, we may write these end factors as $(1 + \tilde{\Sigma})$. They therefore *cancel* the $\tilde{\Sigma}$ dependence from each of the external leg $(Z_2^v)^{-1/2}$ factors. Diagrammatically we thus obtain Fig. 8 and we see that the dressing effects have exponentiated. Since the

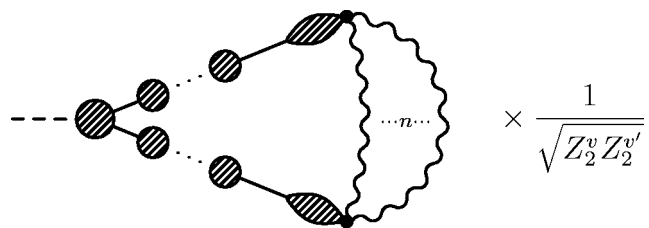


FIG. 6. Diagrams with both poles and IR divergences.

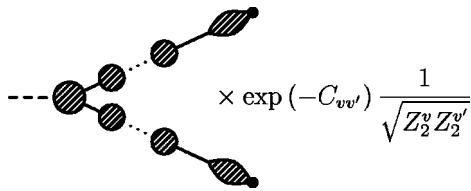


FIG. 7. The preceding figure after factorization.

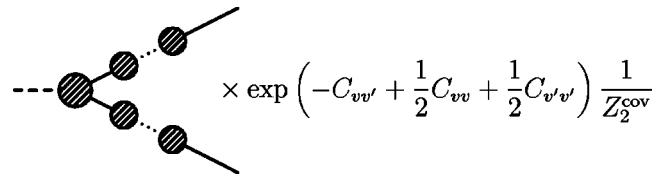


FIG. 8. Exponentiation of the dressings.

remaining covariant diagrams times $1/Z_2^{\text{cov}}$ are known to exponentiate, we see that *all the soft effects exponentiate* in the residue of the double poles. We have seen that they cancel at one loop, this now directly implies that this residue is IR finite at all orders. This demonstration shows the validity of the dressing equation. We also note that the extension of this to other vertices is straightforward: we conclude that *correctly dressed on-shell Green's functions are infrared finite to all orders*. Full details of the proof will be presented elsewhere together with the details of the one-loop calculations.

We now finish with some remarks on the implications of this paper for the non-Abelian theory. The Eqs. (1) and (4) open a way to *deriving* how glue dresses quarks. In Ref. [9] it was shown that there are no global solutions to the dressing equation (1) in an unbroken non-Abelian theory, however perturbative solutions can be constructed and nonperturbative effects can be incorporated into them. Perturbative solutions for the gluonic dressing are relevant for short distance physics such as initial jet formation. At larger scales nonperturbative effects will dominate. The sharpness condition (4) on the dressing leads to sharp quark states in the

region where the heavy mass equation (3) holds. For heavy quarks this will be at a scale significantly greater than Λ_{QCD} and thus for such quarks a non-Abelian extension of the Abelian dressing described above would be expected within a Gribov horizon where the dressing equation can be solved. For the lighter quarks, their heavy sector is at a scale lower than Λ_{QCD} and thus nonperturbative contributions to the dressing, such as condensates [9], would be expected to play a dominant role even before the effects of the horizon become manifest. We note here that the interquark potential can be found rather directly from dressings (see Sect. 7 of Ref. [9] and Ref. [14]) and further that it has been argued [19] that the incorporation of nonperturbative effects associated with the Gribov ambiguity, and hence with the breakdown of solutions to Eq. (1), lead to a linearly rising potential. The incorporation of nonperturbative effects in dressings which solve Eqs. (1) and (4) and their impact on the potential will be studied elsewhere.

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