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One-loop QCD corrections to deeply-virtual Compton scattering: The parton helicity-independent case

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We show that the one-loop QCD correction to deeply virtual Compton scattering can be factorized into finite perturbative contributions and collinearly divergent terms, which correspond to the matrix elements of the off-forward parton distributions. As a by-product, we obtain the next-to-leading order coefficient functions in the generalized operator product expansion of two vector currents. [S0556-2821(98)50203-2]

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In searching for ways to measure the amount of the nucleon spin carried by quark orbital angular momentum, one of us introduced deeply virtual Compton scattering (DVCS) as a probe to a novel class of ''off-forward'' parton distributions (OFPDs) [1]. DVCS is a process in which a highly virtual photon (with virtual mass $Q^2 \gg \Lambda^2_{\rm QCD}$) scatters on a nucleon target (polarized or unpolarized), producing an exclusive final state consisting of a high-energy real photon plus a slightly recoiled nucleon. With the virtual photon in the Bjorken limit, a quantum chromodynamic (QCD) analysis shows that the scattering is dominated by the simple mechanism in which a quark (antiquark) in the initial nucleon absorbs the virtual photon, immediately radiates a real one, and falls back to form the recoiled nucleon.

Several interesting theoretical papers have since appeared in the literature, which made further studies of the DVCS process [2–6] and the OFPDs [7–9]. A process closely related to DVCS, Compton scattering with two-virtual photons, have been studied before from theoretical interests [10,11]. The OFPDs have also been recognized in a number of theoretical studies in the past [12,13]. Moreover, the QCD evolution equations for the OFPDs [2,3,8,9,11] are closely related to the evolution of meson wave functions and light-ray or string operators [14]. However, physical significance and actual measurement of OFPDs have not been thoroughly explored in the literature. It was pointed out in Ref. [1] that the OFPDs can appear in general types of hard diffractive processes. Recent works in Ref. [15–18] have indeed found their use in diffractive meson production [19,20] and Z^0 or muon pair production [21,22].

In this paper, we study the one-loop correction to DVCS in QCD. Here, for simplicity, we consider only the parton helicity-independent amplitude. (The helicity-dependent case will be published separately, together with a more detailed analysis of the present calculation [23].) Our result shows that the infrared divergences in the one-loop diagrams can be entirely factorized into the nonperturbative OFPDs. This result is not immediately obvious from an analogy with a general virtual Compton scattering, because in DVCS the final state photon is real and new infrared divergences can potentially arise from the appearance of a new light-like momentum. It turns out, however, that a factorization theorem does hold at one-loop level with the final state photon real and structureless. We expect the factorization theorem to be true to all orders in perturbation theory, allowing DVCS to be studied in perturbative QCD like other gold-plated examples such as deep-inelastic scattering. The finite part of our one-loop results, together with the two-loop evolution of OFPDs in the modified minimal subtraction ($\overline{\rm MS}$) scheme, provides the necessary ingredients for calculating DVCS at the next-to-leading order.

To carry out our study in a more general setting, we actually consider the non-forward, unequal-mass virtual Compton scattering [5,10,11]. We call the incoming (out-going) virtual photon momentum q^μ ($q'^\mu = q^\mu - \Delta^\mu$), and the incoming (out-going) nucleon momentum P^μ ($P'^\mu = P^\mu + \Delta^\mu$). We introduce the ''average'' photon momentum $\overline{q}^\mu = (q+q')^\mu/2$ and ''average'' proton momentum $\overline{P}^\mu = (P+P')^\mu/2$. The Compton amplitude is defined as

$$T^{\mu\nu} = i\int d^4z e^{-i\overline{q}\cdot z}\left\langle P'\left|TJ^\mu\left(-\frac{z}{2}\right)J^\nu\left(\frac{z}{2}\right)\right|P\right\rangle. \tag{1}$$

We want to study the simplification of the amplitude in the Bjorken limit: $\overline{Q}^2 = -\overline{q}^2 \to \infty$, $\overline{P}\cdot\overline{Q} \to \infty$, and the ratio of the two staying finite.

For convenience, we choose two light-like momenta p^μ and n^μ so that $p^2 = n^2 = 0$ and $p\cdot n = 1$. We let the average photon and nucleon momenta to be parallel to p^μ and n^μ, so that

$$\overline{P}^\mu = p^\mu + \frac{\overline{M}^2}{2}n^\mu,$$

$$\overline{q}^\mu = -\overline{x}_B p^\mu + \frac{\overline{Q}^2}{2\overline{x}_B}n^\mu, \tag{2}$$

where $\overline{M}^2 = M^2 - \Delta^2/4$ and M is the mass of the nucleon. Here, $\overline{x}_B$ is $\overline{Q}^2/(2\overline{P}\cdot\overline{q})$. Clearly, any four-momentum can be expanded in terms of p^μ, n^μ, and two unit vectors in the transverse directions. Large scalars in any Feynman diagram come from the product of the p component of a four-vector and the n component of $\overline{q}$. Therefore, in the leading order, we can safely ignore other components of an external momentum. For instance, the momenta of the initial and final state nucleons can be approximated as $(1+\xi)p^\mu$ and $(1-\xi)p^\mu$, respectively, where ξ is defined from the expansion

$$\Delta^\mu = -2\xi p^\mu + \cdots \tag{3}$$

and is constrained to [0,1] by our choice of coordinates. (Note that the definition of ξ differs from that in Ref. [1] by a factor of 2.) From this, it follows that our $\overline{x}_B$ is related to the conventional $x_B = Q^2/2P\cdot q$ via $\overline{x}_B = (1+\xi)x_B - \xi$ when corrections of order t/Q^2 are ignored. Moreover, the momenta of initial and final partons participating in a hard subprocess can effectively be taken as $(x+\xi)p^\mu$ and $(x-\xi)p^\mu$, respectively, where $-1<x<1$. The longitudinal momentum fractions of the initial and final nucleons carried by partons are $(x+\xi)/(1+\xi)$ and $(x-\xi)/(1-\xi)$, respectively.

We consider only the part of $T^{\mu\nu}$ which is insensitive to helicities of partons in a nucleon target. Like the forward Compton scattering, there are two leading-order Lorentz-invariant amplitudes [5],

$$T^{\mu\nu} = (-g^{\mu\nu} + p^\mu n^\nu + p^\nu n^\mu)T_1 + \left(p^\mu - \frac{q'^\mu p\cdot q'}{q'^2}\right) \times \left(p^\nu - \frac{q^\nu p\cdot q}{q^2}\right) T_L , \tag{4}$$

where T_i are functions of $\overline{x}_B$, ξ, $t=\Delta^2$, and $\overline{Q^2}$. Physically, T_1 represents the amplitude for the transversely polarized photon scattering, and T_L the amplitude for the longitudinally polarized scattering. Since T_L does not contribute to DVCS, we ignore it in the remainder of the paper.

A factorization theorem is believed to exist for the general virtual Compton amplitudes defined above. For instance,

$$T_1(\overline{x}_B, \xi, t, \overline{Q^2}) = \int_{-1}^{1} \frac{dx}{x} \sum_a F_a(x,\xi,t,\overline{Q^2}) \times C_{1a}\left(\frac{x}{\overline{x}_B}, \frac{\xi}{\overline{x}_B}, \alpha_s(\overline{Q^2})\right), \tag{5}$$

where a labels different parton species. The F_a are the helicity independent leading-twist off-forward parton distributions, as defined in Ref. [1]. For quarks, one has

$$F_{a=q}(x) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x} \left\langle P' \left| \overline{\psi}\left(-\frac{\lambda}{2}n\right) \not{n} \psi\left(\frac{\lambda}{2}n\right) \right| P \right\rangle, \tag{6}$$

and similarly for gluons, $F_G(x)$. C_{1a} is the coefficient function calculable in perturbative QCD. At the leading order in α_s, one has

$$C^0_{1a}(x,\xi) = -e_a^2\left(\frac{x}{x-1} + \frac{x}{x+1}\right), \tag{7}$$

where e_a is the electric charge. The second term in the bracket represents a crossing contribution which is always present in Compton processes. For $\xi=t=0$, we go back to the well-known forward virtual Compton scattering.

The factorization formula works for both $\overline{x}_B>1$ and $\overline{x}_B<1$ regions. In fact, it is defined throughout the complex plane of $\overline{x}_B$. The amplitude is analytic at $|\overline{x}_B|>1$, where there is no on-shell propagation. Therefore, one can expand C_{1a} in Taylor series at $\overline{x}_B=\infty$:

$$C_{1a}\left(\frac{x}{\overline{x}_B}, \frac{\xi}{\overline{x}_B}\right) = \sum_{n=2,4\ldots}^{\infty} \sum_{i=0,2\ldots} C_a^{ni}(\alpha_s(\overline{Q^2})) \frac{\overline{x^n \xi^i}}{\overline{x}_B^{n+i}}. \tag{8}$$

The x-integration in Eq. (5) can now be done using

$$\int_{-1}^{1} dx x^{n-1} F_a(x,\xi,t) = a_n(\xi,t) = \tfrac{1}{2} n^{\mu_1} n^{\mu_2} \cdots n^{\mu_n} \times \langle P' | \overline{\psi} i\overleftrightarrow{D}^{\mu_1} \cdots i\overleftrightarrow{D}^{\mu_{n-1}} \gamma^{\mu_n} \psi | P \rangle. \tag{9}$$

Therefore the $i=0$ terms in the amplitude T_1 are just the result of the usual twist-2 operators in the operator product expansion (OPE) for forward virtual Compton scattering. As for the $i\neq 0$ terms, one can translate the ξ factors into total derivatives on the twist-2 operators:

$$n_{\mu_1} \cdots n_{\mu_{n+i}} \langle P' | i\partial^{\mu_{n+1}} \cdots i\partial^{\mu_{n+i}} \overline{\psi} i\overleftrightarrow{D}^{\mu_1} \cdots i\overleftrightarrow{D}^{\mu_{n-1}} \gamma^{\mu_n} \psi | P \rangle = 2(2\xi)^i a_n(\xi,t). \tag{10}$$

Therefore the factorization formula in Eq. (5) reflects a general OPE in terms of twist-2 operators with arbitrary numbers of total derivatives. $C^{n0}(\alpha_s(\overline{Q^2}))$ is the coefficient function relevant for deep-inelastic scattering [24]. A general virtual Compton process also requires the $i\neq 0$ terms.

Deeply virtual Compton scattering is a physical process with the kinematic requirement that the final photon be on its mass shell. This corresponds to the region $\overline{x}_B = \xi < 1$, where the OPE form of the amplitude is not particularly illuminating. In particular, the Compton amplitudes now have both real and imaginary parts. The analytic continuation to the $|\overline{x}_B|<1$ region is made by approaching the real axis from the lower half plane, and hence $\overline{x}_B$ has a small negative imaginary part. The central question we would like to address is this: Does the factorization theorem still hold when $\overline{x}_B = \xi$? One can, of course, study this question using the general factorization techniques [25]. While we believe the answer is affirmative, we present a one-loop calculation to support this.

We have calculated the general one-loop virtual Compton scattering with on-shell quarks and gluons. We present the quark result first. The initial and final momenta of the quark target are taken to be $P^\mu = (x+\xi)p^\mu$ and $P'^\mu = (x-\xi)p^\mu$, respectively. For convenience, we replace the spinor-space matrix element $\overline{u}(P')\Gamma u(P)$ by a trace $\mathrm{Tr}[\not{p}\Gamma]/2$. Although they are not identical, this replacement only affects the interpretation of the result. We call the resulting Compton amplitude, $t^{\mu\nu}$, with Lorentz-scalar amplitudes t_i. At the leading order, one has

$$t_{1a}=-e_a^2\left(\frac{1}{x-\overline{x}_B+i\epsilon}+\frac{1}{x+\overline{x}_B-i\epsilon}\right). \tag{11}$$

A practical aspect of parton calculation lies in the fact that the coefficient function $C_{1a}(x,\xi)$ can be obtained from the infrared subtracted parton scattering amplitude.

We use dimensional regularization to regularize the infrared divergences present in the loop calculations ($d=4+\epsilon,\epsilon>0$). We use the Lehmann-Symanzik-Zimmermann reduction formula to extract on-shell matrix elements from the corresponding Green's functions. The calculation is done in the region where $\overline{x}_B>1$ so that there is no imaginary part. Summing up the standard vertex, self-energy, and box diagrams, we find the following divergent contribution:

$$t_{1a}^{\text{pole}}=e_a^2\frac{\alpha_s}{2\pi}C_F\left(-\frac{2}{\epsilon}\right)\frac{1}{x-\overline{x}_B}\left(\frac{3}{2}+\frac{x^2+\overline{x}_B^2-2\xi^2}{x^2-\xi^2}\times\int_\xi^x\frac{dy}{y-\overline{x}_B}+\frac{(\overline{x}_B+\xi)(x-\overline{x}_B+2\xi)}{2\xi(x+\xi)}\int_{-\xi}^{\xi}\frac{dy}{y-\overline{x}_B}\right)+(\overline{x}_B\to-\overline{x}_B). \tag{12}$$

An examination shows that the above expression is proportional to the perturbative matrix elements of the OFPDs calculated in dimensional regularization at one-loop order [3]. Therefore, the effect of this term is already taken into account in the general factorization formula with a leading-order coefficient function C_{1a}^0 and the nonperturbative OFPDs. In the limit $\overline{x}_B\to\xi$, the factorization is not affected.

We also find the finite part of the one-loop result [taking the renormalization point $\mu^2=Q^2$ in the modified minimal subtraction ($\overline{\text{MS}}$) scheme],

$$\begin{aligned}t_{1a}^{\text{finite}}=e_a^2\frac{\alpha_s}{2\pi}C_F\Bigg\{&-\frac{9\omega}{2(1-\omega x)}+\Bigg[\frac{3x(1-\omega^2\xi^2)}{(x^2-\xi^2)(1-\omega^2x^2)}\\&-\frac{x(1-\omega\xi)}{(x^2-\xi^2)}\left(\frac{1}{2\omega\xi}+\frac{1+\omega\xi}{1-\omega^2x^2}\right)\log(1-\xi\omega)\Bigg]\\&\times\log(1-\omega\xi)+\left(\frac{(1-\omega x)^2+2(\omega x-\omega^2\xi^2)}{2\omega(x^2-\xi^2)(1-\omega x)}\right.\\&\left.\times\log(1-\omega x)-\frac{3}{2}\frac{2x-\omega(x^2+\xi^2)}{(x^2-\xi^2)(1-\omega x)}\right)\log(1-\omega x)\Bigg\}\\&+(\omega\to-\omega),\end{aligned} \tag{13}$$

where we have introduced $\omega=1/\overline{x}_B$. The above expression is manifestly finite in the DVCS limit, $\omega\xi\to1$, although there is a logarithmic branch point there, so the one-loop factorization theorem for DVCS holds for quark scattering. When $\xi=0$, we recover the coefficient function for the forward Compton scattering pertinent to deep-inelastic scattering [24]. The coefficient function coupling with the off-forward quark distributions in the general virtual Compton scattering at order α_s is

$$C_{1a}^1(x,\xi)=xt_{1a}^{\text{finite}}(x,\xi,\omega=1). \tag{14}$$

Next, we consider virtual Compton scattering on an on-shell gluon. Again for convenience, we replace the polarization product $\epsilon^{\mu *}\epsilon^{\nu}$ by $(-g^{\mu\nu}+p^\mu n^\nu+p^\nu n^\mu)/(2+\epsilon)$. We neglect the last two terms because of the color gauge invariance. At one-loop level, there are six quark-box diagrams, three of which have identical results as the other three. Two of the three inequivalent diagrams are related to each other by crossing symmetry; the remaining one is itself crossing symmetric. The infrared divergent part of the amplitude is

$$\begin{aligned}t_{1g}^{\text{pole}}=\frac{\alpha_s}{2\pi}2T_F\sum_a e_a^2\left(-\frac{2}{\epsilon}\right)\Bigg[&\frac{2x\overline{x}_B}{x^2-\xi^2}+\frac{\overline{x}_B^2+(x-\overline{x}_B)^2-\xi^2}{x^2-\xi^2}\\&\times\int_\xi^x\frac{dy}{y-\overline{x}_B}+\left(\frac{(\overline{x}_B+\xi)(x-2\overline{x}_B+\xi)}{2\xi(x+\xi)}\right.\\&\left.+\frac{x(\overline{x}_B^2-\xi^2)}{\xi(x^2-\xi^2)}\right)\int_{-\xi}^{\xi}\frac{dy}{y-\overline{x}_B}\Bigg]+(\overline{x}_B\to-\overline{x}_B),\end{aligned} \tag{15}$$

where $T_F=1/2$. It is not immediately clear that all the terms above can be absorbed into the renormalization mixing of the quark distributions with the gluon. However, notice that the helicity-independent gluon distribution $F_G(x)$ is antisymmetric in $x\to-x$. Therefore, when convoluted with $F_G(x)$ only the x-symmetric part of the amplitude contributes. And the x-symmetric t_{1g}^{pole} is identical to the mixing coefficient in a renormalized quark density [3].

The remaining finite part is considered as the gluon contribution to the Compton scattering in the $\overline{\text{MS}}$ scheme. Our calculation yields

$$\begin{aligned}t_{1g}^{\text{finite}}=\frac{\alpha_s}{2\pi}\left(2T_F\sum_a e_a^2\right)\Bigg\{&\Bigg[\left(1+\frac{2(1-\omega x)}{\omega^2(x^2-\xi^2)}\right)\\&\times\left(1-\frac{1}{2}\log(1-\omega x)\right)+2\frac{1-\omega x}{\omega^2(x^2-\xi^2)}\Bigg]\log(1-\omega x)\\&+\Bigg[\left(\frac{1}{\omega\xi}-\frac{2}{\omega^2(x^2-\xi^2)}\right)\left(1-\frac{1}{2}\log(1-\xi\omega)\right)\\&-\frac{2}{\omega^2(x^2-\xi^2)}\Bigg](1-\omega\xi)\log(1-\xi\omega)\Bigg\}\\&+(\omega\to-\omega).\end{aligned} \tag{16}$$

The above expression is finite in the limit of $\omega\xi\to1$, so the factorization for DVCS on a gluon target holds at one-loop order. For $\xi=0$, our result agrees with the coefficient function calculated in [24]. Using the definition of the off-forward gluon distribution $F_G(x)$, we found the coefficient function in the general factorization formula at the α_s order:

$$C_{1g}^1(x,\xi)=\frac{x^2}{x^2-\xi^2}t_{1g}^{\text{finite}}(x,\xi,\omega=1). \tag{17}$$

In a recent paper, Müller [26] studied the constraints of the spacetime conformal symmetry on the form of the OPE for two vector currents. He found that at the next-to-leading order, the OPE can be expressed in terms of the coefficient functions in the forward scattering ($\xi=0$) and the improved conformal-covariant operators at the leading order. While the result can be checked for the non-singlet case using the two-loop anomalous dimensions in the literature [27], such anomalous dimensions for the singlet case are not yet available. Clearly, they are needed if one is interested in carrying the analysis for DVCS to the full next-to-leading order.

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