

## Bell's locality and $\varepsilon'/\varepsilon$

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We formulate a set of Bell's inequalities for the system of two correlated neutral kaons coming from the decay of a  $\phi$  meson, without assuming  $CP$  and  $CPT$  invariance. We show that a nonvanishing value of the phenomenological parameter  $\varepsilon'$  would violate such inequalities, ruling out Bell's locality. [S0556-2821(98)50303-7]

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The neutral kaon system has proven to be one of the most useful laboratories for studying many aspects of modern particle physics. Indeed, the study of the phenomena associated with kaon production, propagation, and decay has provided strong experimental confirmation for various predictions of the standard model.

Recently, the  $K^0$ - $\bar{K}^0$  system has also been proposed as the natural system to look for possible new phenomena leading to the loss of quantum coherence and  $CPT$  violating effects [1–3]. These arise as natural consequences of effective dynamics that treat the kaon system as an open quantum system, and could be of particular relevance in the study of correlated kaons [4–6].

When a  $\phi$  meson at rest decays into two neutral kaons, the final state has the property of being antisymmetric in the spatial part, due to angular momentum conservation. Although the two kaons fly apart with opposite momenta, they remain quantum mechanically correlated in a way that is very similar to the entanglement of two spin-1/2 particles in a singlet state. Therefore, as in the spin case, by studying the evolution of certain observables of the correlated kaons state, one can perform fundamental tests on the behavior of entangled systems.

In the following we shall focus on the property called Bell's locality [7–10] and discuss whether, by looking at experimentally observable quantities, the system of the two neutral kaons coming from the  $\phi$  decay satisfies this condition. Typically, observables that can be studied in the two-kaon system are the probabilities  $\mathcal{P}(f_1, \tau_1; f_2, \tau_2)$  that one kaon decays into the final state  $f_1$  at proper time  $\tau_1$ , while the other kaon decays into the final state  $f_2$  at proper time  $\tau_2$ . The requirement of Bell's locality can be translated into certain inequalities that the probabilities  $\mathcal{P}$  should satisfy (Bell's inequalities).

We shall see that in the standard quantum mechanical description of the correlated two-kaon system, these inequalities are violated by the phenomenological constant  $\varepsilon'$ , that parametrizes small direct  $CP$  and  $CPT$  violating effects in the decays of the kaons in two pions. Therefore, if in an actual experiment the parameter  $\varepsilon'$  is found to be nonzero, then Bell's locality will be ruled out. It should be noticed

that many *ad hoc* experiments have been proposed in order to test Bell's inequalities using correlated neutral kaons. However, while experiments aiming at the measure of  $\varepsilon'$  are actually under way, the possibility of experimentally revealing violations of Bell's locality through direct investigations of kaon correlations seems at the moment technically very difficult.

For our discussion, we shall adopt a particle description of the phenomena related to the time evolution and decay of the  $K^0$ - $\bar{K}^0$  system. This means that the transition probabilities we shall be dealing with are actually  $S$ -matrix elements, i.e., matrix elements of the scattering operator  $S$  (or better, the transition operator  $T=1-S$ ) between asymptotic particle states. This description is standard in particle physics since it is the closest to the actual experimental situations in which the various decay products are identified in physical detectors. The  $K^0$ - $\bar{K}^0$  system can then be effectively described by means of a two-dimensional Hilbert space [11]. A useful orthonormal basis in this space is given by the  $CP$ -eigenstates  $|K_1\rangle$  and  $|K_2\rangle$ :

$$\begin{aligned} |K_1\rangle &= \frac{1}{\sqrt{2}} [ |K^0\rangle + |\bar{K}^0\rangle ], \\ |K_2\rangle &= \frac{1}{\sqrt{2}} [ |K^0\rangle - |\bar{K}^0\rangle ]. \end{aligned} \quad (1)$$

For the discussion that follows, we find it convenient to describe the states of a physical system by means of density matrices. These are Hermitian matrices  $\rho$ , with positive eigenvalues and unit trace. In the case of the kaon system,  $\rho$  is a two-dimensional matrix. With respect to the basis (1) it can be written as

$$\rho = \begin{pmatrix} \rho_1 & \rho_3 \\ \rho_4 & \rho_2 \end{pmatrix}, \quad (2)$$

where  $\rho_4 \equiv \rho_3^*$ , and \* signifies complex conjugation. Since the kaon system is unstable, its evolution in time is described by

$$\rho \mapsto \rho(t) \equiv \gamma_i[\rho] = e^{-itH} \rho e^{itH^\dagger}, \quad (3)$$

where  $H = M - (i/2)\Gamma$  is the effective (Weisskopf-Wigner) Hamiltonian, with  $M$  and  $\Gamma$  positive Hermitian  $2 \times 2$  matrices. The entries of these matrices can be expressed in terms of the complex parameters  $\varepsilon_S$ ,  $\varepsilon_L$ , appearing in the eigenstates of  $H$ , described by the density matrices [2,3]

$$\rho_L = |N_L|^2 \begin{pmatrix} |\varepsilon_L|^2 & \varepsilon_L \\ \varepsilon_L^* & 1 \end{pmatrix} \quad \rho_S = |N_S|^2 \begin{pmatrix} 1 & \varepsilon_S^* \\ \varepsilon_S & |\varepsilon_S|^2 \end{pmatrix}, \quad (4)$$

and the four real parameters,  $m_S$ ,  $\gamma_S$  and  $m_L$ ,  $\gamma_L$  characterizing the eigenvalues of  $H$ :  $\lambda_S = m_S - (i/2)\gamma_S$ ,  $\lambda_L = m_L - (i/2)\gamma_L$ , with  $N_S = (1 + |\varepsilon_S|^2)^{-1/2}$  and  $N_L = (1 + |\varepsilon_L|^2)^{-1/2}$  normalization factors;  $\gamma_S$ ,  $\gamma_L$  and  $m_S$ ,  $m_L$  are the physical decay widths and masses of the states  $K_S$  and  $K_L$ . The constants  $\varepsilon_S$  and  $\varepsilon_L$  parametrize the so-called indirect  $CP$  and (for  $\varepsilon_S \neq \varepsilon_L$ )  $CPT$  violating effects; if we ignore these small effects,  $\rho_L$  and  $\rho_S$  would correspond to the  $CP$ -eigenstates in Eq. (1).

The time evolution of a system of two correlated neutral kaons, as those coming from the decay of a  $\phi$  meson, can be derived from the single-kaon dynamical map  $\gamma_i$  in Eq. (3). First note that, since the  $\phi$  meson has spin 1, its decay into two spinless bosons produces an antisymmetric spatial state. In the  $\phi$  rest frame, the two neutral kaons are produced flying apart with opposite momenta; in the basis  $|K_1\rangle$ ,  $|K_2\rangle$ , the resulting state can be described by

$$|\psi_A\rangle = \frac{1}{\sqrt{2}} (|K_1, -p\rangle \otimes |K_2, p\rangle - |K_2, -p\rangle \otimes |K_1, p\rangle). \quad (5)$$

The corresponding density matrix  $\rho_A$  is a  $4 \times 4$  matrix that can be conveniently written as

$$\rho_A = \frac{1}{2} [P_1(-p) \otimes P_2(p) + P_2(-p) \otimes P_1(p) - P_3(-p) \otimes P_4(p) - P_4(-p) \otimes P_3(p)], \quad (6)$$

where  $P_1 \equiv |K_1\rangle\langle K_1|$ ,  $P_2 \equiv |K_2\rangle\langle K_2|$ ,  $P_3 \equiv |K_1\rangle\langle K_2|$  and  $P_4 \equiv |K_2\rangle\langle K_1|$ . (Henceforth, the explicit reference to the momentum  $p$  will be dropped.)

Once produced in a  $\phi$  decay, the kaons evolve independently in time each according to the map  $\gamma_i$  in Eq. (3). Therefore, the density matrix that describes a situation in which the first kaon has evolved up to proper time  $\tau_1$  and the second up to proper time  $\tau_2$  is given by:

$$\begin{aligned} \rho_A(\tau_1, \tau_2) &\equiv (\gamma_{\tau_1} \otimes \gamma_{\tau_2})[\rho_A] \\ &= \frac{1}{2} [P_1(\tau_1) \otimes P_2(\tau_2) + P_2(\tau_1) \otimes P_1(\tau_2) \\ &\quad - P_3(\tau_1) \otimes P_4(\tau_2) - P_4(\tau_1) \otimes P_3(\tau_2)], \quad (7) \end{aligned}$$

where  $P_i(\tau_1)$  and  $P_i(\tau_2)$ ,  $i=1,2,3,4$ , represent the evolution according to Eq. (3) of the initial operators  $P_i$ , up to the time  $\tau_1$  and  $\tau_2$ , respectively.

The formula (7) can now be used to compute the time evolution of characteristic observables for the two-kaon system; indeed, any physical property of this system can be extracted from the density matrix  $\rho_A(\tau_1, \tau_2)$  by taking its

trace with suitable Hermitian operators. The typical observables that can be studied are double decay probabilities  $\mathcal{P}(f_1, \tau_1; f_2, \tau_2)$ , i.e., the probabilities that one kaon decays into a final state  $f_1$  at proper time  $\tau_1$ , while the other kaon decays into the final state  $f_2$  at proper time  $\tau_2$  [12]:

$$\mathcal{P}(f_1, \tau_1; f_2, \tau_2) \equiv \text{Tr}[(\mathcal{O}_{f_1} \otimes \mathcal{O}_{f_2})\rho_A(\tau_1, \tau_2)], \quad (8)$$

where  $\mathcal{O}_{f_1}$  and  $\mathcal{O}_{f_2}$  represent the  $2 \times 2$  projector matrices, describing the decay of a single kaon into the final states  $f_1$  and  $f_2$ , respectively. Useful observables are associated with the decay of the neutral kaons into two pions and into semileptonic states. We shall be as general as possible and use matrices  $\mathcal{O}_f$  that encode possible  $CP$  and  $CPT$  violating effects also in the decay amplitudes. Let us stress that the quantities  $\mathcal{P}$  introduced in Eq. (8) are decay probabilities and not decay rates; indeed, normalized expressions for the various observables  $\mathcal{O}$  are used:  $\text{Tr}[\mathcal{O}] = 1$ .

In the case of the decay into  $\pi^+ \pi^-$  final state, the corresponding projector matrix can be written as [3,5]

$$\mathcal{O}_{+-} = N_{+-} \begin{bmatrix} 1 & Y_{+-} \\ Y_{+-}^* & |Y_{+-}|^2 \end{bmatrix}, \quad (9)$$

where  $Y_{+-}$  is the ratio of the decay amplitudes of  $K_2$  and  $K_1$  states into  $\pi^+ \pi^-$ , and  $N_{+-} = (1 + |Y_{+-}|^2)^{-1}$ . Similar expressions hold for the decay into  $2\pi^0$ , which can be obtained from the above formulas by replacing  $Y_{+-}$  with the ratio  $Y_{00}$  between the decay amplitudes of  $K_2$  and  $K_1$  into two neutral pions. The two amplitude ratios  $Y_{+-}$  and  $Y_{00}$  can be expressed in terms of the phenomenological constants  $\varepsilon$  and  $\varepsilon'$  that parametrize the  $K_L$ ,  $K_S$  amplitudes ratio for the decay into  $\pi^+ \pi^-$  and  $2\pi^0$ ; these are directly accessible to the experiment [13]. Up to first order in all small  $CP$  and  $CPT$  violating terms, one explicitly finds [5]

$$Y_{+-} = \varepsilon - \varepsilon_L + \varepsilon', \quad Y_{00} = \varepsilon - \varepsilon_L - 2\varepsilon'. \quad (10)$$

In a similar way one can derive the expressions for the observables that describe the decay of neutral kaons into the semileptonic states  $\pi^- \ell^+ \nu$  and  $\pi^+ \ell^- \bar{\nu}$ . The projector matrices to be used in this case are [3,5]

$$\mathcal{O}_{\ell^+} = N_+ \begin{bmatrix} |1+x|^2 & (1+x^*)(1-x) \\ (1+x)(1-x^*) & |1-x|^2 \end{bmatrix}, \quad (11a)$$

$$\mathcal{O}_{\ell^-} = N_- \begin{bmatrix} |z+1|^2 & (z^*+1)(z-1) \\ (z+1)(z^*-1) & |z-1|^2 \end{bmatrix}, \quad (11b)$$

where the complex parameters  $x$  and  $z$  measure the violation of the  $\Delta S = \Delta Q$  rule, that would forbid the decays  $K^0 \rightarrow \pi^+ \ell^- \bar{\nu}$  and  $\bar{K}^0 \rightarrow \pi^- \ell^+ \nu$  [14], and  $N_+ = [2(1 + |x|^2)]^{-1}$ ,  $N_- = [2(1 + |z|^2)]^{-1}$ .

Once inserted in the general formula (8), the four observables  $\mathcal{O}_{+-}$ ,  $\mathcal{O}_{00}$ ,  $\mathcal{O}_{\ell^+}$ , and  $\mathcal{O}_{\ell^-}$  allow determining various joint probabilities for the system of two kaons coming from the decay of a  $\phi$  meson. These double probabilities enter a class of inequalities that can be derived from the hypothesis of Bell's locality. Before giving explicit expressions for those probabilities, we shall derive and discuss these relations (Bell's inequalities).

In actual experimental setups (the so called  $\phi$  factories), one studies the decay of a  $\phi$  meson into two neutral kaons by counting the occurrence of the various final decay states for the two kaons. For instance, the probability  $\mathcal{P}(f_1, \tau_1; -, \tau_2)$  of finding a certain final state  $f_1$  at proper time  $\tau_1$  for one of the two kaons and any decay mode for the second one at proper time  $\tau_2$  can be experimentally obtained as the ratio of partial  $(f_1, \tau_1; -, \tau_2)$  counts over the total  $\phi$  decays. A similar argument holds for the double probability  $\mathcal{P}(f_1, \tau_1; f_2, \tau_2)$ .

Let us now suppose that we can describe the system of the two kaons by a set of variables that we globally call  $\lambda$ . We assume that the description in terms of the set  $\lambda$  is the best available characterization of the system. In particular, given the variables  $\lambda$ , we expect a well-defined probability  $p_\lambda(f_1, \tau_1; -, \tau_2)$  of detecting a final decay state  $(f_1, \tau_1)$  for one of the two kaons and any state  $(-, \tau_2)$  for the second one, and a probability  $p_\lambda(f_1, \tau_1; f_2, \tau_2)$  of detecting a final decay state  $(f_1, \tau_1)$  for one kaon and a final decay state  $(f_2, \tau_2)$  for the second one. The average probabilities  $\mathcal{P}(f_1, \tau_1; -, \tau_2)$  and  $\mathcal{P}(f_1, \tau_1; f_2, \tau_2)$  are then given by

$$\mathcal{P}(f_1, \tau_1; -, \tau_2) = \int d\lambda \rho(\lambda) p_\lambda(f_1, \tau_1; -, \tau_2), \quad (12a)$$

$$\mathcal{P}(f_1, \tau_1; f_2, \tau_2) = \int d\lambda \rho(\lambda) p_\lambda(f_1, \tau_1; f_2, \tau_2), \quad (12b)$$

where  $\rho(\lambda)$  is a normalized probability density characterizing the ensemble of initial  $\phi$  particles. It should be stressed that this description of the  $\phi$  decay into two neutral kaons is rather general, and surely can be made to agree with ordinary quantum mechanics.

Since the decays of the two kaons coming from a  $\phi$  meson are localized events, usually very well spatially separated, one is led to assume that  $p_\lambda(f_1, \tau_1; -, \tau_2)$  and  $p_\lambda(-, \tau_1; f_2, \tau_2)$  are independent probabilities. In other words, one considers a description of the two-kaon system for which

$$p_\lambda(f_1; \tau_1; f_2, \tau_2) = p_\lambda(f_1, \tau_1; -, \tau_2) p_\lambda(-, \tau_1; f_2, \tau_2). \quad (13)$$

This condition is known as Bell's locality condition. Measurable consequences of this assumption can be easily derived. We will now adapt the arguments developed in Ref. [8] for stationary spin singlets to the case of time-evolving correlated kaon systems.

By noting that for any set of four positive numbers  $x_1, x_2, x_3, x_4$  less or equal to 1 the following inequality holds

$$x_1 x_2 - x_1 x_4 + x_3 x_2 + x_3 x_4 \leq x_3 + x_2, \quad (14)$$

using Eq. (13) and integrating over  $\lambda$  with weight  $\rho(\lambda)$ , one easily deduce the following relation among double probabilities:

$$\begin{aligned} &\mathcal{P}(f_1, \tau_1; f_2, \tau_2) - \mathcal{P}(f_1, \tau_1; f_4, \tau_2) + \mathcal{P}(f_3, \tau_1; f_2, \tau_2) \\ &+ \mathcal{P}(f_3, \tau_1; f_4, \tau_2) \leq \mathcal{P}(f_3, \tau_1; -, \tau_2) + \mathcal{P}(-, \tau_1; f_2, \tau_2). \end{aligned} \quad (15)$$

This inequality for the probabilities  $\mathcal{P}$ , involving four generic decay states  $f_i, i=1,2,3,4$ , of the neutral kaons, is a direct consequence of the condition (13) and is called a (generalized) Bell's inequality.

We shall now discuss whether this relation is satisfied by ordinary quantum mechanics. In doing this, one has to compute the probabilities  $\mathcal{P}$  appearing in Eq. (15) using the expression (8). Actually, we shall study a simplified version of the inequality, obtained from Eq. (15) by setting  $f_1=f_4$  and  $\tau_1=\tau_2=\tau$ . Since the original starting state  $|\psi_A\rangle$  in Eq. (5) is antisymmetric, the probability  $\mathcal{P}(f_1, \tau; f_1, \tau)$  identically vanishes in quantum mechanics, so that Eq. (15) reduces to

$$\begin{aligned} &\mathcal{P}(f_1, \tau; f_2, \tau) + \mathcal{P}(f_3, \tau; f_2, \tau) + \mathcal{P}(f_3, \tau; f_1, \tau) \\ &\leq \mathcal{P}(f_3, \tau; -, \tau) + \mathcal{P}(-, \tau; f_2, \tau). \end{aligned} \quad (16)$$

Furthermore, it is convenient to eliminate the single-kaon probabilities in the right-hand side (RHS) of Eq. (16), by using the identity

$$\begin{aligned} \mathcal{P}(f_3, \tau; -, \tau) &\equiv \text{Tr}[(\mathcal{O}_{f_3} \otimes \mathbf{1}) \rho_A(\tau, \tau)] \\ &= \mathcal{P}(f_3, \tau; f_4, \tau) + \mathcal{P}(f_3, \tau; \tilde{f}_1, \tau), \end{aligned} \quad (17)$$

where  $\mathbf{1}$  is the  $2 \times 2$  unit matrix and  $\mathcal{O}_{f_1}, \mathcal{O}_{\tilde{f}_1}$  are orthogonal projectors corresponding to an orthonormal basis in the two-dimensional kaon Hilbert space, so that  $\mathbf{1} = \mathcal{O}_{f_1} + \mathcal{O}_{\tilde{f}_1}$ . In practice, the use of Eq. (17) allows trading the state  $f_1$  for the state  $\tilde{f}_1$ . In fact, after these manipulations, the inequality (16) takes the form

$$\mathcal{P}(f_3, \tau; f_2, \tau) - \mathcal{P}(f_3, \tau; \tilde{f}_1, \tau) \leq \mathcal{P}(\tilde{f}_1, \tau; f_2, \tau). \quad (18)$$

It should be stressed that Eq. (18) is a weaker condition than the original relation (16). Indeed, the relation (17) is compatible with the expressions (12) for the probabilities  $\mathcal{P}$  only under the assumption that the decay of one kaon is stochastically independent from the decay of the second one. In other words, in using Eq. (17) we assume the absence of variables  $\lambda$  that, without violating Bell's locality (13), might nevertheless correlate the two decays. This is the price one has to pay for using an effective two-dimensional description of the  $K^0 - \bar{K}^0$  system. As already observed, if one aims to identify the final states in Eq. (18) with actual asymptotic particles, this is essentially the only consistent available description.

The three kaon decay states appearing in Eq. (18) can be taken to be the two-pion or the semileptonic states discussed before; in particular, for  $\tilde{f}_1 = \pi^+ \pi^-, f_2 = 2\pi^0, f_3 = \pi^- \ell^+ \nu$ , or  $\tilde{f}_1 = 2\pi^0, f_2 = \pi^+ \pi^-, f_3 = \pi^- \ell^+ \nu$ , one obtains the two independent inequalities:

$$\begin{aligned} &\mathcal{P}(\pi^- \ell^+ \nu, \tau; 2\pi^0, \tau) - \mathcal{P}(\pi^- \ell^+ \nu, \tau; \pi^+ \pi^-, \tau) \\ &\leq \mathcal{P}(\pi^+ \pi^-, \tau; 2\pi^0, \tau), \end{aligned} \quad (19a)$$

$$\begin{aligned} &\mathcal{P}(\pi^- \ell^+ \nu, \tau; \pi^+ \pi^-, \tau) - \mathcal{P}(\pi^- \ell^+ \nu, \tau; 2\pi^0, \tau) \\ &\leq \mathcal{P}(2\pi^0, \tau; \pi^+ \pi^-, \tau); \end{aligned} \quad (19b)$$

since  $\mathcal{P}(f_1, \tau; f_2, \tau)$  is symmetric under the exchange  $f_1 \leftrightarrow f_2$ , see Eq. (8), these can be combined into

$$\begin{aligned} & |\mathcal{P}(\pi^- \ell^+ \nu, \tau; 2\pi^0, \tau) - \mathcal{P}(\pi^- \ell^+ \nu, \tau; \pi^+ \pi^-, \tau)| \\ & \leq \mathcal{P}(2\pi^0, \tau; \pi^+ \pi^-, \tau). \end{aligned} \quad (20)$$

A similar relation holds when the semileptonic state  $\pi^- \ell^+ \nu$  is substituted with  $\pi^+ \ell^- \bar{\nu}$ .

Explicit expressions for the three probabilities appearing in Eq. (20) can be easily obtained using the general formula (8) and the normalized observables  $\mathcal{O}_{+-}$ ,  $\mathcal{O}_{00}$ , and  $\mathcal{O}_{\ell^+}$ . For our purposes, it is sufficient to work with approximate expressions obtained by keeping only the leading terms in the small  $CP$  and  $CPT$  violating parameters  $\varepsilon_L$ ,  $\varepsilon_S$ ,  $Y_{+-}$ ,  $Y_{00}$ , and  $\Delta S = \Delta Q$  violating terms  $x$  and  $z$ . Up to leading order in all small parameters, one finds

$$\begin{aligned} \mathcal{P}(\pi^- \ell^+ \nu, \tau; 2\pi^0, \tau) &= \frac{1}{4} e^{-(\gamma_L + \gamma_S)\tau} [1 - 2\text{Re}(Y_{00}) \\ & \quad - 2\text{Re}(x)], \end{aligned} \quad (21a)$$

$$\begin{aligned} \mathcal{P}(\pi^- \ell^+ \nu, \tau; \pi^+ \pi^-, \tau) &= \frac{1}{4} e^{-(\lambda_L + \gamma_S)\tau} [1 - 2\text{Re}(Y_{+-}) \\ & \quad - 2\text{Re}(x)], \end{aligned} \quad (21b)$$

$$\mathcal{P}(2\pi^0, \tau; \pi^+ \pi^-, \tau) = \frac{1}{2} e^{-(\gamma_L + \gamma_S)\tau} |Y_{+-} - Y_{00}|^2. \quad (21c)$$

Similar expressions hold for  $\mathcal{P}(\pi^+ \ell^- \bar{\nu}, \tau; 2\pi^0, \tau)$  and  $\mathcal{P}(\pi^+ \ell^- \bar{\nu}, \tau; \pi^+ \pi^-, \tau)$ ; they are obtained from Eqs. (21a) and (21b) by changing the signs in front of  $\text{Re}(Y_{+-})$  and  $\text{Re}(Y_{00})$ , and by replacing  $\text{Re}(x)$  with  $\text{Re}(z)$ .

Notice that since we are considering probability correlations at equal time, the time-dependence factorizes. Therefore, inserting Eq. (21) in Eq. (20), the time-dependence drops and one finds

$$|\text{Re}(Y_{+-} - Y_{00})| \leq |Y_{+-} - Y_{00}|^2. \quad (22)$$

Recalling the expressions (10), one sees that the difference  $Y_{+-} - Y_{00}$  is expressible solely in terms of the phenomenological parameter  $\varepsilon'$ . Then, the assumption of Bell's locality for the system of correlated neutral kaons coming from the decay of a  $\phi$  predicts that  $\varepsilon'$  must satisfy the condition:

$$|\text{Re}(\varepsilon')| \leq 3|\varepsilon'|^2. \quad (23)$$

Therefore, a direct measurement of this parameter in any experimental setup, not necessarily involving correlated kaons, would automatically be a check of Bell's locality, within the assumption of stochastic independence of kaon decays.

The complex parameter  $\varepsilon'$  is expected to be very small; theoretical estimates based on the standard model predict

$|\varepsilon'| \leq 10^{-6}$ , while its phase is expected to be close to  $\pi/4$  [13]. Therefore, the inequality (23) can be satisfied only if  $\varepsilon'$  is identically zero (such a prediction is made by the so-called superweak model). The most recent experimental determinations are not very accurate and the measured value of  $\text{Re}(\varepsilon'/\varepsilon)$ , the parameter so far accessible in actual experiments, does not significantly differ from zero [15,16]. However, more refined experimental setups are presently under construction, or are just operative, and soon very precise determination of  $\varepsilon'$  will be available.<sup>1</sup> If these new experiments will confirm the theoretical predictions for a nonzero  $\varepsilon'$ , then the condition (23) will be violated by several orders of magnitude, resulting in one of the best tests of Bell's locality.

Before closing, we would like to comment on some of the earlier approaches to the study of Bell's inequalities in the system of correlated neutral kaons (see [17–19] and references therein). Although the existing literature on the topic is vast, none of these contributions discuss Bell's locality by taking into account  $CP$  and  $CPT$  violating effects both in the mass matrix and in the decay amplitudes. Further, most of the papers deal with a class of Bell's inequalities which are different from the one considered here; they involve correlated decay probabilities at different times, not directly connected with asymptotic states, i.e., with experimentally detectable final particles.

The inequalities (18) have also been discussed in Ref. [19], where they have been derived using different techniques. The aim of that paper is to propose a direct experimental test of those inequalities, using a  $\phi$  factory. Although it is certainly of interest to devise direct tests of Bell's locality in actual experimental setups, we emphasize that our conclusions are independent from these considerations. Indeed, we were able to reduce the relations (18) to an equivalent condition on the phenomenological parameter  $\varepsilon'$ , directly accessible to the experiment. Any measure of this parameter is therefore a test on the inequalities (18): there is no need to construct *ad hoc* quantum interferometers to check directly these conditions. As already observed, such measures of  $\varepsilon'$ , besides being crucial for the confirmation of the standard model, will also constitute automatically one of the cleanest tests of Bell's locality.

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<sup>1</sup>The fixed target experiments KTEV at Fermilab and NA48 at CERN are already collecting data; the KLOE apparatus at the DAΦNE  $\phi$  factory in Frascati is expected to be operative in a year.

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