Non-Abelian Chern-Simons coefficient in the Higgs phase

Hsien-chung Kao*
Department of Physics, Tamkang University, Tamsui, Taiwan 25137, Republic of China
(Received 29 October 1997; published 15 May 1998)

We calculate the one loop corrections to the Chern-Simons coefficient κ in the Higgs phase of Yang-Mills Chern-Simons Higgs theories. When the gauge group is SU(N), we show, by taking into account the effect of the would-be Chern-Simons term, that the corrections are always integer multiples of $1/4\pi$, as they should be for the theories to be quantum-mechanically consistent. In particular, the correction is vanishing for SU(2). The same method can also be applied to the case where the gauge group is SO(N). The result for SO(2) agrees with that found in the Abelian Chern-Simons theories. Therefore, the calculation provides us with a unified understanding of the quantum correction to the Chern-Simons coefficient. [S0556-2821(98)03612-1]

PACS number(s): 11.10.Kk, 11.10.Gh, 11.15.Ex, 12.38.Bx

Chern-Simons theories in 2+1 dimensions provide a field-theoretic description of particle excitations with fractional spin and statistics, and thus can be used to study the fractional quantum Hall effect [1–3]. Furthermore, we can put the theories in a self-dual form by including Higgs fields with a special sixth-order potential. When this happens, the systems admit a so-called Bogomol'nyi bound in energy [4], which is saturated by solutions satisfying a set of first-order self-duality equations [5]. These solutions have a rich structure especially when the gauge symmetry is non-Abelian and they are interesting in their own right [6]. It is also known that the self-duality in these systems is a result of an underlying $N=2$ supersymmetry [7].

The quantum correction to the Chern-Simons coefficient is also interesting. In the Abelian case, when there are neither massless charged particles nor spontaneous symmetry breaking, Coleman and Hill have shown that the correction to the Chern-Simons coefficient can only come from the fermion one-loop effect and is quantized $(1/4\pi)$ [8]. When either of the two conditions is violated, scalar particles may also contribute to the correction and higher-loop effect is generally nonvanishing [9,10]. In particular, the one-loop correction looks complicated and is not quantized. For Abelian Chern-Simons theories, this does not really cause a problem.

When the gauge symmetry is non-Abelian, however, the Chern-Simons coefficient must be an integer multiple of $1/4\pi$ for the systems to be quantum-mechanically consistent. Therefore, it is interesting to see whether the quantization condition survives the quantum correction. In the symmetric phase, this has been shown to one loop [11]. When there is no bare Chern-Simons term, it is also verified up to two loops by considering the fermionic contribution [12]. In the Higgs phase, the situation is more subtle. If there is a remaining symmetry in the Higgs phase, e.g., SU(N) with $N\geqslant 3$, it has been shown that the correction still satisfies the quantization condition [13–15]. In contrast, if the gauge symmetry is completely broken, e.g., SU(2), the correction is again complicated and not quantized [16]. It is usually argued that this arises because there is no well-defined symmetry generator in the system.

One can, however, take an alternative perspective to the whole thing. A more careful analysis suggests that so-called would-be Chern-Simons terms could exist in the effective action which are completely gauge invariant and induce in the Higgs phase terms similar to the Chern-Simons one [16]. In fact, the one-loop correction in the Higgs phase has been shown to be identical to that in the symmetric phase for a general class of renormalizable Abelian Chern-Simons theories [17]. Although the results reported in Ref. [18] look complicated, we believe they can also be incorporated in this picture. Therefore everything will fit together well if we can show how it works in non-Abelian theories. Unfortunately, the calculation seems to be much too complicated, and an explicit demonstration is still lacking.

In this paper, we take up the on-going effort and calculate the one loop corrections to the Chern-Simons coefficient in the Higgs phase of Yang-Mills-Chern-Simons Higgs theories. With the Higgs phase being in the fundamental SU(N), we show that the corrections are always integer multiples of $1/4\pi$ for all N. In particular, the correction is vanishing for SU(2). The nice thing is that we can avoid the tedious calculations encountered in Ref. [17] as will be explained later. We also apply the same method to the case where the gauge group is SO(N). In particular, the correction is vanishing for SO(2), consistent with the result in Ref. [17]. We conclude with some comment on the case where the Higgs field is in the adjoint representation.

Let us consider the following Yang-Mills Chern-Simons theories with a complex Higgs field Φ in the fundamental representation:

$$\mathcal{L}=\frac{1}{g^2}\mathrm{tr}\left\{-\frac{1}{2g^2}F_{\mu\nu}^2-i\kappa\epsilon^{\mu\nu\rho}\left(A_\mu\partial_\nu A_\rho-\frac{2}{3}iA_\mu A_\nu A_\rho\right)\right\}+|D_\mu\Phi|^2+\lambda(|\Phi|^2-v^2)^2. \tag{1}$$

Here $D_\mu=(\partial_\mu-iA_\mu^m T^m)$ and $\epsilon_{012}=1$. To be specific, we choose the gauge group to be SU(N). The generators satisfy $[T^m,T^n]=if^{lmn}T^l$, with the normalization $\mathrm{tr}\{T^mT^n\}=\delta^{mn}/2$. Moreover, $\Sigma_m(T^m)_{\alpha\beta}(T^m)_{\gamma\delta}=\frac{1}{2}\delta_{\alpha\delta}\delta_{\beta\gamma}-(1/2N)\delta_{\alpha\beta}\delta_{\gamma\delta}$.

Because of its conceptual advantage, the background field method will be employed. For this purpose, we separate A_μ

*Email address: hckao@ibm1.phys.tku.edu.tw

into the background part A_μ and the quantum part Q_μ. In the Higgs phase, $\Phi=\phi+\varphi$ with $\varphi^\dagger\varphi=v^2$. The gauge fixing term is given by

$$\mathcal{L}_{gf}=\frac{1}{2\xi}\{(\hat{D}_\mu Q_\mu)^m+i\xi(\varphi^\dagger T^m\phi-\phi^\dagger T^m\varphi)\}^2, \qquad (2)$$

where $\hat{D}_\mu$ is the covariant derivative with the background field. Following standard procedure, one can find the Faddeev-Popov ghost term

$$\mathcal{L}_{\rm FP}=2\,{\rm tr}\{(\hat{D}_\mu\overline{\eta})(\hat{D}_\mu\eta)-i(\hat{D}_\mu\overline{\eta})[Q_\mu,\eta]\}+\xi(\varphi^\dagger\overline{\eta}\eta\varphi-\varphi^\dagger\eta\overline{\eta}\varphi)+\xi(\varphi^\dagger\overline{\eta}\eta\phi-\phi^\dagger\eta\overline{\eta}\varphi). \qquad (3)$$

From Eqs. (1)–(3), we see to quadratic order in Q_μ and ϕ, the relevant terms are

$$\mathcal{L}_0=\frac{1}{2}Q_\mu^m\left\{\left[\frac{-1}{g^2}(\partial^2\delta_{\mu\nu}-\partial_\mu\partial_\nu)-\frac{1}{\xi}\partial_\mu\partial_\nu+i\kappa\epsilon_{\mu\nu\rho}\partial_\rho\right]\delta_{mn}+\delta_{\mu\nu}[(\varphi^\dagger T^mT^n\varphi)+(\varphi^\dagger T^nT^m\varphi)]\right\}Q_\nu^n$$
$$+\frac{1}{2}\phi_a^p\left\{\left[-\partial^2+\frac{1}{2}\xi\varphi^2\right](\delta_{ab}\delta_{pq}-\hat{\varphi}_a^p\hat{\varphi}_b^q)+[-\partial^2+m_H^2](\hat{\varphi}_a^p\hat{\varphi}_b^q)+\left[\frac{(N-2)}{2N}\xi\varphi^2\right](\epsilon_{ac}\epsilon_{bd}\hat{\varphi}_c^p\hat{\varphi}_d^q)\right\}\phi_b^q$$
$$+f^{lmn}\left\{\frac{1}{g^2}(\partial_\mu A_\nu^l)Q_\mu^mQ_\nu^n+\frac{1}{g^2}(\partial_\mu Q_\nu^l)A_\mu^mQ_\nu^n+\frac{1}{g^2}(\partial_\mu Q_\nu^l)Q_\mu^mA_\nu^n+\frac{1}{\xi}(\partial_\mu Q_\mu^l)A_\mu^mQ_\nu^n-\frac{i\kappa}{2}\epsilon_{\mu\nu\rho}A_\mu^lQ_\nu^mQ_\rho^n\right\}$$
$$+2(\varphi^\dagger A_\mu Q_\mu\phi)+2(\phi^\dagger Q_\mu A_\mu\varphi). \qquad (4)$$

Here, $\phi^p=1/\sqrt{2}(\phi_1^p+i\phi_2^p)$, $\varphi^p=(\varphi_1^p+i\varphi_2^p)$, $\hat{\varphi}_a^p=\varphi_a^p/\sqrt{\varphi^2}$, with $p,q=1,2,\ldots,N$ denoting the components of the Higgs field. $m_H^2=4\lambda\varphi^2$, with $\varphi^2=\Sigma_{p,a}(\varphi_a^p)^2=v^2$.

As pointed out in Ref. [16], there could exist in the effective action so-called would-be Chern-Simons terms, which are invariant even under the large gauge transformation and induce terms similar to the Chern-Simons one in the Higgs phase. In fact, one finds there are two such terms relevant to our discussion:

$$O_1=\epsilon^{\mu\nu\rho}i\{\Phi^\dagger T^m(D_\mu\Phi)-(D_\mu\Phi)^\dagger T^m\Phi\}F_{\nu\rho}^m,$$
$$O_2=\epsilon^{\mu\nu\rho}i\{\Phi^\dagger(D_\mu\Phi)-(D_\mu\Phi)^\dagger\Phi\}(\Phi^\dagger F_{\nu\rho}\Phi). \qquad (5)$$

In the Higgs phase, they give rise to

$$\epsilon^{\mu\nu\rho}A_\mu^nF_{\nu\rho}^m\{(\varphi^\dagger T^mT^n\varphi)+(\varphi^\dagger T^nT^m\varphi)\},$$
$$2\epsilon^{\mu\nu\rho}A_\mu^nF_{\nu\rho}^m(\varphi^\dagger T^m\varphi)(\varphi^\dagger T^n\varphi), \qquad (6)$$

respectively.

To extract the correction to the Chern-Simons coefficient, it is helpful to introduce the following projection operators in finding the propagators of the gauge and Higgs fields:

$$(P_1)_{mn}=\delta_{mn}-2[(\hat{\varphi}^\dagger T^mT^n\hat{\varphi})+(\hat{\varphi}^\dagger T^nT^m\hat{\varphi})]+\frac{2(N-2)}{(N-1)}(\hat{\varphi}^\dagger T^m\hat{\varphi})(\hat{\varphi}^\dagger T^n\hat{\varphi}),$$
$$(P_2)_{mn}=2[(\hat{\varphi}^\dagger T^mT^n\hat{\varphi})+(\hat{\varphi}^\dagger T^nT^m\hat{\varphi})]-4(\hat{\varphi}^\dagger T^m\hat{\varphi})(\hat{\varphi}^\dagger T^n\hat{\varphi}),$$
$$(P_3)_{mn}=\frac{2N}{(N-1)}(\hat{\varphi}^\dagger T^m\hat{\varphi})(\hat{\varphi}^\dagger T^n\hat{\varphi}); \qquad (7)$$

$$(R_1)_{ab}^{pq}=\delta_{pq}\delta_{ab}-\hat{\varphi}_a^p\hat{\varphi}_b^q-\epsilon_{ac}\epsilon_{bd}\hat{\varphi}_c^p\hat{\varphi}_d^q,$$
$$(R_2)_{ab}^{pq}=\hat{\varphi}_a^p\hat{\varphi}_b^q,$$
$$(R_3)_{ab}^{pq}=\epsilon_{ac}\epsilon_{bd}\hat{\varphi}_c^p\hat{\varphi}_d^q.$$

It is easy to check that they indeed satisfy

$$P_iP_j=\delta_{ij}P_i;$$
$$R_iR_j=\delta_{ij}R_i. \qquad (8)$$

With these projection operators, it is now straightforward to obtain the propagators of Q_μ and ϕ:

$$\Delta^{mn}_{\mu\nu}(k)=\{[\Delta^1_{\mu\nu}(k)](P_1)_{mn}+[\Delta^2_{\mu\nu}(k)](P_2)_{mn}+[\Delta^3_{\mu\nu}(k)](P_3)_{mn}\},$$

$$D^{pq}_{ab}(k)=\{[D^1(k)](R_1)^{pq}_{ab}+[D^2(k)](R_2)^{pq}_{ab}+[D^3(k)](R_3)^{pq}_{ab}\}. \tag{9}$$

Here,

$$\Delta^1_{\mu\nu}(k)=\frac{g^2(k^2\delta_{\mu\nu}-k_\mu k_\nu)+g^2M\epsilon_{\mu\nu\rho}k^\rho}{k^2(k^2+M^2)}+\frac{\xi k_\mu k_\nu}{k^4},$$

$$\Delta^2_{\mu\nu}(k)=\frac{g^2(k^2+M_{W^+}M_{W^-})(\delta_{\mu\nu}-k_\mu k_\nu/k^2)+g^2M\epsilon_{\mu\nu\rho}k^\rho}{(k^2+M^2_{W^+})(k^2+M^2_{W^-})}+\frac{\xi k_\mu k_\nu}{k^2[k^2+(1/2)\xi\varphi^2]},$$

$$\Delta^3_{\mu\nu}(k)=\frac{g^2(k^2+M_{Z^+}M_{Z^-})(\delta_{\mu\nu}-k_\mu k_\nu/k^2)+g^2M\epsilon_{\mu\nu\rho}k^\rho}{(k^2+M^2_{Z^+})(k^2+M^2_{Z^-})}+\frac{\xi k_\mu k_\nu}{k^2\{k^2+[(N-1)/N]\xi\varphi^2\}}, \tag{10}$$

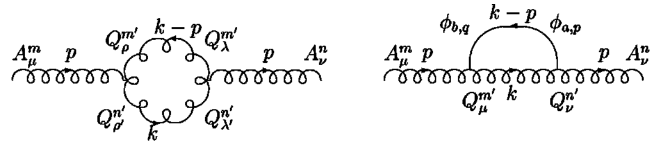
$$D^1(k)=\frac{1}{[k^2+(1/2)\xi\varphi^2]},$$

$$D^2(k)=\frac{1}{(k^2+m_H^2)},$$

$$D^3(k)=\frac{1}{\{k^2+[(N-1)/N]\xi\varphi^2\}},$$

with $M=\kappa g^2$, and

$$M_{W^\pm}=(a_W\pm 1)M/2,\quad a_W=\sqrt{1+\frac{2\varphi^2}{\kappa^2g^2}};$$



(a) (b)

FIG. 1. The one-loop diagrams that contribute to the parity odd part of the vacuum polarization. (a) involves an internal gluon loop, while (b) involves an internal loop with both gluon and Higgs field.

$$M_{Z^\pm}=(a_Z\pm 1)M/2,\quad a_Z=\sqrt{1+\frac{4(N-1)/N\varphi^2}{\kappa^2g^2}}. \tag{11}$$

Note that $\Delta^1_{\mu\nu}$, $\Delta^2_{\mu\nu}$, and $\Delta^3_{\mu\nu}$ correspond to propagators of the unbroken parts W and Z, respectively.

To determine the renormalization of the Chern-Simons coefficient, we must calculate both the quantum corrections of the bilinear and trilinear parts of the Chern-Simons term. Thanks to the background field method, the effective action is expected to have explicit gauge invariance. As a result, it is enough to consider the bilinear part. In fact, to one-loop order only the two graphs in Fig. 1 contribute to the parity odd part of the vacuum polarization. They come from the diagrams with a gauge loop and a gauge-Higgs loop, respectively [15]. Carrying out the algebra we see that

$$[\Pi^{mn}_{\mu\nu}(p)]_{\rm odd}=\epsilon_{\mu\nu\rho}p_\rho\{\Pi_1(p^2)\delta_{mn}+\Pi_2(p^2)[(\hat{\varphi}^\dagger T^mT^n\hat{\varphi})+(\hat{\varphi}^\dagger T^nT^m\hat{\varphi})]+\Pi_3(p^2)(\hat{\varphi}^\dagger T^m\hat{\varphi})(\hat{\varphi}^\dagger T^n\hat{\varphi})\}. \tag{12}$$

It is easy to see that the two would-be Chern-Simons terms only contribute to $\Pi_2(0)$ and $\Pi_3(0)$. Therefore, all we need to calculate is $\Pi_1(0)$ to find the correction to the Chern-Simons coefficient. In the Landau gauge,

$$\Pi_1(p)=\frac{(N-1)}{2}\Pi^{Ia}(p)+\frac{1}{2}\Pi^{Ib}(p),$$

$$\Pi_2(p)=-(N-1)\Pi^{Ia}(p)-\Pi^{Ib}(p)+\frac{N(N-2)}{(N-1)}\Pi^{Ic}(p)+\frac{N}{(N-1)}\Pi^{Id}(p)+\Pi^{IIa}(p)+\Pi^{IIb}(p)+\frac{2N(N-2)}{(N-1)}\Pi^{IIc}(p)+\frac{2}{N(N-1)}\Pi^{IId}(p), \tag{13}$$

$$\Pi_3(p)=\frac{(N-2)}{2}\Pi^{Ia}(p)+\frac{(N+2)}{2}\Pi^{Ib}(p)-\frac{N(N-2)}{(N-1)}\Pi^{Ic}(p)-\frac{N}{(N-1)}\Pi^{Id}(p)-\Pi^{IIa}(p)-\Pi^{IIb}(p)-\frac{2N(N-2)}{(N-1)}\Pi^{IIc}(p)-\frac{2}{N(N-1)}\Pi^{IId}(p)+\frac{2(N-1)}{N}\Pi^{IIe}(p)+2(N-1)\Pi^{IIf}(p).$$

Here,

$$\Pi^{Ia}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[k^2p^2-(k\cdot p)^2][4M^2+10k^2-10k\cdot p+8p^2]}{p^2k^2(k^2+M^2)(k-p)^2[(k-p)^2+M^2]}\right\}+\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[-2k^2p^2-2(k\cdot p)^2+4p^2(k\cdot p)]}{p^2k^2(k^2+M^2)(k-p)^2}\right\},$$

$$\Pi^{Ib}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[k^2p^2-(k\cdot p)^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)[(k-p)^2+M_{W^+}^2][(k-p)^2+M_{W^-}^2]}\right\}$$
$$\times\left\{6M^2+\frac{(k^2+M_{W^+}M_{W^-})[-M^2+8k^2-4k\cdot p+4p^2]}{k^2}+\frac{[(k-p)^2+M_{W^+}M_{W^-}][-M^2+8k^2-12k\cdot p+8p^2]}{(k-p)^2}\right.$$
$$\left.+\frac{(k^2+M_{W^+}M_{W^-})[(k-p)^2+M_{W^+}M_{W^-}][-6k^2+6k\cdot p-4p^2]}{k^2(k-p)^2}\right\}$$
$$+\int\frac{d^3k}{(2\pi)^3}\left\{\frac{-2M(k\cdot p)[M_{W^+}M_{W^-}+2k^2-2p^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}+\frac{M(k^2+M_{W^+}M_{W^-})[-2k^2p^2-2(k\cdot p)^2+4k^2(k\cdot p)]}{p^2k^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}\right\},\tag{14}$$

and all other integrals are given in the Appendix. Note that Π^{Ia} and Π^{Ib} are identical to Eqs. (11) and (12) in Ref. [15], respetively, up to a factor. Since Π^{Ia} and Π^{Ib} only involve the diagram with a gluon loop, the result we obtain here is actually independent of the form of the Higgs potential. In the zero momentum limit,

$$\Pi^{Ia}(0)=\frac{\kappa}{2\pi|\kappa|},$$
$$\Pi^{Ib}(0)=0.\tag{15}$$

In the background field gauge Q_μ does not get renormalized. As a result,

$$\kappa_{\rm ren}=\kappa+\Pi_1(0),$$
$$=\kappa+\frac{(N-1)\kappa}{4\pi|\kappa|}\tag{16}$$

for $N\geqslant 3$, in agreement with the results found in Refs. [13–15]. Although the above result can also be obtained by calculating the parity odd part of the vacuum polarization in the unbroken sector as in Refs. [13–15], this might be particular to the case that the Higgs field is in the fundamental representation.

In the SU(2) case, the gauge symmetry is completely broken and there is no such a thing as unbroken part in the Higgs phase. As a result, all the terms involving $\Delta^1_{\mu\nu}$ should be set to zero and hence the correction to the Chern-Simons coefficient is vanishing consistent with the claim in Ref. [14]. An interesting point is that for SU(2) the group generators T^m's are proportional to the Pauli matrices. Making use of the identity $(\sigma^m\sigma^n+\sigma^n\sigma^m)=2\delta_{mn}$, we see that the first term in Eq. (6) becomes proportional to the Chern-Simons one. This explains why it is impossible to find the right correction to the Chern-Simons coefficient in the conventional calculation.

We can perform similar calculation in the SO(N) case by noting that there $\mathrm{tr}\{T^mT^n\}=2\delta^{mn}$, $\Sigma_m(T^m)_{\alpha\beta}(T^m)_{\gamma\delta}=\delta_{\alpha\delta}\delta_{\beta\gamma}-\delta_{\gamma\alpha}\delta_{\beta\delta}$, and $(\varphi^\dagger T^m\varphi)=0$. The procedure is very similar and we will just give the main result

$$\Pi_1(p)=(N-3)\Pi^{Ia}(p)+\Pi^{Ib}(p).\tag{17}$$

Here, Π^{Ia} and Π^{Ib} are identical to those in Eqs. (14) with $a_W=\sqrt{1+8\varphi^2/\kappa^2g^2}$. Consequently,

$$\kappa_{\rm ren}=\kappa+\frac{(N-3)\kappa}{2\pi|\kappa|}\tag{18}$$

for $N\geqslant 3$. It is interesting to see that for SO(3), which is also the adjoint representation of SU(2), there is no correction in the Higgs phase. For SO(2), the gauge symmetry is again completely broken in the Higgs phase and the correction is also vanishing. This is consistent with the results found in Ref. [17]. Thus, we see that the Abelian result is really just a special case of the non-Abelian ones.

Naturally, it is interesting to see whether this kind of analysis can be applied to the case where the Higgs field is in the adjoint representation. Since there can be several inequivalent vacua in these systems, it is nontrivial to show that the Chern-Simons coefficient is quantized in all the Higgs phases. At this moment, there are at least two difficulties. First, we do not know the form of projection operators such as P_1, P_2, and P_3 in the adjoint representation. Second, there could be an infinite number of would be Chern-Simons terms, e.g., $i\epsilon^{\mu\nu\rho}\mathrm{tr}\{[(\Phi^\dagger)^n,F_{\nu\rho}](D_\mu\Phi^n)-(D_\mu\Phi^n)^\dagger[F_{\nu\rho},\Phi^n]\}$, with n an arbitrary positive integer. As mentioned above, this may also make it impossible for us to find the correction to the Chern-Simons term by calculating only the parity odd part of the vacuum polarization in the unbroken sector.

The author is indebted to K. Lee for helpful communication. This work was supported in part by the National Science Council of the ROC under Grant No. NSC87-2112-M-032-002.

APPENDIX

$$\Pi^{Ic}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[k^2p^2-(k\cdot p)^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2[(k-p)^2+M^2]}\right\}\left\{[5M^2+8k^2-12k\cdot p+8p^2]\right.$$
$$\left.+\frac{(k^2+M_{W^+}M_{W^-})[-M^2+2k^2+2k\cdot p]}{k^2}\right\}+\int\frac{d^3k}{(2\pi)^3}\left\{\frac{-2M(k\cdot p)[k^2-p^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}\right.$$
$$+\frac{M(k^2+M_{W^+}M_{W^-})[-k^2p^2-(k\cdot p)^2+2k^2(k\cdot p)]}{p^2k^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}$$
$$\left.+\frac{MM_{W^+}M_{W^-}[k\cdot p-p^2]+M[-k^2p^2-(k\cdot p)^2+2p^2(k\cdot p)]}{p^2k^2(k-p)^2[(k-p)^2+M^2]}\right\},$$

$$\Pi^{Id}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M[k^2p^2-(k\cdot p)^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)[(k-p)^2+M_{Z^+}^2][(k-p)^2+M_{Z^-}^2]}\right\}$$
$$\times\left\{6M^2+\frac{(k^2+M_{W^+}M_{W^-})[-M^2+8k^2-4k\cdot p+4p^2]}{k^2}+\frac{[(k-p)^2+M_{Z^+}M_{Z^-}][-M^2+8k^2-12k\cdot p+8p^2]}{(k-p)^2}\right.$$
$$\left.+\frac{(k^2+M_{W^+}M_{W^-})[(k-p)^2+M_{Z^+}M_{Z^-}][-6k^2+6k\cdot p-4p^2]}{k^2(k-p)^2}\right\}+\int\frac{d^3k}{(2\pi)^3}\left\{\frac{-M(k\cdot p)[M_{W^+}M_{W^-}+2k^2-2p^2]}{p^2(k^2+M_{Z^+}^2)(k^2+M_{Z^-}^2)(k-p)^2}\right.$$
$$+\frac{-M(k\cdot p)[M_{Z^+}M_{Z^-}+2k^2-2p^2]}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}+\frac{M(k^2+M_{W^+}M_{W^-})[-k^2p^2-(k\cdot p)^2+2k^2(k\cdot p)]}{p^2k^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}$$
$$\left.+\frac{M(k^2+M_{Z^+}M_{Z^-})[-k^2p^2-(k\cdot p)^2+2k^2(k\cdot p)]}{p^2k^2(k^2+M_{Z^+}^2)(k^2+M_{Z^-}^2)(k-p)^2}\right\}. \tag{A1}$$

In each of the above expressions, the first integral is identical to that of the corresponding Feynman diagram in the usual Landau gauge, and the second integral comes from the combining effect of the ghost and unphysical Higgs bosons:

$$\Pi^{IIa}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M(g^2\varphi^2)(k\cdot p)}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)[(k-p)^2+m_H^2]}\right\},$$

$$\Pi^{IIb}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M(g^2\varphi^2)(k\cdot p)}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}\right\},$$

$$\Pi^{IIc}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M(g^2\varphi^2)(k\cdot p)}{p^2k^2(k^2+M^2)(k-p)^2}\right\}, \tag{A2}$$

$$\Pi^{IId}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M(g^2\varphi^2)(k\cdot p)}{p^2(k^2+M_{Z^+}^2)(k^2+M_{Z^-}^2)(k-p)^2}\right\},$$

$$\Pi^{IIe}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M(g^2\varphi^2)(k\cdot p)}{p^2(k^2+M_{Z^+}^2)(k^2+M_{Z^-}^2)[(k-p)^2+m_H^2]}\right\},$$

$$\Pi^{IIf}(p)=\int\frac{d^3k}{(2\pi)^3}\left\{\frac{M(g^2\varphi^2)(k\cdot p)}{p^2(k^2+M_{W^+}^2)(k^2+M_{W^-}^2)(k-p)^2}\right\}.$$

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