Cosmological solutions in M and string theory

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We consider solutions to the cosmological equations of motion in 11 dimensions with and without 4-form charges. We show explicitly the correspondence between some of these solutions and known solutions in 10 dimensional string gravity. New solutions involving combinations of 4-form charges are explored. [S0556-2821(98)03908-3]

I. INTRODUCTION

There is little doubt or discussion that classical Friedmann-Robertson-Walker (FRW) cosmology provides an excellent description of the evolving Universe at late times (say nucleosynthesis and beyond). Indeed, there is little reason to doubt the validity of FRW cosmology back to very early times corresponding to the grand unified theory (GUT) epoch. At some point however, it is reasonable to suppose that Einstein gravity is modified, and at present, the only consistent modification available is due to string theory. A strong argument in favor of such a modification is that it is not possible to bring Einstein's general relativity in full accord with quantum mechanics, and hence, accepting the postulates of quantum mechanics, altering the theory of gravity seems inevitable. In the regime of large curvatures these alterations should be expected to play a very significant role. Since the singularity theorems of Hawking and Penrose [1] state that such large curvature regions are a generic state of the early Universe, it then seems plausible to assume that in this epoch the effects of quantum gravity strongly influence the evolution of the Universe. There have been numerous efforts attempting to explore the effects of string theory in cosmology [2–21]. One could characterize the main aims of many of these studies as either an attempt to utilize the additional degrees of freedom in the massless sector to induce inflation [2,3,6,7,8,10,11] or developing arguments on the type of modifications to Einstein gravity which are necessary to avoid a cosmological singularity [9,13,14,16,18,19,20].

Much of the emphasis in previous work on string cosmology has been on the modifications to Einstein gravity [22]. The modifications were accommodated either by enlarging the zero mass sector with the inclusion of the dilaton and axion/moduli fields or by considering higher curvature terms described by the low energy string action. Dilaton/axion cor-

rections do indeed have an effect on the equations of motion at early times and lead to new cosmological solutions. They are not however very conducive to a de Sitter, or a more general inflationary phase, at least not without resorting to supersymmetry breaking potentials for trapping the dilaton [3]. As an alternative, much effort has been invested in trying to resolve the standard cosmological problems with a pre-big-bang phase [11], which appears in the general space of solutions to the string dilaton-gravity system. It still remains to be seen, whether or not such models can successfully solve all of the problems normally associated with inflation and produce density perturbations consistent with the Cosmic Background Explorer (COBE) measurements of the microwave background anisotropy.

Another interest of string gravity is the problem of an initial (or final) cosmological singularity. The attempts to address it included the use of winding modes wrapped around spatial directions [9], higher-derivative–higher-genus induced corrections [13], decompactification to higher dimensions with simultaneous insertion of D-brane type matter sources [23], instanton-like constructions [24], and models with generalized scalar-tensor couplings, [25–33]. Though we know that the simple types of dilaton-axion modifications to Einstein gravity considered up to now are not capable of removing singularities [18,19,33], progress has been made concerning the form of the corrections needed [20]. Any such solution (at least from the 10D point of view) must rely on non-perturbative features of the gravitational action or come from some more complete theory of gravity at the string scale.

Out of the morass of different weakly, and strongly, coupled string theories, M theory is emerging as the single underlying theory capable of unifying all particle interactions [34,35]. At this time, our understanding of M theory is still incomplete. While its various low energy limits, and the links between them, are known (which are the consistent string theories and the 11D supergravity, related by the web of dualities), the full description of the theory is still being sought for. A candidate that has been proposed recently is the M(atrix) theory, formulated as a large N-limit supersym-

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metric matrix quantum mechanics [36]. An interesting shift of the point of view that has emerged out of these developments is that the dilaton scalar field, present in all known string theories, has been demoted to merely another modulus field in the 11D supergravity. This could have important consequences for cosmological applications. The troubles with implementing conventional inflationary scenarios in string theory arise because of the dilaton and its couplings to the other modes in the string spectrum. In the arena of 11D supergravity, such couplings are absent. Some of the obstacles for inflation in dilaton-plagued string theories could perhaps be resolved by way of M theory. Let us be more specific: the extreme weak and strong coupling limits in string theory formulations correspond to the regimes where the size of the eleventh dimension becomes very small or very large. These limits sit in rather special portions of the phase space of the full theory, and perhaps should be viewed as unnatural. Indeed, there seems to be no reason why at a given very large energy scale the dynamics should treat any direction in the Universe any differently than the others. On the other hand, the present knowledge of the low-energy limits of M theory does not seem to prefer one construction over the other. Since the limits where the moduli attain their extremes do exist within reach of solutions in the phase space, it then seems logical to see if they can be dynamically understood from the M-theory point of view. For example, we can imagine a scenario in string theory where in the limit when the moduli converge towards their extrema, the effective stringy description must be lifted to 11 dimensions, where some intrinsically M-theoretic (and as yet unspecified) mechanism saturates moduli evolution. It might be possible to find some inflationary scenario in this limit. This scenario could work in string theories essentially because of duality, albeit the mechanism might take a different guise there.[1]

At his moment, we are far from being able to address comprehensively the questions we pose above. However, we can at least consider the known string cosmologies from the advantage of the 11$^{\text{th}}$ dimension. We will therefore concentrate here on the oxidation of 10D type IIA superstring theory cosmologies to 11D supergravity theory with the geometric reinterpretation of the string coupling. There have been several interesting papers directly exploring cosmological solutions based on the M-theory-inspired action [29] and making use of string dualities [30]. Furthermore, many of the earlier investigations of cosmological models with higher-rank form-field charges in superstring models [31] can be directly incorporated into the framework of the 11D supergravity by dimensionally oxidizing the solutions. In this way, one obtains a description of the known string cosmologies, which treats the dilaton field on equal footing with the other

moduli fields. An immediate consequence of this approach is that by means of U duality one can flow between different M-theory configuration, as exemplified in [32]. The possible advantages of such a *modular democracy* remain to be investigated further.

Below we will survey several classes of cosmological solutions of the 11D theory which can be reduced to the solutions of string dilaton gravity. We will give the explicit relationship of the string and M-theory solutions where applicable. We will also study a case when the 4-form field strength carries two different charges, a magnetic and an electric one, which do not correspond to any of the combinations of form-field charges studied in reduced models so far (but can be given a string theory interpretation via Scherk-Schwarz dimensional reduction). Most of the solutions still feature the unattractive properties of their lower-dimensional stringy relatives, in that they are singular and have running moduli, and hence cannot be used for building an inflationary scenario by themselves. However, we will show a special example where a flat space in 11 dimensions, viewed by an observer accelerated along one of the circles, reduces to a singular dilatonic string cosmology. This example is not really inflation, but it moderates the singularity by dimensional uplift.

II. STRING THEORY ACTIONS AND EQUATIONS OF MOTIONS

Before we begin our study of the general M-theory inspired, 11D supergravity action, it will be useful to review some of the salient features of the effective field theory formulation of string gravity as it pertains to cosmological solutions. Neglecting for now the contribution to dynamics from the 6D Calabi-Yau space, we can begin with the lowest order 4D effective action of the Neveu-Schwarz–Neveu-Schwarz (NS-NS) sector of any string construction [22]:

$$S = \int d^4x \sqrt{g} e^{-2\phi} \left\{ R + 4(\nabla\phi)^2 - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} + 2\Lambda \right\}. \quad (1)$$

The 3-form $H = dB$ is the field strength of the Kalb-Ramond 2-form $B_{\mu\nu}$. The stringy cosmological constant Λ can arise from central charge deficit in conformal field theory constructions or by the reduction of higher rank form fields, as we will see later. In four dimensions, this 3-form is dynamically dual to a pseudoscalar axion field. The correspondence is given by

$$H_{\mu\nu\lambda} = \sqrt{2} e^{2\phi} \sqrt{g} \epsilon_{\mu\nu\lambda\rho} \partial^\rho \chi \quad (2)$$

resulting in the replacement of the 3-form kinetic term coupled to the inverse string coupling $e^{-2\phi}$ by the pseudoscalar kinetic term coupled to the string coupling itself. By means of a simple conformal rescaling, $g_{\mu\nu} \to e^{2\phi} g_{\mu\nu}$, the action can be put into the Einstein frame, where the Planck mass is constant (for simplicity, we set $2\kappa^2 = 1$):

$$S = \int d^4x \sqrt{\bar{g}} \{ \bar{R} - 2(\nabla\phi)^2 - 2e^{4\phi}(\nabla\chi)^2 + 2\Lambda e^{2\phi} \}. \quad (3)$$

With the FRW spatially flat ansatz for the metric,

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[1]In spirit, this would be similar to the proposed (but to date unknown) scenario for solving the cosmological constant problem in string theory. The idea is to find a duality relationship between a string vacuum with unbroken supersymmetries and a vacuum with all supersymmetries broken. Then duality would guarantee that the cosmological constant, which is zero in the vacuum with unbroken SUSY's must also vanish in the vacuum without manifest SUSY's. Perhaps it is possible to hide inflation in string theory in this sense too.

$$ds^2 = -n^2(t)dt^2 + a^2(t)d\vec{x}^2 \qquad (4)$$

where n is a gauge parameter (lapse function), we can easily derive the equations of motion from the action (1) (see e.g. [19]). They are

$$\dot{h} = 2h\dot{\phi} - 3h^2 + \rho$$

$$\dot{\rho} + 6h\rho = 0$$

$$2\dot{\phi}^2 + 3h^2 - 6h\dot{\phi} - \rho/2 + \Lambda = 0 \qquad (5)$$

where $h = \dot{a}/a$ is the string-frame Hubble parameter, and $\rho = e^{4\phi}\dot{\chi}^2$ is the effective energy density of the pseudo-scalar axion field, and in the gauge $n = 1$.

These equations of motion are straightforward to solve. Many authors have already considered various aspects of the solutions, both in the Einstein frame and in the string frame. The simplest case is certainly the pure metric-dilaton solution with vanishing cosmological constant [4,5,7,8,11,15], which is given by two classes of solutions separated by the curvature singularity, used in [11] to construct the pre-big-bang scenario:

$$ds_+^2 = -dt^2 + a_0^2\left|\frac{t}{t_0}\right|^{\mp 2/\sqrt{3}}d\vec{x}^2, \quad e^{-2\phi} = e^{-2\phi_0}\left|\frac{t}{t_0}\right|^{1\pm\sqrt{3}}$$

$$(t<0)$$

$$ds_-^2 = -dt^2 + a_0^2\left(\frac{t}{t_0}\right)^{\pm 2/\sqrt{3}}d\vec{x}^2, \quad e^{-2\phi} = e^{-2\phi_0}\left(\frac{t}{t_0}\right)^{1\mp\sqrt{3}}$$

$$(t>0). \quad (6)$$

In the string frame, both branches consist of two classes of solutions: expanding and contracting. The solutions for $t<0$ are by now widely referred to as the $(+)$ branch, and those for $t>0$ as the $(-)$ branch. The proper definition of branches is derived from solving the quadratic constraint equation in (5) for $\dot{\phi}$. The sign of each branch is determined by the sign of the square root which arises in the solution, where the discriminant is not zero. If the discriminant vanishes anywhere on the phase space trajectory, the branches connect there. The solutions are isotropic T duals of each other. A goal of the pre-big-bang scenario is the connection of the expanding solutions in the two branches, in such a way that the $(+)$ branch chronologically precedes the $(-)$ branch and hence the singularity would be removed. In spite of some recent results [20], it still remains to be seen if a coherent and fully consistent description of branch-changing can be found. It is interesting to note that in the E frame, both the expanding and contracting metrics degenerate to a single Einstein frame metric, and that the only difference between the two subclasses of solutions is the sign of the dilaton field. Since in the Einstein frame the switch of the sign of the dilaton corresponds to the classical form of the S-duality map, these solutions are also S duals of each other.

Further generalizations of these solutions can be easily obtained with the help of generating techniques. For example, if we add the cosmological constant, we can find new solutions starting from (6) and applying a solution-generating technique described in [37]. Moreover, we can obtain solutions with the axion field if we apply an SL(2,R) duality rotation to (6), as described in [38,17]. These solutions can be written in terms of the functions a and $\exp(-2\phi)$ describing the axion-less case. Following Copeland *et al.* [17], they are

$$d\bar{s}^2 = (s^2 + r^2 e^{-4\phi})(-dt^2 + a^2(t)d\vec{x}^2)$$

$$e^{2\bar{\phi}} = s^2 e^{2\phi} + r^2 e^{-2\phi}, \quad \bar{\chi} = \frac{qs + pre^{-4\phi}}{s^2 + r^2 e^{-4\phi}} \qquad (7)$$

where p, q, r and s are real numbers satisfying $ps - qr = 1$. The 3-form axion field is $H = Q_A d^3\vec{x}$, in form notation, where the constant charge is determined by the integration constants. The solutions can be readily generalized to include additional scalar moduli, which arise from the reduction of the 10D string theories. Note that for these solutions each branch now contains only one congruence of the system trajectories. This is because as the string frame scale factor approaches zero, the dominant source in the equations of motion is the axion, since its contribution to the total stress energy goes as $1/a^6(t)$. This term then forces the Universe to bounce away from zero volume and start expanding again. Hence the qualitative picture of evolution in both branches is that the Universe begins in a stage of contraction, reaches its minimal volume and starts expanding again to infinity. One should note however, that such an axion-driven bounce does not allow one to evade the cosmological singularity. Perhaps the easiest way to see this is to note that the bounce occurs at some small but finite value of the scale factor and some large but still finite value of the coupling. When it occurs, the bounce changes only the sign of the Hubble parameter and not the sign of the $\dot{\phi}$ and therefore the coupling continues to grow. The curvature singularities reside in the regime of very large coupling which therefore can still be attained in the axionic cosmologies. Since the Universe is now expanding, the axion's contribution to ρ is red-shifted away, and eventually the Universe becomes dilaton dominated and therefore must inevitably run away towards the singularity. In other words, the bounce occurs at finite but negative t and still evolves towards the singularity at $t = 0$.

Finally, we can obtain solutions with spatial curvature, either by directly solving differential equations for models with spatially curved sections [8,15,21] or by using a Wick rotation and a dimensional reduction of the 5D Schwarzschild black hole solutions [28]. The equations of motion including spatial curvature (but for simplicity excluding the stringy cosmological term and the axion contribution) are

$$\dot{h} = 2h\dot{\phi} - 3h^2 - 2\frac{k}{a^2}, \quad 2\dot{\phi}^2 + 3h^2 - 6h\dot{\phi} + 3\frac{k}{a^2} = 0 \qquad (8)$$

and the generic solutions are given [8,28,21]

$$ds^2 = \mu(\mathcal{C}^{1\pm\sqrt{3}}(\vartheta + \vartheta_0))(\mathcal{S}^{1\mp\sqrt{3}}(|\vartheta + \vartheta_0|))(-d\vartheta^2 + d\Omega_k)$$

$$e^\phi = \left(\frac{\mathcal{C}(\vartheta + \vartheta_0)}{\mathcal{S}(|\vartheta + \vartheta_0|)}\right)^{\pm\sqrt{3}} \qquad (9)$$

where $d\Omega_k$ is the metric on the maximally symmetric 3D spaces with constant curvature k, and $\mathcal{C}$ and $\mathcal{S}$ are the trigonometric or hyperbolic cosine and sine, depending on whether $k=1$ or $k=-1$, respectively. The parameters μ and ϑ_0 are integration constants. We should mention here that there also exist special solutions for $k=1$ cases, when the string frame scale factor depends linearly on the comoving time [5]. Note here that in contrast to the spatially flat models, the curved ones do not have an infinite amount of time available for pole expansion. Rather, the closed $k=1$ solutions emerge out of the spatial curvature-controlled singularity [(+) branch] or end up in it [(−) branch], while the open $k=-1$ solutions begin in a contracting phase, and only rebound later, pole-expanding for a finite amount of time before hitting the curvature singularity. This has been used recently to argue that pre-big-bang viewed as inflation suffers from a fine tuning problem [21].

As we have seen above, the curvature singularity which separates the (+) and (−) branches shows that near it the cosmological evolution is dominated by the dilaton field [33], where the string coupling $\lambda_s = \exp(\phi)$ diverges, and all other degrees of freedom become irrelevant. Hence, all the solutions in this regime are extremely well approximated by the pure metric-dilaton configuration. Recently attempts have been made to dampen this singularity with the higher derivative and/or higher genus contributions to the equations of motion [13,20,39,40]. However, it has also been noticed [18] that there exist solutions in the model proposed by Damour and Polyakov [14] where the effective coupling function also diverges, but the strong coupling limit string metric remains completely smooth. In this context, the Damour-Polyakov universality ansatz amounts to replacing the factor $e^{-2\phi}$ by a function $B(\phi)=e^{-2\phi}+c_0+c_1e^{2\phi}+\cdots$ in Eq. (1). An action of this form was considered in [18] in an attempt to achieve a graceful exit from a pre-big-bang phase in the dilaton-gravity cosmological evolution. Such a solution was indeed found, however, with the unpleasant aspect that though there were no space-time singularities in the solution, there was a point in the evolution in which the function $B(\phi)$ changes sign corresponding to a signature change in the metric.

From another perspective, extensions of the standard cosmological solutions to higher dimensions have also been considered [41,42]. There is a wide variety of motivations for such considerations which we will not attempt to review here. Most of them (in a cosmological setting) are based on an ansatz for the metric of the form

$$ds^2=-dt^2+a^2(t)g_{ij}dx^idx^j+b^2(t)g_{mn}dx^mdx^n \quad (10)$$

where g_{ij} is assumed to be a maximally symmetric 3-space and g_{mn} some other metric describing the d compactifed dimensions. Considerable effort was expended to investigate the possibilities that such a system could account for inflation, whereby the FRW portion of the metric expands exponentially (or fast enough) at the expense of the remaining d dimensions. Also, if we assume that the original theory which we want to study (10) is devoid of the dilaton field, we could retrieve the dilaton in lower dimensions. Upon compactification, it can be seen that the moduli from higher dimensions can play the role of the dilaton, which was hoped to be identified with the inflaton. An approach of this kind,

which most closely resembles the system we will study below, is that of [43] based on 10D supergravity with the action Eq. (3). If we assume g_{mn} to be maximally symmetric as well, and work in the Einstein frame, where the effective dilaton field has the canonical kinetic term, the correct cosmological equations of motion with the dilaton as the only matter field can be simply written as

$$3\frac{\ddot{a}}{a}+d\frac{\ddot{b}}{b}=-2\dot{\phi}^2 \quad (11)$$

$$\frac{\ddot{a}}{a}+2\frac{\dot{a}^2}{a^2}+\frac{2k}{a^2}+d\frac{\dot{a}}{a}\frac{\dot{b}}{b}=0 \quad (12)$$

$$\frac{\ddot{b}}{b}+(d-1)\frac{\dot{b}^2}{b^2}+\frac{(d-1)k_d}{b^2}+3\frac{\dot{a}}{a}\frac{\dot{b}}{b}=0 \quad (13)$$

$$\ddot{\phi}+\left(3\frac{\dot{a}}{a}+d\frac{\dot{b}}{b}\right)\dot{\phi}=0. \quad (14)$$

A more detailed inspection of these equations shows that regardless of the frame, the solutions for the scale factors behave as powers of the comoving time [17,44]. In addition, they are all singular, and hence can still be grouped into different branches, much like when the internal space is constant. Thus, the system (11)–(14) does not admit conventional de Sitter inflationary solutions. [The de Sitter solution in [43] stems from a simple sign mistake in (11).]

III. TRANS-DIMENSIONAL COSMOLOGY

As we have indicated above, our main goal in this paper is to examine the cosmological implications of the oxidation of 10D string theory to 11D M theory in which we will interpret the string coupling $e^{-2\phi}$ as the scale factor of the 11$^{\text{th}}$ dimension [34,35]. Our starting point therefore, will be the 11D supergravity action

$$S=\int d^{11}x\sqrt{g}$$

$$\times\left\{R-\frac{1}{48}F^2_{\mu_1\ldots\mu_4}\right.$$

$$-\frac{1}{(4!)^2(3!)^2}\frac{\epsilon^{\mu_1\ldots\mu_3\nu_1\ldots\nu_4\lambda_1\ldots\lambda_4}}{\sqrt{g}}$$

$$\left.\times A_{\mu_1\ldots\mu_3}F_{\nu_1\ldots\nu_4}F_{\lambda_1\ldots\lambda_4}\right\} \quad (15)$$

where R is the scalar curvature of the 11D metric, and $A_{\mu_1\ldots\mu_3}$ and $F_{\nu_1\ldots\nu_4}=4\partial_{[\nu_1}A_{\nu_2\ldots\nu_4]}$ are the 3-form potential and its 4-form field strength. The last term is the Chern-Simons term for A. Our conventions are $g_{\mu\nu}=\text{diag}(-1,\vec{1}_{10})$, $R^\mu_{\nu\lambda\sigma}=\partial_\lambda\Gamma^\mu_{\nu\sigma}-\ldots$, $A=(1/3!)A_{\mu_1\ldots\mu_3}dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_3}$ and $F=dA=(1/4!)F_{\mu_1\ldots\mu_4}dx^{\mu_1}\wedge\ldots\wedge dx^{\mu_4}$. We choose units such that in the E frame we have $16\pi G_N=1$.

Before we investigate the equations of motion coming from the action (15), let us first reduce to 10D to further clarify our notation which will follow closely that of Witten [34]. Assuming that the 11th direction is compact, we can carry out Kaluza-Klein reduction of (15) to find

$$S = \int d^{10}x \sqrt{g_{10}} \mathcal{R}_{11} \left\{ R_{10} - \mathcal{R}_{11}^2 \frac{1}{4} F_{KK}^2 \right.$$
$$- \frac{1}{48} \bar{F}_{\mu_1 \ldots \mu_4}^2 - \frac{1}{12} \mathcal{R}_{11}^{-2} H_{\mu\nu\lambda}^2$$
$$\left. - \frac{1}{384} \frac{\epsilon^{\mu_1\mu_2\nu_1 \ldots \nu_4\lambda_1 \ldots \lambda_4}}{\sqrt{g}} B_{\mu_1\mu_2} F_{\nu_1 \ldots \nu_4} F_{\lambda_1 \ldots \lambda_4} \right\}$$

(16)

where $F_{KK\,\mu\nu} = 2\partial_{[\mu} V_{\nu]}^{11}$ is the field strength of the Kaluza-Klein gauge field coming from the metric and the reduced 2-form is $B_{\mu\nu} = A_{\mu\nu 11}$, and its 3-form field strength is $H_{\mu\nu\lambda} = \nabla_\mu B_{\nu\lambda} +$ cyclic permutations. The reduced 4-form field strength $\bar{F}$ acquires Chern-Simons type couplings to the reduced 2- and 1-forms: $\bar{F}_{\mu\nu\lambda\sigma} = F_{\mu\nu\lambda\sigma} + (A_\mu H_{\nu\lambda\sigma} +$ cyclic permutations). After a conformal rescaling $g_{10} = \mathcal{R}_{11}^{-1} g_s$, and defining the dilaton by $\exp(2\phi/3) = \mathcal{R}_{11}$, we find

$$S = \int d^{10}x \sqrt{g_s} \left\{ e^{-2\phi} \left(R_s + 4(\nabla\phi)^2 - \frac{1}{12} H^2 \right) \right.$$
$$- \frac{1}{48} \bar{F}_{\mu_1 \ldots \mu_4}^2 - \frac{1}{4} F_{KK}^2$$
$$\left. - \frac{1}{384} \frac{\epsilon^{\mu_1\mu_2\nu_1 \ldots \nu_4\lambda_1 \ldots \lambda_4}}{\sqrt{g}} B_{\mu_1\mu_2} F_{\nu_1 \ldots \nu_4} F_{\lambda_1 \ldots \lambda_4} \right\}.$$

(17)

This is precisely the effective action which describes the low energy limit of the IIA superstring. We can recognize the first three terms as the NS-NS sector of the theory, and the remaining ones as the Ramond-Ramond (RR) sector. It is easy to rewrite this action in the ten-dimensional Einstein frame, by a further conformal rescaling $g_s = e^{\phi/2} g_E$. The action (17) can be reduced further to make contact with type IIB and heterotic theories.

Since we want to relate the M-theory cosmological solutions to the stringy cosmologies studied so far, we will assume that the base manifold is split into

$$\mathcal{M}_{11} = \mathcal{R}_t \times \mathcal{M}_3^{k=0} \times \mathcal{S}^1 \times \mathcal{M}_6^{CY} \qquad (18)$$

where $\mathcal{R}_t \times \mathcal{M}_3^{k=0}$ is the spatially flat 4D FRW Universe, $\mathcal{S}^1$ is a circle corresponding to the 11th dimension, and $\mathcal{M}_6^{CY}$ is some Calabi-Yau manifold, whose specifics are not necessary for our purposes here. Here we will ignore all graviphotons which could arise from a generic dimensional reduction of the metric. First, such degrees of freedom cannot arise from mixing the Calabi-Yau sector with the space-time, since the topology of Calabi-Yau spaces does not support harmonic 1-forms, that would be needed to carry the reduced

gauge fields. Furthermore, while we in principle can obtain an U(1) field from the metric, which describes D0 branes in the resulting type IIA theory, at this time we wish to consider only gauge-neutral cosmological solutions. This is because our focus here is on making the M-theory reinterpretation of the known string cosmological solutions. Indeed, such setting of the gauge field to zero is consistent with the equations of motion, which are homogeneous in the cross terms. The ansatz for the metric is then

$$ds^2 = -n^2(t)dt^2 + a^2(t)d\vec{x}^2 + b^2(t)G_{MN}^{CY}(y)dy^M dy^N$$
$$+ \mathcal{R}_{11}^2(t)d\varphi^2 \qquad (19)$$

where we again retain the lapse function $n^2(t)$ since we will work in the action. The functions $a(t)$, $b(t)$ and $\mathcal{R}_{11}(t)$ are the radii of the three subspaces $\mathcal{M}_3^{k=0}$, $\mathcal{M}_6^{CY}$ and S^1 of (18), respectively, from the point of view of the 11D observer. After the reduction to 10D, $\mathcal{R}_{11}$ is related to the string coupling constant. $G_{MN}^{CY}(y)$ in (19) is the metric on the Calabi-Yau 3-fold, which depends only on the Calabi-Yau coordinates y^M. We will not need the explicit dependence of G^{CY} on y in this work. It will be sufficient to keep in mind that this metric is Ricci-flat, i.e. that $\mathcal{R}_{MN} = 0$. Still, this subspace influences the overall dynamics because it is nontrivially warped in 11D, via the scale factor $b(t)$.

We will also ignore the possibility that the Calabi-Yau factor in (18) can have nontrivial harmonic two-forms which can support additional reduced U(1) gauge fields. Here F will be completely supported by the remainder of the base manifold, i.e. the $\mathcal{R}_t \times \mathcal{M}_3^{k=0} \times \mathcal{S}^1$ subspace. The Bianchi identity for F is $dF = 0$, which follows from the definition $F = dA$, and in terms of the components, becomes $\partial_{[\rho} F_{\mu\nu\lambda\sigma]} = 0$. By varying the action (15) with respect to $A_{\nu\sigma}$ we find the propagation equation for F. The gauge dynamics is therefore determined by

$$\nabla_\mu F^{\mu\nu\lambda\sigma} = \frac{1}{32(3!)^2} \frac{\epsilon^{\nu\lambda\sigma\mu_1 \ldots \mu_4\rho_1 \ldots \rho_4}}{\sqrt{g}} F_{\mu_1 \ldots \mu_4} F_{\rho_1 \ldots \rho_4}$$

$$\partial_{[\rho} F_{\mu\nu\lambda\sigma]} = 0. \qquad (20)$$

If we restrict our attention to those configurations where F lives only in the $\mathcal{R}_t \times \mathcal{M}_3^{k=0} \times \mathcal{S}^1$ subspace, and require that the three-space $\mathcal{M}_3^{k=0}$ is isotropic, we see that F must be proportional to an exterior product of the volume form on $\mathcal{M}_3^{k=0}$ and a one-form. The two linearly independent possibilities for this one-form are $d\varphi$ and dt. Hence we are left with F_{0ijk} and $F_{\varphi ijk}$. It is quite clear that one cannot consider a case of non-zero $F_{0\varphi jk}$, because of the symmetry of the three-space $\mathcal{M}_3^{k=0}$. There are no invariant antisymmetric tensors of rank 2 (as well as rank 1) in $\mathcal{M}_3^{k=0}$ and because of this any non-zero value of $F_{0\varphi jk}$ will destroy the symmetry of the three-dimensional space. If we take into account the 6D Calabi-Yau indices, we can write down objects like $F_{0\varphi AB}$ where A, B are in the Calabi-Yau subspace, which has harmonic two-forms and hence admit such terms. However, from the point of view of 4 dimensions, terms of this type behave just like scalar fields, after we reduce on φ. Their contributions are in principle consistent with the presence of a maximally symmetric subspace in four dimensions, and

there is no *a priori* reason to rule out such terms. Nevertheless, here we will for simplicity set these terms equal to zero.

With this, we see that in form notation, we can write down the 4-form as follows:

$$F = F_{0123} dt \wedge \Omega_3 + F_{\varphi 123} d\varphi \wedge \Omega_3. \tag{21}$$

Here $\Omega_3 = d^3\vec{x}$ is the comoving volume form of the three-space $\mathcal{M}_3^{k=0}$. Since the Chern-Simons source in the first of Eq. (20) is proportional to $F \wedge F$, it is always zero for the backgrounds we consider, and hence we will ignore it from now on [30].

Let us now solve the equations for F. For F_{0ijk}, since we assume that it depends only on t, the Bianchi identity is vacuous, and so are the equations $\nabla_\mu F^{\mu 0jk} = 0$. The remaining Euler-Lagrange equation yields

$$\nabla_\mu F^{\mu ijk} = \frac{1}{\sqrt{g}} \partial_\mu(\sqrt{g} F^{\mu ijk}) = -\frac{1}{a^3 b^6 \mathcal{R}_{11}} \frac{d}{dt}\left(\frac{b^6 \mathcal{R}_{11}}{a^3} F_{0ijk}\right)$$

$$= 0 \tag{22}$$

and so,

$$F_{0ijk} = P \frac{a^3}{b^6 \mathcal{R}_{11}} \epsilon_{0ijk} \tag{23}$$

where P is a constant of integration and ϵ_{0ijk} is the 4D Levi-Civita symbol. We recognize this as the monopole ansatz of Freund and Rubin [45], and also Englert [46]. Similar cosmological backgrounds were considered in [42]. We can now look at the other mode, $F_{\varphi ijk}$. The Euler-Lagrange equations in (20) are trivial, since they contain only derivatives with respect to φ and x^k. However, the Bianchi identity for this mode gives

$$\dot{F}_{\varphi ijk} dt \wedge d\varphi \wedge \Omega_3 = 0 \tag{24}$$

and hence

$$F_{\varphi 123} = Q \tag{25}$$

where Q is another integration constant. In terms of the potential A_{ijk}, we can write this mode as $A = Q\varphi\Omega_3$. The linear dependence of the potential on the compact coordinate means that this solution is the Scherk-Schwarz mode of A, which corresponds to a 4-brane wrapped around the circle φ. These generalized reductions were considered as means of breaking supersymmetry [47], and in contemporary developments have found the interpretation of p-branes wrapped around longitudinal tori [48]. The combined solution therefore is

$$F = \frac{P}{b^6 \mathcal{R}_{11}} a^3 dt \wedge \Omega_3 + Q d\varphi \wedge \Omega_3. \tag{26}$$

The Einstein equations of motion which are obtained by the variation of the action (15) with respect to the 11D metric $g_{\mu\nu}$ are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{12} F_{\mu\lambda\sigma\rho} F_\nu{}^{\lambda\sigma\rho} - \frac{1}{96} g_{\mu\nu} F^2_{\lambda\sigma\rho\omega} \tag{27}$$

because the Chern-Simons term in (15) does not depend on the metric, and hence does not contribute to the stress-energy tensor of F. However, it is much simpler to work in the action, since the background (19), (26) depends nontrivially only on time t. We therefore dimensionally reduce the 11D action (15) to a 1D one, and then vary it with respect to the independent degrees of freedom a, b and $\mathcal{R}_{11}$, and the lapse n. We can ignore the Chern-Simons term, since it does not contribute to either the equations of motion for F [by virtue of (21)] or the gravitational equations of motion, as we see from (27). To proceed, we first need the Ricci scalar of $g_{\mu\nu}$. The easiest way to find it is to use the tangent space representation, which is given in terms of the 11-bein

$$ds^2 = \eta_{ab} e^a e^b, \quad e^0 = n dt, \quad e^k = a(t) dx^k,$$

$$e^M = b(t) E^M, \quad e^\varphi = \mathcal{R}_{11} d\varphi \tag{28}$$

and E^M are the internal 6-bein of the Calabi-Yau 3-fold $\mathcal{M}^{CY}$, such that $G^{CY}_{MN} = \delta_{KL} E^K{}_M E^L{}_N$. The capital latin indices run from 1 to 6 and δ_{KL} is just the 6×6 unit matrix. This "bastard" split of the 11-bein is similar to what is used in the studies of the more complicated non-diagonal Bianchi models [49]. The next step is to determine the spin connection 1-forms ω_{ab}. Since we assume that the background (19) is torsion-less, we can use $de^a = -\omega^a{}_b \wedge e^b$ to find the spin connection. This gives $\omega^k{}_0 = (a'/na)e^k$, $\omega^M{}_0 = (b'/nb)e^M$, $\omega^\varphi{}_0 = (\mathcal{R}'_{11}/n\mathcal{R}_{11})e^\varphi$ where the prime denotes derivatives with respect to time. Defining the internal Calabi-Yau spin connection ζ_{MN} (details of which are not necessary for our purposes), we find that $\omega^M{}_N = \zeta^M{}_N$. The set of connection 1-forms with all indices lowered for convenience is simply

$$\omega_{k0} = \frac{a'}{na} e^k, \quad \omega_{M0} = \frac{b'}{nb} e^M,$$

$$\omega_{MN} = \zeta_{MN}, \quad \omega_{\varphi 0} = \frac{\mathcal{R}'_{11}}{n\mathcal{R}_{11}} e^\varphi. \tag{29}$$

The next step is to work out the curvature 2-forms, using $R_{ab} = d\omega_{ab} + \omega_{ac} \wedge \omega^c{}_b$. We recall here that in the Calabi-Yau sector, we will have the intrinsic curvature $\mathcal{R}_{KL} = (1/2)\mathcal{R}_{KLMN} E^M \wedge E^N$. Using the curvature forms, we can obtain the curvature components in the tangent basis. The tangent space curvature components are [normalized by $R_{ab} = (1/2)R_{abcd} e^c \wedge e^d$]

$$R_{k0j0} = -\left[\frac{1}{n}\left(\frac{a'}{na}\right)' + \frac{a'^2}{n^2 a^2}\right]\delta_{kj},$$

$$R_{M0L0} = -\left[\frac{1}{n}\left(\frac{b'}{nb}\right)' + \frac{b'^2}{n^2 b^2}\right]\delta_{ML}$$

$$R_{\varphi 0\varphi 0} = -\left[\frac{1}{n}\left(\frac{\mathcal{R}'_{11}}{n\mathcal{R}_{11}}\right)' + \frac{\mathcal{R}'^2_{11}}{n^2 \mathcal{R}^2_{11}}\right],$$

$$R_{\varphi k\varphi j} = \frac{\mathcal{R}'_{11} a'}{n^2 \mathcal{R}_{11} a}\delta_{kj}$$

$$R_{\varphi M\varphi L}=\frac{\mathcal{R}_{11}'b'}{n^2\mathcal{R}_{11}b}\delta_{ML} \quad R_{jklm}=\frac{a'^2}{n^2a^2}(\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl})$$

$$R_{kMjL}=\frac{a'b'}{n^2ab}\delta_{kj}\delta_{ML}$$

$$R_{MNLK}=\frac{1}{b^2}\,\mathcal{R}_{MNLK}+\frac{b'^2}{n^2b^2}(\delta_{ML}\delta_{NK}-\delta_{MK}\delta_{NL}). \quad (30)$$

The contraction of indices is the interior product, and hence a tensor operation, so it does not depend on the basis. Hence we can contract the indices of these components, using the flat tangent space metric η_{ab}, to get the tangent space Ricci tensor components: $R_{ab}=\eta^{cd}R_{acbd}$. Contracting again, we get the Ricci scalar, which of course is basis independent: $R=\eta^{ab}\eta^{cd}R_{abcd}$. The tangent space Ricci tensor is

$$R_{00}=-3\left[\frac{1}{n}\left(\frac{a'}{na}\right)'+\frac{a'^2}{n^2a^2}\right]-\left[\frac{1}{n}\left(\frac{\mathcal{R}_{11}'}{n\mathcal{R}_{11}}\right)'+\frac{\mathcal{R}_{11}'^2}{n^2\mathcal{R}_{11}^2}\right]$$

$$-6\left[\frac{1}{n}\left(\frac{b'}{nb}\right)'+\frac{b'^2}{n^2b^2}\right]$$

$$R_{kj}=\left[\frac{1}{n}\left(\frac{a'}{na}\right)'+3\frac{a'^2}{n^2a^2}+\frac{\mathcal{R}_{11}'a'}{n^2\mathcal{R}_{11}a}+6\frac{a'b'}{n^2ab}\right]\delta_{kj}$$

$$R_{\varphi\varphi}=\frac{1}{n}\left(\frac{\mathcal{R}_{11}'}{n\mathcal{R}_{11}}\right)'+\frac{\mathcal{R}_{11}'^2}{n^2\mathcal{R}_{11}^2}+3\frac{\mathcal{R}_{11}'a'}{n^2\mathcal{R}_{11}a}+6\frac{\mathcal{R}_{11}'b'}{n^2\mathcal{R}_{11}b}$$

$$R_{MN}=\mathcal{R}_{MN}+\left[\frac{1}{n}\left(\frac{b'}{nb}\right)'+6\frac{b'^2}{n^2b^2}+\frac{\mathcal{R}_{11}'b'}{n^2\mathcal{R}_{11}b}+3\frac{a'b'}{n^2ab}\right]\delta_{MN}. \quad (31)$$

Recalling that $\mathcal{R}_{MN}=0$ for Calabi-Yau spaces, we see that the equations of motion do not discern any intrinsic properties of the Calabi-Yau spaces. Now, the Ricci scalar is

$$R=\frac{2}{n}\left(\frac{\mathcal{R}_{11}'}{n\mathcal{R}_{11}}+3\frac{a'}{na}+6\frac{b'}{nb}\right)'+2\frac{\mathcal{R}_{11}'^2}{n^2\mathcal{R}_{11}^2}+12\frac{a'^2}{n^2a^2}$$

$$+42\frac{b'^2}{n^2b^2}+12\frac{b'\mathcal{R}_{11}'}{n^2b\mathcal{R}_{11}}+6\frac{a'\mathcal{R}_{11}'}{n^2a\mathcal{R}_{11}}+36\frac{a'b'}{n^2a'b'}. \quad (32)$$

It contains a term of second order in derivatives. We will eliminate it from the effective reduced Lagrangian by a partial integration, and omission of the ensuing boundary term, since we are only interested in the bulk equations of motion here. The gravitational Lagrangian is $\mathcal{L}=\sqrt{g}R=\sqrt{G_{CY}}n\mathcal{R}_{11}a^3b^6R=\sqrt{G_{CY}}L$. The action (15) then is

$$S=\int d^{11}x\sqrt{g}R=\int dt d^3\vec{x}d^6y d\varphi n\mathcal{R}_{11}a^3b^6\sqrt{G_{CY}}R$$

$$=\mu_R V\int dt L=\mu_R V\int dt n\mathcal{R}_{11}a^3b^6R \quad (33)$$

where μ_R contains a finite renormalization of the mass scale M_{11} by the volume of the Calabi-Yau 3-fold $\mathcal{V}_6$ and the period 2π of φ. V is the comoving volume of the 3-space $\mathcal{M}_3^{k=0}$.

In addition to the gravitational part of the action, the gauge terms give

$$F_{\mu\nu\lambda\sigma}^2=-\frac{24}{a^6}\left(\frac{\mathcal{F}_1^2}{n^2}+\frac{\mathcal{F}_2^2}{\mathcal{R}_{11}^2}\right) \quad (34)$$

with

$$\mathcal{F}_1^2=P^2\frac{a^6}{b^{12}\mathcal{R}_{11}^2} \quad \mathcal{F}_2^2=Q^2. \quad (35)$$

When we substitute $-(1/48)F_{\mu\nu\lambda\sigma}^2$ into the action along with the explicit form of the Ricci scalar (32), we find that after omitting a boundary term, and using $I=S/(\mu_R V)$, the action becomes

$$I=\int\frac{dt}{n}\left\{\frac{\mathcal{R}_{11}b^6}{2a^3}\mathcal{F}_1^2+\frac{n^2b^6}{2\mathcal{R}_{11}a^3}\mathcal{F}_2^2-\mathcal{R}_{11}a^3b^6\right.$$

$$\left.\times\left(6\frac{a'^2}{a^2}+30\frac{b'^2}{b^2}+6\frac{a'\mathcal{R}_{11}'}{a\mathcal{R}_{11}}+12\frac{\mathcal{R}_{11}'b'}{\mathcal{R}_{11}b}+36\frac{a'b'}{ab}\right)\right\}. \quad (36)$$

To find the equations of motion for the remaining gravitational degrees of freedom, we first vary this action with respect to $n,a,b,\mathcal{R}_{11}$ and then insert the expressions for $\mathcal{F}$ in (35)—i.e. we treat $\mathcal{F}$ as a constant under variations. This reproduces the correct equations of motion, as is easy to check. Choosing the gauge $n=1$, and introducing the minisuperspace ''particle coordinates'' $\alpha=\ln(a)$, $\beta=\ln(b)$ and $\gamma=\ln(\mathcal{R}_{11})$, we obtain the following equations of motion:

$$6\alpha'^2+30\beta'^2+6\alpha'\gamma'+12\beta'\gamma'+36\alpha'\beta'$$

$$=\frac{P^2}{2}e^{-2\gamma-12\beta}-\frac{Q^2}{2}e^{-2\gamma-6\alpha}$$

$$\alpha''+3\alpha'^2+6\alpha'\beta'+\alpha'\gamma'=-\frac{P^2}{3}e^{-2\gamma-12\beta}-\frac{Q^2}{3}e^{-2\gamma-6\alpha}$$

$$\beta''+6\beta'^2+3\alpha'\beta'+\beta'\gamma'=\frac{P^2}{6}e^{-2\gamma-12\beta}+\frac{Q^2}{6}e^{-2\gamma-6\alpha}$$

$$\gamma''+\gamma'^2+3\alpha'\gamma'+6\beta'\gamma'=\frac{P^2}{6}e^{-2\gamma-12\beta}-\frac{Q^2}{3}e^{-2\gamma-6\alpha}. \quad (37)$$

These equations resemble the equations of motion of a mechanical system evolving with friction (the terms bilinear in first derivatives). The constraint equation can be thought of as a generalized energy integral. To make the mechanical analogy for (37) more precise, we will introduce a new gauge below, which will remove the friction terms.

At this point, however, it is illustrative to review the 11D Kasner solutions, defined by setting $P=Q=0$. The Kasner ansatz corresponds to choosing $\alpha'=\alpha_0/t$, $\beta'=\beta_0/t$ and $\gamma'=\gamma_0/t$. So the equations (37) give

$$\alpha_0^2 + 5\beta_0^2 + \alpha_0\gamma_0 + 2\beta_0\gamma_0 + 6\alpha_0\beta_0 = 0$$

$$\alpha_0 = 3\alpha_0^2 + 6\alpha_0\beta_0 + \alpha_0\gamma_0$$

$$\beta_0 = 6\beta_0^2 + 3\alpha_0\beta_0 + \beta_0\gamma_0$$

$$\gamma_0 = \gamma_0^2 + 3\alpha_0\gamma_0 + 6\beta_0\gamma_0. \tag{38}$$

The solutions come in several varieties: (1) if none of the α_0, β_0 and γ_0 are zero, they must satisfy $3\alpha_0 + 6\beta_0 + \gamma_0 = 1$ (as one can easily see from the latter three equations, which degenerate to this single equation) and $3\alpha_0^2 + 6\beta_0^2 + \gamma_0^2 = 1$ (which arises after taking the square the equation above and then subtracting from it the first equation from (38); (2) a few degenerate cases, where the possibilities for $(\alpha_0, \beta_0, \gamma_0)$ are (i) $(0,0,1)$, (ii) $(1/2,0,-1/2)$, (iii) $(-1/3,1/3,0)$, (iv) $(5/9,-1/9,0)$ and (v) $(0,2/7,-5/7)$. In fact, case (i) is locally just the 11D flat space in Milne coordinates, as can be seen by applying a simple coordinate transformation. To make the contact with string models we have discussed earlier, in particular with the action (17), we note that solutions (iii) and (iv) correspond to 10D modular string cosmology solutions with the constant dilaton field, while (ii) and (v) can be understood as particular solutions with both rolling dilaton and rolling moduli fields. The generic Bianchi models where all the scale factors depend on time also correspond to string cosmologies with rolling dilaton and moduli fields. However, the two cases where $2\beta_0 + \gamma_0 = 0$, and $0 \neq \alpha_0 \neq \beta_0 \neq 0$ produce precisely the metric-dilaton string solutions (6), as can be immediately verified by following the procedure outlined above, in the discussion leading to (17). Namely, first dimensionally reduce from 11D to 10D type IIa string theory and then down to 4D assuming that the internal six dimensions span an isotropic six-torus.

Finally, perhaps the most curious interpretations of these solutions arise in the following way. There may exist reduction procedures which map 11D solutions to lower dimensional stringy ones, and in particular 4D solutions, but in a way which involves a phase of M theory not belonging to known string theories. For example, if we take the apparently trivial case (i) or the curved case (ii), we can map them both onto the same metric-dilaton solutions (6). The solutions, however, acquire the guise of string theory only at the very end—in both cases we first perform the dimensional reduction on the Calabi-Yau space, producing a simple Einstein theory in five dimensions as a result. To check that this is a consistent truncation of the 11D theory one only need recall that the equations of motion for the Calabi-Yau scale factor are homogeneous [see e.g. (37)]. Then, in a manner similar to that discussed by Behrndt and Förste in [28], we go one dimension down to four, obtaining an action of a scalar-tensor theory of gravity in a Brans-Dicke frame:

$$S = \int d^4x \sqrt{\hat{g}} \, \mathcal{R}_{11} \hat{R}. \tag{39}$$

The hats denote the Brans-Dicke frame quantities. In this frame, our solutions (i) and (ii) are $\mathcal{R}_{11} = t$, $d\hat{s}^2 = -dt^2 + d\vec{x}^2$ and $\mathcal{R}_{11} = 1/\sqrt{t}$, $d\hat{s}^2 = -dt^2 + t d\vec{x}^2$. To see that both of these solutions are conformal to the solution (6), we perform another conformal transformation, to the string

frame. For both of these two cases, the conformal transformation and the field redefinition of the scalar field look formally the same. They are $g_{\mu\nu} = \mathcal{R}_{11}^{1 \pm \sqrt{3}} \hat{g}_{\mu\nu}$ and $\phi = \pm (\sqrt{3}/2) \ln \mathcal{R}_{11}$ [note that these field redefinitions are slightly different from those used to obtain Eq. (17) because we are now reducing from 5D to 4D as opposed to from 11D to 10D]. Since in the two cases the radius of the eleventh dimensions depends on time differently, we need to carry out coordinate transformations separately. In case (i), we thus find the string-frame comoving time to be $\tau \sim t^{(3 \pm \sqrt{3})/2}$, while for case (ii) the transformation is $\tau \sim t^{(3 \pm \sqrt{3})/4}$. When we substitute the field redefinitions and these coordinate transformations into the solutions, both cases lead precisely to (6). As one can now see, this solution (6) is highly degenerate from the point of view of M theory as the same solution in string theory can be obtained from several different solutions in M theory.

Reductions of this type have not been given much attention before, since they employ a dimensional descent inside M theory, making contact with strings only at the very end. However, they are none the less interesting, since by being solutions of the 11D supergravity equations of motion they certainly belong to the general phase space of M theory. An interesting feature of the case (i) discussed above is that it offers a reinterpretation of the pre-big-bang curvature singularity in (6) as a Rindler horizon in 11D, in a way very similar to what has been discussed in [23]. However, since the horizon involves the compact coordinate along the circle $\mathcal{S}^1$ of (18), the singularity has not been completely removed from the 11D geometry. Instead, the periodicity of φ implies that the Rindler wedges of the 11D manifold contain closed time-like curves, and moreover that the manifold is not Hausdorff, as observed in [23]. A difference between our example and those of [23] is that we use the 11$^{\text{th}}$ direction to define the horizon, and hence lift the singularity, thus going outside of the realm of string theory constructions. In this case, we regulate the 4D string coupling by decompactifying directly into the 11D supergravity phase, rather than trying to stabilize the coupling within the string framework. The benefit of this approach is the softening of the singularity, namely the curvature singularity is absent, but the space-time exhibits geodesic incompleteness. However, this still cannot be taken as an example of graceful exit in pre-big-bang. Such an extension of the solution involves an ascent to 11D supergravity which does not belong to the original pre-big-bang scenario. Furthermore, at the transition, the function which corresponds to the effective string coupling is really very small, as opposed to very large—which is in contrast to the generic situation in the pre-big-bang scenario. The flow of the coupling in this example is opposite to the flow of the coupling in various implementations of pre-big-bang. In a sense, this example appears to accomplish one of the goals of pre-big-bang—singularity softening—while failing to attain the other—pole inflation.

IV. COSMIC BRANES

Returning to the general case with two charges, we will use a more suitable gauge to examine solutions of (37). We note that the terms which are bilinear in the first derivatives of the fields α, β and γ in (37) are always proportional to

$(3\alpha' + 6\beta' + \gamma')$. In fact, this is the reason why the three equations of motion in the Kasner case degenerated to just one. This implies that we can gauge away all such terms by using a different time coordinate. So let $dt = n\,d\tau$, where $n = \exp(3\alpha + 6\beta + \gamma)$. Then we can rewrite the equations of motion (37) as (where the overdots denote τ derivatives)

$$6\dot\alpha^2 + 30\dot\beta^2 + 6\dot\alpha\dot\gamma + 12\dot\beta\dot\gamma + 36\dot\alpha\dot\beta = \frac{P^2}{2}e^{6\alpha} - \frac{Q^2}{2}e^{12\beta}$$

$$\ddot\alpha = -\frac{P^2}{3}e^{6\alpha} - \frac{Q^2}{3}e^{12\beta}, \quad \ddot\beta = \frac{P^2}{6}e^{6\alpha} + \frac{Q^2}{6}e^{12\beta}$$

$$\ddot\gamma = \frac{P^2}{6}e^{6\alpha} - \frac{Q^2}{3}e^{12\beta}. \tag{40}$$

These equations admit the mechanical analogy we have indicated at the end of the previous section. The constraint can be thought of as the conservation of energy—it is just the Hamiltonian, with the requirement that $E = 0$. If we take the derivative-dependent terms in the constraint to denote the kinetic energy, and the exponentials to be the potential, such that $H = T + W$, we can define the Lagrangian as $L = T - W$. One can then show that the second order equations of motions follow from the variation of this Lagrangian. Similar equations of motion were considered recently in [31].

Before continuing with the investigation of the general solutions, we will first review the cases when one of the charges is zero. These cases were considered in [29], [30], [17], and we include them here for completeness. We will see that the two possibilities lead to different subclasses of solutions. Let us begin with the case $Q = 0$. This case corresponds to the axionic cosmology extended to 10D by the addition of rolling moduli, which has been investigated recently by Copeland, Lidsey and Wands [44]. The equations of motion reduce to

$$6\dot\alpha^2 + 30\dot\beta^2 + 6\dot\alpha\dot\gamma + 12\dot\beta\dot\gamma + 36\dot\alpha\dot\beta = \frac{P^2}{2}e^{6\alpha}$$

$$\ddot\alpha = -\frac{P^2}{3}e^{6\alpha}, \quad \ddot\beta = \frac{P^2}{6}e^{6\alpha}, \quad \ddot\gamma = \frac{P^2}{6}e^{6\alpha} \tag{41}$$

and it is clear that they are easily integrable—all we need to do is solve the α equation, and the rest reduce to simple double integrals. The α equation is the familiar Liouville equation. It can be integrated once to give the first integral

$$\dot\alpha^2 = \theta_0^2 - \frac{P^2}{9}e^{6\alpha} \tag{42}$$

where θ_0^2 is a positive integration constant (if it were zero or negative, we would have found $\dot\alpha = 0$ and so $P = 0$). This separates variables, and we can rewrite it as the integral

$$\int \frac{d(e^{-3\alpha})}{\sqrt{\theta_0^2 e^{-6\alpha} - P^2/9}} = \mp 3(\tau + \tau_0) \tag{43}$$

where τ_0 is another integration constant. We can set it to zero by a time translation. After all the integrations, we find

$$e^{-3\alpha} = \left|\frac{P}{3\theta_0}\right|\cosh 3\theta_0\tau, \quad e^{6\beta} = e^{6\beta_0 + 6\beta_1\tau}\cosh 3\theta_0\tau$$

$$e^{6\gamma} = e^{6\gamma_0 + 6\gamma_1\tau}\cosh 3\theta_0\tau. \tag{44}$$

The constraint relates the integration constants according to $30\beta_1^2 + 12\beta_1\gamma_1 = 9\theta_0^2/2$. These solutions are valid for both $\tau > 0$ and $\tau < 0$, and $\tau = 0$ is the singularity. Note that we can take the limit when $\gamma = \beta$, by adjusting the integration constants, in which case this reduces to the solutions discussed very recently by Lukas and Ovrut [32].

Consider now the $P = 0$ cases. Here there is more variety, which may appear slightly puzzling since one might expect that there should be a kind of duality correspondence between the $P = 0$ and $Q = 0$ cases. The equations of motion are

$$6\dot\alpha^2 + 30\dot\beta^2 + 6\dot\alpha\dot\gamma + 12\dot\beta\dot\gamma + 36\dot\alpha\dot\beta = -\frac{Q^2}{2}e^{12\beta}$$

$$\ddot\alpha = -\frac{Q^2}{3}e^{12\beta}, \quad \ddot\beta = \frac{Q^2}{6}e^{12\beta}, \quad \ddot\gamma = -\frac{Q^2}{3}e^{12\beta}. \tag{45}$$

Note that now we need to solve the β equation, which again is a Liouville equation, and the rest follows easily. However, also note that since the RHS of the β equation is positive, there are more possibilities for the value of the integration constant θ_0^2. First, we again get the integral of motion,

$$\dot\beta^2 = \theta_1 + \frac{Q^2}{36}e^{12\beta} \tag{46}$$

where θ_1 is the integration constant, which now can be any real number. When $\theta_1 = 0$, the solution is

$$e^{6\beta} = \frac{1}{|Q\tau|}, \quad e^{3\alpha} = e^{3\alpha_0 + 3\alpha_1\tau}|\tau|, \quad e^{3\gamma} = e^{3\gamma_0 + 3\gamma_1\tau}|\tau| \tag{47}$$

with $6\alpha_1(\alpha_1 + \gamma_1) = 0$. When $\theta_1 > 0$, the solutions are

$$e^{-6\beta} = \left|\frac{Q}{6\sqrt{\theta_1}}\sinh 6\sqrt{\theta_1}\tau\right|, \quad e^{3\alpha} = e^{3\alpha_0 + 3\alpha_1\tau}|\sinh 6\sqrt{\theta_1}\tau|$$

$$e^{3\gamma} = e^{3\gamma_0 + 3\gamma_1\tau}|\sinh 6\sqrt{\theta_1}\tau| \tag{48}$$

and the constraint gives $\alpha_1^2 + \alpha_1\gamma_1 = 3\theta_1$. This case should be the dual of the case $Q = 0$ considered before. Finally, when $\theta_1 < 0$, the solutions are

$$e^{-6\beta} = \left|\frac{Q}{6\sqrt{|\theta_1|}}\sin 6\sqrt{|\theta_1|}\tau\right|,$$

$$e^{3\alpha} = e^{3\alpha_0 + 3\alpha_1\tau}|\sin 6\sqrt{|\theta_1|}\tau|$$

$$e^{3\gamma} = e^{3\gamma_0 + 3\gamma_1\tau}|\sin 6\sqrt{|\theta_1|}\tau|. \tag{49}$$

The constraint now gives $\alpha_1^2 + \alpha_1\gamma_1 = -3\theta_1$. Because the solutions are given in terms of trigonometric functions, they are without an analogue in the $Q = 0$ case discussed above. Hence, this sub-family has a larger phase space.

What can we do with the equations (37) when both charges are nonzero? Consider the equations for $\ddot{\alpha}$ and $\ddot{\beta}$. If we look at their linear combinations, we can see that $\ddot{\alpha} + 2\ddot{\beta} = 0$. This gives us one integral of motion: $\alpha + 2\beta = c(\tau - \tau_0)$. Here c and τ_0 are integration constants. We can choose $\tau_0 = 0$, by a time translation. This allows us to rewrite the equations (40) in terms of only two variables: α and γ. We find

$$15c^2 - 9\dot{\alpha}^2 + 6c\dot{\alpha} + 12c\dot{\gamma} = P^2 e^{6\alpha} - Q^2 e^{6c\tau - 6\alpha}$$

$$\ddot{\alpha} = -\frac{P^2}{3}e^{6\alpha} - \frac{Q^2}{3}e^{6c\tau - 6\alpha}, \quad \ddot{\gamma} = \frac{P^2}{6}e^{6\alpha} - \frac{Q^2}{3}e^{6c\tau - 6\alpha}.$$
$$(50)$$

These equations are not easily integrable. For example, consider $c = 0$. (We can choose this since c is arbitrary.) Now, all the reference to γ in the constraint disappears. It becomes

$$9\dot{\alpha}^2 = Q^2 e^{-6\alpha} - P^2 e^{6\alpha} \qquad (51)$$

and we can verify that this is just the first integral of the $\ddot{\alpha}$ equation. This equation separates variables, so we can write the solutions as

$$\int \frac{d\alpha}{\sqrt{Q^2 e^{-6\alpha} - P^2 e^{6\alpha}}} = \pm \frac{\tau}{9}. \qquad (52)$$

(Note that we can still shift the time to get rid of another integration constant, since when $c = 0$ our previous shift was not necessary.) The integral can be simplified using $z = \sqrt{P/Q}\,\exp(3\alpha)$, and we can rewrite it as

$$\int \frac{dz}{\sqrt{1 - z^4}} = \pm \frac{\sqrt{PQ}\,\tau}{3}. \qquad (53)$$

This integral belongs to a type of binomial differentials which cannot be integrated in terms of elementary functions, as shown over a hundred years ago by Chebyshev. The analysis becomes even more complicated when we consider $c \neq 0$. In order to obtain information about these cases, we have to resort to numerical methods.

It should be clear that the initial sizes of the three subspaces in (19) are not independent parameters on their own. Rather, they combine with the charges P and Q and the eleven-dimensional Planck mass (which enters by defining the time scale of the evolution) to give the relevant parameters for numerical integration. Hence we can simply set all of them to one at the beginning and vary the two charges. Here it is useful to begin by first classifying cosmological solutions according to the relative signs of time derivatives of the three scale factors in (19). According to this classification there are *a priori* eight possibilities (since each initial "scale velocity" can be either positive or negative). However, by time reversal we can correlate the subclasses, and arrive at the conclusion that we only need to consider four types of initial conditions, which we denote by ordered triplets $[\mathrm{sgn}(\dot{a}_0), \mathrm{sgn}(\dot{b}_0), \mathrm{sgn}(\dot{\mathcal{R}}_{11\,0})$: $(+,+,+)$, $(+,+,-)$, $(+,-,+)$ and $(-,+,+)$. The solutions however turn out to be connected further by dynamics, as can be seen in the figures.
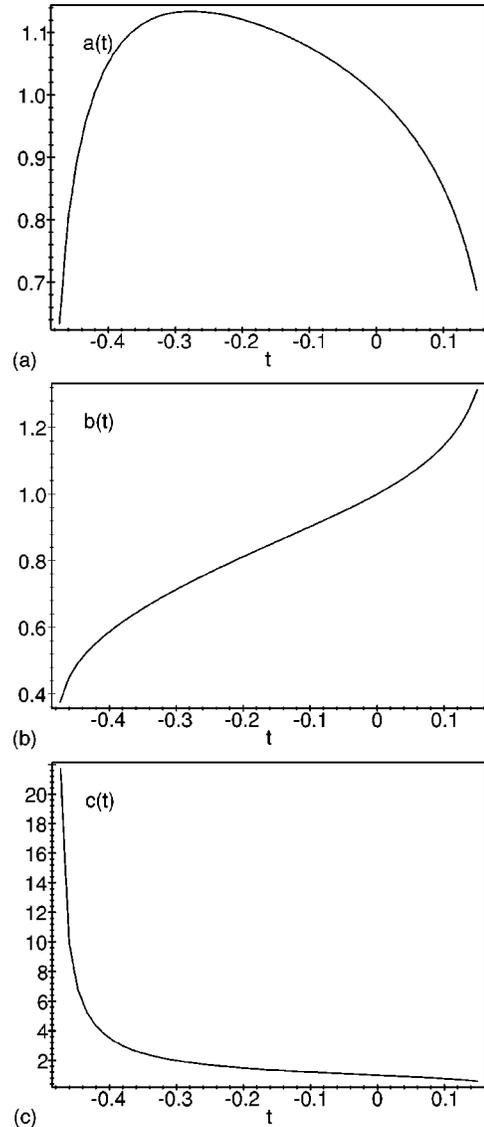

FIG. 1. The scale factors of the 3-space (a), Calabi-Yau 6-fold (b) and the circle S^1 (c). The Calabi-Yau space always expands while the circle shrinks, corresponding to the flow of the coupling from the strong to weak regime, in contrast to the conventional pre-big-bang solutions. The 3-space begins with a zero radius and expands to its maximum size, after which it recollapses.

The cases $(+,-,+)$ and $(+,+,+)$ evolve into cases $(-,+,+)$, in a way effectively similar to the generic idea of branch-changing (which in this case does not correspond to the exit, since all the solutions where "branch-changing" occurs possess singularities in the future)]. This however does not mean that the generic behavior of a cosmology with two charges can be completely reduced down to only two cases. Given the signs of initial values of "scale velocities," we can find different types of evolution depending on the ratios of the derivatives. We present several typical cases where we show the scale factors of the 3-space (a), Calabi-Yau 6-fold (b) and the circle S^1 (c). In Fig. 1, we show a $(+,+,-)$ case where the Calabi-Yau space always expands while the circle shrinks, corresponding to the flow of the coupling from the strong to weak regime, in contrast to the conventional pre-big-bang solutions. The 3-space begins with a zero radius and expands to its maximum size, after which it recollapses,
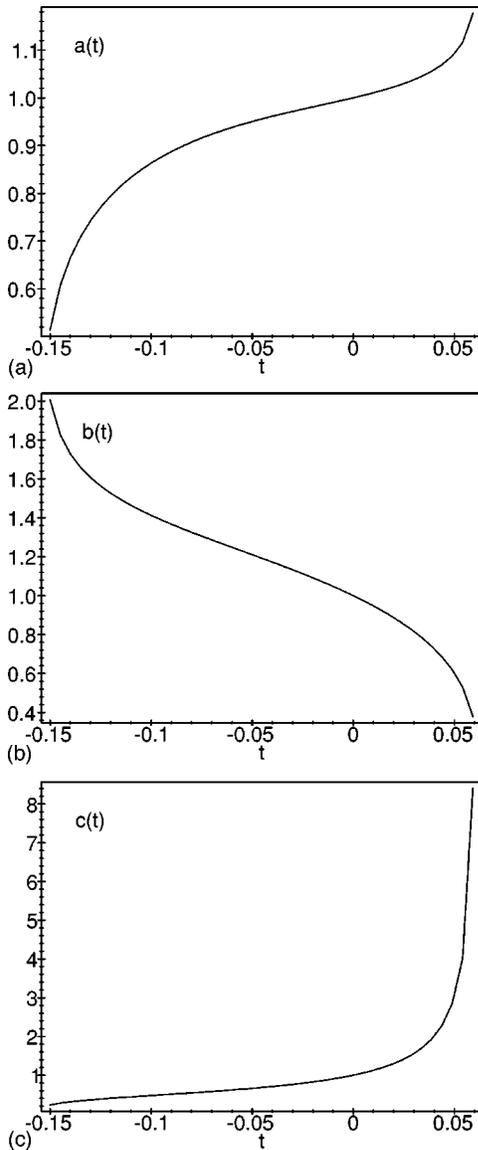
FIG. 2. In this example the effect of the Calabi-Yau on the 3-space is similar to spatial curvature: the 3-space begins again with zero radius, and expands toward a pole-like inflation while the Calabi-Yau 6-fold shrinks to zero size. The coupling always flows from weak to strong.

FIG. 3. While the evolution in this case initially looks like that in Fig. 4, eventually the flows of the internal subspaces are interchanged and the dynamics becomes similar to that in Fig. 1, with a future 3-space big crunch.

characteristic of a positively curved FRW space-time, though here we have taken $^3k=0$. In the example of Fig. 2, we look at a $(+,-,+)$ case where the effect of the Calabi-Yau on the 3-space is similar to spatial curvature: the 3-space begins again with zero radius, and expands toward a pole-like inflation while the Calabi-Yau 6-fold shrinks to zero size. The coupling always flows from weak to strong. The situation in Figure 3 can describe several cases (depending where we choose the initial time)—for example, $(+,-,+)$, but also $(+,+,+)$ and $(-,+,+)$. While the evolution in this case initially looks like that in Fig. 2, eventually the flows of the internal subspaces are interchanged and the dynamics becomes similar to Fig. 1, with a future 3-space big crunch. In Fig. 4, we show a solution similar to that in Fig. 5, but with a more complex flow of the coupling scale. In Fig. 5 we find an example which in some respect is the most similar to the original pre-big-bang solutions (6), since it does not have a
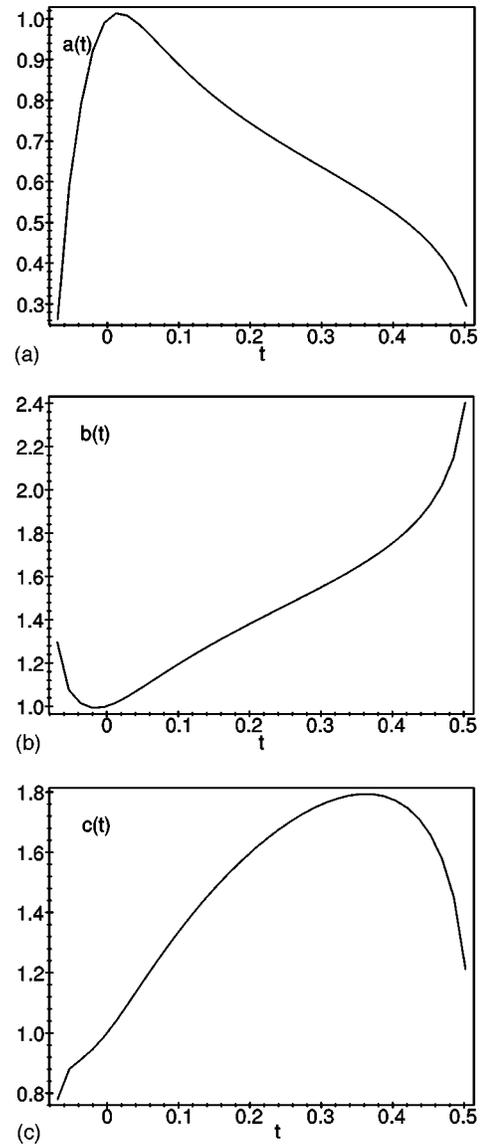
past singularity. Here the solution for the 3-space starts out very small and grows while the Calabi-Yau scale and the coupling scale start large and decrease. However, at some moment the flows of the scale factors a and b reverse, and the 3-space shrinks to zero size while the Calabi-Yau space pole inflates. The coupling always flows from strong to weak.

Aside from disclosing a number of solutions, our numerical investigation suggests that the solutions in Eq. (6) when viewed as cosmological backgrounds of type IIA string theory and M theory are really rather special in that they have infinitely old inflating past branches. Most charged solutions appear to have spatial sections which evolve out of a past spatial singularity, much like the $k=1$ solutions described in Eq. (9). This is clearly the effect of the 4-form charges, and therefore it seems that the efficiency of the pre-big-bang scenario to produce inflation is very limited from the point of view of type IIA string theory, because of the
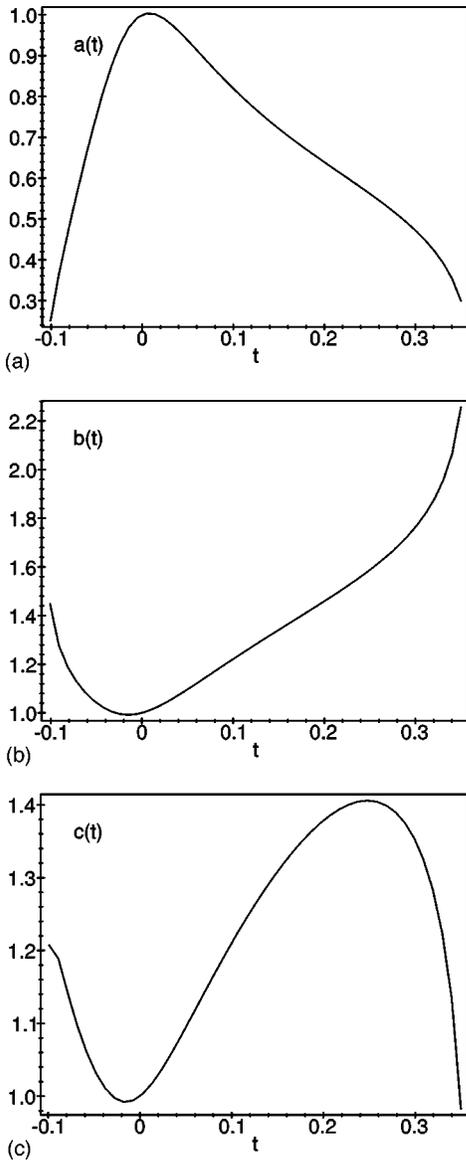
FIG. 4. Similar to example of Fig. 3, but with a more complex flow of the coupling scale.
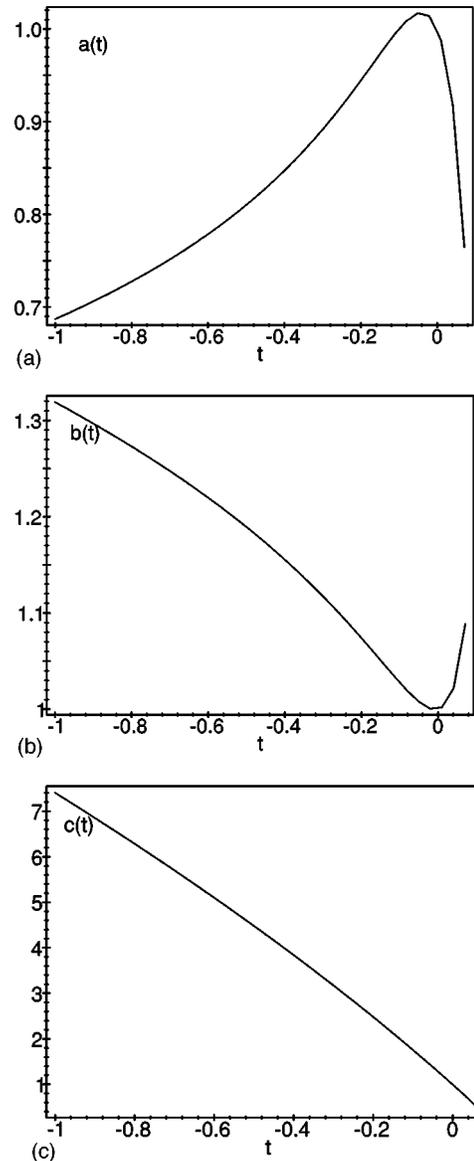

FIG. 5. This case does not have a past singularity. Instead, the solution for the 3-space starts out very small and grows while the Calabi-Yau scale and the coupling scale start large and decrease. However, at some moment the flows of the scale factors a and b reverse, and the 3-space shrinks to zero size while the Calabi-Yau space pole inflates. The coupling always flows from strong to weak in this case.

fine tuning problems discussed in [21]. In these cases, many of the solutions either recollapse too soon or emerge out of an initial singularity. (While there may be some doubt as to what happens in four dimensions after dimensional reduction, note that the doubly singular such as those presented in Figs. 1, 3, and 4 clearly will remain doubly singular after reduction, hence providing an example of our argument here.)

V. CONCLUSION

In this article, we have considered several aspects of the application of M theory in cosmology. Our main aim has been to cast the known string cosmological backgrounds, as well as some of their more straightforward generalizations, as 11-dimensional metric-4-form configurations. This point of view is useful in order to study the large coupling limits of string cosmologies, which is ill-defined in string perturbation theory, but can be completely understood in terms of the

decompactification of the 11th dimension. Further, this allows for an egalitarian description of the string moduli fields, which arise due to compactification, and can be related by U-duality maps. In the process of this 11-dimensional reinterpretation of solutions, we have shown that in some simple cases the cosmological curvature singularities can be moderated. In particular, some of the known scalar-field dominated cosmological solutions are equivalent to a flat 11-dimensional space-time, which however has a pathological topology that could involve closed time-like curves in its maximal extension. The acausal domain is separated from the physical sector of the space-time by a horizon, which upon dimensional reduction produces the coupling and curvature singularities.

Before closing, we would like to note some aspects re-

garding the initial singularity in the context of M theory. As we have discussed earlier, one goal of the pre-big-bang scenario is the smooth transition from a (+)-branch solution (which evolves towards singularities) to a (−)-branch solution (which evolves away from singularities). In [18], a solution to the gravitational equations of motion based on the Damour-Polyakov ansatz showed such a smooth transition. By an appropriate choice of the corrected coupling function $B(\phi)$, a non-singular solution has been found. However, the solution was not without its peculiarities. In the two solutions presented in [18], the evolution of the dilaton caused the function $B(\phi)$ to pass through zero [in fact this is a general requirement of any such solution as it must utilize an ''egg'' with $B<0$ (see [18] for details)], essentially indicating a signature change in 10 dimensions. However, the ten-dimensional picture may be misleading, because in the context of M mtheory, we should only look at the function $B(\phi)$ as being related to the radius of the 11th dimension. There, it appears that we could view the evolution as a process of decompactification, where the eleventh dimension blows up and then shrinks again. From this point of view, we need not see any change of signature at all. In this way, the compact radius would ''bounce'' at infinity. At this moment, in support of this we can only offer an analogy with the bounce picture in Liouville field theory of non-critical strings proposed by one of us some time ago [50], which was used later in the study of a non-equilibrium temporal flow and a closed-time-like path formalism for non-critical strings by [51]. Because the Liouville field is ultimately connected with time in non-critical string theory, it is very tempting to think about possible connections between these two pictures of evolution in M theory and non-critical string theory. We hope to return to this issue in the future.

Finally, we should mention the possibility of relating cosmological evolution in M theory we have discussed here with renormalization group flows. As we have seen, during cosmological expansion not only is the spatial scale $a(t)$ evolving with time, but so is the coupling constant $\lambda(t)$. This means that one can consider the evolution of the coupling constant λ not as a function of time, but as a function of the scale factor a, $\lambda=\lambda(a)$, leading to $d\lambda(a)/d\ln a=\beta(\lambda)$. The β function $\beta(\lambda)$ can be easily calculated for any particular cosmological solution. It is interesting to ask whether or not this ''renormalization group'' (RG) flow could correspond to an actual quantum RG flow in a ten-dimensional theory or even in a four-dimensional one after compactification. For example, it would be very interesting to find an example featuring logarithmic behavior rather than the above scaling, which could give inflationary expansion, $a(t)\sim\exp(ct)$, in the lower-dimensional manifold. Unfortunately, we could not find any M-theoretic cosmological solution with exponential inflation in the physical three-dimensional space and power-law evolution in the eleventh dimension. However, this posibillity does not seem excluded at this point. Indeed, solutions with such behavior would be of great interest, as they could be used for inflation simultaneous with running coupling. Hence, an affirmative answer to the question of existence of such solutions would be an extremely interesting result. A conceptual difficulty to surpass here is that the cosmological evolution under consideration is purely *classical* by definition—no quantization of *M* theory was performed in our analysis (simply because it does not exist yet)—while the ordinary RG flow is an entirely *quantum* phenomenon. In a way, the situation is reminiscent of the anomaly treatment in Wess-Zumino-Witten-Novikov σ models, where in the fermionic picture, the anomaly is quantum while in the bosonic is appears purely classical. Perhaps a similar connection can be established here. Indeed, the fact that in the classical formulation of M theory one may have a flow of coupling constant is quite interesting and certainly deserves further investigation.

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